

Metaprogramming

Statically verified code

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Can metaprogramming generate statically verified code reusing common verification techniques?

Example of verification (Spec#)

```
Nat pow(Nat x, Nat y) ensures result = x^y; {  
  if (y==0) { return 1; }  
  return x*pow(x, y-1); }
```

- OO languages supporting static verification usually extends method declaration with syntax supporting pre/post conditions. During compilation, directly after typing, an automatic theorem prover checks the constraints. Very slow even on fast hardware!
- `pow(x,y)` execution is slower than an hand written `pow3(x)`

```
Nat pow3(Nat x) ensures result = x^3; { return x*x*x; }
```

- Manually declaring `pown` methods: repetitive, boring
- Running the static verifier on all `pown` versions: time consuming

Metaprogramming and verification

```
Nat pow3(Nat x) ensures result = x^3; {return x*x*x;}
```

What if we could programmatically generate code at compile time?
For example, we could write `MakePow.ofExp(y)` as a method generating a class body with a method `pow(x)` computing `pow(x, y)`

```
Pow3=MakePow.ofExp(3)  
Pow3.pow(4) == pow3(4)
```

- There is very little literature on generating statically verified code with metaprogramming. Metaprogramming usually relies on quasi quotation, an AST based technique that do not consider contracts

Iteratively building up pow

- Verification supports code reuse / class inheritance.
What if metaprogramming was based on code reuse?
For example, here we use inheritance to encode statically verified pows while reusing part of the already verified contract
- A code optimizer could inline nested calls, producing the same efficient code of the hand written versions of before

```
class Pow0{
  Nat pow0(Nat x) ensures result=x^0;{return 1;}
}
class Pow1 : Pow0{
  Nat pow1(Nat x) ensures result=x^1;{return x*pow0(x);}
}
class Pow2 : Pow1{
  Nat pow2(Nat x) ensures result=x^2;{return x*pow1(x);}
}
class Pow3 : Pow2{
  Nat pow3(Nat x) ensures result=x^3;{return x*pow2(x);}
}
```

Metaprogramming

class bodies as first class values

code reuse operators

can be used to iteratively compose arbitrary behavior

- In the following, we rely on a language where class bodies (unnamed classes with methods) are first class values, and:
 - `code1 + code2` composes code similar to inheritance;
 - `code[rename a->b]` renames methods;
 - `code[hide a]` renames methods to private names.

Sum: code1 + code2 = code3

```
class{  
  abstract String hello();  
  String helloWorld(){return hello()+" World";}  
}  
+  
class{String hello(){return "Hi";} }  
=  
class{  
  String hello(){return "Hi";}  
  String helloWorld(){return hello()+" World";}  
}
```

- The result of the sum contains the methods of both arguments.
- If a method is present in both arguments, they need to have the same type signature, and at least one of the two needs to be abstract.

Sum and verification

```
class{
  abstract String hello() ensures result.size()>1;
  String helloWorld(){return hello()+" World";}
}
+
class{String hello() ensures result.size()>1;{return "Hi";}}
=
class{
  String hello() ensures result.size()>1;{return "Hi";}
  String helloWorld(){return hello()+" World";}
}
```

- Contracts are matched: **abstract** $c1 + c1 \& c2 = c1 \& c2$
- This example would produce an error if the implemented contract did not contained '**result.size()>1**'
- Contract matching is syntactical: '**result.size()==2**'
would not work, it would need to be in the (equivalent) form
'**result.size()>1 & result.size()==2**'

Example revised

```
Pow0 = class{  
  Nat pow(Nat x) ensures result=x^0;{return 1;}  
}  
Pow1 = (Pow0[rename pow->_pow] + class{  
  abstract Nat _pow(Nat x) ensures result=x^0;  
  Nat pow(Nat x)ensures result=x^1;{return x*_pow(x);}  
})[hide _pow]  
Pow2 = (Pow1[rename pow->_pow] + class{  
  abstract Nat _pow(Nat x) ensures result=x^1;  
  Nat pow(Nat x) ensures result=x^2;{return x*_pow(x);}  
})[hide _pow]  
//and so on
```

- This code is more explicit then before: **abstract** _pow!
- + composes the **abstract** _pow with the concrete _pow
- The exponents are still **hard coded** in the contracts!!

