Using Capabilities for Strict Runtime Invariant Checking

Isaac Oscar Gariano, Marco Servetto*, Alex Potanin

Victoria University of Wellington, Kelburn, 6012, Wellington, New Zealand

Abstract

In this paper we use pre-existing language support for both reference and object capabilities to enable sound runtime verification of representation invariants. Our invariant protocol is stricter than the other protocols, since it guarantees that invariants hold for all objects involved in execution. Any language already offering appropriate support for reference and object capabilities can support our invariant protocol with minimal added complexity. In our protocol, invariants are simply specified as methods whose execution is statically guaranteed to be deterministic and to not access any externally mutable state. We formalise our approach and prove that our protocol is sound, in the context of a language supporting mutation, dynamic dispatch, exceptions, and non-deterministic I/O. We present case studies showing that our system requires a much lower annotation burden compared to Spec#, and performs orders of magnitude less runtime invariant checks compared to the widely used 'visible state semantics' protocols of D and Eiffel.

Keywords: reference capabilities, object capabilities, runtime verification, class invariants

1. Introduction

Representation invariants (sometimes called class invariants or object invariants) are a useful concept when reasoning about software correctness in OO (Object Oriented) languages. Such invariants are predicates on the state of an object and its ROG (Reachable Object Graph). They can be presented as documentation, checked as part of static verification, or, as we do in this paper, monitored for violations using runtime verification. In our system, a class specifies its invariant by defining a method called `invariant()` that returns a boolean. We say that an object's invariant holds when its `invariant()` method would return `true`.[1]

Invariants are designed to hold most of the time, however it is commonly required to (temporarily) violate invariants while performing complex sequences of mutations. To support this behaviour, most invariant protocols present in the literature allow invariants to be broken and observed broken. The two main forms of invariant protocols are *visible state semantics* [2] and the *Pack-Unpack/Boogie methodology* [3]. In visible state semantics, invariants can be broken when a method on the object is active (that is, currently executing). Some interpretations of the visible state are more permissive, requiring the invariants of receivers to hold only before and after every public method call, and after constructors. In the pack-unpack approach, objects are either in a 'packed' or 'unpacked' state, the invariant of 'packed' objects must hold, whereas unpacked objects can be broken.

In this paper we propose a much stricter invariant protocol: at all times, the invariant of every object involved in execution must hold; thus they can be broken when the object is not (currently) involved in execution. An object is *involved in execution* when it is in the ROG of any of the objects mentioned in the

*Corresponding Author

Email addresses: `isaac@ecs.vuw.ac.nz` (Isaac Oscar Gariano), `marco.servetto@ecs.vuw.ac.nz` (Marco Servetto), `alex@ecs.vuw.ac.nz` (Alex Potanin)

[1] We do this (as in Dafny [1]) to minimise the special treatment of invariants, whereas other approaches often treat invariants as a special annotation with its own syntax.

method call, field access, or field update that is about to be reduced; we state this more formally later in the paper.

Our strict protocol supports easier reasoning: an object can never be observed broken. However at first glance it may look overly restrictive, preventing useful program behaviour. Consider the iconic example of a `Range` class, with a `min` and `max` value, where the invariant requires that `min<=max`:

```
class Range{ private field min; private field max;
  method invariant(){return min<max;}
  method set(min, max){
    if(min>=max){throw new Error(/**/);}
    this.min = min;
    this.max = max; }}
```

In this example we omit types to focus on the runtime semantics. The code of `set` does not violate visible state semantics: `this.min = min` may temporarily break the invariant of `this`, however it will be fixed after executing `this.max = max`. Visible state allows such temporary breaking of invariants since we are inside a method on `this`, and by the time it returns, the invariant will be re-established. However, if `min` is $\geq$ `this.max`, `set` will violate our stricter approach. The execution of `this.min = min` will break the invariant of `this` and `this.max = max` would then involve a broken object. If we were to inject a call `Do.stuff(this);` between the two field updates, arbitrary user code could observe a broken object; adding such a call is however allowed by visible state semantics.

Using the *box pattern*, we can provide a modified `Range` class with the desired client interface, while respecting the principles of our strict protocol:

```
class BoxRange{//no invariant in BoxRange
  field min; field max;
  BoxRange(min, max){ this.set(min, max); }
  method Void set(min, max){
    if(min>=max){throw new Error(/**/);}
    this.min = min; this.max = max;
    } }
class Range{ private field box; //box contains a BoxRange
  Range(min, max){ this.box = new BoxRange(min, max); }
  method invariant(){ return this.box.min < this.box.max; }
  method set(min, max){ return this.box.set(min,max); }
  }
```

The code of `Range.set(min,max)` does not violate our protocol. The call to `BoxRange.set(min,max)` works in a context where the `Range` object is unreachable, and thus not involved in execution. That is, the `Range` object is not in the ROG of the receiver or the parameters of `BoxRange.set(min,max)`. Thus `Range.set(min,max)` can temporarily break the `Range`'s invariant. By using the `box` field as an extra level of indirection, we restrict the set of objects involved in execution while the state of the object `Range` is modified.[2] With appropriate type annotations, the code of `Range` and `BoxRange` is accepted as correct by our system: no matter how `Range` objects are used, a broken `Range` object will never be involved in execution.

Contributions

Invariant protocols allow for objects to make necessary changes that might make their invariant temporarily broken. In visible state semantics any object that has an active method call anywhere on the call stacks is potentially invalid; arguably not a sufficent guarantee as observed by Gopinathan *et al.*'s. [5] Approaches such as *pack/unpack* [3] represent potentially invalid objects in the type system; this encumbers the type system and the syntax with features whose only purpose is to distinguish objects with broken invariants.

[2] Due to its simplicity and versatility, we do not claim this pattern to be a contribution of our work, as we expect others to have used it before. We have however not been able to find it referenced with a specific name in the literature, though technically speaking, it is a simplification of the Decorator, but with a different goal. While in very specific situations the overhead of creating such additional box object may be unacceptable, we designed our work for environments where such fine performance differences are negligible. Also note that many VMs and compilers can optimize away wrapper objects in many circumstances. [4]

The core insight behind our work is that we can use a small number of decorator-like design patterns to avoid exposing those potentially invalid objects in the first place, thus avoiding the need of representing them at the type level.

In the remainder of this paper, we discuss how to combine runtime checks and capabilities to soundly enforce our strict invariant protocol. Our solution only requires that all code is well-typed, and works in the presence of mutation, I/O, non-determinism, and exceptions, all under an open world assumption.

We formalise our approach and, in Appendix A, prove that our use of Reference and Object Capabilities soundly enforces our invariant protocol.

We have fully implemented our protocol in L42[3], we used this implementation to implement many case studies, showing that our protocol is more succinct than the pack/unpack approach and much more efficient then the visible state semantic. It is important to note that unlike most prior work, we soundly handle catching of invariant failures and I/O. We describe our case studies in Section 6. Our approach may seem very restrictive; the programming patterns in Section 7 show how our approach does not hamper expressiveness; in particular we show how batch mutation operations can be performed with a single invariant check, and how the state of a 'broken' object can be safely passed around.

We proposed our approach for integration in the L42 language, and after minor reworking it has been accepted, and the current version of L42 integrated our invariant checking protocol in a cohesive way with the support for caching and parallelism. Section **??** discusses the details of this integration.

2. Background on Reference and Object Capabilities

Reasoning about imperative OO programs is a non-trivial task, made particularly difficult by mutation, aliasing, dynamic dispatch, I/O, and exceptions. There are many ways to perform such reasoning; instead of using automated theorem proving, it is becoming more popular to verify aliasing and immutability properties using a type system. For example, three languages: L42 [6, 7, 8, 9], Pony [10, 11], and the language of Gordon *et al.* [12] use RCs (Reference Capabilities)[4] and OCs (Object Capabilities) to statically ensure deterministic parallelism and the absence of data races. While studying those languages, we discovered an elegant way to enforce invariants: we use capabilities to restrict how/when the result of invariant methods changes; this is done by restricting I/O, and how mutation through aliases can affect the state seen by invariants.

Reference Capabilities

RCs, as used in this paper, are a type system feature that allows reasoning about aliasing and mutation. Recently a new design for them has emerged that radically improves their usability; three different research languages are being independently developed relying on this new design: the language of Gordon *et al.*, Pony, and L42. These projects are quite large: several million lines of code are written in Gordon *et al.*'s language and are used by a large private Microsoft project; Pony and L42 have large libraries and are active open source projects. In particular the RCs of these languages are used to provide automatic and correct parallelism [12, 10, 11, 7].

Reference capabilities are a well known mechanism [13, 14, 15, 10, 9, 12] that allow statically reasoning about the mutability and aliasing properties of objects. Here we refer to the interpretation of [12], that introduced the concept of recovery/promotion. This concept is the basis for L42, Pony, and Gordon *et al.*'s type systems [12, 7, 6, 10, 11]. With slightly different names and semantics, those languages all support the following RCs for object references:

- Mutable (`mut`): the referenced object can be mutated and shared/aliased without restriction; as in most imperative languages without reference capabilities.

[3] Our implementation is implemented by checking that a given class conforms to our protocol, and injecting invariant checks in the appropriate places. An anonymised version of L42, supporting the protocol described in this paper, together with the full code of our case studies, is available at `http://l42.is/InvariantArtifact.zip`.

[4] RCs are called *Type Modifiers* in former works on L42.

- Immutable (imm): the referenced object cannot mutate, not even through other aliases. An object with any imm aliases is an *immutable object*. Any other object is a *mutable object*. All objects are born mutable and may later become immutable.

- Readonly (read): the referenced object cannot be mutated by such references, but there may also be mutable aliases to the same object, thus mutation can be observed. Readonly references can refer to both mutable and immutable objects, as read types are supertypes of both their imm and mut variants. There are only two kinds of objects: mutable and immutable, but there are more kinds of RCs.

- Encapsulated (capsule): every mutable object in the ROG of a capsule reference (including itself) is only reachable through that reference. Immutable objects in the ROG of a capsule reference are not constrained, and can be freely referred to without passing through that reference.

RCs are different to field or variable qualifiers like Java's final: RCs apply to references, whereas final applies to fields themselves. Unlike a variable/field of a read type, a final variable/field cannot be reassigned, it always refers to the same object, however the variable/field can still be used to mutate the referenced object. On the other hand, an object cannot be mutated through a read reference, however a read variable can still be reassigned.[5]

As you can see, RC are applied to all types. This of course includes types in method parameters and the method receiver. A mut method is a method where this is typed mut; An imm method is a method where this is typed imm, and so on for all the other RCs.

Consider the following example usage of mut, imm, and read, where we can observe a change in rp caused by a mutation inside mp.

```
 mut Point mp = new Point(1, 2);
 mp.x = 3; // ok
 imm Point ip = new Point(1, 2);
 //ip.x = 3; // type error
read Point rp = mp;
//rp.x = 3; // type error
mp.x = 5; // ok, now we can observe rp.x == 5
ip = new Point(3, 5); // ok, ip is not final
```

RCs influence the access to the whole ROG; not just the referenced object itself, as in the full/deep interpretation of type modifiers [16, 17]:

- A mut field accessed from a read reference produces a read reference; thus a read reference cannot be used to mutate the ROG of the referenced object.

- Any field accessed from an imm reference produces an imm reference; thus all the objects in the ROG of an immutable object are also immutable.

A common misconception of this line of work is that a mut field would always refer to a mutable object. Classes declares RCs for their methods and fields types, but what kinds of objects are stored in an object fields depend also on the kind of the object: a mut field of a mutable object will contain a mutable object; but a mut field of an immutable object will contain an immutable object. This is different with respect to many other approaches, where the declarations determine what to expect, and any information from the context must be explicit passed to the type using, for example, a generic reference capability parameter.

Another common misconception is the belief that capsule fields and capsule local variables always hold capsule references. How capsule local variables are handled differs widely in the literature:

In L42, a capsule local variable always holds a capsule reference: this is ensured by allowing them to be used only once (similar to linear and affine types [18]). Pony and Gordon *et al.* follow a more complicated approach: capsule variables can be accessed multiple times, however the result will not be a capsule reference

[5] In C, this is similar to the difference between A* const (like final) and const A* (like read), where const A* const is like final read.

and can only be used in limited ways. Pony and Gordon also provide destructive reads, where the variable's old value is returned as `capsule`. Like `capsule` variables, how `capsule` fields are handled differs widely in the literature, however they must always be initialised and updated with `capsule` references. In order for access to a `capsule` field to safely produce a `capsule` reference, Gordon *et al.* only allows them to be read destructively (i.e. by replacing the field's old value with a new one, such as `null`). In contrast, Pony does not guarantee that `capsule` fields contain a `capsule` reference at all times, as it provides non-destructive reads. L42 is even more radical: an L42 `capsule` field never contains a `capsule` reference; it is simply initialized with one. [7, 19] Pony and L42's `capsule` fields are useful for safe parallelism but not invariant checking. [6]

In Section 3 we present a novel kind of `capsule` field useful for invariant checking; we added support for these fields to L42, and believe they could be easily added to Pony and Gordon *et al.*'s language.

Promotion and Recovery

Many different techniques and type systems handle the RCs above [16, 20, 21, 12, 6]. The main progress in the last few years is with the flexibility of such type systems: where the programmer should use `imm` when representing immutable data and `mut` nearly everywhere else. The system will be able to transparently promote/recover [12, 10, 6] the reference capability, adapting them to their use context. To see a glimpse of this flexibility, consider the following:

```
    mut Circle mc = new Circle(new Point(0, 0), 7);
capsule Circle cc = new Circle(new Point(0, 0), 7);
    imm Circle ic = new Circle(new Point(0, 0), 7);
```

Here `mc`, `cc`, and `ic` are all syntactically initialised with the same exact expression. All `new` expressions return a `mut` [10, 19], so `mc` is well typed. The declarations of `cc` and `ic` are also well typed, since any expression (not just `new` expressions) of a `mut` type that has no `mut` or `read` free variables can be implicitly promoted to `capsule` or `imm`. This requires the absence of `read` and `mut` *global/static* variables, as in L42, Pony, and Gordon *et al.*'s language. This is the main improvement on the flexibility of RCs in recent literature [7, 6, 12, 10, 11]. From a usability perspective, this improvement means that these RCs are opt-in: a programmer can write large sections of code simply using `mut` types and be free to have rampant aliasing. Then, at a later stage, another programmer may still be able to encapsulate those data structures into an `imm` or `capsule` reference.

Exceptions

In most languages exceptions may be thrown at any point; combined with mutation this complicates reasoning about the state of programs after exceptions are caught: if an exception was thrown while mutating an object, what state is that object in? Does its invariant hold? The concept of *strong exception safety* [22, 8] simplifies reasoning: if a `try–catch` block caught an exception, the state visible before execution of the `try` block is unchanged, and the exception object does not expose any object that was being mutated; this prevents exposing objects whose invariant was left broken in the middle of mutations. L42 enforces strong exception safety for unchecked exceptions using RCs[7] in the following way:[8]

- Code inside a `try` block that captures unchecked exceptions is typed as if all `mut` variables declared outside of the block are `read`.
- Only `imm` objects may be thrown as unchecked exceptions.

This strategy does not restrict when exceptions can be *thrown*, but only restricts when unchecked exceptions can be *caught*. Strong exception safety allows us to throw invariant failures as unchecked exceptions: if an object's ROG was mutated into a broken state within a try block, when the invariant failure is caught, the mutated object will be unreachable/garbage-collectable. This works since strong exception safety guarantees that no object mutated within a try block is visible when it catches an unchecked exception.[9]

[6] It may seem surprising that those weaker forms of encapsulation are still suffcient to ensure safe parallelism. The detailed way L42 parallelism works is unrelated to the presented work. Please refer to `L42.is/tutorial.xhtml` (sections 5 and 6) for more information on parallelism in L42.

[7] This is needed to support safe parallelism. Pony takes a drastic approach and not support exceptions. We are not aware of how Gordon *et al.* handles exceptions, however to have sound unobservable parallelism it must have some restrictions.

[8] Formal proof that these restriction are sufficient is in the work of Lagorio [8].

[9] Transactions are another way of enforcing strong exception safety, but they require specialised and costly run time support.

Object Capabilities

OCs, which L42, Pony, and Gordon *et al.*'s work have, are a widely used [23, 24, 25] programming technique where access rights to resources are encoded as references to objects. When this style is respected, code unable to reach a reference to such an object cannot use its associated resource. Here, as in Gordon *et al.*'s work, we enforce the OC pattern with RCs in order to reason about determinism and I/O. To properly enforce this, the OC style needs to be respected while implementing the primitives of the standard library, and when performing foreign function calls that could be non-deterministic, such as operations that read from files or generate random numbers. Such operations would not be provided by static methods, but instead by instance methods of classes whose instantiation is kept under control by carefully designing their implementation.

For example, in Java, `System.in` is a *capability object* that provides access to the standard input resource. However, since it is globally accessible it completely prevents reasoning about determinism. In contrast, if Java were to respect the object capability style, the `main` method could take a `System` parameter, as in

```
public static void main(System s){... s.in.read() ...}
```

Calling methods on that `System` instance would be the only way to perform I/O; moreover, the only `System` instance would be the one created by the runtime system before calling `main(s)`. This design has been explored by Joe-E [26].

OCs are typically not part of the type system nor do they require runtime checks or special support beyond that provided by a memory safe language. However, since L42 allows user code to perform foreign calls without going through a predefined standard library, the OC pattern is enforced by the type system:

- Foreign methods (which have not been whitelisted as deterministic) and methods whose names start with `#$` are *capability operations*.
- Constructors of *capability classes* are also *capability operations*.
- Capability operations can only be called by other capability operations or `mut`/`capsule` methods of capability classes.
- In L42 there is no `main` method, rather it has several *main expressions*; such expressions can also call capability operations, thus they can instantiate OCs and pass them around to the rest of the program.

3. Our Invariant Protocol

All classes contain a `read method Bool invariant() {..}`, if no `invariant()` method is explicitly present, a trivial one returning `true` is assumed.

Our protocol guarantees that the whole ROG of any object involved in execution (formally, in a redex) is *valid*: if you can use an object, calling `invariant()` on it is guaranteed to return `true` in a finite number of steps.

As the `invariant()` is used to determine whether `this` is broken, it may receive a broken `this`; however this will only occur for calls to `invariant()` inserted by our approach. User written calls to `invariant()` are guaranteed to receive a valid `this`.

We restrict `invariant()` methods so that they represent a predicate over the receiver's `imm` and `capsule` fields. To ensure that `invariant()` methods do not expose a potentially broken `this` to the other objects, we require that all occurrences of `this`[10] in the `invariant()`'s body are the receiver of a field access (`this.f`) of an `imm`/`capsule` field, or the receivers of a method call (`this.m(..)`) of a final (non-virtual) method that in turn satisfies these restrictions. No other uses of `this` are allowed, such as as the right hand side of a variable declaration, or an argument to a method. An equivalent alternative design could instead rely on static `invariant(..)` methods taking each `imm` and `capsule` field as a parameter.

