On the Turing Completeness of Typed Feature Structure Unification

Guy Emerson

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- Proof: any Turing Machine can be simulated as unification in a Delph-in type system

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- Proof: any Turing Machine can be simulated as unification in a Delph-in type system
  - … using only two features
- Demo: untyped lambda calculus (Turing-complete)
- Recipe: how to add "relational constraints" to a Delph-in grammar
- Recipe (experimental): how to add *nondeterministic* "relational constraints" to a Delph-in grammar

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- Previous work demonstrated Turing completeness of unification
- BUT: Delph-in formalism more restricted
  - Copestake (2002): "the type inference system is essentially non-recursive"
- Different kind of recursion from previous work!

Didn’t you present this at the 2019 summit?

- More precisely, how Turing completeness comes in

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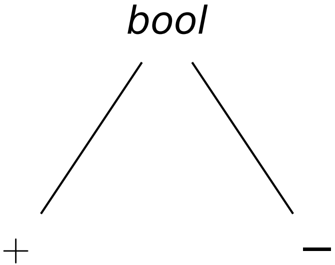
- More precisely, how Turing completeness comes in
- How to be more systematic (more user-friendly?)

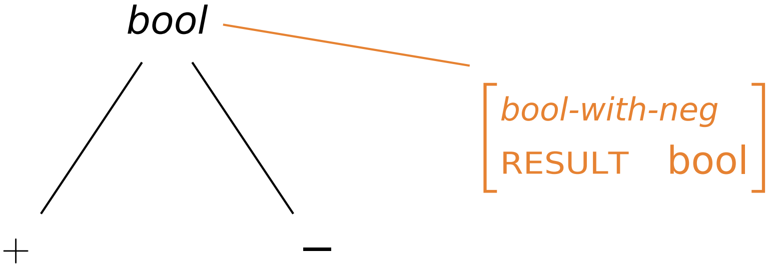
Path to Turing Completeness

- Computation types
- Recursive computation types
- Turing-complete computation types

Computation Types



Computation Types



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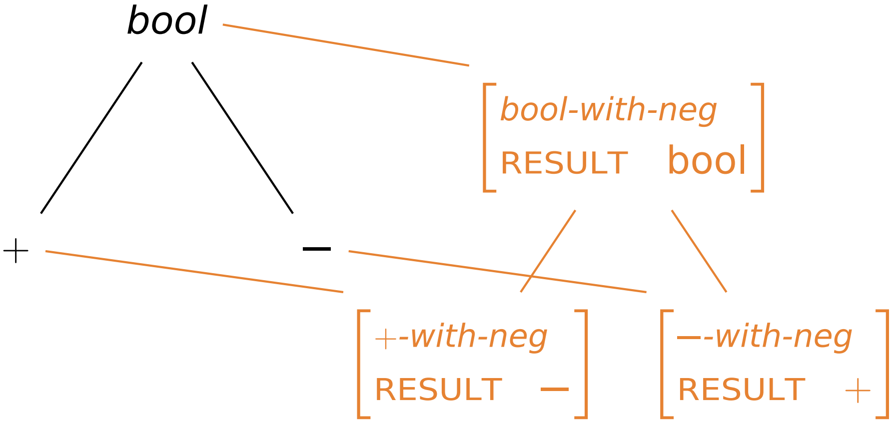


Computation Types

- Data type (e.g. *bool*)
- Computation type (e.g. *bool-with-neg*)
  - Subtype of data type
  - Additional feature (e.g. RESULT)
- Common subtypes define computation
  (e.g. $+$-*with-neg*, $-$-*with-neg*)

Recursive Computation Types

- Recursive data type (e.g. *list*)
- Recursive computation type (e.g. *list-with-diff-list*)
  - Subtype of data type
  - Additional feature (e.g. RESULT)
- Common subtypes define computation (e.g. *cons-with-diff-list*, *null-with-diff-list*)

