Dijkstra's Algorithm Verification

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1 Dijkstra's Algorithm

1.1 Pseudocode(Idris)

```
−− data structures
Node : (n : Nat) → Fin n
NodeSet : List Node          −− 'NodeSet' can represents adjacent edges for each node
Graph : (List Node, List NodeSet)

sortNodes : (weight : Type) →
(gtW : weight → weight → Bool) →
(add : weight → weight → weight) →
(size : Nat) →
(nodes : Vect size (Node m)) →
(dist : Vect m weight) →
(Vect size (Node m))
sortNodes w gtW add Z Nil dist = Nil
sortNodes w gtW add (S s') (x :: xs) dist
      = insertSort x (sortNodes w gtW add s xs dist)

updateDist : (weight : Type) →
            (gtW : weight → weight → Bool) →
            (add : weight → weight → weight) →
            (size : Nat) →
            (cur : Fin size) →
            (adj : NodeSet size weight) →
            (dist : Vect size weight) →
            (Vect size weight)
updateDist w gtW add size cur adj dist
= for Node n ∈ adj:
      if $dist[cur] + weight[cur-> n] < dist[n]$
```

then $dist[n] = dist[cur] + weight[cur- > n]$
        else continue to the next nodde

−− if unexplored is Nil, then we have calculated the min distance for all nodes
runDijkstras : (weight : Type) →
            (gtW: weight → weight → Bool) →
            (add : weight → weight → weight) →
            (size : Nat) →
            (size' : Nat) → −− number of unexplored nodes
            (graph : Graph size weight) →
            (dist : Vect size weight) →
            (lte size' size = True) →
            (unexplored : Vect size' (Node size)) →
            (Vect size weight)
runDijkstras _ _ _ _ Z g dist _ Nil = dist
runDijkstras w gtW add _ (S s') g dist refl ((MKNode x) :: xs)
        = updateDist w gtW add _ x adj_x dist
        call (**runDijkstras** _ _ _ _ s' g dist' refl (sortNodes w gtW add s' xs dist))

dijkstras : (weight : Type) →
            (gtW : weight → weight → Bool) →
            (add : weight → weight → weight) →
            (size : Nat) →
            (source : Node size) →
            (graph : Graph size weight) →
            (Vect size weight)
dijkstras weight gtW add size source g@(nodes, edges)
        = **runDijkstras** weight gtW add size size g dist reflProof (sortNodes weight gtW
add size nodes)
            where
                dist = list of nodes and their distance to source. $dist[source] = 0$
                reflProof = proof of (lte size size = True)

2   Proof of Correctness

2.1   Proof of Termination

As the size of list unexplored decreases by one during each call to runDijkstras, the
function runDijkstras is guaranteed to terminate, thus function dijkstras terminates.

2.2 Proof of Correctness

Lemma.

Proof. Base Case:
Inductive Hypothesis:
Inductive Step □