[10] Some languages allow the `this` receiver to be implicit. For clarity in this work we require `this` to be always used explicit.

Invariants can only refer to immutable and encapsulated state. Thus while we can easily verify that a doubly linked list of immutable elements is correctly linked up, we can not do the same for a doubly linked lists of mutable elements. We do not make it harder to correctly implement such data structures, but the `invariant()` method is unable to access the list's nodes, since they may contain `mut` references to shared/unencapsulated objects. There is a line of work [27] striving to allow invariants over other forms of state. We have not tried to integrate such solutions into our work as we believe it would make our system more complex and ad hoc, probably requiring numerous specialised kinds of RCs. Thus we have traded some expressive power in order to preserve safety and simplicity.

Purity

L42's enforcement of RCs and OCs statically guarantees that any method with only `read` or `imm` parameters (including the receiver) is *pure*; we define pure as being deterministic and not mutating existing memory. This holds because (1) the ROG of the parameters (including `this`) is only accessible as `read` (or `imm`), thus it cannot be mutated (2) if a capability object is in the ROG of any of the arguments (including the receiver), then it can only be accessed as `read`, preventing calling any non-deterministic (capability) methods; (3) no other pre-existing objects are accessible (as L42 does not have global variables). In particular, this means that our `invariant()` methods are pure, since their only parameter (the receiver) is `read`.

Capsule Fields

Section 7 'ownership' of [19]describes how in L42 "depending on how we expose the owned data, we can closely model [...]owners-as-dominators[...]owners-as-qualifiers[...]a third variant". Those informal considerations have then influenced the L42 language design, bringing to the creation of syntactic sugar and programming patterns to represent various kinds of `capsule` fields aimed to model various forms of ownership. Under the hood, all those forms of `capsule` fields are just private `mut` fields with some extra restrictions. Describing in the details those restrictions would be outside of the scope of this paper.

Here we present a novel kind of `capsule` field[11], that can coexists with those other kinds of `capsule` fields, enforcing the following key property: the ROG of a capsule field $o.f$ can only be mutated under the control of a `mut` method of o, and during such mutation, o itself cannot be seen. This is similar of owner-as-modifier [28, 29], where we could consider an object to be the 'owner' of all the mutable objects in the ROG of its `capsule` fields; but with the extra restriction that the owner is unobservable during the mutation process.

More precisely, if a reference to an object in the ROG of a capsule field $o.f$ is involved in execution as `mut`, then: (1) no reference to o is involved in execution, (2) a call to a `mut` method for o is above the current stack frame, (3) mutable references to the ROG of $o.f$ are not leaked out of such method execution, either as return values, exception values, or stored in the ROG of a parameter, or in any other field of the method's receiver.

To show how our `capsule` fields ensure these properties, we first define some terminology: `x.f` is a *field access*, `x.f=e` is a *field update*,[12] a `mut` method with a field access on a capsule field of `this` is a *capsule mutator*. Note that a field *update* of a `capsule` field (instead of a field access) does not make a method a capsule mutator.

The following rules define our novel `capsule` fields:

- A `capsule` field can only be initialised/updated with a `capsule` expression.
- A `capsule` field access will return a:
  - `mut` reference, when accessed on `this` within a capsule mutator,
  - `read` reference, when accessed on any other `mut` receiver,
  - `imm` if the receiver is `imm`, `read` if the receiver is `read`, or `capsule` if the receiver is `capsule`. This last case is safe since a `capsule` receiver object will then be garbage collectable, so do not need to preserve its invariant.

[11] As for the other kinds of `capsule` fields, our new kind is also just a private `mut` fields with extra restrictions.

[12] Thus a field update `x.f=e` is not a field access followed by an assignment.

- A capsule mutator must:
  - use `this` exactly once: to access the `capsule` field,
  - have no `mut` or `read` parameters (except the `mut` receiver),
  - not have a `mut` return type,
  - not throw any checked exceptions[13].

The above rules ensures that capsule mutators controls the mutation of the ROG of capsule fields, and ensure our points (1), (2), and (3): o will not be in the ROG of $o.f$ and only a capsule mutator on o can see $o.f$ as `mut`; this means that the only way to mutate the ROG of $o.f$ is through such methods. If execution is (indirectly) in a capsule mutator, then o is only used as the receiver of the `this.f` expression in the capsule mutator. Thus we can be sure that the ROG of $o.f$ will only be mutated within a capsule mutator, and only after the single use of o to access $o.f$. Since such mutation could invalidate the invariant of o, we call the `invariant()` method at the end of the capsule mutator body; before o can be used again. Provided that the invariant is re-established before a capsule mutator returns, no invariant failure will be thrown, even if the invariant was temporarily broken *during* the body of the method.

These properties are stronger then those of the pre-existing `capsule` fields of L42, but still *weaker* than those of `capsule` *references*: we do not need to prevent arbitrary `read` aliases to the ROG of a `capsule` field, and we do allow arbitrary `mut` aliases to exist during the execution of a capsule mutator. In particular, our rules allow unrestricted read only access to our `capsule` fields.

Runtime Monitoring

The language runtime will automatically perform calls to `invariant()`, if such a call returns `false`, an unchecked exception will be thrown. Such calls are performed at the following points:

- After a constructor call, on the newly created object.
- After a field update, on the receiver.
- After a capsule mutator method returns, on the receiver of the method[14].

In Section 5, we show that these checks, together with our aforementioned restrictions, are sufficient to ensure our guarantee that the invariants of all objects involved in execution hold.

Traditional Constructors and Subclassing

L42 constructors directly initialise all the fields using the parameters, and L42 does not provide traditional subclassing. This works naturally with our invariant protocol. We can support traditional constructors as in Pony and Gordon *et al.*'s language, by requiring that constructors only use `this` as the receiver of a field initialisation. Subclassing can be supported by forcing that a subclass invariant method implicitly starts with a check that `super.invariant()` returns `true`. We would also perform invariant checks at the end of `new` expressions, as happens in [30], and not at the end of `super(..)` constructor calls.

4. Essential Language Features

Our invariant protocol relies on many different features and requirements. In this section we will show examples of using our system, and how relaxing any of our requirements would break the soundness of our protocol. In our examples and in L42, the reference capability `imm` is the default, and so it can be omitted. Many verification approaches take advantage of the separation between primitive/value types and objects, since the former are immutable and do not support reference equality. However, our approach works in a

[13] To allow capsule mutators to leak checked exceptions, we would need to check the invariant when such exceptions are leaked. However, this would make the runtime semantics of checked exceptions inconsistent with unchecked ones.

[14] The invariant is not checked if the call was terminated via an an unchecked exception, since strong exception safety guarantees the object will be unreachable.

pure OO setting without such a distinction. Hence we write all type names in `BoldTitleCase` to emphasise this. To save space we omit the bodies of constructors that simply initialise fields with the values of the constructor's parameters, but we show their signature in order to show any annotations.

First we consider `Person`: it has a single immutable (and non final) field `name`.

```
class Person {
  read method Bool invariant() { return !name.isEmpty(); }
  private String name;//the default RC imm is applied here
  read method String name() { return this.name; }
  mut method Void name(String name) { this.name = name; }
  Person(String name) { this.name = name; }  }
```

The `name` field is not final: `Person`s can change state during their lifetime. The ROGs of all of a `Person`'s fields are immutable, but `Person`s themselves may be mutable. We enforce `Person`'s invariant by generating checks on the result of calling `this.invariant()`: immediately after each field update, and at the end of the constructor. Such checks are generated/injected, and not directly written by the programmer.

```
class Person { .. // Same as before
  mut method String name(String name) {
    this.name = name; // check after field update
    if (!this.invariant()) { throw new Error(...); }}
  Person(String name) {
    this.name = name; // check at end of constructor
    if (!this.invariant()) { throw new Error(...); }}  }
```

We now show how if we were to relax (as in Rust), or even eliminate (as in Java), the support for OCs, RCs, or strong exception safety, the above checks would not be sufficient to enforce our invariant protocol.

Unrestricted Access to Capability Objects?

Allowing `invariant()` methods to (indirectly) perform non-deterministic operations by creating new capability objects or mutating existing ones would break our guarantee that (manually) calling `invariant()` always returns `true`. Consider this use of person; where `myPerson.invariant()` may randomly return `false`:

```
class EvilString extends String {//INVALID EXAMPLE
  @Override read method Bool isEmpty() {//Creates a new
    return new Random().bool(); }}//capability out of thin air
...
method mut Person createPersons(String name) {
  // we can not be sure that name is not an EvilString
  mut Person schrodinger = new Person(name); // exception here?
  assert schrodinger.invariant(); // will this fail?
  ...}
```

Despite the code for `Person.invariant()` intuitively looking correct and deterministic (`!name.isEmpty()`), the above call to it is not. Obviously this breaks any reasoning and would make our protocol unsound. In particular, note how in the presence of dynamic class loading, we have no way of knowing what the type of `name` could be. Since our system allows non-determinism only through capability objects, and restricts their creation, the above example is prevented.

Moreover, since our systems allows non-determinism only through `mut` methods on capability objects, even if an object has a `capsule` field referring to a file IO object, it would be unable to read such file during an invariant, since a `mut` reference would be required, but only a `read` reference would be available.

Allowing Internal Mutation Through Back Doors?

Rust [31] and Javari [13] allow interior mutability: the ROG of an 'immutable' object can be mutated through back doors. Such back doors would allow `invariant()` methods to store and read information about previous calls. The example class `MagicCounter` breaks determinism by remotely breaking the invariant of `person` without any interaction with the `person` object itself:

```
class MagicCounter {//INVALID EXAMPLE
  method Int incr(){/*return counter++; using internal mutability*/}}
```

```
class NastyS extends String {..
  MagicCounter c = new MagicCounter(0);
  @Override read method Bool isEmpty(){return this.c.incr()!=2;}}
...
NastyS name = new NastyS(); //RCs believe name's ROG is immutable
Person person = new Person(name); // person is valid, counter=1
name.incr(); // counter == 2, person is now broken
person.invariant(); // returns false, counter == 3
person.invariant(); // returns false, counter == 4
```

Such back doors are usually motivated by performance reasons, however in [12] they discuss how a few trusted language primitives can be used to perform caching and other needed optimisations, without the need for back doors.

No Strong Exception Safety?

The ability to catch and recover from invariant failures allows programs to take corrective action. Since we represent invariant failures by throwing unchecked exceptions, programs can recover from them with a conventional **try–catch**. Due to the guarantees of strong exception safety, any object that has been mutated during a **try** block is now unreachable, as happens in alias burying [18]. This property ensures that an object whose invariant fails will be unreachable after the invariant failure has been captured. If instead we were to not enforce strong exception safety, an invalid object could be made reachable. The following code is ill-typed since we try to mutate `bob` in a **try-catch** block that captures all unchecked exceptions; thus also including invariant failures:

```
mut Person bob = new Person("bob");//INVALID EXAMPLE
// Catch and ignore invariant failure:
try { bob.name(""); } catch (Error t) { }// bob mutated
assert bob.invariant(); // fails!
```

The following variant is instead well typed, since `bob` is now declared inside of the **try**, it is guaranteed to be garbage collectable after the **try** is completed.

```
try {mut Person bob = new Person("bob");    bob.name("");}
catch (Error t) { }
```

Relaxing restrictions on capsule fields?

Capsule fields allow expressing invariants over mutable object graphs. Consider managing the shipment of items, where there is a maximum combined weight:

```
class ShippingList {
  capsule Items items;
  read method Bool invariant(){return this.items.weight()<=300;}
  ShippingList(capsule Items items) {
    this.items = items;
    if (!this.invariant()){throw Error(...);}}//injected check
  mut method Void addItem(Item item) {
    this.items.add(item);
    if (!this.invariant()){throw Error(...);}}}//injected check
```

We inject calls to `invariant()` at the end of the constructor and the `addItem(item)` method. This is safe since the `items` field is declared **capsule**. Relaxing our system to allow a **mut** RC for the `items` field and the corresponding constructor parameter would make the above checks insufficient: it would be possible for external code with no knowledge of the **ShippingList** to mutate its items. In order to write correct library code in mainstream languages like Java and C++, defensive cloning [32] is needed. For performance reasons, this is hardly done in practice and is a continuous source of bugs and unexpected behaviour.

```
mut Items items = ...;//INVALID EXAMPLE
mut ShippingList l = new ShippingList(items); // l is valid
items.addItem(new HeavyItem()); // l is now invalid!
```

If we were to allow `x.items` to be seen as **mut**, where `x` is not **this**, then even if the **ShippingList** has full control of `items` at initialisation time, such control may be lost later, and code unaware of the `ShippingList` could break it:

```
//INVALID EXAMPLE: l.items can be exposed as mut
mut ShippingList l = new ShippingList(new Items()); // l is ok
mut Items evilAlias = l.items; // here l loses control
evilAlias.addItem(new HeavyItem()); // now l is invalid!
```

Relaxing our requirements for capsule mutators would break our protocol: if capsule mutators could have a `mut` return type the following would be accepted:

```
//INVALID EXAMPLE: capsule mutator expose(c) return type is mut
mut method mut Items expose(C c) {return c.foo(this.items);}
```

Depending on dynamic dispatch, `c.foo()` may just be the identity function, thus we would get in the same situation as the former example.

Allowing `this` to be used more than once would allow the following code, where `this` may be reachable from `f`, thus `f.hi()` may observe an object that does not satisfying its invariant:

```
mut method Void multiThis(C c) {//INVALID EXAMPLE: two 'this'
  read Foo f = c.foo(this);
  this.items.add(new HeavyItem());
  f.hi(); }//'this' could be observed here if it is in ROG(f)
```

In order to ensure that a second reference to `this` is not reachable through arguments to such methods, we only allow `imm` and `capsule` parameters. Accepting a `read` parameter, as in the example below, would cause the same problems as before, where `f` may contain a reference to `this`:

```
mut method Void addHeavy(read Foo f) {//INVALID EXAMPLE
  this.items.add(new HeavyItem());
  f.hi(); }//'this' could be observed here if it is in ROG(f)
...
mut ShippingList l = new ShippingList(new Items());
read Foo f = new Foo(l);
l.addHeavy(f); // We pass another reference to 'l' through f
```

5. Formal Language Model

To model our system we need to formalise an imperative OO language with exceptions, object capabilities, and type system support for RCs and strong exception safety. Formal models of the runtime semantics of such languages are simple, but defining and proving the correctness of such a type system would require a paper of its own, and indeed many such papers exist in the literature [7, 6, 12, 10, 8]. Thus we are assuming that we already have an expressive and sound type system enforcing the properties we need, and instead focus on invariant checking. We clearly list in Appendix A the assumptions we make on such a type system, so that any language satisfying them, such as L42, can soundly support our invariant protocol. To keep our small step semantics as conventional as possible, we follow Pierce [33] and Featherweight Java [34]; we model an OO language where receivers are always specified explicitly, and the receivers of field accesses and updates in method bodies are always `this`; that is, all fields are instance-private. Constructors are all of the form $C(T_1 x_1, \ldots, T_n x_n)$`{this.`$f_1$`=`$x_1$`;`$\ldots$`;this.`$f_n$`=`$x_n$`;}`, where the fields of C are $T_1 f_1; \ldots; T_n f_n;$. We do not model custom constructors and traditional subclassing since this would make the proof more involved without adding any additional insight.

We additionally assume the following:

- An implicit program/class table; we use the notation $C.m$ to get the method declaration for m within class C, similarly we use $C.f$ to get the declaration of field f, and $C.i$ to get the declaration of the i^{th} field.

- Memory, $\sigma : l \rightarrow C\{\overline{v}\}$, is a finite map from locations, l, to annotated tuples, $C\{\overline{v}\}$, representing objects; where C is the class name and $\overline{v}$ are the field values. We use the notation $\sigma[l.f = v]$ to update

$$
\begin{array}{lcll}
e & ::= & x \mid \texttt{true} \mid \texttt{false} \mid e.m(\overline{e}) \mid \texttt{this}.f \mid \texttt{this}.f\ \texttt{=}\ e \mid \texttt{new}\ C(\overline{e}) \mid \texttt{try}\,\{e_1\}\,\texttt{catch}\,\{e_2\} & \text{expression} \\
 & & \mid l \mid l.f \mid l.f\ \texttt{=}\ e \mid \texttt{M}(l;e_1;e_2) \mid \texttt{try}^{\sigma}\{e_1\}\ \texttt{catch}\ \{e_2\} & \text{runtime expr.} \\
v & ::= & l & \text{value} \\
\mathcal{E}_v & ::= & [] \mid \mathcal{E}_v.m(\overline{e}) \mid v.m(\overline{v}_1,\mathcal{E}_v,\overline{e}_2) \mid v.f\ \texttt{=}\ \mathcal{E}_v & \text{eval. context} \\
 & & \mid \texttt{new}\ C(\overline{v}_1,\mathcal{E}_v,\overline{e}_2) \mid \texttt{M}(l;\mathcal{E}_v;e) \mid \texttt{M}(l;v;\mathcal{E}_v) \mid \texttt{try}^{\sigma}\{\mathcal{E}_v\}\ \texttt{catch}\ \{e\} & \\
\mathcal{E} & ::= & [] \mid \mathcal{E}.m(\overline{e}) \mid e.m(\overline{e}_1,\mathcal{E},\overline{e}_2) \mid \mathcal{E}.f \mid \mathcal{E}.f\ \texttt{=}\ e \mid e.f\ \texttt{=}\ \mathcal{E} \mid \texttt{new}\ C(\overline{e}_1,\mathcal{E},\overline{e}_2) & \text{full context} \\
 & & \mid \texttt{M}(l;\mathcal{E};e) \mid \texttt{M}(l;e;\mathcal{E}) \mid \texttt{try}^{\sigma?}\{\mathcal{E}\}\ \texttt{catch}\ \{e\} \mid \texttt{try}^{\sigma?}\{e\}\ \texttt{catch}\ \{\mathcal{E}\} & \\
CD & ::= & \texttt{class}\ C\ \texttt{implements}\ \overline{C}\{\overline{F}\,\overline{M}\} \mid \texttt{interface}\ C\ \texttt{implements}\ \overline{C}\{\overline{M}\} & \text{class decl.} \\
F & ::= & T\ f\texttt{;} & \text{field} \\
M & ::= & \mu\,\texttt{method}\ T\ m(T_1\,x_1,...,T_n\,x_n)\ e? & \text{method} \\
\mu & ::= & \texttt{mut} \mid \texttt{imm} \mid \texttt{capsule} \mid \texttt{read} & \text{reference capability} \\
T & ::= & \mu\,C & \text{type} \\
\mathcal{E}_r & ::= & \mathcal{E}_v[[].m(\overline{v})] \mid \mathcal{E}_v[v.m(\overline{v}_1,[],\overline{v}_2)] \mid \mathcal{E}_v[[].f] \mid \mathcal{E}_v[[].f\ \texttt{=}\ v] \mid \mathcal{E}_v[v.f\ \texttt{=}\ []] & \text{redex context} \\
 & & \mid \mathcal{E}_v[\texttt{new}\,C(\overline{v}_1,[],\overline{v}_2)] & \\
\mathit{error} & ::= & \mathcal{E}_v[\texttt{M}(l;v;\texttt{false})],\ \text{ where } \mathcal{E}_v \text{ not of form } \mathcal{E}_v{}'[\texttt{try}^{\sigma?}\{\mathcal{E}_v{}''\}\ \texttt{catch}\ \{\_\}] & \text{validation error}
\end{array}
$$

Figure 1: Grammar

a field of l, $\sigma[l.f]$ to access one, and $\sigma \setminus l$ to delete l.

