

Using nested classes as associated types.

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Abstract

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2012 ACM Subject Classification Dummy classification

Keywords and phrases Dummy keyword

Digital Object Identifier 10.4230/LIPIcs.CVIT.2016.23

1 Introduction

Associated types are a powerful form of generics, now integrated in both Scala and Rust. They are a new kind of member, like methods fields and nested classes. Associated types behave as 'virtual' types: they can be overridden, can be abstract and can have a default. However, the user has to specify those types and their concrete instantiations manually; that is, the user have to provide a complete mapping from all virtual type to concrete instantiation. When the number of associated types is small this poses no issue, but it hinders designs where the number of associated types is large. In this paper we examine the possibility of completing a partial mapping in a desirable way, so that the resulting mapping is sound and also robust with respect to code evolution.

The core of our design is to reuse the concept of nested classes instead of relying of a new kind of member for associated types. An operation, call Redirect, will redirect some nested classes in some external types. To simplify our formalization and to keep the focus on the core of our approach, we present our system on top of a simple Java like languages, with only final classes and interfaces, when code reuse is obtained by trait composition instead of conventional inheritance. We rely on a simple nominal type system, where subtyping is induced only by implementing interfaces; in our approach we can express generics without having a polymorphic type system. To simplify the treatment of state, we consider fields to be always instance private, and getters and setters to be automatically generated, together with a `static` method `of(..)` that would work as a standard constructor, taking the value of the fields and initializing the instance. In this way we can focus our presentation to just (static) methods, nested classes and implements relationships. Expanding our presentation to explicitly include visible fields, constructors and sub-classing would make it more complicated without adding any conceptual underpinning. In our proposed setting we could write:

```
String=...
SBox={String inner;
  method String inner(){..} //implicit
  static method SBox of(String inner){..} //implicit
myTtrait={
  Box={Elem inner} //implicit Box(Elem inner) and Elem inner()
  Elem={Elem concat(Elem that)}
  static method Box merge(Box b, Elem e){return Box.of(b.inner().concat(e));}
}
Result=myTrait<Box=SBox> //equivalent to trait<Box=SBox, Elem=String>
...Result.merge(SBox.of("hello "), "world");//hello world
```



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42nd Conference on Very Important Topics (CVIT 2016).

Editors: John Q. Open and Joan R. Access; Article No. 23; pp. 23:1–23:10

Leibniz International Proceedings in Informatics



LIPICs Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

23:2 Using nested classes as associated types.

48 Here class **SBox** is just a container of **Strings**, and **myTrait** is code encoding **Boxes** of any kind
49 of **Elem** with a **concat** method. By instantiating **myTrait<Box=SBox>**, we can infer **Elem=String**,
50 and obtain the following flattened code, where **Box** and **Elem** has been removed, and their
51 occurrences are replaced with **SBox** and **String**.

```
52 Result={static method SBox merge(SBox b,String e){  
53     return SBox.of(b.inner().concat(e));}  
54 }
```

56 Note how **Result** is a new class that could have been written directly by the programmer,
57 there is no trace that it has been generated by **myTrait**. We will represent trait names with
58 lower-case names and class/interface names with upper-case names. Traits are just units of
59 code reuse, and do not induce nominal types.

60 We could have just written **Result=myTrait<Elem=String>**, obtaining

```
61 Result={  
62     Box={String inner}  
63     static method Box merge(Box b,String e){  
64         return Box.of(b.inner().concat(e));}  
65 }
```

67 Note how in this case, class **Result.Box** would exist. Thanks to our decision of using nested
68 classes as associated types, the decision of what classes need to be redirected is not made
69 when the trait is written, but depends on the specific redirect operation. Moreover, our
70 redirect is not just a way to show the type system that our code is correct, but it can change
71 the behaviour of code calling static methods from the redirected classes.

72 This example shows many of the characteristics of our approach:

- 73 ■ (A) We can redirect mutually recursive nested classes by redirecting them all at the
74 same time, and if a partial mapping is provided, the system is able to infer the complete
75 mapping.
- 76 ■ (B) **Box** and **Elem** are just normal nested classes inside of **myTrait**; indeed any nested
77 class can be redirected away. In case any of their (static) methods was implemented, the
78 implementation is just discarded. In most other approaches, abstract/associated/generic
79 types are special and have some restriction; for example, in Java/Scala static methods
80 and constructors can not be invoked on generic/associated types. With redirect, they are
81 just normal nested classes, so there are no special restrictions on how they can be used.
82 In our example, note how **merge** calls **Box.of(..)**.
- 83 ■ (C) While our example language is nominally typed, nested classes are redirected over
84 types satisfying the same structural shape. We will show how this offers some advantages
85 of both nominal and structural typing.

