# Calculus

Max Kasperowski

January 11, 2018

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# 1 Derivatives

## 1.1 Definition

If the derivative is defined to be $y = f(x)$, then its derivative is $f'(x) = \lim_{h\to\infty} \frac{f(x+h)-f(x)}{h}$.

Equivalent notations for the derivative of $y = f(x)$ are:

$$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = Df(x)$$

Equivalent notations for the the derivative of $y = f(x)$ evaluated at $x = a$ are:

$$f'(a) = y'\Big|_{x=a} = \frac{df}{dx}\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a)$$

## 1.2 Properties

The following properties hold where $f(x)$ and $g(x)$ are differential functions and $c$ and $n$ are any real numbers.

1. $(cf)' = cf'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(fg)' = f'g + fg'$
4. $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$
5. $\frac{d}{dx}(c) = 0$
6. $\frac{d}{dx}(x^n) = nx^{n-1}$
7. $\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$

## 1.3 Common Derivatives

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

$\frac{d}{dx}(a^x) = a^x \ln(a)$

$\frac{d}{dx}(e^x) = e^x$

$(\ln(x)) = \frac{1}{x}, x > 0$

$\frac{d}{dx}(ln|x|) = \frac{1}{x}, x \neq 0$

$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a}, x > 0$

## 1.4 Higher Order Derivatives

#### Second Order Derivative

$$f''(x) = f^{(2)}(x) = \frac{d^2 f}{dx^2} = (f'(x))'$$

**n**[th] **Order Derivative**

$$f^{(n)} = \frac{d^n f}{dx^n} = \left(f^{(n-1)}(x)\right)'$$

## 1.5 Implicit Differentiation

Implicit differentiation is done just as regular differentiation just that care must be taken to keep in mind that $y$ is a function and therefore the chainrule must be applied when deriving. Finally the equation needs to be solved for $y'$.

## 1.6 Analysis

**Critical Points** $c$ is a critical point of $f(x)$ if $f'(c) = 0$ or $f'(c)$ doesn't exist.

**Increasing/Decreasing** $f(x)$ is increasing on an interval $I$ if $f'(x) > 0$ for all $x$ on the interval $I$. $f(x)$ is decreasing on an interval $I$ if $f'(x) < 0$ for all $x$ on the interval $I$. $f(x)$ is constant on an interval $I$ if $f'(x) = 0$ for all $x$ on the interval $I$.

**Concave Up/Concave Down** $f(x)$ is concave up on an interval $I$ if $f''(x) > 0$ for all $x$ on the interval $I$. $f(x)$ is concave down on an interval $I$ if $f''(x) < 0$ for all $x$ on the interval $I$.

**Inflection Points** $c$ is an inflection point of $f(x)$ if the concavity changes at $f(c)$.