- The main expression is reduced in the context of a memory and program.
- A typing relation, $\Sigma;\Gamma;\mathcal{E} \vdash e : T$, where the expression e can contain locations and free variables. The types of locations are encoded in a memory environment, $\Sigma : l \to C$, while the types of free variables are encoded in a variable environment, $\Gamma : x \to T$. $\mathcal{E}$ encodes the location, relative to the top-level expression we are typing, where e was found; this is needed so that locations can be typed with different reference capabilities when in different positions.
- We use Σ^{σ} to trivially extract the corresponding Σ from a σ.

To encode object capabilities and I/O, we assume a special location c of class `Cap`. This location would refer to an object with methods that behave non-deterministically, such methods would model operations such as file reading/writing. In order to simplify our proof, we assume that:

- `Cap` has no fields,
- instances of `Cap` cannot be created with a `new` expression,
- `Cap`'s `invariant()` method is defined to have a body of '`true`', and
- all other methods in the `Cap` class must require a `mut` receiver; such methods will have a non-deterministic body, i.e. calls to them may have multiple possible reductions.

For simplicity, we do not formalise actual exception objects, rather we have *error*s, which correspond to expressions which are currently 'throwing' an exception; in this way there is no value associated with an *error*. Our L42 implementation instead allows arbitrary `imm` values to be thrown as (unchecked) exceptions, formalising exceptions in such way would not cause any interesting variation of our proof.

Grammar

The grammar is defined in Figure 1. Most of our expressions are standard. *Monitor expressions* are the syntactic representation of our injected invariant checks. They are of the form $\texttt{M}(l;e_1;e_2)$, they are runtime expressions and thus are not present in method bodies, rather they are generated by our reduction rules inside the main expression. Here, l refers to the object being monitored, e_1 is the expression which is being monitored, and e_2 denotes the evaluation of l`.invariant()`; e_1 will be evaluated to a value, and the e_2 will be further evaluated, if e_2 evaluated to `false` or an *error*, then l's invariant failed to hold; such a monitor

(UPDATE)
$$\frac{}{\sigma|l.f\ \texttt{=}\ v \to \sigma[l.f = v]|\texttt{M(}l\texttt{;}l\texttt{;}l\texttt{.invariant())}}$$

(NEW)
$$\frac{}{\sigma|\texttt{new}\ C\texttt{(}\overline{v}\texttt{)} \to \sigma, l \mapsto C\{\overline{v}\}|\texttt{M(}l\texttt{;}l\texttt{;}l\texttt{.invariant())}}$$

(MCALL)
$$\frac{}{\sigma|l.m\texttt{(}v_1,\ldots,v_n\texttt{)} \to \sigma|e'[\texttt{this} \coloneqq l, x_1 \coloneqq v_1, \ldots, x_n \coloneqq v_n]}$$
$\sigma(l) = C\{\_\}$
$C.m = \mu\,\texttt{method}\ T\ m\texttt{(}T_1\ x_1 \ldots T_n x_n\texttt{)}\ e$
if $\mu = \texttt{mut}$ and $\exists f$ such that
$\quad C.f = \texttt{capsule}\ \_$ and $e = \mathcal{E}[\texttt{this}.f]$
then $e' = \texttt{M(}l\texttt{;}e\texttt{;}l\texttt{.invariant())}$
otherwise $e' = e$

(MONITOR EXIT)
$$\frac{}{\sigma|\texttt{M(}l\texttt{;}v\texttt{;true)} \to \sigma|v}$$

(CTXV)
$$\frac{\sigma_0|e_0 \to \sigma_1|e_1}{\sigma_0|\mathcal{E}_v[e_0] \to \sigma_1|\mathcal{E}_v[e_1]}$$

(TRY ENTER)
$$\frac{}{\sigma|\texttt{try}\ \{e_1\}\ \texttt{catch}\ \{e_2\} \to \sigma|\texttt{try}^{\sigma}\{e_1\}\ \texttt{catch}\ \{e_2\}}$$

(TRY OK)
$$\frac{}{\sigma,\sigma'|\texttt{try}^{\sigma}\{v\}\ \texttt{catch}\ \{\_\} \to \sigma,\sigma'|v}$$

(TRY ERROR)
$$\frac{}{\sigma,\sigma'|\texttt{try}^{\sigma}\{error\}\ \texttt{catch}\ \{e\} \to \sigma,\sigma'|e}$$

(ACCESS)
$$\frac{}{\sigma|l.f \to \sigma|\sigma[l.f]}$$

Figure 2: Reduction rules

expression corresponds to the throwing of an unchecked exception. In addition, our reduction rules will annotate `try` expressions with the original state of memory. This is used in our type-system assumptions (see Appendix A) to model the guarantee of strong exception safety, that is, the annotated memory will not be mutated by executing the body of the `try`. Note: this strong limitation is only needed for unchecked exceptions, in particular, invariant failures. Our calculus only models unchecked exceptions/errors, however L42 also supports checked exceptions, and `try-catch`es over them impose no limits on object mutation during the `try`.

Well-Formedness Criteria and Reduction Rules

We additionally restrict the grammar with the following well-formedness criteria:

- `invariant()`s and capsule mutators follow the requirements of Section 3.
- Method bodies do not contain runtime expressions (i.e. l, $l.f$, $l.f\ \texttt{=}\ e$, $\texttt{M}$, or $\texttt{try}^{\sigma}$).

Our reduction rules are defined in Figure 2. They are standard, except for our handling of monitor expressions. Monitor expressions are added after all field updates, `new` expressions, and calls to capsule mutators. Monitor expressions are only a proof device, they need not be implemented directly as presented. For example, in L42 we implement them by statically injecting calls to `invariant()` at the end of setters (for `imm` and `capsule` fields), factory methods, and capsule mutators; this works as L42 follows the uniform access principle, so it does not have primitive expression forms for field updates and constructors, rather they are uniformly represented as method calls.

The failure of a monitor expression, $\texttt{M(}l\texttt{;}e_1\texttt{;}e_2\texttt{)}$, will be caught by our TRY ERROR rule, as will any other uncaught monitor failure in e_1 or e_2.

Statement of Soundness

We define a deterministic reduction arrow to mean that exactly one reduction is possible:
$\sigma_0|e_0 \Rightarrow \sigma_1|e_1$ iff $\{\sigma_1|e_1\} = \{\sigma|e$, where $\sigma_0|e_0 \to \sigma|e\}$

We say that an object is *valid* iff calling its `invariant()` method would deterministically produce `true` in a finite number of steps, i.e. it does not evaluate to `false`, fail to terminate, or produce an *error*. We also require evaluating `invariant()` to preserve existing memory (σ), however new objects (σ') can be created and freely mutated:

$\mathit{valid}(\sigma, l)$ iff $\sigma | l.\texttt{invariant()} \Rightarrow^{+} \sigma, \sigma' | \texttt{true}$.

To allow the `invariant()` method to be called on an invalid object, and access fields on such object, we define the set of trusted execution steps as the call to `invariant()` itself, and any field accesses inside its evaluation:

$\mathit{trusted}(\mathcal{E}_r[l])$ iff, either:

- $\mathcal{E}_r = \mathcal{E}_v[\texttt{M(}l\texttt{;}v\texttt{;}[].\texttt{invariant())}]$, or
- $\mathcal{E}_r = \mathcal{E}_v[\texttt{M(}l\texttt{;}v\texttt{;}\mathcal{E}_v'[[].f]\texttt{)}]$, and this $[].f$ expression came from the body of the **`invariant()`** method itself.

[Isaac: bassically this has $\mathcal{E}_v, r_L$ before, I've simplified it greatly now] Note that $\mathit{trusted}(\mathcal{E}_r[l])$ only holds when the very next reduction we are about to perform is $l.\texttt{invariant()}$ or $l.f$.

We define a *validState* as one that was obtained by any number of reductions from a well typed initial expression and memory, containing no monitors and with only the c memory location available:

$\mathit{validState}(\sigma, e)$ iff $c \mapsto \texttt{Cap\{\}} | e_0 \rightarrow^{+} \sigma | e$, for some e_0 with:

$$c : \texttt{Cap}; \emptyset; [] \vdash e_0 : T,\ \texttt{M(\_;\_;\_)} \notin e_0, \text{ and if } l \in e_0 \text{ then } l = c.$$

Finally, we define what it means to soundly enforce our invariant protocol:

Theorem 1 (Soundness)**.** If $\mathit{validState}(\sigma, \mathcal{E}_r[l])$, then either $\mathit{valid}(\sigma, l)$ or $\mathit{trusted}(\mathcal{E}_r[l])$.

Except for the injected invariant checks (and fields directly accessed), any redex in the execution of a well typed program takes in input only valid objects. In particular, no method call (other than *injected* invariant checks themselves) can see an object which is being checked for validity.

This is a very strong statement because $\mathit{valid}(\sigma, l)$ requires the invariant of l to deterministically terminate, and termination is a difficult property to ensure. Our setting does ensure termination of the invariant of any l in a redex. This works because non terminating `invariant()` methods would cause the monitor expression to never terminate. Thus, an l with a non terminating `invariant()` is never involved in an untrusted redex. Invariants are deterministic computations in function of the state of l. If l is in a redex, a monitor expression must have terminated after the object instantiation and after any update to the state of l. Thus, the very existence of an l outside of a monitor expression is a witness of the invariant termination.

6. Case Studies

To perform compelling case studies, we used our system on many examples, including one designed to be a worst case scenario for our approach. We also replicate many examples originally proposed by other papers, so that not all the code examples come from us.

6.1. An interactive GUI

We start by presenting our GUI example; a program that interacts with the real world using I/O. It demonstrates how to verify invariants over cyclic mutable object graphs. Our example is particularly relevant since, as with most GUI frameworks, it uses the *composite* programming pattern; arguably one of the most fundamental patterns in OO.

Our case study involves a GUI with containers (`SafeMovables`) and `Buttons`; the `SafeMovable` class has an invariant to ensure that its children are graphically contained within it and do not overlap. The `Buttons` move their `SafeMovable` when pressed. We have a `Widget` interface which provides methods to get `Widgets`' size and position as well as children (a list of `Widgets`). Both `SafeMovables` and `Buttons` implement `Widget`. Crucially, since the children of `SafeMovable` are stored in a list of `Widgets` it can contain other `SafeMovables`, and all queries to their size and position are dynamically dispatched; such queries are also used in `SafeMovable`'s invariant. Here we show a simplified version[15], where `SafeMovable` has just one `Button` and certain sizes and positions are fixed. Note that `Widgets` is a class representing a mutable list of `mut Widgets`.

[15]The full version, written in L42, which uses a different syntax, is available in our artifact at `http://l42.is/InvariantArtifact.zip`

```
class SafeMovable implements Widget {
  capsule Box box; Int width = 300; Int height = 300;
  @Override read method Int left() { return this.box.l; }
  @Override read method Int top() { return this.box.t; }
  @Override read method Int width() { return this.width; }
  @Override read method Int height() { return this.height; }
  @Override read method read Widgets children() { return this.box.c; }
  @Override mut method Void dispatch(Event e) {
    for (Widget w:this.box.c) { w.dispatch(e); }}
  read method Bool invariant() {..}
  SafeMovable(capsule Widgets c) { this.box = makeBox(c); }
  static method capsule Box makeBox(capsule Widgets c) {
    mut Box b = new Box(5, 5, c);
    b.c.add(new Button(0, 0, 10, 10, new MoveAction(b));
    return b; }} // mut b is soundly promoted to capsule
class Box { Int l; Int t; mut Widgets c;
  Box(Int l, Int t, mut Widgets c) {..}}
class MoveAction implements Action { mut Box outer;
  MoveAction(mut Box outer) { this.outer = outer; }
  mut method Void process(Event e) { this.outer.l += 1; }}
... //main expression
//#$ is a capability operation making a Gui object
Gui.#$().display(new SafeMovable(...));
```

As you can see, `Box`es encapsulate the state of the `SafeMovable`s that can change over time: `left`, `top`, and `children`. Also note how the ROG of `Box` is cyclic: since the `MoveAction`s inside `Button`s need a reference to the containing `Box` in order to move it. Even though the children of `SafeMovable`s are fully encapsulated, we can still easily dispatch events to them using `dispatch(e)`. Once a `Button` receives an `Event` with a matching ID, it will call its `Action`'s `process(e)` method.

Our example shows how to encode interactive GUI programs, where widgets may circularly reference other widgets. In order to perform this case study we had to first implement a simple GUI Library in L42. This library uses object capabilities to draw the widgets on screen, as well as fetch and dispatch events. Importantly, neither our application, nor the underlying GUI library requires back doors, into either RCs or OCs.

The Invariant

`SafeMovable` is the only class in our GUI that has an invariant, our system automatically checks it in two places: the end of its constructor and the end of its `dispatch(e)` method (which is a capsule mutator). There are no other checks inserted since we never do a direct field update on a `SafeMovable`. The code for the invariant is just a couple of simple nested loops:

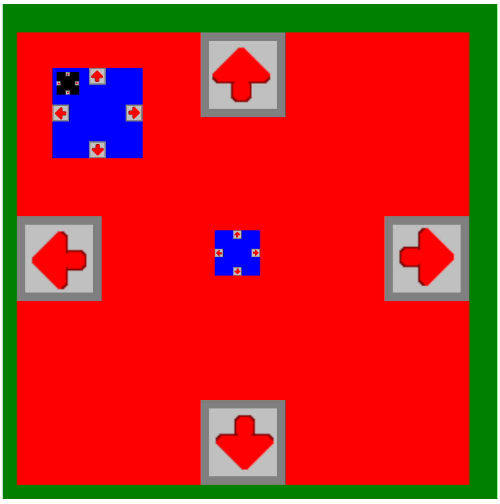
```
read method Bool invariant() {
  for(Widget w1 : this.box.c) {
    if(!this.inside(w1)) { return false; }
    for(Widget w2 : this.box.c) {
      if(w1!=w2 && SafeMovable.overlap(w1, w2)){return false;}}}
  return true;}
```

Here `SafeMovable.overlap` is a static method that simply checks that the bounds of the widgets don't overlap. The call to `this.inside(w1)` similarly checks that the widget is not outside the bounds of `this`; this instance method call is allowed as `inside(w)` only uses `this` to access its `imm` and `capsule` fields.

Our Experiment

As shown in the figure below, counting both `SafeMovable`s and `Button`s, our main method creates 21 widgets: a top level (green) `SafeMovable` without buttons, containing 4 (red, blue, and black) `SafeMovable`s with 4 (gray) buttons each. When a button is pressed it moves the containing `SafeMovable` a small amount in the corresponding direction. This set up is not overly complicated, the maximum nesting level of `Widgets` is 5.

Our main method automatically presses each of the 16 buttons once. In L42, using our invariant protocol, this resulted in 77 calls to `SafeMovable`'s invariant.



Comparison With Visible State Semantics

As an experiment, we set our implementation to generate invariant checks following the visible state semantics approaches of D and Eiffel [35, 36], where the invariant of the receiver is instead checked at the start and end of *every* public (in D) and qualified[16] (in Eiffel) method call. In our `SafeMovable` class, all methods are public, and all calls (outside the invariant) are qualified, thus this difference is irrelevant. Neither protocol performs invariant checks on field accesses or updates, however due to the 'uniform access principle' [36], Eiffel allows fields to directly implement methods, allowing the `width` and `height` *fields* to directly implement `Widget`'s `width()` and `height()` *methods*. On the other hand in D, one would have to write getter *methods*, which would perform invariant checks. When we ran our test case following the D approach, the `invariant()` method was called 52,734,053 times, whereas the Eiffel approach 'only' called it 14,816,207 times;[17]in comparison our invariant protocol only performed 77 calls. The number of checks is exponential in the depth of the GUI: the invariant of a `SafeMovable` will call the `width()`, `height()`, `left()`, and `top()` methods of its children, which may themselves be `SafeMovable`s, and hence such calls may invoke further invariant checks. Note that `width()` and `height()` are simply getters for fields, whereas the other two are non-trivial *methods*. Concluding, we have shown that when an invariant check queries other objects with invariants the visible state semantics may cause an exponential explosion in the number of checks.

Spec# Comparison

We also encoded our example in Spec#[18]; that relies on pack/unpack; also called inhale/exhale or the boogie methodology. In pack/unpack, an object's invariant is checked only by the explicit pack operations. In order for this to be sound, some form of aliasing and/or mutation control is necessary. Spec# uses a theorem prover, together with source code annotations. Spec# can be used for full static verification, but it conveniently allows invariant checks to be performed at runtime, whilst statically verifying aliasing, purity and other similar standard properties. This allows us to closely compare our approach with Spec#.

As the back-end of the L42 GUI library is written in Java, we did not port it to Spec#, rather we just simulated it, and don't actually display a GUI in Spec#. We ran our code through the Spec# verifier (powered by Boogie [37]), which only gave us 2 warnings[19]: that the invariant of `SafeMovable` was not known to hold at the end of its constructor and `dispatch(e)` method. Thus, like our system, Spec# checks the invariant at those two points at runtime. Thus the code is equivalently verified in both Spec# and L42; in particular it performed exactly the same number (77) of runtime invariant checks.

While the same numbers of checks are performed, we do not have the same guarantee provided by our approach: Spec#/Boogie does not soundly handle the non-deterministic impact of I/O, thus it does not properly prevent us from writing unsound invariants that may be non-deterministic. We also encoded our GUI in Microsoft Code Contracts [38], whose unsound heuristic also calls the invariant 77 times; however Code Contract does not enforce the encapsulation of `children()`, thus this approach is even less sound than Spec#.

