455 midterm review

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Chapter 1:

- Law of total prob. $P(A) = \sum P(A|B_i)P(B_i)$ $\quad \Omega = \cup B_i$
- conditional distribution. $\begin{cases} P(Y=y|X=x) = \dfrac{P(X=x, Y=y)}{P(X=x)} \\ f_{Y|X}(y|x) = \dfrac{f(x,y)}{\int f(x,y)dy} \end{cases}$
- conditional expectation $\begin{cases} E(Y|X=x) = \sum y P(Y=y|X=x) \\ E(Y|X=x) = \int y f_{Y|X}(y|x)dy \end{cases}$
- properties of cond. exp. $E(g(Y)|X=x) = \begin{cases} \sum g(y)P(Y=y|X=x) \\ \int g(y) f_{Y|X}(y|x)dy \end{cases}$
- Law of total expectation $E(Y) = \sum E(Y|A_i)P(A_i)$

$$E(Y) = \sum_x E(Y|X=x)P(X=x)$$

$$E(Y) = E(E(Y|X))$$

- conditional variance $Var(Y|X=x) = E((Y-\mu_x)^2|X=x)$
- Law of total variance. $Var(Y) = E(Var(Y|X)) + Var(E(Y|X))$

Chapter 2

- Markov Chain: $P(X_{n+1}=j|X_0=i_0, X_1=i_1, \dots, X_{n-1}=i_{n-1}, X_n=i) = P(X_{n+1}=j|X_n=i)$

$$= P(X_1=j|X_0=i)$$

- stochastic matrix. $\mathbb{P} = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1k} \\ \vdots & & & \\ P_{1k} & \cdots & & P_{kk} \end{pmatrix}$ $\quad \sum_{j=1}^{k} P_{ij} = 1$
- graphs (weighted, directed)

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- n step transition matrix / initial dist. α | dist of X_n : $\alpha \mathbb{P}^n$ / Joint dist.
  $\mathbb{P}^n$

Chapter 3 long term

- Limiting dist. $\lim_{n\to\infty} \mathbb{P}^n_{ij} = \lambda_j$ for all i. $\Leftrightarrow$ ① For any α. $\lim \alpha \mathbb{P}^n = \lambda$
  ②
  ③

- Interpretation of λ_j : long term proportion of time that the chain visits j.

- How to find limiting dist ? check $\mathbb{P}^n$ for large n.

- Stationary dist. $\pi = \pi \mathbb{P}$

- If the initial dist is π, then the chain stays in the dist all the time.

$$\begin{array}{ccccccc} X_1 & \xrightarrow{\mathbb{P}} & X_2 & \xrightarrow{\mathbb{P}} & X_3 & \cdots \xrightarrow{\mathbb{P}} & X_n \to \cdots \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \pi & & \pi & & \pi & & \pi \end{array}$$

- how to find π ?

→ • limiting dist are stationary dist ? why ?
  stationary dist are limiting dist ? why ?

- regular MC Thm.

- stationary dist for graphs.

- communication class / Irreducible MC / recurrent & transient states. (class property)

$$\begin{array}{cc} \text{recurrent} & \text{transient} \\ \downarrow & \downarrow \\ f_j = P(T_j < \infty \mid X_0 = j) = 1 & f_j < 1 \\ \sum \mathbb{P}^n_{jj} = \infty & \sum \mathbb{P}^n_{jj} < \infty \end{array}$$

- How to check recurrent & transient ?

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- Canonical decomposition

$$
\underline{\mathbb{P}} = \begin{array}{c c} & \begin{array}{ccccc} T & R_1 & R_2 & \cdots & R_m \end{array} \\ \begin{array}{c} T \\ R_1 \\ R_2 \\ \vdots \\ R_m \end{array} & \begin{pmatrix} * & * & * & \cdots & * \\ & \mathbb{P}_1 & & & 0 \\ & & \mathbb{P}_2 & & \\ & 0 & & \ddots & \\ & & & & \mathbb{P}_m \end{pmatrix} \end{array}
$$

- Finite irreducible MC theorem.

stationary $\pi_j = \dfrac{1}{\mu_j}$. $\mu_j = E(T_j \mid X_0 = j)$

$$\pi_j = \lim_{n \to \infty} \frac{1}{n} \sum_{0}^{n-1} \mathbb{P}_{jj}^{n}$$

- First step analysis to find μ_j $\longrightarrow$ $\boxed{\text{correction on example 3.17}}$

- periodicity. $d(i) = \gcd\{ n > 0 : \mathbb{P}_{jj}^{n} > 0 \}$

  $\downarrow$

  class property

  periodic / aperiodic $\longrightarrow$ ergodic. (irreducible, aperiodic, $\mu_j < \infty$)

- Ergodic MC Thm. $\pi_j = \lim \underline{\mathbb{P}}_{ij}^{n}$ for all i, j.

- Absorbing chain.

$$\underline{\mathbb{P}} = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix} \qquad \mathbb{P}^n \longrightarrow \begin{pmatrix} 0 & (I-Q)^{-1}R \\ 0 & I \end{pmatrix}$$