86 A variation of redirect, able to only redirect a single nested class, was already presented
87 in literature. While points (B) and (C) already apply to such redirect, we will show how
88 supporting (A) greatly improves their value.

89 The formal core of our work is in defining

- 90 ■ **ValidRedirect**, a computable predicate telling if a mapping respects the structural shapes
91 and nominal subtype relations.
- 92 ■ A formal definition of what properties a procedure expanding a partial mapping into a
93 complete one should respect.
- 94 ■ **ChoseRedirect**, an efficient algorithm respecting those properties.

95 We first formally define our core language, then we define our redirect operator and its
96 formal properties. Finally we motivate our model showing how many interesting examples of
97 generics and associated types can be encoded with redirect. Finally, as an extreme application,
98 we show how a whole library can be adapted to be injected in a different environment.

2 Language grammar and well formedness

$e ::= x \mid e.m(es) \mid T.m(es) \mid e.x \mid \text{new } T(es)$	expression	$T ::= \text{This}n.Cs$	types
$L ::= \{\text{interface } Tz; M\} \mid \{Tz; Mz; K\}$	code literal	$Tx ::= T \ x$	parameter
$M ::= \text{static? } T \ m(Txs) \ e? \mid C=E$	member	$D ::= id=E$	declaration
$K ::= \langle Txz \rangle?$	state	$id ::= C \mid t$	class/trait id
$E ::= L \mid t \mid E_1 <+ E_2 \mid E <Cs=T \rangle$	Code Expr.	$v ::= \text{new } T(vs)$	value

We apply our ideas on a simplified object oriented language with nominal typing and (nested) interfaces and final classes. Instead of inheritance, code reuse is obtained by trait composition, thus the source code would be a sequence of top level declarations D followed by a main expression; a lower-case identifier t is a trait name, while an upper case identifier C is a class name. To simplify our terminology, instead of distinguishing between nested classes and nested interfaces, we will call *nested class* any member of a code literal named by a class identifier C . Thus, the term *class* may denote either an *interface class* (interface for short) or a *final class*.

In the context of nested classes, types are paths. Syntactically, we represent them as relative paths of form $\text{This}n.Cs$, where the number n identify the root of our path: $\text{This}0$ is the current class, $\text{This}1$ is the enclosing class, $\text{This}2$ is the enclosing enclosing class and so on. $\text{This}n.Cs$ refers to the class obtained by navigating throughout Cs starting from $\text{This}n$. Thus, $\text{This}0$ is just the type of the directly enclosing class. Note how there can be multiple different types referring to the same class.

Code literals L serve the role of class/interface bodies; they contain the set of implemented interfaces Tz , the set of members Mz and their (optional) state. In the concrete syntax we will use **implements** in front of a non empty list of implemented interfaces and we will omit parenthesis around a non empty set of fields. A class member M can be a nested class or a (static) method. Abstract methods are just methods without a body. Well formed interface methods can only be abstract and non-static. To facilitate code reuse, classes can have (static) abstract methods, code composition is expected to provide an implementation for those or, as we will see, redirect away the whole class.

Expressions are used as body of (static) methods and for the main expression. They are variables x (including **this**) and conventional (static) method calls. Field access and **new** expressions are included but with restricted usage: well formed field accesses are of form **this**. x in method bodies and $v.x$ in the main expression, while well formed **new** expressions have to be of form **new This0**(xs) in method bodies and of form v in the main expression. Those restrictions greatly simplify reasoning about code reuse, since they require different classes to only communicate by calling (static) methods. Supporting unrestricted fields and constructors would make the formalism much more involved without adding much of a conceptual difficulty. Values are of form **new** $T(vs)$.

For brevity, in the concrete syntax we assume a syntactic sugar declaring a static of method (that serve as a factory) and all fields getters; thus the order of the fields would induce the order of the factory arguments. In the core calculus we just assume such methods to be explicitly declared.