Note how both our L42 and Spec# code required us to use the box pattern for our `SafeMovable`, due

[16] That is, the receiver is not `this`.
[17] This difference is caused by Eiffel treating getters specilly, and skipping invariant checks when calling a getter. Thus, even ignoring getter methods, the visible state semantic would still run 14 millions of invariant checks.
[18] We compiled Spec# using the latest available source (from 19/9/2014). The verifier available online at `rise4fun.com/SpecSharp` behaves differently.
[19] We used `assume` statements, equivalent to Java's `assert`, to dynamically check array bounds. This aligns the code with L42, which also performs such checks at runtime.

to the cyclic object graph caused by the `Actions` of `Buttons` needing to change their enclosing `SafeMovable`'s position. We found it quite difficult to encode the GUI in Spec#, due to its unintuitive and rigid ownership discipline. In particular we needed to use many more annotations, which were larger and had greater variety. The following table shows the annotation burden, for the *program* that defines and displays the `SafeMovables` and our GUI; as well as the *library* which defines `Buttons`, `Widget`, and event handling. We only count constructs Spec# adds over C# as annotations, we also do not count annotations related to array bounds or null checks:

	Spec# program	Spec# library	L42 program	L42 library
Total number of annotations	40	19	19	18
Tokens (except `.,;(){}[]` and whitespace)	106	34	19	18
Characters (with minimal whitespace)	619	207	74	60

To encode the GUI example in L42, the only annotations we needed were the 3 reference capabilities: `mut`, `read`, and `capsule`. Our Spec# code requires purity, immutability, ownership, method pre/post-conditions and method modification annotations. In addition, it requires the use of 4 different ownership functions including explicit ownership assignments. In total we used 18 different kinds of annotations in Spec#. In the table we present token and character counts to compare against Spec#'s annotations, which can be quite long and involved, whereas ours are just single keywords. Consider for example the Spec# pre-condition on `SafeMovable`'s constructor:

```
requires Owner.Same(Owner.ElementProxy(children), children);
```

The Spec# code also required us to deviate from the code style shown in our simplified version: we could not write a usable `children()` method in `Widget` that returns a list of children, instead we had to write `children_count()` and `children(int i)` methods; we also needed to create a trivial class with a `[Pure]` constructor (since `Object`'s one is not marked as such). In contrast, the only indirection we had to do in L42 was creating `Boxes` by using an additional variable in a nested scope. This is needed to delineate scopes for promotions. Based on these results, we believe our system is significantly simpler and easier to use.

6.2. A Comparison of a Simple Example in Spec#

Suppose we have a `Cage` class which contains a `Hamster`; the `Cage` will move its `Hamster` along a path. We would like to ensure that the `Hamster` does not deviate from the path. We can express this as the invariant of `Cage`: the position of the `Cage`'s `Hamster` must be within the path (stored as a field of `Cage`). This example is interesting since it relies on `Lists` and `Points` that are not designed with `Hamster/Cages` in mind.

```
class Point { Double x; Double y; Point(Double x, Double y) {..}
  @Override read method Bool equals(read Object that) {
    return that instanceof Point &&
      this.x == ((Point)that).x && this.y == ((Point)that).y; }}
class Hamster {Point pos; //pos is imm by default
  Hamster(Point pos) {..}}
class Cage {
  capsule Hamster h;
  List<Point> path; //path is imm by default
  Cage(capsule Hamster h, List<Point> path) {..}
  read method Bool invariant() {
    return this.path.contains(this.h.pos); }
  mut method Void move() {
    Int index = 1 + this.path.indexOf(this.pos());
    this.moveTo(this.path.get(index % this.path.size())); }
  read method Point pos() { return this.h.pos; }
  mut method Void moveTo(Point p) { this.h.pos = p; }}
```

The `invariant()` method on `Cage` simply verifies that the `pos` of `this.h` is within the `this.path` list. This is accepted by our invariant protocol since `path` is an `imm` field (hence deeply immutable) and `h` is a `capsule` field (hence fully encapsulated). The `path.contains` call is accepted by our type system as it only needs read access: it merely needs to be able to access each element of the list and call Point's equal method, which takes a read receiver and parameter. The move method actually moves the hamster along the path, but to ensure that our restrictions on capsule fields are respected we forwarded some of the behaviour to separate methods: pos() which returns the position of h and moveTo(p) which updates the position of h. The pos method is needed since move() is a mut method, and so any direct this.h access would cause it to be a capsule mutator, which would make the program erroneous as move() uses this multiple times. Similarly, we need the moveTo(p) method to modify the ROG of the h field, this must be done within a capsule mutator that uses this only once.

As our path and h fields are never themselves updated, the only point where the ROG of our Cage can mutate is in the moveTo(p) capsule mutator, thus our invariant protocol will insert runtime invariant checks only here and at the end of the constructor.

Note: since only Cage has an invariant, only the code of Cage needs to be handled carefully; allowing the code for Point and Hamster to be unremarkable. This contrasts with Spec#: all code involved in verification needs to be designed with verification in mind [39].

Comparison with Spec#

We now show our hamster example in Spec#; the system most similar to ours:

```
// Note: assume everything is 'public'
class Point { double x; double y; Point(double x, double y) {..}
  [Pure] bool Equal(double x, double y) {
    return x == this.x && y == this.y; }}
class Hamster{[Peer] Point pos;
  Hamster([Captured] Point pos){..}}
class Cage {
  [Rep] Hamster h; [Rep, ElementsRep] List<Point> path;
  Cage([Captured] Hamster h, [Captured] List<Point> path)
    requires Owner.Same(Owner.ElementProxy(path), path); {
      this.h = h; this.path = path; base(); }
  invariant exists {int i in (0 : this.path.Count);
    this.path[i].Equal(this.h.pos.x, this.h.pos.y) };
  void Move() {
    int i = 0;
    while(i<path.Count && !path[i].Equal(h.pos.x,h.pos.y)){i++;}
    expose(this) {this.h.pos = this.path[i%this.path.Count];}}}
```

In both this and our original version, we designed Point and Hamster in a general way, and not solely to be used by classes with an invariant: thus Point is not an immutable class.

The Spec# approach uses ownership: the Rep attribute on the h and path fields means its value is owned by the enclosing Cage, similarly the ElementsRep attribute on the path field means its *elements* are owned by the Cage. Conversely, in the Hamster class, the Peer annotation on the pos field means its value is owned by the owner of the enclosing Hamster, thus if a Cage owns a Hamster, it also owns the Hamster's pos. The Captured annotations on the constructor parameters of Cage and Hamster means that the passed in values must be un-owned and the body of the constructor may modify their owners (the owner is automatically updated when the parameter is assigned to a Rep or Peer field).

Though we don't want either pos or path to ever mutate, Spec# currently has no way of enforcing that an *instance* of a non-immutable class is itself immutable.[20] In Spec#, an invariant() can only access fields on owned or immutable objects, thus necessitating our use of the Peer and Rep annotations on the pos and path fields.

Note that this prevents multiple Cages from sharing the same point instance in their path. Had we made Point an immutable class, we would get no such restriction. A similar problem applies to our pos field: the

[20] There is a paper [40] that describes a simple solution to this problem: assign ownership of the object to a special predefined 'freezer' object, which never gives up mutation permission, however this does not appear to have been implemented; this would provide similar flexibility to the RC system we use, which allows an initially mutable object to be promoted to immutable.

`pos` of `Hamsters` in different `Cages` cannot be the same `Point` instance. Note how if we consider being in the ROG of an object's capsule fields as being 'owned' by the object, our `capsule` fields behave like `Rep` fields; similarly, `mut` fields (that are in the ROG of a `capsule` field) behave like `Peer` fields.

The `expose(this)` block is needed, since in Spec# in order to modify a field of an object (like `this.h.pos`), we must first "expose" its owner (the `Cage`). During an `expose` block, Spec# will not assume the invariant of the exposed object, but will ensure it is re-established at the end of the block. This is similar to our concept of capsule mutators (like our `moveTo` method above), however it is supported by adding an extra syntactic construct (the `expose` block), which we avoid.

Finally, note the custom `Equal(x,y)` method on `Point`: this is needed since we can't overload the usual `Object.Equals(other)` method because is marked as `Reads(ReadsAttribute.Reads.Nothing)`, which requires the method not read any fields, even those of its receiver. We resorted to making our own `Equal(x,y)` method. Since it is called in `Cage`'s invariant, Spec# requires it to be annotated as `Pure`, this requires that it can only read fields of objects owned by the *receiver* of the method, so a method `[Pure] bool Equal(Point that)` can read the fields of `this`, but not the fields of `that`. Of course this would make the method unusable in `Cage` since the `Points` we are comparing equality against do not own each other. As such, the simplest solution is to just pass the fields of the other point to the method. Sadly this mean we can no longer use `List`'s `Contains(elem)` and `IndexOf(elem)` methods, rather we have to expand out their code manually.

Even with all the above annotations, we needed special care in creating `Cages`:

```
List<Point> pl = new List<Point>{new Point(0,0),new Point(0,1)};
Owner.AssignSame(pl, Owner.ElementProxy(pl));
Cage c = new Cage(new Hamster(new Point(0, 0)), pl);
```

In Spec# objects start their life as un-owned, so each `new` instruction above returns an unowned object; however when the `Points` are placed inside the `pl` list, Spec# loses track of this. Thus the `AssignSame` call is needed to mark the elements of `pl` as still being unowned (since `pl` itself is unowned). Contrast this with our system which requires no such operation; we can simply write:

```
Cage c = new Cage(new Hamster(new Point(0, 0)),
  List.of(new Point(0, 0), new Point(0, 1)));
```

In Spec# we had to add 10 different annotations, of 8 different kinds; some of which were quite involved. In comparison, our approach requires only 8 simple keywords, of 3 different kinds; however we needed to write separate `pos()` and `moveTo(p)` methods.

6.3. A Worst Case for the Number of Invariant Checks

The following test case was designed to produce a worst case in the number of invariant checks. We have a `Family` that (indirectly) contains a list of `parents` and `children`. The `parents` and `children` are of type `Person`. Both `Family` and `Person` have an invariant, the invariant of `Family` depends on its contained `Persons`.

```
class Person {
  final String name;
  Int daysLived;
  final Int birthday;
  Person(String name, Int daysLived, Int birthday) { .. }
  mut method Void processDay(Int dayOfYear) {
    this.daysLived += 1;
    if (this.birthday == dayOfYear) {
      Console.print("Happy birthday " + this.name + "!"); }}
  read method Bool invariant() {
    return !this.name.equals("") && this.daysLived >= 0 &&
      this.birthday >= 0 && this.birthday < 365; }
}
class Family {
  static class Box {
    mut List<Person> parents;
    mut List<Person> children;
```

```
    Box(mut List<Person> parents, mut List<Person> children){..}
    mut method Void processDay(Int dayOfYear) {
      for(Person c : this.children) { c.processDay(dayOfYear); }
      for(Person p : this.parents) { p.processDay(dayOfYear); }}
  }
  capsule Box box;
  Family(capsule List<Person> ps,capsule List<Person> cs) {
    this.box = new Box(ps, cs); }
  mut method Void processDay(Int dayOfYear) {
    this.box.processDay(dayOfYear); }
  mut method Void addChild(capsule Person child) {
    this.box.children.add(child); }
  read method Bool invariant() {
    for (Person p : this.box.parents) {
      for (Person c : this.box.children) {
        if (p.daysLived <= c.daysLived) {
          return false; }}}
    return true; }
}
```

Note how we created a `Box` class to hold the `parents` and `children`. Thanks to this pattern, the invariant only needs to hold at the end of `Family.processDay(dayOfYear)`, after all the `parents` and `children` have been updated. Thus `processDay(dayOfYear)` is atomic: it updates all its contained `Persons` together. Had we instead made the `parents` and `children` `capsule` fields of `Family`, the invariant would be required to also hold between modifying the two lists. This could cause semantic problems if, for example, a child was updated before their parent.

We have a simple test case that calls `processDay(dayOfYear)` on a `Family` 1,095 (3×365) times.

```
// 2 parents (one 32, the other 34), and no children
var fam = new Family(List.of(new Person("Bob", 11720, 40),
    new Person("Alice", 12497, 87)), List.of());

for (Int day = 0; day < 365; day++) { // Run for 1 year
  fam.processDay(day);
}
for (Int day = 0; day < 365; day++) { // The next year
  fam.processDay(day);
  if (day == 45) {
    fam.addChild(new Person("Tim", 0, day)); }}

for (Int day = 0; day < 365; day++) { // The 3rd year
  fam.processDay(day);
  if (day == 340) {
    fam.addChild(new Person("Diana", 0, day)); }}
```

The idea is that everything we do with the `Family` is a mutation; the `fam.processDay` calls also mutate the contained `Persons`.

This is a worst case scenario for our approach compared to visible state semantics since it reduces our advantages: our approach avoids invariant checks when objects are not mutated but in this example most operations are mutations; similarly, our approach prevents the exponential explosion of nested invariant checks when deep object graphs are involved, but in this example the object graph of `fam` is very shallow.

We ran this test case using several different languages: L42 (using our protocol) performs 4,000 checks, D and Eiffel perform 7,995, and finally, Spec# performs only 1,104.

Our protocol performs a single invariant check at the end of each constructor, `processDay(dayOfYear)` and `addChild(child)` call (for both `Person` and `Family`).

The visible state semantics of both D and Eiffel perform additional invariant checks at the beginning of each call to `processDay(dayOfYear)` and `addChild(child)`.

The results for Spec# are very interesting, since it performs fewer checks than L42. This is the case since `processDay(dayOfYear)` in `Person` just does a simple field update, which in Spec# do not invoke runtime invariant checks. Instead, Spec# tries to statically verify that the update cannot break the invariant; if it is unable to verify this, it requires that the update be wrapped in an `expose` block, which will perform a runtime invariant check.

Spec# relies on the absence of arithmetic overflow, and performs runtime checks to ensure this[21], as such the verifier concludes that the field increment in `processDay(dayOfYear)` cannot break the invariant. Spec# is able to avoid some invariant checks in this case by relying on all arithmetic operations performing runtime overflow checks; whereas integer arithmetic in L42 has the common wrap around semantics.

The annotations we had to add in the Spec# version[22] were similar to our previous examples, however since the fields of `Person` all have immutable classes/types, we only needed to add the invariant itself. In order to implement the `addChild(child)` method we were forced to do a shallow clone of the new child (this also caused a couple of extra runtime invariant checks). Unlike L42 however, we did not need to create a box to hold the `parents` and `children` fields, instead we wrapped the body of the `Family.processDay(dayOfYear)` method in an `expose (this)` block. In total we needed 16 annotations, worth a total of 45 tokens, this is worse than the code following our approach that we showed above, which has 14 annotations and 14 tokens.

6.4. Encoding Examples from Spec# Papers

There are many published papers about the pack/unpack methodology used by Spec#. To compare against their expressiveness we will consider the three main ones that introduced their methodology and extensions:

- *Verification of Object-Oriented Programs with Invariants:* [3] this paper introduces their methodology. In their examples section (pages 41–47), they show how their methodology would work in a class hierarchy with `Reader` and `ArrayReader` classes. The former represents something that reads characters, whereas the latter is a concrete implementation that reads from an owned array. They extend this further with a `Lexer` that owns a `Reader`, which it uses to read characters and parse them into tokens. They also show an example of a `FileList` class that owns an array of file names, and a `DirFileList` class that extends it with a stronger invariant. All of these examples can be represented in L42[23]. The most interesting considerations are as follow:
  - Their `ArrayReader` class has a `relinquishReader()` method that 'unpacks' the `ArrayReader` and returns its owned array. The returned array can then be freely mutated and passed around by other code. However, afterwards the `ArrayReader` will be 'invalid', and so one can only call methods on it that do not require its invariant to hold. However, it may later be 'packed' again (after its invariant is checked). In contrast, our approach requires the invariant of all usable objects to hold. We can still relinquish the array, but at the cost of making the `ArrayReader` forever unreachable. This can be done by declaring `relinquishReader()` as a `capsule method`, this works since our type modifier system guarantees that the receiver of such a method is not aliased, and hence cannot be used again. Note that Spec# itself cannot represent the `relinquishReader()` method at all, since it does not provide explicit pack and unpack operations, rather its `expose` statement performs both an unpack and a pack, thus we cannot unpack an `ArrayReader` without repacking it in the same method.
  - Their `DirFileList` example inherits from a `FileList` which has an invariant, and a final method, this is something their approach was specifically designed to handle. As L42 does not have traditional subclassing, we are unable to express this concept fully, but L42 does have code reuse via trait composition, in which case `DirFileList` can include the methods from `FileList`, and they will automatically enforce the invariant of `DirFileList`.

[21] Runtime checks are enabled by a compilation option; when they fail, unchecked exceptions are thrown.
[22] The Spec# code is in the artifact.
[23] Our encodings are in the artifact.

- *Object Invariants in Dynamic Contexts:* [41] this paper shows how one can specify an invariant for a doubly linked list of `int`s (here `int` is an immutable value type). Unlike our protocol however, it allows the invariant of `Node` to refer to sibling `Node`s which are not owned/encapsulated by itself, but rather the enclosing `List`. Our protocol can verify such a linked list[24] (since its elements are immutable), however we have to specify the invariant inside the `List` class. We do not see this as a problem, as the `Node` type is only supposed to be used as part of a `List`, thus this restriction does not impact users of `List`.

- *Friends Need a Bit More: Maintaining Invariants Over Shared State:* [27] this paper shows how one can verify invariants over interacting objects, where neither owns/contains the other. They have multiple examples which utilise the 'subject/observer' pattern, where a 'subject' has some state that an 'observer' wants to keep track of. In their `Subject`/`View` example, `View`s are created with references to `Subject`s, and copies of their state. When a `Subject`'s state is modified, it calls a method on its attached `View`s, notifying them of this update. The invariant is that a `View`'s copy of its `Subject`'s state is up to date. Their `Master`/`Clock` example is similar, a `Clock` contains a reference to a `Master`, and saves a copy of the `Master`'s time. The `Master` has a `Tick` method that increases its time, but unlike the `Subject`/`View` example, the `Clock` is not notified. The invariant is that the `Clock`'s time is never ahead of its `Master`'s. Our protocol is unable to verify these interactions, because the interacting objects are not immutable or encapsulated by each other.

7. Patterns

In this section we show programming patterns that allow various kinds of invariants. Our goal is not to verify existing code or patterns, but to create a simple system that allows soundly verifying the correctness of data structures. In particular, as we show, in order to use our approach to ensure invariants, one has to program in an uncommon and very defensive style.

The SubInvariant Pattern

We showed how the box pattern can be used to write invariants over cyclic mutable object graphs, the latter also shows how a complex mutation can be done in an 'atomic' way, with a single invariant check. However the box pattern is much more powerful.