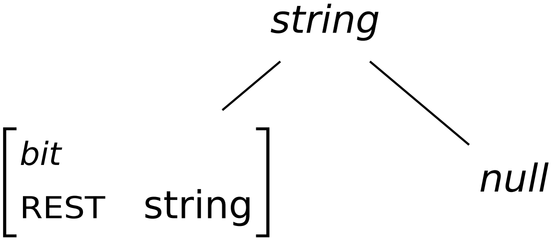
Recursive Data Types

list

$$\begin{bmatrix} \textit{cons} & \\ \text{FIRST} & \text{*top*} \\ \text{REST} & \text{list} \end{bmatrix}$$

null

Recursive Data Types



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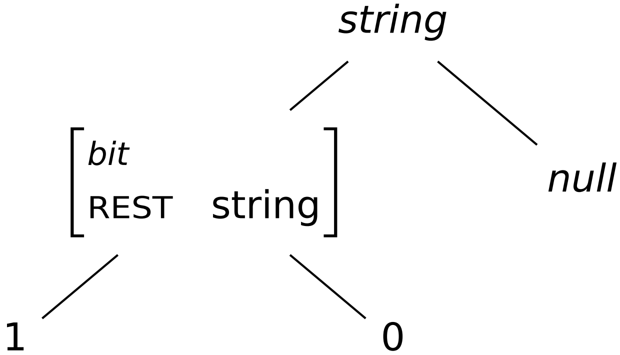
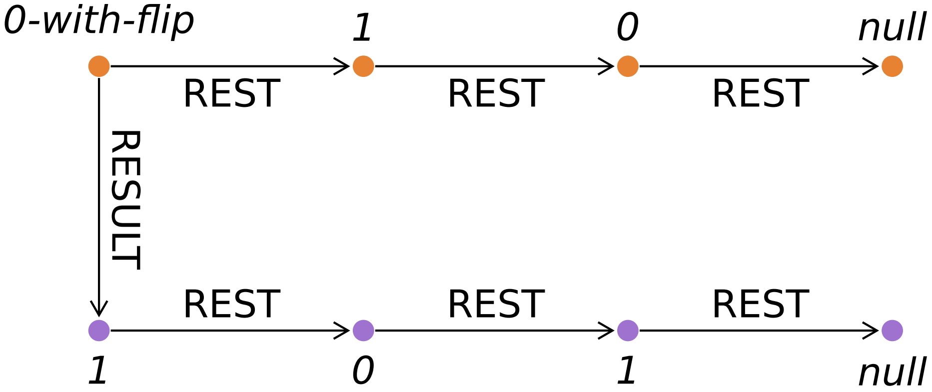


Recursive Data Types

Recursive Computation Types



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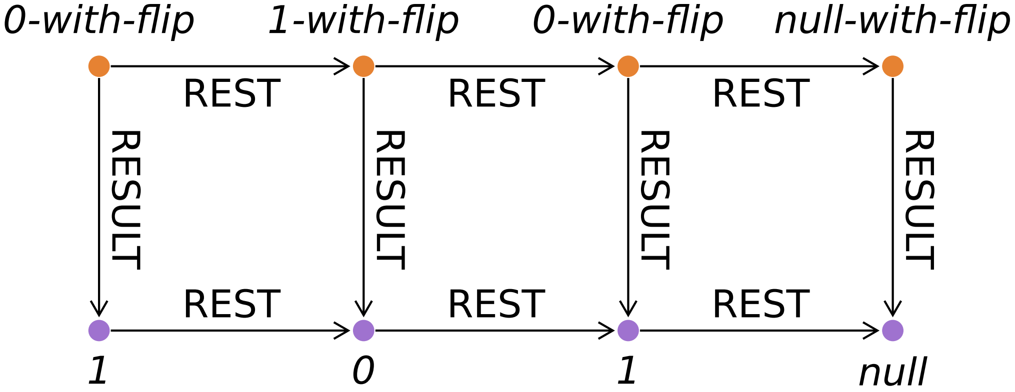

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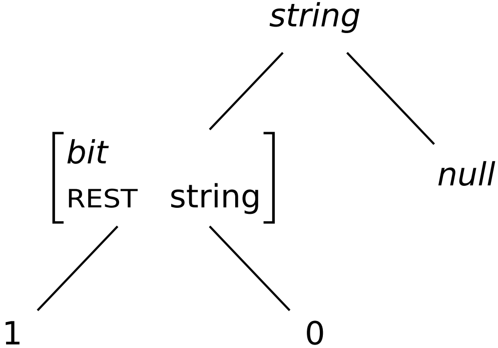