Finally, we examine the shape of a nested class: $C=E$. The right hand side is not just a code literal but a code composition expression E . In trait composition, the code expression will be reduced/flattened to a code literal L during compilation. Code expressions denote an algebra of code composition, starting from code literal L and trait names t , referring to a literal declared before by $t=E$. We consider two operators: conventional preferential sum $E_1 <+ E_2$ and our novel redirect $E <Cs=T \rangle$.

The compilation process consists in flattening all the E into L , starting from the innermost leftmost E . This means that sum and redirect work on LV s: a kind of L , where all the nested classes are of form $C=LV$. The execution happens after compilation and consist in the conventional execution of the main expression e in the context of the fully reduced declarations, where all trait composition has been flatted away. Thus, execution is very simple and standard and behaves like a variation of FJ \square with interfaces instead of inheritance, and where nested classes are just a way to hierarchically organize code names. On the other side, code composition in this setting is very interesting and powerful, where nested classes are much more than name organization: they support in a simple and intuitive way expressive code reuse patterns. To flatten an E we need to understand the behaviour of the two operators, and how to load the code of a trait: since it was written in another place, the syntactic representation of the types need to be updated. For each of those points we will first provide some informal explanation and then we will proceed formalizing the precise behaviour.

2.1 Redirect

Redirect $LV<Cs=T>CsT$ will

2.2 Preferential sum

$\mathcal{E}_V ::= \square \mid \mathcal{E}_V <+ E \mid LV <+ \mathcal{E}_V \mid \mathcal{E}_V <Cs=T>$ context of library-evaluation

$LV ::= \{\text{interface } Tz; amt\} \mid \{Tz; MVs ; K\}$ literal value

$MV ::= C=LV \mid mt$

$\mathcal{E}_v ::= \square \mid \mathcal{E}_v.m(es) \mid v.m(vs \mathcal{E}_v es) \mid T.m(vs \mathcal{E}_v es)$

$DL ::= id=L$ partially-evaluated-declaration

$DV ::= id=LV$ evaluated-declaration

$Mid ::= C \mid m$ member-id

$p ::= DLs; DVs$ program

We use t and C to syntactically distinguish between trait and class names. An E is a top-level class expression, which can contain class-literals, references to traits, and operations on them, namely our sum $E <+ E$ and redirect $e(Cs = T)$. A declaration D is just an $id = E$, representing that id is declared to be the value of E , we also have CD, CV, DL , and DV that constrain the forms of the LHS and RHS of the declaration. A literal L has 4 components, an optional interface keyword, a list of implemented interfaces, a list of members, and an optional constructor. For simplicity, interfaces can only contain abstract-methods (amt) as members, and cannot have constructors. A member M , is either an (potentially abstract) method mt or a nested class declaration (CD). A member value MV , is a member that has been fully compiled. An mid is an identifier, identifying a member. Constructors, K , contain a Txs indicating the type and names of fields. An e is normal featherweight-java style expression, it has variables x , method calls $e.m(es)$, field accesses $e.x$ and object creation $newes$. $CtxV$ is the evalation context for class-expressions E , and $ctxv$ is the usuall one for e 's.

An S represents what the top-level source-code form of our language is, it's just a sequence of declarations and a main expression. The most interesting form of the grammer is a p , it is a 'program', used as the context for many reductions and typing rules, on the LHS of the ; is a stack representing which (nested) declaration is currently being processed, the bottom (rightmost) DL represents the D of the source-program that is currently being processed.

179 Th RHS of the ; represents the top-level declarations that have already been compiled, this
 180 is necessary to look up top-level classes and traits.

181 To look up the value of a type in the program we will use the notation $p(T)$, which is defined
 182 by the following, but only if the RHS denotes an LV :

$$(\text{; } _, C = L, _)(\text{This0}.C.Cs) := L(Cs)$$

$$183 \quad (id = L, p)(\text{This0}.Cs) := L(Cs)$$

$$184 \quad (id = L, p)(\text{This}n + 1.Cs) := p(\text{This}n.Cs)$$

185 To get the relative value of a trait, we define $p[t]$:

$$186 \quad (DLs; _, t = LV, _)[t] := LV[\text{This}\#DLs]$$

187

188 To get a the value of a literal, in a way that can be understand from the current location
 189 (This0) , we define:

$$190 \quad p[T] := p(T)[T]$$

191

192 And a few simple auxiliary definitions:

$$Ts \in p := \forall T \in Ts \bullet p(T) \text{ is defined}$$

$$L(\emptyset) := L$$

$$193 \quad L(C.Cs) := L(Cs) \text{ where } L = \text{interface? } \{_, _, C = L, _, _ \}$$