Suppose we want to pass a temporarily 'broken' object to other code as well as perform multiple field updates with a single invariant check. Instead of adding new features to the language, like an `invalid` modifier (denoting an object whose invariant need not hold), and an `expose` statement like Spec#, we can use a 'box' class and a capsule mutator to the same effect:

```
interface Person{mut method Bool accept(read Account a,read Transaction t);}
interface Transaction{mut method ImmList<Transfer> compute();}
//Here ImmList<T> represents a list of immutable Ts.
class Transfer{Int money;
  method Void execute(mut AccountBox that){
    // Gain some money, or lose some money
    if(this.money>0){that.income+=money;}
    else{that.expenses-=money;}
  }}
class AccountBox{
  UInt income=0; UInt expenses=0;
  read method Bool subInvariant(){return this.income>=this.expenses;}
  // An 'AccountBox' is like a 'potentially invalid Account':
  // we may observe income >= expenses
  }
```

[24]Our protocol allows for encoding this example, but to express the invariant we would need to use reference equality, which the L42 language does not support.

```
class Account{
  capsule AccountBox box; mut Person holder;
  read method Bool invariant(){return this.box.subInvariant();}
  // 'h' could be aliased elsewhere in the program
  Account(mut Person h){this.holder=h; this.box=new AccountBox();}
  mut method Void transfer(mut Transaction ts){
    if(this.holder.accept(this, ts)){this.transferInner(ts.compute());}}
  // capsule mutator, like an 'expose(this)' statement
  private mut method Void transferInner(ImmList<Transfer> ts){
     mut AccountBox b = this.box;
     for (Transfer t : ts) { t.execute(b); }
     // check the invariant here
  }}
```

The idea here is that `transfer(ts)` will first check to see if the account holder wishes to accept the transaction, it will then compute the full transaction (which could cache the result and/or do some I/O), and then execute each transfer in the transaction. We specifically want to allow an individual `Transfer` to raise the `expenses` field by more than the `income`, however we don't want an entire `Transaction` to do this. Our capsule mutator (`transferInner`) allows this by behaving like a Spec# `expose` block: during its body (the `for` loop) we don't know or care if `this.invariant()` is `true`, but at the end it will be checked. For this to make sense, we make `Transfer.execute` take an `AccountBox` instead of an `Account`: it cannot assume that the invariant of `Account` holds, and it is allowed to modify the fields of `that` without needing to check it. Though capsule mutators can be used to perform batch operations like the above, they can only take immutable and capsule objects. This means that they can perform no non-deterministic I/O (due to our OCs system), and other externally accessible objects (such as a `mut Transaction`) cannot be mutated during such a batch operation.

As you can see, adding support for features like `invalid` and `expose` is unnecessary, and would likely require making the type system significantly more complicated as well as burdening the language with more core syntactic forms.

In particular, the above code demonstrates that our system can:

- Have useful objects that are not entirely encapsulated: the `Person holder` is a `mut` field; this is fine since it is not mentioned in the `invariant()` method.
- Wrap normal methods over capsule mutators: `transfer` is not a capsule mutator, so it can use `this` multiple times and take a `mut` parameter.
- Perform multiple state updates with only a single invariant check: the loop in `transferInner(ts)` can perform multiple field updates of `income` and `expenses`, however the `invariant()` will only be checked at the end of the loop.
- Temporarily break an invariant: it is fine if during the `for` loop, `expenses > income`, provided that this is fixed before the end of the loop.
- Pass the state of an 'invalid' object around, in a safe manner: an `AccountBox` contains the state of `Account`, but not the invariant method.

Under our strict invariant protocol, the invariant holds for all reachable objects. The sub invariant pattern allows to control when an object is required to be valid. Instead, other protocols strive to allow the invariant to be observed broken in controlled conditions defined by the protocol itself.

The sub invariant pattern offers interesting guarantees: any object '`a`' with a `subInvariant()` method that is checked by the `invariant()` method of an object '`b`' will respect its `subInvariant()` in all contexts where '`b`' is involved in execution. This is because whenever '`b`' is involved in execution, its invariant holds. Moreover, `a`'s `subInvariant()` can be observed as `false` only if a capsule mutator of '`b`' is currently active (that is, being executed), or `b` is now garbage collectable. Thus, even when there is no reachable reference to `b` in the current stack frame, if no capsule mutator on `b` is active, `a`'s `subInvariant()` will hold.

In the former example, this means that if you can refer to an `Account`, you can be sure that its `income >= expenses`; if you have an `AccountBox` then you can be sure that either `income >= expenses` or a capsule mutator of the corresponding `Account` object is currently active. This closely resemble some visible state semantic protocols, aiming to ensure that either an object's invariant holds, or one of its methods is currently active.

Another interesting and natural application of the sub invariant pattern would be to support a version of the GUI such that when a `Widget`'s position is updated, the `Widget` can in turn update the coordinates of its parent `Widgets`, in order to re-establish their `subInvariants`. This would also make the GUI follow the versions of the composite pattern were objects have references to their 'parent' nodes. The main idea is to define an interface `HasSubInvariant`, that denotes `Widgets` with a `subInvariant()` method. Then, `WidgetWithInvariant` is a decorator over a `Widget`; the invariant method of a `WidgetWithInvariant` checks the `subInvariant()` of each widget in its ROG.

We define `SafeMovable` as a `Widget` and `HasSubInvariant`; since `subInvariant()` methods don't have the restrictions of invariant methods, it allows `SafeMovable` to be significantly simpler than the version shown before in Section 6.1.

```
interface HasSubInvariant{read method Bool subInvariant();}
class SafeMovable implements Widget,HasSubInvariant {
  Int width = 300; Int height = 300;
  Int left; Int top;  // Here we do not use a box, thus all the state
  mut Widgets c;      // is in SafeMovable.
  mut Widget parent;//We add a parent field
  @Override read method Int left(){return this.left;}
  @Override read method Int top(){return this.top;}
  @Override read method Int width(){return this.width;}
  @Override read method Int height(){return this.height;}
  @Override read method read Widgets children(){return this.c;}
  @Override mut method Void dispatch(Event e){
    for(mut Widget w :this.c){w.dispatch(e);}
    }
  @Override read method Bool subInvariant(){/*same of original GUI*/}
  SafeMovable(mut Widget parent,mut Widgets c){
    this.c=c;           //SafeMovable no longer has an invariant,
    this.left=5;        //so we impose no restrictions on its constructor
    this.top=5;
    this.parent=parent;
    c.add(new Button(0,0,10,10,new MoveAction(this));
  }}
class MoveAction implements Action{
  mut SafeMovable o;
  MoveAction(mut SafeMovable o){this.o=o;}
  mut method Void process(Event e){
    this.o.left+=1;
    Widget p = this.o.parent;
    ... // mutate p to re-establish its subInvariant
  }}
class WidgetWithInvariant implements Widget{
  capsule Widget w;
  @Override read method Int left(){return this.w.left;}
  @Override read method Int top(){return this.w.top;}
  @Override read method Int width(){return this.w.width;}
  @Override read method Int height(){return this.w.height;}
  @Override read method read Widgets children(){return this.w.c;}
  @Override mut method Void dispatch(Event e){w.dispatch(e);}
  @Override read method Bool invariant(){return wInvariant(w);}
  static method Bool wInvariant(read Widget w){
```

```
    for(read Widget wi:w.children()){        //Check that the subInvariant of
      if(!wInvariant(wi)){return false;}//all of w's descendants holds
    }
    if(!(w instanceof HasSubInvariant)){return true;}
    HasSubInvariant si=(HasSubInvariant)w;
    return si.subInvariant();
  }
  WidgetWithInvariant(capsule Widget w){this.w=w;}
}
... // main expression
//#$ is a capability operation making a Gui object
mut Widget top=new WidgetWithInvariant(new SafeMovable(...))
Gui.#$().display(top);
```

In this way, the method `WidgetWithInvariant.dispatch()` is the only capsule mutator, hence the only invariant checks will be at the end of `WidgetWithInvariant`'s constructor and dispatch methods.

Importantly, this allows the graph of widgets to be cyclic and for each to freely mutate each other, even if such mutations (temporarily) violate their `subInvariant`'s. In this way a widget can access its parent (whose `subInvariant()` may not hold) in order to re-establish it. Note that this trade off is logically unavoidable: in order to manipulate a parent in order to fix it, the parent must be reachable, but by mutating a `Widget`'s position, its parent may become invalid. Thus if `Widgets` were to encode their validity in their `invariant()` methods they could not have access to their parents. Instead, by encoding their validity in a `subInvariant()` method, they can access invalid widgets, but this comes at a cost: the programmer must reason as to when `Widgets` are valid, as we described above.

The Transform Pattern

Recall the GUI case study from Section 6.1, where we had a `Widget` interface and a `SafeMovable` (with an invariant) that implements `Widget`. Suppose we want to allow `Widgets` to be scaled, we could add `mut` setters for `width()`, `height()`, `left()`, and `top()` in the `Widget` interface. However, if we also wish to scale its children we have a problem, since `Widget.children()` returns a `read Widgets`, which does not allow mutation. We could of course add a `mut` method `zoom(w)` to the `Widget` interface, however this does not scale if more operations are desired. If instead `Widget.children` returned a `mut Widgets`, it would be difficult for `Widget` implementations, such as `SafeMovable`, to mention their `children()` in their `invariant()`. A simple and practical solution would be to define a `transform(t)` method in `Widget`, and a `Transformer` interface like so:

```
interface Transformer<T> { method Void apply(mut T elem); }
interface Widget { ...
  mut method Void top(Int that); // setter for immutable data
  // transformer for possibly encapsulated data
  mut method read Void transform(Transformer<Widgets> t);
}
class SafeMovable { ...
  // A well typed capsule mutator
  mut method Void transform(Transformer<Widgets> t) {t.apply(this.box.c);}}
```

The `transform` method offers an expressive power similar to `mut` getters, but prevents `Widgets` from leaking out. With a `Transformer`, a `zoom(w)` function could be simply written as:

```
static method Void zoom(mut Widget w) {
  w.transform(ws -> { for (wi : ws) { zoom(wi); }});
  w.width(w.width() / 2); ...; w.top(w.top() / 2); }
```

Using Patterns Together: A general and flexible Graph class

Here we rely on all the patterns shown above to encode a general library for `Graphs` of `Nodes`. Users of this library can define personalised kinds of nodes, with their own personalised sub invariant. The library will ensure that no matter how the library is used, for any accessible `Graph`, each user defined sub invariant of its `Nodes` holds. Note that those sub invariants are not restricted to be only about the local state of a node; since they can explore the state of all of the reachable nodes, they may even depend upon the whole graph.

The `Nodes` are guaranteed to be encapsulated by the `Graph`, however they can be arbitrarily modified by user defined transformations using the Transform Pattern.

```
interface Transform<T>{method read T apply(mut Nodes nodes);}

interface Node{
  read method Bool subInvariant(read Nodes nodes)
  mut method mut Nodes directConnections()
}
class Nodes{//just an ordered set of nodes
  mut method Void add(mut Node n){..}
  read method Int indexOf(read Node n){..}
  mut method Void remove(read Node n){..}
  mut method mut Node get(Int index){..}
}
class Graph{
  capsule Nodes nodes; //box pattern
  Graph(capsule Nodes nodes){..}
  read method read Nodes getNodes(){return this.nodes;}
  <T> mut method read T transform(Transform<T> t){
    mut Nodes ns=this.nodes;//capsule mutator with a single use of 'this'
    return t.apply(ns);
  }
  read method Bool invariant(){
    for(read Node n: this.nodes){
      if(!n.subInvariant(this.nodes)){return false;}
    }
    return true;
  }
}
```

We now show how our `Graph` library allows the invariant of the various `Nodes` to be customized by the library user, and arbitrary transformations can be performed on the `Graphs`. This is a generalization of the example proposed by [42](section 4.2) as one of the hardest problems when it comes to enforcing invariants.

Note how there are only a minimal set of operations defined in the above code, others can be freely defined by the user code, as demonstrated below:

```
class MyNode{
  mut Nodes directConnections;
  mut method mut Nodes directConnections(){return this.directConnections;}
  MyNode(mut Nodes directConnections){..}
  read method Bool subInvariant(read Nodes nodes){
    /* any condition on this or nodes */}
  capsule method read MyNode addToGraph(mut Graph g){..}
  read method Void connectWith(read Node other, mut Graph g){..}
}
...
mut Graph g=new Graph(new Nodes());
read MyNode n1=new MyNode(new Nodes()).addToGraph(g);
read MyNode n2=new MyNode(new Nodes()).addToGraph(g);
//lets connect our two nodes
n1.connectWith(n2,g);
```

Here we define a `MyNode` class, where the `subInvariant(nodes)` can express any property over `this` and `nodes`, such as properties over their direct connections, or any other reachable node.

We can define methods in `MyNode` to add our nodes to graphs and to connect them with other nodes.

Note that the method `addToGraph(g)` is marked as `capsule`; this ensures that the node is not in any other graph. In contrast, the method `connectWith(other, g)` is marked as `read`, even though it is clearly intend to modify the ROG of `this`. It works by recovering a `mut` reference to `this` from the `mut Graph`.

These methods can be implemented like this:

```
read method Void connectWith(read Node other,mut Graph g){
  Int i1=g.getNodes().indexOf(this);
  Int i2=g.getNodes().indexOf(other);
  if(i1==-1 || i2==-1){throw /*error nodes not in g*/;}
  g.transform(ns->{
    mut Node n1=ns.get(i1);
    mut Node n2=ns.get(i2);
    n1.directConnections().add(n2);
  });
}
capsule method read MyNode addToGraph(mut Graph g){
  return g.transform(ns->{
    mut MyNode n1=this;//single usage of capsule 'this'
    ns.add(n1);
    return n1;
  });
}
```

As you can see, both methods rely on the transform pattern.

These transformation operations are very general since they can access the `mut Nodes` of the `Graph` and any `capsule` or `imm` data from outside. Note how in the lambda in `connectWith(other,g)`, we can neither see the `read this` nor the `read other`, but we get their (immutable) indexes and recover the concrete objects from the `mut Nodes ns` object. In this way, we also obtain more useful `mut` references to those nodes. On the other hand, note how in `addToGraph(g)` we use the reference to the `capsule this` within the lambda.

8. Integration in L42

In the last version of L42, invariants have been integrated with caching and automatic parallelism; it would be out of this articles scope to explain in detail this integration, but the overall idea is that an invariant is seen as a `Void` cached value that is always kept up to date whenever the object is visible. L42 also supports eager cached methods, which get computed in parallel when an instance of the corresponding class is created.

L42 libraries rely on a very expressive form of metaprogramming to generate a lot of boilerplate/redundant code. In L42 many tasks can be either manually performed by writing code directly, or partially automated by code generation. L42 allows writing `class` methods (similar to a static method in Java) with appropriate parameters instead of `invariants` method and capsule mutators. The bodies of such methods don't have special restrictions as they cannot see `this`, instead the meta-programming generates appropriate instance methods, conforming to our restrictions, which call the user provided `class` methods.

Our restrictions are also checked by the type system, so even if the user manually writes these methods, instead of relying on the metaprogramming, they still cannot break our invariant protocol.

To make this work more accessible to programmers familiar with Java/C#, we have shown our examples in a more Java-like syntax. Here you can see our `ShippingList` example from Section 4 in the full L42 Syntax:

```
ShippingList = Data:{
  capsule Items items
  @Cache.Now
  class method Void invariant(read Items items) =
    X[items.weight()<=300Num]
  @Cache.Clear
  class method Void addItem(mut Items items,Item item) =
    items.add(item)
```

```
}
```

In this example, the `Data` decorator generates a factory method, a method `addItem(Item item)` and a lot of other utility methods, including equality and conversion to string. The `@Cache.Now` annotation causes the `invariant` method to be automatically computed, and recomputed every time a `@Cache.Clear` method is called. Please refer to `L42.is/tutorial.xhtml` for more information.

9. Related Work

Reference Capabilities

We rely on a combination of RCs supported by at least 3 languages/lines of research: L42 [6, 7, 8, 9], Pony [10, 11], and Gordon *et al.* [12]. They all support full/deep interpretation (see page 5), without back doors. Former work [43, 44, 45, 46, 47] (which eventually enabled the work of Gordon *et al.*) does not consider promotion and infers uniqueness/isolation/immutability only when starting from references that have been tracked with restrictive annotations along their whole lifetime. Other approaches like Javari [13, 48] and Rust [31] provide back doors, which are not easily verifiable as being used properly.

Ownership [49, 16, 50] is a popular form of aliasing control often used as a building block for static verification [51, 39]. However, ownership does not require the whole ROG of an object to be 'owned'. This complicates restricting the data accessible by invariants.

Object Capabilities

In the literature, OCs are used to provide a wide range of guarantees, and many variations are present. Object capabilities [52], in conjunction with reference capabilities, are able to enforce purity of code in a modular way, without requiring the use of monads. L42 and Gordon use OCs simply to reason about I/O and non-determinism. This approach is best exemplified by Joe-E [26], which is a self-contained and minimalistic language using OCs over a subset of Java in order to reason about determinism. However, in order for Joe-E to be a subset of Java, they leverage a simplified model of immutability: immutable classes must be final and have only final fields that refer to immutable classes. In Joe-E, every method that only takes instances of immutable classes is pure. Thus their model would not allow the verification of purity for invariant methods of mutable objects. In contrast our model has a more fine grained representation of mutability: it is *reference-based* instead of *class-based*. Thanks to this crucial difference, in our work every method taking only `read` or `imm` *references* is pure, regardless of their class type; in particular, we allow the parameter of such a method to be mutated later on by other code.

Invariant protocols

Invariants are a fundamental part of the design by contract methodology. Invariant protocols differ wildly and can be unsound or complicated, particularly due to re-entrancy and aliasing [41, 53, 54].

While invariant protocols all check and assume the invariant of an object after its construction, they handle invariants differently across object lifetimes; popular approaches include:

- The invariants of objects in a *steady* state are known to hold: that is when execution is not inside any of the objects' public methods [5]. Invariants need to be constantly maintained between calls to public methods.
- The invariant of the receiver before a public method call and at the end of every public method body needs to be ensured. The invariant of the receiver at the beginning of a public method body and after a public method call can be assumed [55, 53]. Some approaches ensure the invariant of the receiver of the *calling* method, rather than the *called* method [56]. JML [57] relaxes these requirements for helper methods, whose semantics are the same as if they were inlined.
- The same as above, but only for the bodies of 'selectively exported' (i.e. not instance-private) methods, and only for 'qualified' (i.e. not `this`) calls [54].
- The invariant of an object is assumed only when a contract requires the object be 'packed'. It is checked after an explicit 'pack' operation, and objects can later be 'unpacked' [3].