Example revised

```
Pow0 = class{
  Nat exp() ensures result=0;{return 0;}
  Nat pow(Nat x) ensures result=x^exp();{return 1;}
}

Pow1 = (Pow0[rename pow->_pow, exp->_exp] + class{
  abstract Nat _exp()/*empty contract */;
  abstract Nat _pow(Nat x)ensures result=x^_exp();
  Nat exp() ensures result=1+_exp();{return 1+_exp();}
  Nat pow(Nat x)ensures result=x^exp();{return x*_pow(x);}
})[hide _pow, _exp]

Pow2 = (Pow1[rename pow->_pow, exp->_exp] + class{
  abstract Nat _exp()/*empty contract */;
  abstract Nat _pow(Nat x)ensures result=x^_exp();
  Nat exp() ensures result=1+_exp();{return 1+_exp();}
  Nat pow(Nat x)ensures result=x^exp();{return x*_pow(x);}
})[hide _pow, _exp]
```

- Note the only difference between Pow2 and Pow1
- We can use a loop to get metaprogramming!

Example Metaprogramming

```
MakePow = class{
  static Code ofExp(Nat y) {
    ClassLit base=class{
      Nat exp() ensures result=0; {return 0;}
      Nat pow(Nat x) ensures result=x^exp(); {return 1;}};
    ClassLit inductive=class{
      abstract Nat _exp() /*empty contract */;
      abstract Nat _pow(Nat x) ensures result=x^_exp();
      Nat exp() ensures result=1+_exp(); {return 1+_exp();}
      Nat pow(Nat x) ensures result=x^exp(); {return x*_pow(x);}};
    ClassLit res=base;
    for(Nat i in 0..y){
      res=res[rename pow->_pow, exp->_exp];
      res=res+inductive;
      res=res[hide _pow, _exp];
    }
    return res;
  }
}

...
Pow7 = MakePow.ofExp(7); //example usage
```

Example Metaprogramming

- How does static verification flow? what do we know and when do we know it?

```
static Code ofExp(Nat y) {  
  Code base={..};//SV during compilation of MakePow.ofExp  
  Code inductive={..};//SV during compilation of MakePow.ofExp  
  Code res=base;//SV since base is SV  
  for (Nat i in 1..y) {  
    res=res[rename pow->_pow, exp->_exp];  
    //res is still SV since contracts are resilient to renaming  
    res=res+inductive;  
    //+ sums contracts in a way that preserves SV  
    res=res[hide _pow, _exp];  
    //res is still SV since contracts are resilient to hiding  
  }  
  return res;//res is still SV  
  //However, statically, we do not know what its contracts are  
}
```

Example Metaprogramming

```
MakePow = class{  
  static Code ofExp(Nat y)  
  ensuresRV result.exp.ensures equivTo (result = y)  
    & result.pow.ensures = (result = x^exp());  
  { .. }  
}  
...  
Pow7 = MakePow.ofExp(7); //mini verification here for equivTo
```

- Code composition ensure contract matching, but the resulting contract is unknown
- Thus, the contract for MakePow.ofExp(y) can only be checked at run-time. The contract of pow(x) is checked to be identical to $x^{\text{exp}()}$; but checking exp() is harder: @result.exp().ensures **equivTo** (@result = y)
- EquivTo checks that the contract $1 + _ \text{exp}\$342()$ is equivalent to y, this needs the contract of private method $_ \text{exp}\$342()$

Concluding

- Iterative composition is a disciplined metaprogramming technique that reasons at class level
- It allows holes in the behavior using abstract methods
- The static verifier runs only once on each piece of source code
- Metaprogramming reuses already SV code, which does not need to be SV again
- Only few and cheap checks are performed when code is composed

Example of metaprogramming: QQ

```
fun powerAux(n : Nat) : Expr< x:Nat |- Nat > =  
  if (n = 0) then [| 1 |] else [| x * $( powerAux (n -1) ) |];  
  
fun powerGen(n : Nat) : Nat -> Nat =  
  compile ( [| λ x . $( powerAux (n) ) |] );  
  
power7 = powerGen 7;  
power7 = λ x . x*x*x*x*x*x*x*1 //equivalent!
```

- Quasi Quotations are expressions not first class values.
- The Value of type **Expr** have no holes any more.
- Holes (denote by **\$(e)**) do not have a name.
- There is no obvious way to map verification on QQ.