$$L[C = E'] := \text{interface? } \{Tz; MVs C = E' Ms; K?\}$$

$$194 \quad \text{where } L = \text{interface? } \{Tz; MVs C = _ Ms; K?\}$$

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195 We have two-top level reduction rules defining our language, of the form $Dse^{\sim\sim} > Ds'e$
196 which simply reduces the source-code. The first rule (*compile*) ‘compiles’ each top-level
197 declaration (using a well-typed subset of already compiled top-level declarations), this
198 reduces the defining expression. The second rule, (*main*) is executed once all the top-level
199 declarations have compiled (i.e. are now fully evaluated class literals), it typechecks the
200 top-level declarations and the main expression, and then proceeds to reduce it. In principle
201 only one-typechecking is needed, but we repeat it to avoid declaring more rules.

```
202 Define Ds e --> Ds' e'
203 =====
204 DVs' |- Ok
205 empty; DVs'; id | E --> E'
206 (compile)----- DVs' subsetof DVs
207 DVs id = E Ds e --> DVs id = E' Ds e
208
209 DVs |- Ok
210 DVs |- e : T
211 DVs |- e --> e'
212 (main)----- for some type T
213 DVs e --> DVs e'
```

214 3 Compilation

215 Aside from the redirect operation itself, compilation is the most interesting part, it is defined
216 by a reduction arrow $p; id | E \rightarrow E'$, the *id* represents the id of the type/trait that we
217 are currently compiling, it is needed since it will be the name of *This0*, and we use that fact
218 that that is equal to *This1.id* to compare types for equality. The (*CtxV*) rule is the standard
219 context, the (*L*) rule propagates compilation inside of nested-classes, (*trait*) merely evaluates
220 a trait reference to its defined body, (*sum*) and (*redirect*) perform our two meta-operations.

```
221 Define p; id | E --> E'
222 =====
223 p; id | E --> E'
224 (CtxV) -----
225 p; id | CtxV[E] --> CtxV[E']
226
227 id = L[C = E], p; C | E --> E'
228 (L) ----- // TODO use fresh C?
229 p; id | L[C = E] --> L[C = E']
230
231 (trait) -----
232 p; id | t -> p[t]
233
234 LV1 <+p' LV2 = LV3 p' = C' = LV3, p
235 (sum) ----- for fresh C'
236 p; id | LV1 <+ LV2 --> LV3
237
238 // TODO: Inline and de-42 redirect formalism
239 (redirect) -----LV'=redirect(p, LV, Cs, P)
240 p; id | LV(Cs=P) -> LV'
```

4 The Sum operation

The sum operation is defined by the rule $L1 < +p L2 = L3$, it is unconventional as it assumes we already have the result ($L3$), and simply checks that it is indeed correct. We believe (but have not proved) that this rule is unambiguous, if $L1 < +p L2 = L3$ and $L1 < +p L2 = L3'$, then $L3 = L3'$ (since the order of members does not matter for Ls).

The main rule for summing of non-interfaces, sums the members, unions the implemented interfaces (and uses *minimize* to remove any duplicates), it also ensures that at most one of them has a constructor. For summing an interface with a interface/class we require that an interface cannot 'gain' members due to a sum. The actual L42 implementation is far less restrictive, but requires complicated rules to ensure soundness, due to problems that could arise if a summed nested-interface is implemented. Summing of traits/classes with state is a non-trivial problem and not the focus of our paper, there are many prior works on this topic, and our full L42 language simply uses ordinary methods to represent state, however this would take too much effort to explain here.

```

Define L1 <+p L2 = L3
=====
{Tz1; Mz1; K?1} <+p {Tz2; Mz2; K?2} = {Tz; Mz; K?}
Tz = p.minimize(Tz1 U Tz2)
Mz1 <+p Mz1 = Mz
{empty, K?1, K?2} = {empty, K?} //may be too sophisticated?

interface{Tz1; amtz,amtz';} <+p interface?{Tz2;amtz;} = interface {Tz;amtz,amtz';}
Tz = p.minimize(Tz1 U Tz2)
if interface? = interface then amtz'=empty