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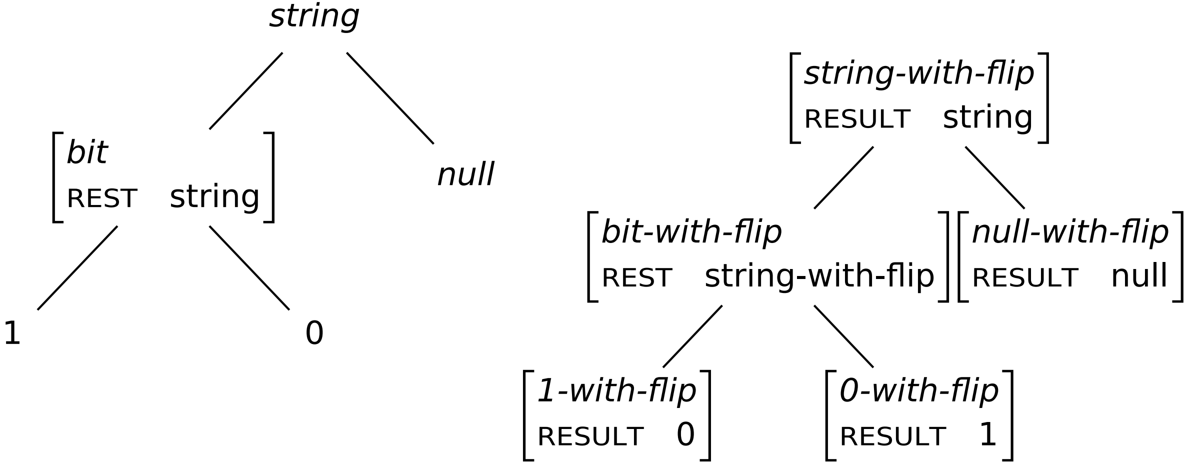

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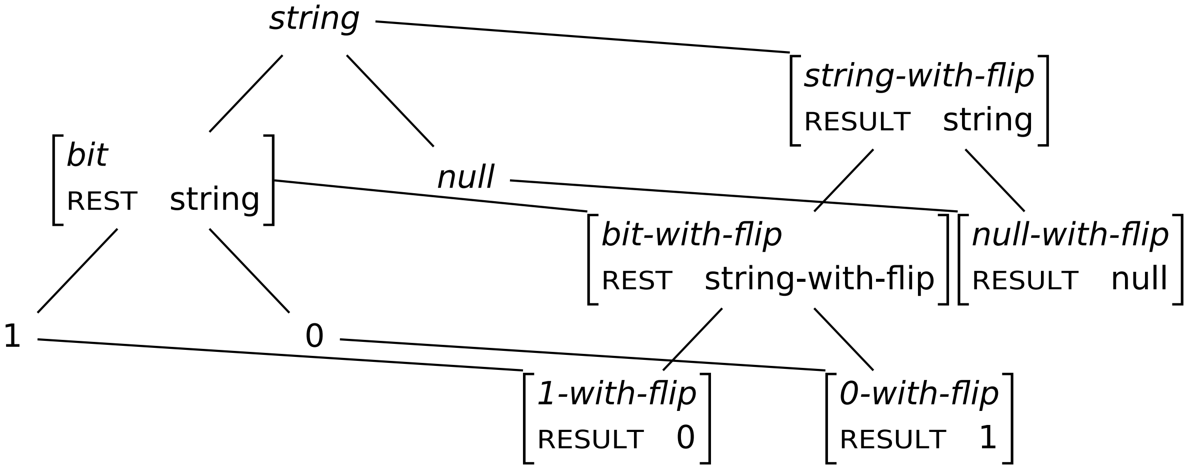