These different protocols can be deceivingly similar. Note that all those approaches fail our strict requirements and allow for broken objects to be observed. Some approaches like JML suggest verifying a simpler approach (that method calls preserve the invariant of the *receiver*) but assume a stronger one (the invariant of *every* object, except `this`, holds).

Security and Scalability

Our approach allows verifying an object's invariant independently of the execution context. This is in contrast to the main strategy of static verification: to verify a method, the system assumes the contracts of other methods, and the content of those contracts is the starting point for their proof. Thus, static verification proceeds like a mathematical proof: a program is valid if it is all correct, but a single error invalidates all claims. This makes it hard to perform verification on large programs, or when independently maintained third party libraries are involved. Static verification has more flexible and fine-grained annotations and often relies on a fragile theorem prover as a backend.

To soundly verify code embedded in an untrusted environment, as in gradual typing [58, 59], it is possible to consider a verified core and a runtime verified boundary. One can see our approach as an extremely modularized version of such a system: every class is its own verified core, and the rest of the code could have Byzantine behaviour. Our formal proofs show that every class that compiles/type checks is soundly handled by our protocol, independently of the behaviour of code that uses such class or any other surrounding code.

Our approach works both in a library setting and with the open world assumption. Consider for example the work of Parkinson [60]: he verified a property of the `Subject/Observer` pattern. However, the proof relies on (any override of) the `Subject.register(Observer)` method respecting its contract. Such assumption is unrealistic in a real-world system with dynamic class loading, and could trivially be broken by a user-defined `EvilSubject`: checking contracts at load time is impractical and is not done by any verification systems we know of.

Static Verification

AutoProof [61] is a static verifier for Eiffel that also follows the Boogie methodology, but extends it with *semantic collaboration* where objects keep track of their invariants' dependencies using ghost state.

Dafny [1] is a new language where all code is statically verified. It supports invariants with its `{:autocontracts}` annotation, which treats a class's `Valid()` function as the invariant and injects pre and post-conditions following visible state semantics; however it requires objects to be newly allocated (or cloned) before another object's invariant may depend on it. Dafny is also generally highly restrictive with its rules for mutation and object construction, it also does not provide any means of performing non-deterministic I/O.

Spec# [62] is a language built on top of C#. It adds various annotations such as method contracts and class invariants. It primarily follows the Boogie methodology [63] where (implicit) annotations are used to specify and modify the owner of objects and whether their invariants are required to hold. Invariants can be *ownership* based [3], where an invariant only depends on objects it owns; or *visibility* based [27, 64], where an invariant may depend on objects it doesn't own, provided that the class of such objects know about this dependence. Unlike our approach, Spec# does not restrict the aliases that may exist for an object, rather it restricts object mutation: an object cannot be modified if the invariant of its owner is required to hold. This allows invariants to query owned mutable objects whose ROG is not fully encapsulated. However as we showed in Section 6.1, it can become much more difficult to work with and requires significant annotation, since merely having an alias to an object is insufficient to modify it or call its methods. Spec# also works with existing .NET libraries by annotating them with contracts, however such annotations are not verified. Spec#, like us, does perform runtime checks for invariants and throws unchecked exceptions on failure. However Spec# does not allow soundly recovering from an invariant failure, since catching unchecked exceptions in Spec# is intentionally unsound. [65]

Specification languages

Using a specification language based on the mathematical metalanguage and different from the programming language's semantics may seem attractive, since it can express uncomputable concepts, has no mutation or non-determinism, and is often easier to formally reason about. However, a study [66] discovered that developers expect specification languages to follow the semantics of the underling language, including short-circuit semantics and arithmetic exceptions; thus for example `1/0 || 2>1` should not hold, while `2>1 || 1/0` should, thanks to short circuiting. This study was influential enough to convince JML to change its interpretation of logical expressions accordingly [67]. Dafny [1] uses a hybrid approach: it has mostly the same language for both specification and execution. Specification ('ghost') contexts can use uncomputable constructs such as universal quantification over infinite sets, whereas runtime contexts allow mutation, object allocation and print statements. The semantics of shared constructs (such as short circuiting logic operators) is the same in both contexts. Most runtime verification systems, such as ours, use a metacircular approach: specifications are simply code in the underlying language. Since specifications are checked at runtime, they are unable to verify uncomputable contracts.

Ensuring determinism in a non-functional language is challenging. Spec# recognizes the need for purity/determinism when method calls are allowed in contracts [68] '*There are three main current approaches: a) forbid the use of functions in specifications, b) allow only provably pure functions, or c) allow programmers free use of functions. The first approach is not scalable, the second overly restrictive and the third unsound*'. They recognize that many tools unsoundly use option (c), such as AsmL [69]. Spec# aims to follow (b) but only considers non-determinism caused by memory mutation, and allows other non deterministic operations, such as I/O and random number generation. In Spec# the following verifies:

```
[Pure] bool uncertain() {return new Random().Next() % 2 == 0;}
```

And so `assert uncertain() == uncertain();` also verifies, but randomly fails with an exception at runtime. As you can see, failing to handle non-determinism jeopardises reasoning. A simpler and more restrictive solution to these problems is to restrict 'pure' functions so that they can only read final fields and call other pure functions. This is the approach used by [70]. One advantage of their approach is that invariants (which must be 'pure') can read from a chain of final fields, even when they are contained in otherwise mutable objects. However their approach completely prevents invariants from mutating newly allocated objects, thus greatly restricting how computations can be performed.

Runtime Verification Tools

By looking to a survey by Voigt *et al.* [71] and the extensive MOP project [72], it seems that most runtime verification tools (RV) empower users to implement the kind of monitoring they see fit for their specific problem at hand. This means that users are responsible for deciding, designing, and encoding both the logical properties and the instrumentation criteria [72]. In the context of class invariants, this means the user defines the invariant protocol and the soundness of such protocol is not checked by the tool.

In practice, this means that the logic, instrumentation, and implementation end up connected: a specific instrumentation strategy is only good to test certain logic properties in certain applications. No guarantee is given that the implemented instrumentation strategy is able to support the required logic in the monitored application. Some of these tools are designed to support class invariants: for example InvTS [73] lets you write Python conditions that are verified on a set of Python objects, but the programmer needs to be able to predict which objects are in need of being checked and to use a simple domain specific language to target them. Hence if a programmer makes a mistake while using this domain specific language, invariant checking will not be triggered. Some tools are intentionally unsound and just perform invariant checking following some heuristic that is expected to catch most failures: such as jmlrac [55] and Microsoft Code Contracts [74].

Many works attempt to move out of the 'RV tool' philosophy to ensure RV monitors work as expected, as for example the study of contracts as refinements of types [75]. However, such work is only interested in pre and post-conditions, not invariants.

Our invariant protocol is much stricter than visible state semantics, and keeps the invariant under tight control. Gopinathan *et al.*'s. [5] approach keeps a similar level of control: relying on powerful aspect-oriented support, they detect any field update in the whole ROG of any object, and check all the invariants that such update may have violated. We agree with their criticism of visible state semantics, where methods still have to assume that any object may be broken; in such case calling any public method would trigger an error, but while the object is just passed around (and for example stored in collections), the broken state will not be detected; Gopinathan *et al.* says "*there are many instances where o's invariant is violated by the programmer inadvertently changing the state of p when o is in a steady state. Typically, o and p are objects exposed by the API, and the programmer (who is the user of the API), unaware of the dependency between* *o and p, calls a method of p in such a way that o's invariant is violated. The fact that the violation occurred is detected much later, when a method of o is called again, and it is difficult to determine exactly where such violations occur.*"

However, their approach addresses neither exceptions nor non-determinism caused by I/O, so their work is unsound if those aspects are taken into consideration.

Their approach is very computationally intensive, but we think it is powerful enough that it could even be used to roll back the very field update that caused the invariant to fail, making the object valid again. We considered a rollback approach for our work, however rolling back a single field update is likely to be completely unexpected, rather we should roll back more meaningful operations, similarly to what happens with transactional memory, and so is likely to be very hard to support efficiently. Using RCs to enforce strong exception safety is a much simpler alternative, providing the same level of safety, albeit being more restrictive.

Chaperones and impersonators [76] lifts the techniques of gradual typing [77, 58, 59] to work on general purpose predicates, where values can be wrapped to ensure an invariant holds. This technique is very powerful and can be used to enforce pre and post-conditions by wrapping function arguments and return values. This technique however does not monitor the effects of aliasing, as such they may notice if a contract has been broken, but not when or why. In addition, due to the difficulty of performing static analysis in weakly typed languages, they need to inject runtime checking code around every user-facing operation.

10. Conclusions and Future Work

In this paper we (1) identified the essential language features that support representation invariants in object-oriented verification; (2) presented a full formalism for our approach with capabilities that is proved to be sound and guarantees that all objects involved in execution are valid; (3) conducted extensive case studies showing that we require many order of magnitude less runtime checking than *visible state semantics* and three times less annotation burden than an equivalent version in Spec#. We hope that as a result of this work, the software verification community will make more use of the advanced general purpose language features, such as capabilities, appearing in modern languages to achieve its goals.

Our approach follows the principles of *offensive programming* [78] where no attempt to fix or recover an invalid object is performed. Failures (unchecked exceptions) are raised close to their cause: at the end of constructors creating invalid objects and immediately after field updates and instance methods that invalidate their receivers.

Our work builds on a specific form of RCs and OCs, whose popularity is growing, and we expect future languages to support some variation of these. Crucially, any language already designed with such support can also support our invariant protocol with minimal added complexity.

For an implementation of our work to be sound, catching exceptions like stack overflows or out of memory cannot be allowed in `invariant()` methods, since they are not deterministically thrown. L42 allows catching them only as a capability operation, which thus can't be used inside an invariant.

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Appendix A. Proof and Axioms

As previously discussed, instead of providing a concrete set of typing rules, we provide a set of properties that the type system needs to ensure. We will express such properties using type judgements of the form $\Sigma; \Gamma; \mathcal{E} \vdash e : T$. This judgement form allows an l to be typed with different types based on how it is used, e.g. we might have $\Sigma; \Gamma; [].m(l) \vdash l : \texttt{mut}\ C$ and $\Sigma; \Gamma; l.m([]) \nvdash l : \texttt{mut}\ C$, where m is a `mut` method taking a `read` parameter. Importantly, we allow types to change during reduction (such as to model promotions), but do not allow the type of any sub-expression of a method's body to change when they are called (see the Method Consistency assumption below). We could additionally extend the judgement form to take additional information (such as the past history of reduction), so that a main expression that was produced from one program can have a different RC than that produced by another, but this would just add noise to our assumptions and proofs without making them more interesting.

Auxiliary Definitions

To express our type system assumptions, we first need some auxiliary definitions. We define what it means for an l to be *reachable* from an expression or context:

$\mathit{reachable}(\sigma, e, l)$ iff $\exists l' \in e$ such that $l \in \mathit{rog}(\sigma, l')$,

$\mathit{reachable}(\sigma, \mathcal{E}, l)$ iff $\exists l' \in \mathcal{E}$ such that $l \in \mathit{rog}(\sigma, l')$.

We now define what it means for an object to be *immutable*: it is in the *rog* of an `imm` reference or a *reachable*

imm field:[Isaac: minor quibble, but the colour of keywords in the math and text is different, can we fix that?]

$\mathit{immutable}(\sigma, e, l)$ iff $\exists \mathcal{E}, l'$ such that:

- $e = \mathcal{E}[l']$, $\Sigma^\sigma; \emptyset; \mathcal{E} \vdash l' : \mathtt{imm}\,\_$, and $l \in \mathit{rog}(\sigma, l')$, or
- $\mathit{reachable}(\sigma, e, l')$, $\exists f$ such that $\Sigma^\sigma(l').f = \mathtt{imm}\,\_$, and $l \in \mathit{rog}(\sigma, \sigma[l'.f])$.

We define the *mrog* of an l to be the locations reachable from l by traversing through any number of `mut` and `capsule` fields:

$l' \in \mathit{mrog}(\sigma, l)$ iff:

- $l' = l$ or
- $\exists f$ such that $\Sigma^\sigma(l).f \in \{\mathtt{capsule}\,\_, \mathtt{mut}\,\_\}$, and $l' \in \mathit{mrog}(\sigma, \sigma[l.f])$

Now we can define what it means for an l to be *mutatable*[25] by a sub-expression e found in $\mathcal{E}$: something in l is reachable from a `mut` reference in e, by passing through any number of `mut` or `capsule` fields:

$\mathit{mutatable}(\sigma, \mathcal{E}, e, l)$ iff $\exists \mathcal{E}', l'$ such that:

- $e = \mathcal{E}'[l']$, $\Sigma^\sigma; \emptyset; \mathcal{E}[\mathcal{E}'] \vdash l' : \mathtt{mut}\,\_$, and
- $\mathit{mrog}(\sigma, l')$ not disjoint $\mathit{rog}(\sigma, l)$.

Finally, we model the *encapsulated* property of `capsule` references:

$\mathit{encapsulated}(\sigma, \mathcal{E}, l)$ iff $\forall l' \in \mathit{rog}(\sigma, l)$, if $\mathit{mutatable}(\sigma, [], \mathcal{E}[l], l')$, then not $\mathit{reachable}(\sigma, \mathcal{E}, l')$.

That is, a location l found in a context $\mathcal{E}$ is encapsulated if all *mutatable* objects in its *rog* would be unreachable with that single use of l removed. That single use of l is the connection preventing those *mutatable* objects from being garbage collectable.

Axiomatic Type Properties

Here we assume a slight variation of the usual Subject Reduction: a (sub) expression obtained using any number of reductions, from a well-typed and well-formed initial $\sigma_0|e_0$, is also well-typed:

Assumption 1 (Subject Reduction)**.** If $\mathit{validState}(\sigma, \mathcal{E}[e])$, then $\Sigma^\sigma; \emptyset; \mathcal{E} \vdash e : T$.

As we do not have a concrete type system, we need to assume some properties about its derivations. First we require that `new` expressions only have field initialisers with the appropriate type, fields are only updated with expressions of the appropriate type, methods are only called on receivers with the appropriate RC, method parameters have the appropriate type, and method calls are typed with the return type of the method:

Assumption 2 (Type Consistency)**.**

1. If $\Sigma; \Gamma; \mathcal{E} \vdash \mathtt{new}\ C\mathtt{(}e_1, ..., e_n\mathtt{)} : T$ then $\Sigma; \Gamma; \mathcal{E}[\mathtt{new}\ C\mathtt{(}e_1, ..., e_{i-1}, [], e_{i+1}, ..., e_n\mathtt{)}] \vdash e_i : T_i$ where $T_i = C.i$.
2. If $\Sigma; \Gamma; \mathcal{E}[[].f\ \mathtt{=}\ e'] \vdash e : \_C$ and $C.f = T'\ f$, then $\Sigma; \Gamma; \mathcal{E}[e.f\ \mathtt{=}\ []] \vdash e' : T'$.
3. If $\Sigma; \Gamma; \mathcal{E}[[].m\mathtt{(}e_1, ..., e_n\mathtt{)}] \vdash e : \_C$ and $C.m = \mu\,\mathtt{method}\ T\ m\mathtt{(}T_1\ x_1, ..., T_n x_n\mathtt{)}\ \_$, then:
   (a) $\Sigma; \Gamma; \mathcal{E}[[].m\mathtt{(}e_1, ..., e_n\mathtt{)}] \vdash e : \mu\, C$,
   (b) $\Sigma; \Gamma; \mathcal{E}[e.m\mathtt{(}e_1, ..., e_{i-1}, [], e_{i+1}, ..., e_n\mathtt{)}] \vdash e_i : T_i$, and
   (c) $\Sigma; \Gamma; \mathcal{E} \vdash e.m\mathtt{(}e_1, ..., e_n\mathtt{)} : T$.

We also assume that any expression inside a method body can be typed with the same reference capabilities as when it is expanded by our MCALL rule (in particular any x that is now an l will have the same μ):

Assumption 3 (Method Consistency)**.** If $\mathit{validState}(\sigma, \mathcal{E}_v[l.m\mathtt{(}v_1, ..., v_n\mathtt{)}])$ where:

- $\Sigma^\sigma; \emptyset; \mathcal{E}_v[[].m\mathtt{(}v_1, ..., v_n\mathtt{)}] \vdash l : \_C$, $C.m = \mu\,\mathtt{method}\,\_\,m\mathtt{(}T_1\ x_1, ...T_n\ x_n\mathtt{)}\ \mathcal{E}[e]$,
- $\mathcal{E}' = \mathtt{M(}l\mathtt{;}\mathcal{E}\mathtt{;}l\mathtt{.invariant())}$ if $C.m$ is a capsule mutator, otherwise $\mathcal{E}' = \mathcal{E}$,

[25] We use the term *mutatable* and not '*mutable*' as an object might be neither *mutatable* nor im*mutable*, e.g. if there are only `read` references to it.

- $\Gamma = \texttt{this} : \mu\, C, x_1 : T_1, ..., x_n : T_n$, and $e' = e[\texttt{this} := l, x_1 := v_1, ..., x_n := v_n]$,

  then $\Sigma^\sigma; \emptyset; \mathcal{E}_v[\mathcal{E}'[\texttt{this} := l, x_1 := v_1, ..., x_n := v_n]] \vdash e' : \mu\, \_$ implies $\emptyset; \Gamma; \mathcal{E} \vdash e : \mu\, \_$.

Now we define formal properties about our RCs, thus giving them meaning. First we require that an *immutable* object can not also be *mutatable*: i.e. an object reachable from an `imm` reference/field cannot also be reached from a `mut`/`capsule` reference and through `mut`/`capsule` fields:

Assumption 4 (Imm Consistency)**.**
If *validState*(σ, e) and *immutable*(σ, e, l), then not *mutatable*$(\sigma, [], e, l)$.

Note that this does not prevent *promotion* from a `mut` to an `imm`: a reduction step may change the type of an l from `mut` to `imm`, provided that in the new state there are no longer any `mut` references to the l's *rog*.

We require that if something was not *mutatable*, that it remains that way; this prevents, for example, runtime promotions from `read` to `mut`, as well as field accesses returning a `mut` from a receiver that was not `mut`:

Assumption 5 (Mut Consistency)**.** If *validState*$(\sigma, \mathcal{E}_v[e])$,
not *mutatable*$(\sigma, \mathcal{E}_v, e, l)$, and $\sigma|\mathcal{E}_v[e] \rightarrow^+ \sigma'|\mathcal{E}_v[e']$, then not *mutatable*$(\sigma', \mathcal{E}_v, e', l)$.