```

The rules for summing members are simple, we take two sets of members collect all the ones with unique names, and sum those with duplicates. To sum nested classes we merely sum their bodies, to sum two methods we require their signatures to be identical, if they both have bodies, the result has the body of the RHS, otherwise the result has the body (if present) of the LHS.

```

Define Mz <+p Mz' = Mz"
-----
M, Mz <+p M', Mz' = M <+p M', Mz <+p Mz
//note: only defined when M.Mid = M'.Mid

Mz <+p Mz' = Mz, Mz':
dom(Mz) disjoint dom(Mz')

Define M <+p M' = M"
-----
T' m(Txs') e? <+p T m(Txs) e = T m(Txs) e
T', Txs'.Ts =p Ts, Txs

T' m(Txs') e? <+p T m(Txs) = T m(Txs) e?
T', Txs'.Ts =p Ts, Txs

(C = L) <+p (C = L') = L <+p.push(C) L'

```

287 **5** Type System

288 The type system is split into two parts: type checking programs and class literals, and the
 289 typechecking of expressions. The latter part is mostly conventional, it involves typing judgments
 290 of the form $p; Txs \vdash e : T$, with the usual program p and variable environment Txs (often
 291 called Γ in the literature). rule $(Dsok)$ type checks a sequence of top-level declarations by
 292 simply push each declaration onto a program and typecheck the resulting program. Rule pok
 293 typechecks a program by check the topmost class literal: we type check each of it's members
 294 (including all nested classes), check that it properly implements each interface it claims to,
 295 does something weird, and finanly check check that it's constructor only referenced existing
 296 types,

297

298

299 Define $p \vdash Ok$

300 =====

301

302 $D1; Ds \vdash Ok \dots Dn; Ds \vdash Ok$ 303 $(Ds \text{ ok}) \text{ ----- } Ds = D1 \dots Dn$ 304 $Ds \vdash Ok$

305

306 $p \vdash M1 : Ok \dots p \vdash Mn : Ok$ 307 $p \vdash P1 : Implemented \dots p \vdash Pn : Implemented$ 308 $p \vdash \text{implements}(Pz; Ms) \text{ /*WTF?*/} \quad \text{if } K? = K: p.\text{exists}(K.Txs.Ts)$ 309 $(p \text{ ok}) \text{ ----- } p.\text{top}() = \text{interface? } \{P1 \dots Pn; M1, \dots, Mn$ 310 $p \vdash Ok$

311

312 $p.\text{minimize}(Pz) \text{ subseq } p.\text{minimize}(p.\text{top}().Pz)$ 313 $\text{amt1 } _ \text{ in } p.\text{top}().Ms \dots \text{amtn } _ \text{ in } p.\text{top}().Ms$ 314 $(P \text{ implemented}) \text{ ----- } p[P] = \text{interface } \{Pz; \text{amt1 } \dots$ 315 $p \vdash P : Implemented$

316

317 $(\text{amt-ok}) \text{ ----- } p.\text{exists}(T, Txs.Ts)$ 318 $p \vdash T \text{ m}(Tcs) : Ok$

319

320 $p; \text{This0 this}, Txs \vdash e : T$ 321 $(\text{mt-ok}) \text{ ----- } p.\text{exists}(T, Txs.Ts)$ 322 $p \vdash T \text{ m}(Tcs) e : Ok$

323

324 $C = L, p \vdash Ok$ 325 $(\text{cd-Ok}) \text{ -----}$ 326 $p \vdash C = L : OK$

327

328 Rule $(Pimplemented)$ checks that an interface is properly implemented by the program-
 329 top, we simply check that it declares that it implements every one of the interfaces super-
 330 interfaces and methods. Rules $(amt - ok)$ and $(mt - ok)$ are straightforward, they both
 331 check that types mentioned in the method signature exist, and ofcourse for the latter case,
 332 that the body respects this signature.

333 To typecheck a nested class declaration, we simply push it onto the program and typecheck
 334 the top-of the program as before.