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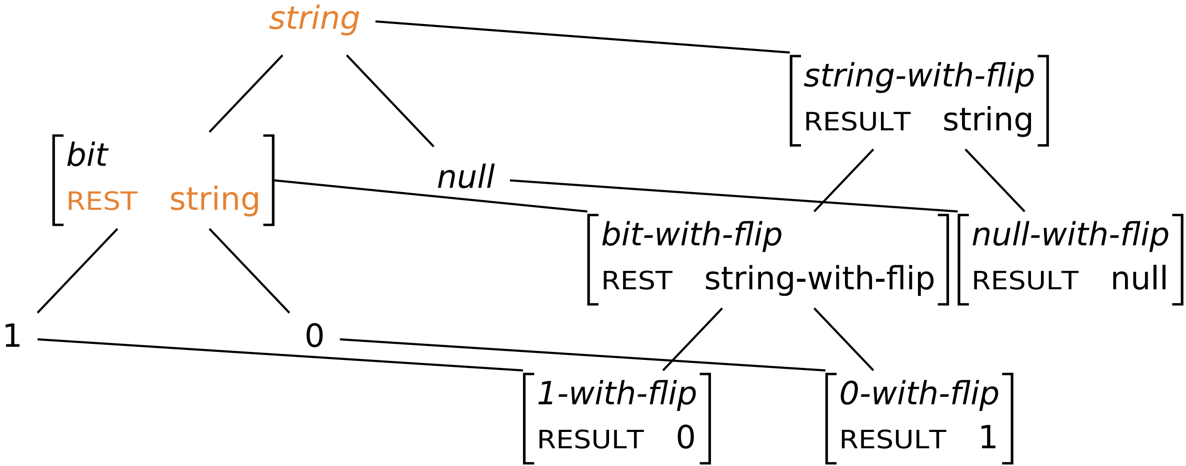

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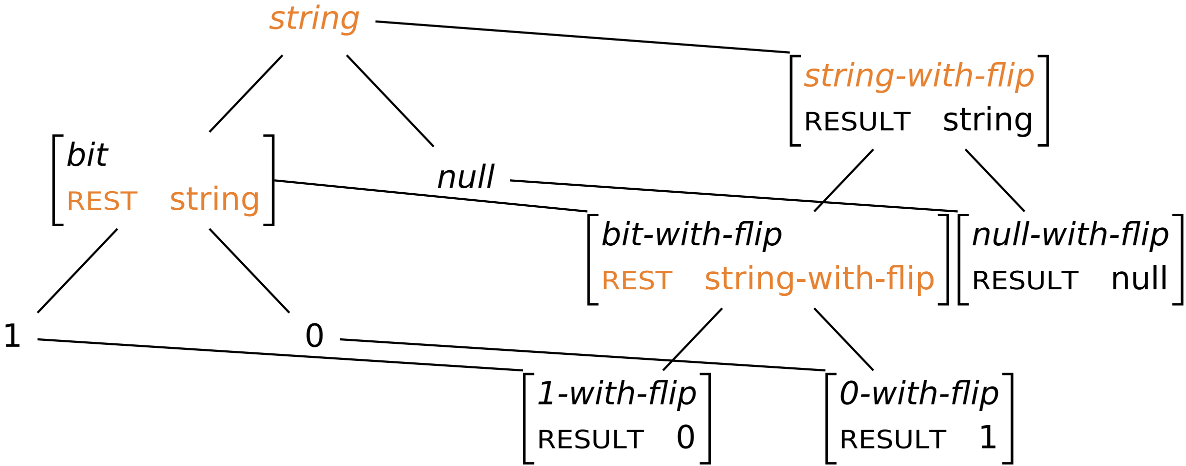

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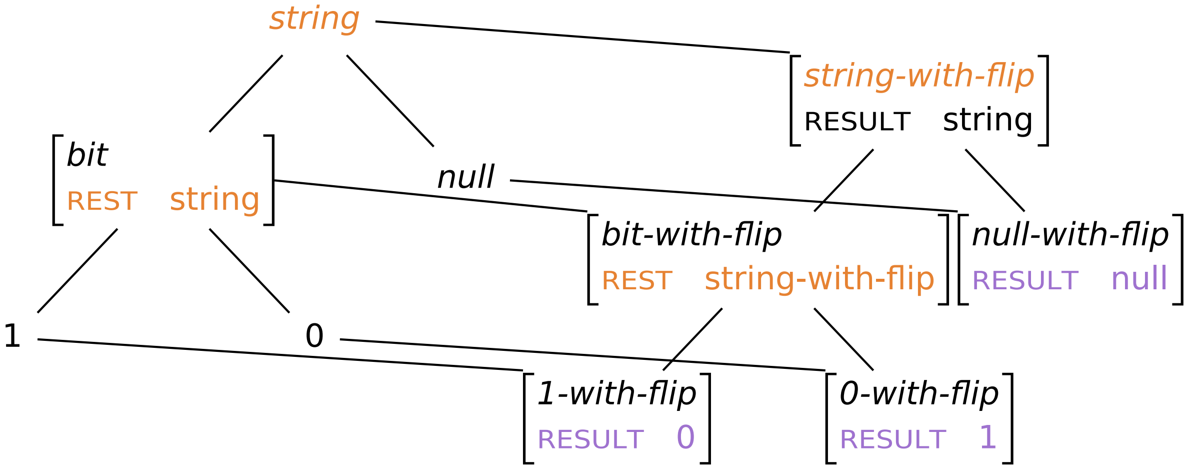

