Discrete Mathematics

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1 Propositional Logic—Zeroth Order Logic

In propositional logic, propositions are denoted by letters (p, q) and are formed by connecting other propositions using logical connectives. Propositions can either be true (T) or false (F).

1.1 Logical Connectives

The logical connectives listed below are the basic connectives available in propositional logic in order of their precedence. Below are the truthtables corresponding to each of the connectives.

1. $\neg$, not
2. $\wedge$, and, $\bigwedge_{i=1}^{n} p_i$
3. $\vee$, or, $\bigvee_{i=1}^{n} p_i$
4. $\rightarrow$, $\Rightarrow$, implies (only if) defined as: $p \rightarrow q \equiv \neg p \vee q$
5. $\leftrightarrow$, $\Leftrightarrow$, is equivalent to (if and only if, iff) defined as: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Negation

p	$\neg$	p
T	F	T
F	T	F

Implication

p	q	p	$\rightarrow$	q
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

Logical And

p	q	p	$\wedge$	q
T	T	T	T	T
T	F	T	F	F
F	T	F	F	T
F	F	F	F	F

Equivalence

p	q	p	$\leftrightarrow$	q
T	T	T	T	T
T	F	T	F	F
F	T	F	F	T
F	F	F	T	F

Logical Or

p	q	p	$\vee$	q
T	T	T	T	T
T	F	T	T	F
F	T	F	T	T
F	F	F	F	F

1.2 Definitions

When given the proposition $p \rightarrow q$, $q \rightarrow p$ is its converse, $\neg q \rightarrow \neg p$ is its contrapositive and $\neg p \rightarrow \neg q$ is its inverse. The contrapositive is equivalent to the original proposition and the converse and inverse are also equivalent.

Tautology A proposition that is always true ($p \vee \neg p$).

Contradiction A proposition that is always false ($p \wedge \neg p$).

Contingency A proposition that is neither a tautology nor a contradiction.

Logical Equivalence p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. The notation for equivalence is typically $\equiv$.

1.3 Properties

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Identity Laws

$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

Domination Laws

$$p \wedge F \equiv F$$
$$p \vee T \equiv T$$

Idempotent Laws

$$p \wedge p \equiv p$$
$$p \vee p \equiv p$$

Negation Laws

$$p \wedge \neg p \equiv F$$
$$p \vee \neg p \equiv T$$

Commutative Laws

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Associative Laws

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

Distributive Laws

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Absorption Laws

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

1.4 Equivalence Proof

This is an example of how to perform an equivalence proof. The aim is to show that $\neg(p \vee (\neg p \wedge q))$ is logically equivalent to $\neg p \wedge \neg q$. We can prove this by forming a series of logical equivalences.

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q) \qquad \text{2}^{\text{nd}} \text{ De Morgan's law} \qquad (1)$$
$$\neg p \wedge \neg(\neg p \wedge q) \equiv \neg p \wedge (p \vee \neg q) \qquad \text{1}^{\text{st}} \text{ De Morgan's law} \qquad (2)$$
$$\neg p \wedge (p \vee \neg q) \equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \qquad \text{Associative law} \qquad (3)$$
$$(\neg p \wedge p) \vee (\neg p \wedge \neg q) \equiv F \vee (\neg p \wedge \neg q) \qquad \text{Negation law} \qquad (4)$$
$$F \vee (\neg p \wedge \neg q) \equiv \neg p \wedge \neg q \qquad \text{Identity law} \qquad (5)$$

2 Predicate Logic —First Order Logic