335 The expression typesystem is mostly straightforward and similar to feartherwiegth Java,
 336 notable we we use $p[T]$ to look up information about types, as it properly ‘from’s paths, and
 337 use a classes constructor definitions to determine the types of fields.

```

338 Define p; Txs |- e : T
339 =====
340 (var)
341 ----- T x in Txs
342 p; Txs |- x : T
343
344 (call)
345 p; Txs |- e0 : T0
346 ...
347 p; Txs |- en : Tn
348 ----- T' m(T1 x1 ... Tn xn) _ in p[T0].Ms
349 p; Txs |- e0.m(e1 ... en) : T'
350
351 (field)
352 p; Txs |- e : T
353 ----- p[T].K = constructor(_ T' x _)
354 p; Txs |- e.x : T'
355
356
357 (new)
358 p; Txs |- e1 : T1 ... p; Txs |- en : Tn
359 ----- p[T].K = constructor(T1 x1 ... Tn xn)
360 p; Txs |- new T(e1 ... en)
361
362
363 (sub)
364 p; Txs |- e : T
365 ----- T' in p[T].Pz
366 p; Txs |- e : T'
367
368
369 (equiv)
370 p; Txs |- e : T
371 ----- T =p T'
372 p; Txs |- e : T'
373
374 - towel1:.. //Map: towel2:.. //Map: lib: T:towel1 f1 ... fn
375   MyProgram: T:towel2 Lib:lib[T=This0.T] ... -

```

375 6 extra

376 Features: Structural based generics embedded in a nominal type system. Code is Nominal,
 377 Reuse is Structural. Static methods support for generics, so generics are not just a trik to

378 make the type system happy but actually change the behaviour Subsume associate types.
 379 After the fact generics; redirect is like mixins for generics Mapping is inferred-> very large
 380 maps are possible -> application to libraries

381 In literature, in addition to conventional Java style F-bound polymorphism, there is
 382 another way to obtain generics: to use associated types (to specify generic paramaters) and
 383 inheritance (to instantiate the paramaters). However, when parametrizing multiple types,
 384 the user to specify the full mapping. For example in Java interface A B m(); inteface
 385 BString f(); class G<TA extends A<TB>, TB>//TA and TB explicitly listed String g(TA
 386 a TB b)return a.m().f(); class MyA implements A<MyB>.. class MyB implements B ..
 387 G<MyA,MyB>//instantiation Also scala offers genercs, and could encode the example in
 388 the same way, but Scala also offers associated types, allowing to write instead....

389 Rust also offers generics and associated types, but also support calling static methods
 390 over generic and associated types.

391 We provide here a fundational model for genericity that subsume the power of F-bound
 392 polimorphisms and associated types. Moreover, it allows for large sets of generic parameter
 393 instantiations to be inferred starting from a much smaller mapping. For example, in our
 394 system we could just write g= A= method B m() B= method String f() method String g(A a
 395 B b)=a.m().f() MyA= method MyB m()= new MyB(); .. MyB= method String f()="Hello";
 396 .. g<A=MyA>//instantiation. The mapping A=MyA,B=MyB

397 We model a minimal calculus with interfaces and final classes, where implementing an
 398 interface is the only way to induce subtyping. We will show how supporting subtyping
 399 constitute the core technical difficulty in our work, inducing ambiguity in the mappings.
 400 As you can see, we base our generic matches the structor of the type instead of respect-
 401 ing a subtype requirement as in F-bound polymorphis. We can easily encode subtype
 402 requirements by using implements: Print=interface method String print(); g= A:implements
 403 Print method A printMe(A a1,A a2) if(a1.print().size())>a2.print.size())return a1; return a2;
 404 MyPrint=implements Print .. g<A=MyPrint> //instantiation g<A=Print> //works too

405 ————— example showing ordering need to strictly improve EI1: interface EA1: imple-
 406 ments EI1

407 EI2: interface EA2: implements EI2

408 EB: EA1 a1 EA1 a1

409 A1: A2: B: A1 a1 A2 a2 [B = EB] // A1 -> EI1, A2 -> EA2 a // A1 -> EA1, A2 ->
 410 EI2 b // A1 -> EA1, A2 -> EA2 c

411 a <=b b <=a c<= a,b a <= c

412 hi **Hi class**

a ::= b c

413 aahi**Hi class**qaq a ::= b c

a ::= b c

414 }][()]
 (TOP)

a → c ∀i < 3a ⊢ b : OK

415
$$\frac{\frac{a \rightarrow c \quad \forall i < 3a \vdash b : OK}{b} \quad a}{1 + 2 \rightarrow 3} \quad c$$