Unification can create paths

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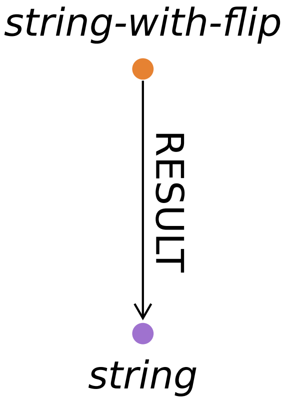

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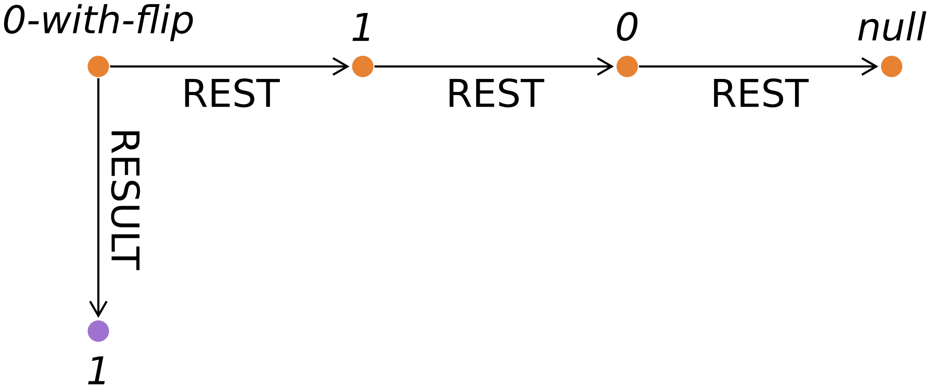

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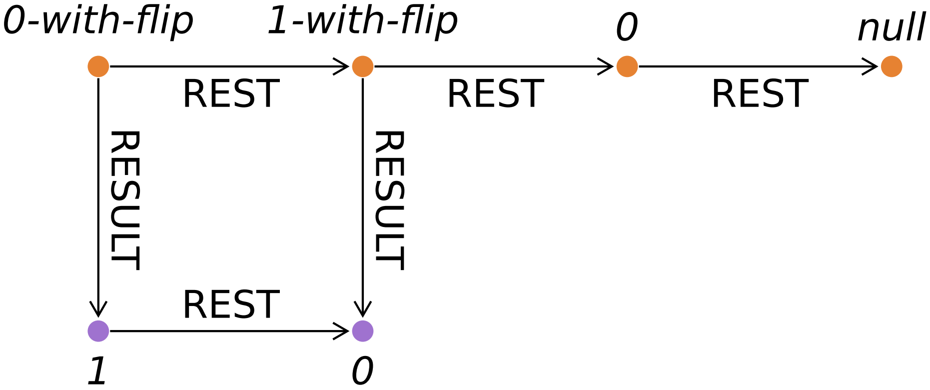

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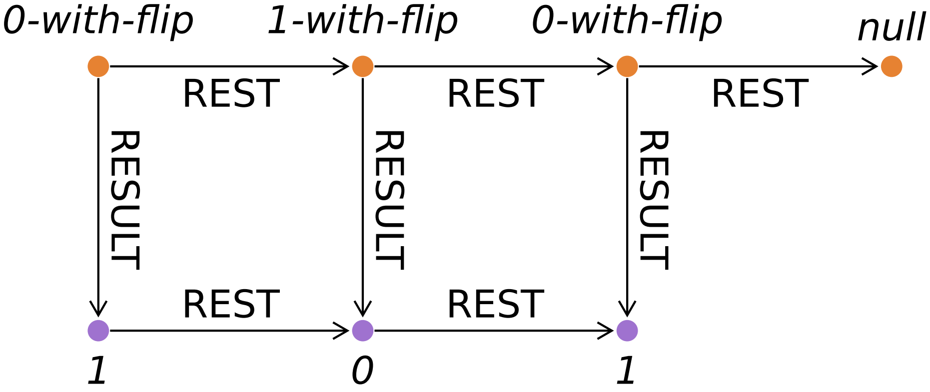