We require that a `capsule` reference be *encapsulated*; and require that `capsule` is a subtype of `mut`:

Assumption 6 (Capsule Consistency)**.**

1. If $\Sigma^\sigma; \emptyset; \mathcal{E} \vdash l : \texttt{capsule}\, \_$, then *encapsulated*$(\sigma, \mathcal{E}, l)$.
2. If $\Sigma; \Gamma; \mathcal{E} \vdash e : \texttt{capsule}\, C$, then $\Sigma; \Gamma; \mathcal{E} \vdash e : \texttt{mut}\, C$.

We require that field updates only be performed on `mut` receivers:

Assumption 7 (Mut Update)**.** If $\Sigma; \Gamma; \mathcal{E} \vdash e.f\ \texttt{=}\ e' : T$, then $\Sigma; \Gamma; \mathcal{E}[[].f\ \texttt{=}\ e'] \vdash e : \texttt{mut}\, \_$.

We additionally require that field accesses only be typed as `mut`, if their receiver is also `mut`:

Assumption 8 (Mut Access)**.** If $\Sigma; \Gamma; \mathcal{E} \vdash e.f : \texttt{mut}\, \_$, then $\Sigma; \Gamma; \mathcal{E}[[].f] \vdash e : \texttt{mut}\, \_$.

Finally, we require that a `read` variable or method result not be typeable as `mut`; in conjunction with Mut Consistency, Mut Update, and Method Consistency, this allows one to safely pass or return a `read` without it being used to modify the object's *rog*:

Assumption 9 (Read Consistency)**.**

1. If $\Gamma(x) = \texttt{read}\, \_$, then $\Sigma; \Gamma; \mathcal{E} \nvdash x : \texttt{mut}\, \_$.
2. If $\Sigma; \Gamma; \mathcal{E}[[].m\texttt{(}\overline{e}\texttt{)}] \vdash e : \_\, C$ and $C.m = \mu\, \texttt{method}\, \texttt{read}\, C'\, \_$, then $\Sigma; \Gamma; \mathcal{E} \nvdash e.m\texttt{(}\overline{e}\texttt{)} : \texttt{mut}\, \_$.

Note that Mut Consistency prevents an access to a `read` field from being typed as `mut`.

Strong Exception Safety

Finally we assume strong exception safety: the memory preserved by each `try–catch` execution is not *mutatable* within the `try`:

Assumption 10 (Strong Exception Safety)**.** If *validState*$(\sigma', \mathcal{E}[\texttt{try}^{\sigma_0}\{e_0\}\ \texttt{catch}\ \{e_1\}])$, then
$\forall l \in dom(\sigma_0)$, not *mutatable*$(\sigma, \mathcal{E}[\texttt{try}^{\sigma_0}\{[]\}\ \texttt{catch}\ \{e_1\}], e_0, l)$.

Note that this *only* needs to hold because our `try–catch` can catch invariant failures: in L42, `try–catch`'s that catch *checked* exceptions doe not this restriction.

We use strong exception safety to prove that locations preserved by `try` blocks are never monitored (this is important as it means that a `catch` that catches a monitor failure will not be able to see the responsible object),

Lemma 1 (Unmonitored Try)**.** If *validState*(σ, e), then $\forall \mathcal{E}$, $e = \mathcal{E}[\texttt{try}^{\sigma_0}\{\mathcal{E}'[\texttt{M(}l\texttt{;}\_\texttt{;}\_\texttt{)}]\}\, \_]$ implies $l \notin \sigma_0$

Proof. By *validState* we have $c \mapsto \texttt{Cap}\{\}|e_0 \rightarrow^+ \sigma|e$, so we proceed by induction on the number of "$\rightarrow$"s: in the base case, $e = e_0$ and so it cannot contain a monitor expression by the definition of *validState*. If this property holds for $\mathit{validState}(\sigma, e)$ but not for $\sigma'|e'$ with $\sigma|e \rightarrow \sigma'|e'$, we must have applied the UPDATE, MCALL, or NEW rules; since no other reduction steps introduce a monitor expression. If the reduction was a NEW, l will be fresh, so it could not have been in σ_0. If the reduction was an UPDATE, by Mut Update, l must have been `mut`, similarly (by our well-formedness rules on method bodies) MCALL will only introduce a monitor over a call to a `mut` method, so by Type Consistency, l was `mut`; either way we have that l was *mutatable*, since our reductions never change the σ_0 annotation, by Strong Exception Safety, we have that $l \notin \sigma_0$.

Determinism

We can use our object capability discipline (described in Section 5) to prove that the `invariant()` method is deterministic and does not mutate existing memory:

Lemma 2 (Determinism)**.** If $\mathit{validState}(\sigma, \mathcal{E}_v[l.\texttt{invariant()}])$ and

$$\sigma|\mathcal{E}_v[l.\texttt{invariant()}] \rightarrow \sigma'|\mathcal{E}_v[e'] \rightarrow^+ \sigma''|\mathcal{E}_v[e''],$$

then $\sigma'' = \sigma, \_$, $\sigma|\mathcal{E}_v[l.\texttt{invariant()}] \Rightarrow^+ \sigma''|\mathcal{E}_v[e'']$, and $\forall l' \in \mathit{dom}(\sigma)$, not $\mathit{mutatable}(\sigma'', \mathcal{E}_v, e'', l')$.

Proof. To prove this, we will use induction on the number of "$\rightarrow$"s, and additionally prove that $\forall l' \in \mathit{dom}(\sigma)$, not $\mathit{mutatable}(\sigma', \mathcal{E}_v, e', l')$, i.e. no part of the original σ will become *mutatable* from the body of the call $l.\texttt{invariant()}$.

Base case: If $\sigma|\mathcal{E}_v[l.\texttt{invariant()}] \rightarrow \sigma'|\mathcal{E}_v[e']$, then the reduction was performed by MCALL. By our well-formedness rules, the `invariant()` method takes a `read this`, so by Method Consistency and Read Consistency, we have that l is not *mutatable* in e' (since l is typeable as `read`). By our well-formedness rules on method bodies and MCALL, we have that no other l' was introduced in e', thus nothing is *mutatable* in e'.

The only non-deterministic single reduction steps are for calls to `mut` methods on a `Cap`; however `invariant()` is a `read` method, so even if $l = c$, we have $\sigma|\mathcal{E}_v[l.\texttt{invariant()}] \Rightarrow \sigma'|\mathcal{E}_v[e']$. Since MCALL does not mutate σ', we also have $\sigma' = \sigma$. Additionally, since l' is the only location reference in the expression $l.\texttt{invariant()}$ and it is not *mutatable*, we additionally have $\forall l' \in \mathit{dom}(\sigma)$, not $\mathit{mutatable}(\sigma', \mathcal{E}_v, e', l')$.

Inductive case: We inductively assume that $\sigma|\mathcal{E}_v[l.\texttt{invariant()}] \Rightarrow^+ \sigma'|\mathcal{E}_v[e'] \rightarrow \sigma''|\mathcal{E}_v[e'']$, $\sigma' = \sigma, \_$, and $\forall l' \in \mathit{dom}(\sigma)$, not $\mathit{mutatable}(\sigma', \mathcal{E}_v, e', l')$. Thus by Mut Consistency, we have that each such $l' \in \mathit{dom}(\sigma)$ is not *mutatable* in e''. Since nothing in σ was *mutatable*: by Mut Update, our reduction can't have modified anything in σ, i.e. $\sigma'' = \sigma', \_ = \sigma, \_$. As our reduction rules never remove things from memory, $c \in \mathit{dom}(\sigma)$, so it can't be *mutatable* in e'. By definition of `Cap`, no other instances of `Cap` exist, thus by Type Consistency, no `mut` methods of `Cap` can be called; since calling such a method is the only way to get a non-deterministic reduction, we have $\sigma'|\mathcal{E}_v[e'] \Rightarrow \sigma''|\mathcal{E}_v[e'']$.

Capsule Field Soundness

Now we define and prove important properties about our novel `capsule` fields. We first start with a few core auxiliary definitions. We define a notation to easily get the `capsule` field declarations for an l:

$f \in \mathit{capsuleFields}(\sigma, l)$ iff $\Sigma^\sigma(l).f = \texttt{capsule}\ \_$.

An l is *capsuleNotCircular* if it is not reachable from its `capsule` fields:

$\mathit{capsuleNotCircular}(\sigma, l)$ iff $\forall f \in \mathit{capsuleFields}(\sigma, l)$, $l \notin \mathit{rog}(\sigma, \sigma[l.f])$.

We say that an l is *wellEncapsulated* if none of its `capsule` fields is *mutatable* without passing through l:

$\mathit{wellEncapsulated}(\sigma, e, l)$ iff $\forall f \in \mathit{capsuleFields}(\sigma, l)$, not $\mathit{mutatable}(\sigma \setminus l, [], e, \sigma[l.f])$.

We say that an l is *notCapsuleMutating* if we aren't in a monitor for l which must have been introduced by MCALL, and we don't access any of it's `capsule` fields as `mut`:

$\mathit{notCapsuleMutating}(\sigma, e, l)$ iff $\forall \mathcal{E}$:

- if $e = \mathcal{E}[\texttt{M}(l;e';\_)]$, then $e' = l$, and
- if $e = \mathcal{E}[l.f]$, $f \in \mathit{capsuleFields}(\sigma, l)$, and $\Sigma^\sigma; \emptyset; \mathcal{E}[[].f] \not\vdash l : \texttt{capsule}\ \_$, then $\Sigma^\sigma; \emptyset; \mathcal{E} \not\vdash l.f : \texttt{mut}\ \_$.

Finally we say that l is *headNotObservable* if we are in a monitor introduced for a call to a capsule mutator, and l is not reachable from inside this monitor, except perhaps through a single `capsule` field access.

$\mathit{headNotObservable}(\sigma, e, l)$ iff $e = \mathcal{E}_v[\texttt{M}(l;e';\_)]$, and either:

- $e' = \mathcal{E}[l.f]$, $f \in \mathit{capsuleFields}(\sigma, l)$, and not $\mathit{reachable}(\sigma, \mathcal{E}, l)$ or
- not $\mathit{reachable}(\sigma, e', l)$.

Now we formally state the core properties of our `capsule` fields (informally described in Section 3):

Theorem 2 (Capsule Field Soundness)**.** If $\mathit{validState}(\sigma, e)$ then $\forall l$, if $\mathit{reachable}(\sigma, e, l)$, then: $\mathit{capsuleNotCircular}(\sigma, l)$ and either:

- $\mathit{wellEncapsulated}(\sigma, e, l)$ and $\mathit{notCapsuleMutating}(\sigma, e, l)$, or
- $\mathit{headNotObservable}(\sigma, e, l)$.

Proof. By *validState* we have $c \mapsto \mathtt{Cap}\{\}|e_0 \rightarrow^+ \sigma|e$, so we proceed by induction on the number of "$\rightarrow$"s. The base case is trivial, since `Cap` has no `capsule` fields and the initial main expression e_0 cannot contain monitors.

In the inductive case, we assume our theorem holds for a *validState* and prove it for the next *validState*. We then proceed by cases on the non-CTXV reduction rule applied:

1. (NEW) $\sigma|\mathcal{E}_v[\mathtt{new}\ C(v_1, ..., v_n)] \rightarrow \sigma'|\mathcal{E}_v[\mathtt{M(}l\mathtt{;}l\mathtt{;}l\mathtt{.invariant())}]$, where $\sigma' = \sigma, l \mapsto C\{v_1, ..., v_n\}$:
   (a) Since the pre-existing σ was not modified, by *validState*, $l \notin \mathit{rog}(\sigma, v_i) = \mathit{rog}(\sigma', \sigma'[l.f])$; thus *capsuleNotCircular* holds for l. In addition, since this reduction didn't modify the fields of any pre-existing l', by the inductive hypothesis, we have *capsuleNotCircular* for l and each such l'.
   (b) Consider any l' in $\mathit{dom}(\sigma')$:
      - Suppose $l' \neq l$ and was *wellEncapsulated* and *notCapsuleMutating*. Suppose we have made it so that it is no longer *wellEncapsulated*, then we must have made some $f' \in \mathit{capsuleFields}(\sigma, l')$ *mutatable*. Since the *rog* of l' can't have been modified, nor could the *rog* of any other pre-existing l'', Since we didn't modify the *rog* of l' nor the *rog* of any other pre-existing l'', we must have that $\sigma[l'.f']$ is now *mutatable* through some $l.f$. This requires that v_i is an initialiser for a `mut` or `capsule` field, which by Type Consistency and Capsule Consistency, means that v_i must also be typeable as `mut`. But then the $\sigma[l'.f']$ was already *mutatable* through v_i, so l' can't have already been *wellEncapsulated*, a contradiction.
      - We can't have broken *notCapsuleMutating* either since we haven't introduced any monitor expressions or field accesses, and by Mut Consistency and Mut Access we can't have made any pre-existing field accesses in EV typeable as `mut`.
      - Now suppose $l' = l$ and consider each i with $C.i = \mathtt{capsule}\,\_\,f$. By Type Consistency and Capsule Consistency, v_i was *encapsulated* and $\mathit{rog}(\sigma, v_i)$ is not *mutatable* from $\mathcal{E}_v$, and so v_i is not $\mathit{mutatable}(\sigma' \setminus l, [], \mathcal{E}_v[\mathtt{M(}l\mathtt{;}l\mathtt{;}l\mathtt{.invariant())}], v_i)$; thus *wellEncapsulated* holds for l and each of its `capsule` fields.
      - We trivially have that l is *notCapsuleMutating* since l was fresh, there can't be any monitor expressions or field accesses for it in $\mathcal{E}_v$.
   (c) By the inductive hypothesis, any other pre-existing l' must be *headNotObservable*, but we haven't removed any monitor expression or field-accesses (because the arguments to the constructor are all of form v), in addition, we haven't made any pre-existing object *reachable* (everything reachable through l was already reachable through some v_i anyway). Thus each such l' is still *headNotObservable*.
2. (ACCESS) $\sigma|\mathcal{E}_v[l.f] \rightarrow \sigma|\mathcal{E}_v[\sigma[l.f]]$:
   (a) As this rule doesn't mutate memory, by the inductive hypothesis, every l' must be *capsuleNotCircular*.
   (b) Consider any l' that was *wellEncapsulated* and *notCapsuleMutating*:
      - Suppose $l' = l$ and $f \in \mathit{capsuleFields}(\sigma, l)$, by Mut Access, either $\Sigma^\sigma; \emptyset; \mathcal{E}_v \not\vdash \sigma[l.f] : \mathtt{mut}\,\_$ or $\Sigma^\sigma; \emptyset; \mathcal{E}_v[[].f] \vdash l : \mathtt{capsule}\,\_$. If l was `capsule`, then by Capsule Consistency and *capsuleNotCircular*, l is not *reachable* from $\mathcal{E}_v[\sigma[l.f]]$, so it is irrelevant if l is no longer *wellEncapsulated*. Otherwise, if l was not `capsule`, $\sigma[l.f]$ will not be `mut`, so *wellEncapsulated* is preserved for l.

- Now consider any l' that was not *headNotObservable*, then by the inductive hypothesis, it was *wellEncapsulated* and *notCapsuleMutating*. Since this reduction doesn't modify memory, by Mut Consistency, there is no other way to make the *rog* of a `capsule` field f' of l' *mutatable* without going through l', so *wellEncapsulated* is preserved for l'.
- Since this reduction doesn't modify memory, and by Mut Consistency, we can't have made the *rog* of any other `capsule` field f' of any l' *mutatable* without going through l', so *wellEncapsulated* is preserved.
- As in the above NEW case, *notCapsuleMutating* is preserved as we haven't introduced any monitor expressions or field accesses.

(c) Consider any l' that was not *wellEncapsulated* and *notCapsuleMutating*, then by the inductive hypothesis it was *headNotObservable* and:

- If $l' = l$, then $\mathcal{E}_v = \mathcal{E}_v{}'[\texttt{M(}l\texttt{;}\mathcal{E}[l.f]\texttt{;\_)}]$, with l not *reachable* from $\mathcal{E}$, and $l.f$ is an access to a `capsule` field. By *capsuleNotCircular*, l is not in the *rog* of $\sigma[l.f]$, and so l is not *reachable* from $\mathcal{E}[\sigma[l.f]]$, and so it is still *headNotObservable*.
- If $l' \neq l$, it is still *headNotObservable* l is as this reduction dosen't make anything *reachable* that wasn't already *reachable* through l or $\mathcal{E}_v$.

3. (UPDATE) $\sigma|\mathcal{E}_v[l.f\ \texttt{=}\ v] \rightarrow \sigma[l.f = v]|\mathcal{E}_v[\texttt{M(}l\texttt{;}l\texttt{;}l\texttt{.invariant())}]$:

(a) By the inductive hypothesis we have that *capsuleNotCircular* holds for l. If $f \in \mathit{capsuleFields}(\sigma, l)$, by Mut Update, we have that l is *mutatable*, so by Type Consistency and Capsule Consistency, $\mathit{encapsulated}(\sigma, \mathcal{E}_v[l.f = []], v)$, hence l is not *reachable* from v, and so after the update, *capsuleNotCircular* still holds for l. In addition, if $f \notin \mathit{capsuleFields}(\sigma, l)$ we obviously must still have *capsuleNotCircular*.

(b) Otherwise, by the inductive hypothesis, l' was *headNotObservable*, and so $l' \notin \mathit{rog}(\sigma, v)$, so we can't have added l' to the *rog* of anything, thus *capsuleNotCircular* still holds.

(c) By the inductive hypothesis we have that *capsuleNotCircular* holds for each l':

- If $f \in \mathit{capsuleFields}(\sigma, l)$: by Mut Update, we have that l is *mutatable*, so by Type Consistency and Capsule Consistency, $\mathit{encapsulated}(\sigma, \mathcal{E}_v[l.f = []], v)$, hence l is not *reachable* from v, and so after the update, *capsuleNotCircular* still holds for l.
- Now consider any l' and $f' \in \mathit{capsuleFields}(\sigma, l')$, with $l'.f' \neq l.f$:
  - If l' was *wellEncapsulated*, by Mut Update, l is `mut`. By *wellEncapsulated*, the *rog* of $l'.f'$ is not *mutatable* (except through a field *access* on l'), thus we have that $l \notin \mathit{rog}(\sigma, \sigma[l'.f'])$, in addition, since $l'.f' \neq l.f$, we can't have modified the *rog* of $l'.f'$, hence l' is still *capsuleNotCircular*.
  - Otherwise, by the inductive hypothesis, l' was *headNotObservable*, and so $l' \notin \mathit{rog}(\sigma, v)$, so we can't have added l' to the *rog* of anything, thus l' is still *capsuleNotCircular*.

(d) Consider any l' that was *wellEncapsulated* and *notCapsuleMutating*:

- If $l' = l$ and $f \in \mathit{capsuleFields}(\sigma, l)$, by Type Consistency and Capsule Consistency, v is *encapsulated*, thus v is not *mutatable* from $\mathcal{E}_v$, and l is not *reachable* from v, thus v is still *encapsulated* and *wellEncapsulated* still holds for l and f.
- Now consider any $f' \in \mathit{capsuleFields}(\sigma, l')$, with $l'.f' \neq l.f$; by the above, l is *capsuleNotCircular* and so $l \notin \mathit{rog}(\sigma, \sigma[l'.f'])$. If f was a `mut` or `capsule` field, by Type Consistency and Capsule Consistency, v was `mut`, so by *wellEncapsulated*, $v \notin \mathit{rog}(\sigma, \sigma[l'.f'])$; thus we can't have made $\mathit{rog}(\sigma, \sigma[l'.f'])$ *mutatable* through $l.f$; so $l'.f'$ can't now be *mutatable* through l. By Mut Consitency, we couldn't have have made $l'.f'$ *mutatable* some other way, so l' is still *wellEncapsulated*.
- As in the above cases, *notCapsuleMutating* is preserved as we haven't introduced any monitor expressions or field accesses.

(e) By the inductive hypothesis, any l' which was not *wellEncapsulated* and *notCapsuleMutating*, was *headNotObservable*; we haven't removed any monitor expression or field-accesses, nor have we made anything *reachable* that wasn't before, so each such l' is still *headNotObservable*.

4. (MONITOR EXIT) $\sigma|\mathcal{E}_v[\texttt{M(}l\texttt{;}v\texttt{;true)}] \rightarrow \sigma|\mathcal{E}_v[v]$:
   (a) As this rule doesn't mutate memory, by the inductive hypothesis, every l' must be *capsuleNotCircular*.
   (b) Any l' that was *wellEncapsulated*, still is by Mut Consistency, since we haven't modified memory; and as with the above cases, if l' was *notCapsuleMutating* it still is we haven't introduced any monitor expressions or field accesses.
   (c) Now consider any l' that was not *wellEncapsulated* and *notCapsuleMutating*, then by the inductive hypothesis it was *headNotObservable*:
      - If $l' \neq l$, then as with the ACCESS case above, we can't have broken *headNotObservable*.
      - Otherwise, suppose $l' = l$. If this monitor was introduced by NEW or UPDATE, then $v = l$. And so *headNotObservable* can't have held for l since $l = v$, and v was not the receiver of a field access.
      - Thus this monitor must have been introduced by MCALL, due to a call to a capsule mutator on l. Consider the state $\sigma_0|\mathcal{E}_v[e_0]$ immediately before that MCALL:
         – We must not have had that l was *headNotObservable*, since e_0 would contain l as the receiver of a method call. Thus, by our inductive hypothesis, l was originally *wellEncapsulated* and *notCapsuleMutating*.
         – Because *notCapsuleMutating* held in $s_0|\mathcal{E}_v[e_0]$, and v contains no field accesses or monitor, it also holds in $\mathcal{E}_v[v]$.
         – Since a capsule mutator cannot have any `mut` parameters, by Type Consistency, Mut Consistency, and Mut Update, the body of the method can't have modified σ_0: thus $\sigma = \sigma_0, \_$. Since no pre-existing memory has changed since the MCALL, and a capsule mutator cannot have a `mut` return type, by Type Consistency, we must have $\Sigma^\sigma; \emptyset; \mathcal{E}_v \vdash v : \mu\,\_$ where $\mu \neq \texttt{mut}$:
            * If $\mu = \texttt{capsule}$, by Capsule Consistency, the value of any `capsule` field of l can't be in the *rog* of v (unless l is no longer *reachable*), so we haven't made such a field *mutatable*.
            * Otherwise, $\mu \in \{\texttt{read}, \texttt{imm}\}$, by Read Consistency, Imm Consistency, and Mut Consistency, we have that v is not *mutatable*.

         Either way, the MONITOR EXIT reduction has restored *wellEncapsulated*$(\sigma_0, \mathcal{E}_v[e_0], l)$.

5. (TRY ERROR) $\sigma|\mathcal{E}_v[\texttt{try}^{\sigma_0}\{error\}\ \texttt{catch}\ \{e\}] \rightarrow \sigma|\mathcal{E}_v[e]$, where $error = \mathcal{E}_v{}'[\texttt{M(}l\texttt{;}\_\texttt{;}\_\texttt{)}]$:
   (a) As above, since this rule doesn't mutate memory, by the inductive hypothesis, every l' must still be *capsuleNotCircular*.
   (b) As above, since we didn't modify memory, or introduce any monitor expressions or field accesses, any l' that was *wellEncapsulated* and *notCapsuleMutating* is still *wellEncapsulated* and *notCapsuleMutating*.
   (c) Consider any l' that was not *wellEncapsulated* and *notCapsuleMutating*, by the inductive hypothesis it must have been *headNotObservable*:
      - If $l' \neq l$. If l' was in σ_0, then by Strong Exception Safety we haven't modified σ_0, and so *headNotObservable* is preserved. Otherwise, by Strong Exception Safety, l' is no longer *reachable*, and so this theorem imposes no requirements on it.
      - If $l' = l$, then by our reduction rules, we were previously in state $\sigma_0|\mathcal{E}_v[\texttt{try}\ \{e_0\}\ \texttt{catch}\ \{e\}]$. By Unmonitored Try, $l \notin dom(\sigma_0)$, and so l was not *reachable* from $\mathcal{E}_v[\texttt{try}\ \{e_0\}\ \texttt{catch}\ \{e\}]$. By Strong Exception Safety, we have that nothing in σ_0 has changed, so we must still have that l is not *reachable* from $\mathcal{E}_v[e]$: thus it's status is irrelevant.

6. (MCALL) $\sigma|\mathcal{E}_v[l.m(v_1, ..., v_n)] \rightarrow \sigma|\mathcal{E}_v[e]$:
   (a) As above, since this rule doesn't mutate memory, by the inductive hypothesis, every l' must still be *capsuleNotCircular*.
   (b) Consider any l' that was *wellEncapsulated* and *notCapsuleMutating*:
      - Suppose m is not a capsule mutator, by our well-formedness rules for method bodies, e doesn't contain a monitor.

- Suppose $l' = l$. Since m is not a capsule mutator, if $e = \mathcal{E}[l.f]$, for some $f \in \mathit{capsuleFields}(\sigma, l)$, we must have that m was not a `mut` method. So by Mut Access and Method Consistency, we have that $\Sigma^\sigma; \emptyset; \mathcal{E}_v[\mathcal{E}] \nvdash l.f : \texttt{mut}\,\_$ only if m was a `capsule` method, which by Method Consistency, would mean that $\Sigma^\sigma; \emptyset; \mathcal{E}_v[\mathcal{E}[[].f]] \vdash l : \texttt{capsule}\,\_$. So regardless of what fields e accesses on l, we can't have broken *notCapsuleMutating* for l.
- Otherwise, $l' \neq l$, and since fields are instance-private, by our well-formedness rules on method bodies, $l' \notin e$, thus we can't have introduced any field accesses on l. As e doesn't contain monitors either, we haven't broken *notCapsuleMutating* for l'.

- Otherwise, suppose $l' = l$, $e = \texttt{M(}l\texttt{;}e'\texttt{;}l\texttt{.invariant())}$. By our rules for capsule mutators, m must be a `mut` method with only `imm` and `capsule` parameters, thus by Type Consistency, l must have been `mut`, and each v_i must be `imm` or `capsule`. By Imm Consistency and Capsule Consistency, l can't be reachable from any v_i. Since capsule mutators use `this` only once, to access a `capsule` field, $e' = \mathcal{E}[l.f]$, for some $f \in \mathit{capsuleFields}(\sigma, l)$. Since l is not *reachable* from any v_i, $l \notin \mathcal{E}$, and by our well-formedness rules for method bodies, l is not *reachable* from any $l' \in \mathcal{E}$, thus *headNotObservable* now holds for l.
- If $l' = l$ then we haven't introduced a monitor expression, and as above, we haven't introduce a field access on it either, we can't have broken *wellEncapsulated*.
- Finally, since we didn't modify memory, l' is still *notCapsuleMutating*.

(c) As above since we haven't removed any monitor expression or field-accesses, nor have we made anything *reachable*, by the inductive hypothesis, every other l' is still *headNotObservable*.

7. (TRY ENTER and TRY OK) these are trivial, since as used in the above cases:
   (a) These rules don't mutate memory, thus by the inductive hypothesis, every l' must still be *capsuleNotCircular*.
   (b) These rules don't modify memory, or introduce any monitor expressions or field accesses, so any l' that was *wellEncapsulated* and *notCapsuleMutating* is still *wellEncapsulated* and *notCapsuleMutating*.
   (c) As these rules don't remove any monitor expression or field-accesses, nor do they make anything *reachable*, by the inductive hypothesis, every other l' is still *headNotObservable*.

Stronger Soundness

It is hard to prove Soundness directly, so we first define a stronger property, called Stronger Soundness.

An object is *monitored* if execution is currently inside of a monitor for that object, and the monitored expression e_1 does not contain l as a *proper* sub-expression:

$\mathit{monitored}(e, l)$ iff $e = \mathcal{E}_v[\texttt{M(}l\texttt{;}e_1\texttt{;}e_2\texttt{)}]$ and either $e_1 = l$ or $l \notin e_1$.

A monitored object is associated with an expression that cannot observe it, but may reference its internal representation directly. In this way, we can safely modify its representation before checking its invariant. The idea is that at the start the object will be valid and e_1 will reference l; but during reduction, l will be used to modify the object; only after that moment, the object may become invalid.

Stronger Soundness says that starting from a well-typed and well-formed $\sigma_0|e_0$, and performing any number of reductions, every *reachable* object is either *valid* or *monitored*:

Theorem 3 (Stronger Soundness)**.** If *validState* (σ, e) then $\forall l$, if $\mathit{reachable}(\sigma, e, l)$ then $\mathit{valid}(\sigma, l)$ or $\mathit{monitored}(e, l)$.

Proof. We will prove this inductively on the number of "$\rightarrow$"s, in a similar way to how we proved Capsule Field Soundness. In the base case, we have $\sigma = c \mapsto \texttt{Cap}\{\}$, since `Cap` is defined to have the trivial invariant, we have that c (the only thing in σ), is *valid*.

Now we assume that everything reachable from the previous *validState* was *valid* or *monitored*, and proceed by cases on the non-CTXV rule that gets us to the next *validState*.

1. (UPDATE) $\sigma|\mathcal{E}_v[l.f\ \texttt{=}\ v] \rightarrow \sigma'|\mathcal{E}_v[e']$, where $e' = \texttt{M(}l\texttt{;}l\texttt{;}l\texttt{.invariant())}$:
   - Clearly l is now *monitored*.

- Consider any other l', where $l \in \mathit{rog}(\sigma, l')$ and l' was *valid*; now suppose we just made l' not *valid*. By our well-formedness criteria, `invariant()` can only accesses `imm` and `capsule` fields, thus by Imm Consistency and Mut Update, we must have that l was in the *rog* of $l'.f'$, for some $f' \in \mathit{capsuleFields}(\sigma, l')$. Since $l \neq l'$, l' can't have been *wellEncapsulated*. Thus, by Capsule Field Soundness, l' was *headNotObservable*, and $\mathcal{E}_v = \mathcal{E}_v{}'[\mathtt{M(}l'\mathtt{;}\mathcal{E}_v{}''\mathtt{;\_)}]$:
  - If $\mathcal{E}_v{}''[l.f\ \mathtt{=}\ v] = \mathcal{E}[l'.f']$, then by *headNotObservable*, l' is not reachable from $\mathcal{E}$. The monitor must have been introduced by an MCALL, on a capsule mutator for l'. Since a capsule mutator can take only `imm` and `capsule` parameters, by Type Consistency, Imm Consistency, and Capsule Consistency, l cannot be in their *rog*s (since l was in the *rog* of l', and l is `mut`). Thus the only way for the body of the monitor to access l is by accessing $l'.f'$. Since capsule mutators can access `this` only once, and by the proof of Capsule Field Soundness, there is no other $l'.f'$ in $\mathcal{E}[l'.f']$, nor was there one in a previous stage of reduction: hence l is not *reachable* from $\mathcal{E}$. This is in contradiction with us having just updated l.
  - Thus, by *headNotObservable*, we must have $\mathcal{E}_v{}''[l.f\ \mathtt{=}\ v] = e$, with l' not *reachable* from e; so l' was, and still is, *monitored*.
- Since we don't remove any monitors, we can't have violated *monitored*. In addition, if an l was not in the *rog* of a *valid* l', by Determinism, l' is still *valid*.

2. (MONITOR EXIT) $\sigma|\mathtt{M(}l\mathtt{;}v\mathtt{;true)} \rightarrow \sigma|v$:

   By *validState* and our well-formedness requirements on method bodies, the monitor expression must have been introduced by UPDATE, MCALL, or NEW. In each case the 3rd expression started off as l`.invariant()`, and it has now (eventually) been reduced to `true`, thus by Determinism l is *valid*. This rule does not modify pre-existing memory, introduce pre-existing ls into the main expression, nor remove monitors on other ls, thus every other pre-existing l' is still *valid* (due to Determinism), or *monitored*.

3. (NEW) $\sigma|\mathcal{E}_v[\mathtt{new}\ C\mathtt{(}\overline{v}\mathtt{)}] \rightarrow \sigma, l \mapsto C\{\overline{v}\}|\mathcal{E}_v[\mathtt{M(}l\mathtt{;}l\mathtt{;}l\mathtt{.invariant())}]$:

   Clearly the newly created object, l, is *monitored*. As with the case for MONITOR EXIT above, every other *reachable* l is still *valid* or *monitored*.

4. (TRY ERROR) $\sigma|\mathcal{E}_v[\mathtt{try}^{\sigma_0}\{\mathit{error}\}\ \mathtt{catch}\ \{e\}] \rightarrow \sigma|\mathcal{E}_v[e]$, where $\mathit{error} = \mathcal{E}_v{}'[\mathtt{M(}l\mathtt{;\_;\_)}]$:

   By the proof of Capsule Field Soundness, we must have that l is no longer *reachable*, it is ok that it is now no longer *valid* or *monitored*. As with the case for MONITOR EXIT above, every other *reachable* l is still *valid* or *monitored*.

None of the other reduction rules modify memory, the memory locations reachable inside of the main expression, or any pre-existing monitor expressions; thus regardless of the reduction performed, we have that each *reachable* l is *valid* or *monitored*.

Proof of Soundness

First we need to prove that an object is not reachable from one of its `imm` fields; if it were, `invariant()` could access such a field and observe a potentially broken object:

Lemma 3 (Imm Not Circular)**.**
If $\mathit{validState}(\sigma, e)$, $\forall f, l$, if $\mathit{reachable}(\sigma, e, l)$, $\Sigma^\sigma(l).f = \mathtt{imm}\ \_$, then $l \notin \mathit{rog}(\sigma, \sigma[l.f])$.

Proof. The proof is by induction; obviously the property holds in the initial $\sigma|e$, since $\sigma = c \mapsto \mathtt{Cap}\{\}$. Now suppose it holds in a *validState* (σ, e) and consider $\sigma|e \rightarrow \sigma'|e'$.

1. Consider any pre-existing *reachable* l and f with $\Sigma^\sigma(l).f = \mathtt{imm}\ \_$, by Imm Consistency and Mut Update, the only way $\mathit{rog}(\sigma, \sigma[l.f])$ could have changed is if $e = \mathcal{E}_v[l.f = v]$, i.e. we just applied the UPDATE rule. By Mut Update we must have that l was `mut`, by Type Consistency, v must have been `imm`, so by Imm Consistency, $l \notin \mathit{rog}(\sigma, v)$. Since $v = \sigma'[l.f]$, we now have $l \notin \mathit{rog}(\sigma', \sigma'[l.f])$.
2. The only rule that makes an l *reachable* is NEW. So consider $e = \mathcal{E}_v[\mathtt{new}\ C\mathtt{(}v_1, ..., v_n\mathtt{)}]$ and each i with $C.i = \mathtt{imm}\ \_$, so by Type Consistency, $1 \leq i \leq n$, and hence v_i is well-defined. But v_i existed in the previous state and $l \notin \mathit{dom}(\sigma)$; so by *validState* and our reduction rules, $l \notin \mathit{rog}(\sigma, v_i) = \mathit{rog}(\sigma', \sigma'[l.f])$.

Note that the above only applies to `imm` *fields*: `imm` *references* to cyclic objects can be created by promoting a `mut` reference, however the cycle must pass through a *field* declared as `read` or `mut`, but such fields cannot be referenced in the invariant method.

We can now finally prove the soundness of our invariant protocol:

Theorem 1 (Soundness)**.** If $\mathit{validState}(\sigma, \mathcal{E}_r[l])$, then either $\mathit{valid}(\sigma, l)$ or $\mathit{trusted}(\mathcal{E}_r[l])$.

Proof. Suppose $\mathit{validState}(\sigma, e)$, and $e = \mathcal{E}_r[l]$. Suppose l is not *valid*; since l is *reachable*, by Stronger Soundness, $\mathit{monitored}(e, l)$, $e = \mathcal{E}[\texttt{M(}l\texttt{;}e_1\texttt{;}e_2\texttt{)}]$, and either:

- $\mathcal{E}_r = \mathcal{E}[\texttt{M(}l\texttt{;}\mathcal{E}'\texttt{;}e_2\texttt{)}]$, that is l was found inside of e_1, but by definition of $\mathcal{E}_r$, we can't have $e_1 = l$, this contradicts the definition of *monitored*, or
- $\mathcal{E}_r = \mathcal{E}[\texttt{M(}l\texttt{;}e_1\texttt{;}\mathcal{E}'\texttt{)}]$, and thus l was found inside e_2. By our reduction rules, all monitor expressions start with $e_2 = l\texttt{.invariant()}$; if this has yet to be reduced, then $\mathcal{E}' = \mathcal{E}''[[]\texttt{.invariant()}]$, thus $\mathcal{E}_r[l]$ is *trusted*. The next execution step will be an MCALL, so by our well-formedness rules for `invariant()`, e_2 will only contain l as the receiver of a field access; so if we just performed said MCALL, $\mathcal{E}' = \mathcal{E}''[[].f]$: hence $\mathcal{E}_r[l]$ is trusted. Otherwise, by Imm Not Circular, Capsule Field Soundness, and *capsuleNotCircular*, no further reductions of e_2 could have introduced an occurrence of l, so we must have that l was introduced by the MCALL to `invariant()`, and so it is *trusted*.

Thus either l is *valid* or $\mathcal{E}_r[l]$ is *trusted*.