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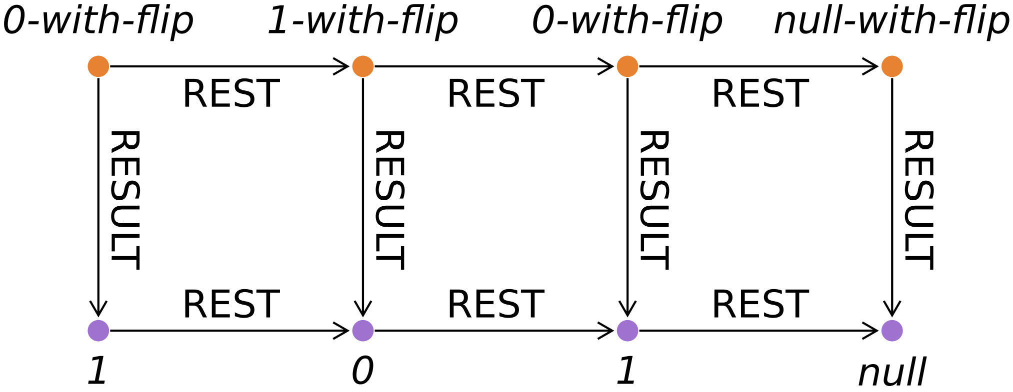
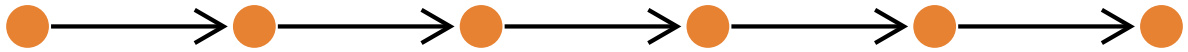


Doubly recursive computation



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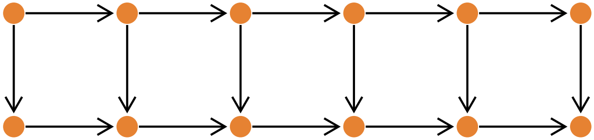

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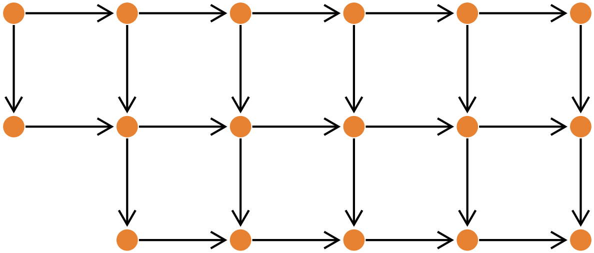

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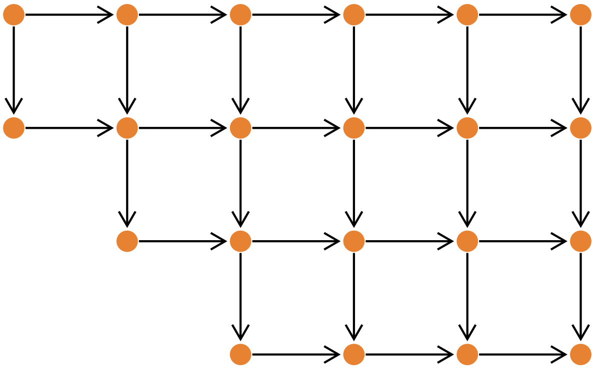

Turing-complete computation

- Recursion with one feature: finite state machine
- Recursion with two features: Turing machine

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- Recursion with one feature: finite state machine
- Recursion with two features: Turing machine
  - One feature for the tape
  - One feature for transitions

TDL Turing Machine

$$\begin{bmatrix} \textit{turing-machine} & \\ \text{TAPE-LEFT} & \text{string} \\ \text{TAPE-RIGHT} & \text{string} \end{bmatrix}$$

TDL Turing Machine

$$\begin{bmatrix} \textit{turing-machine} & \\ \text{TAPE-LEFT} & \text{string} \\ \text{TAPE-RIGHT} & \text{string} \end{bmatrix} \quad \begin{bmatrix} \textit{turing-machine-with-final-state} & \\ \text{TAPE-LEFT} & \text{string} \\ \text{TAPE-RIGHT} & \text{string} \\ \text{NEXT-RESULT} & \text{turing-machine} \\ \text{FINAL-RESULT} & \text{turing-machine} \end{bmatrix}$$

TDL Turing Machine

- Need to define:
  - Push and pop operations for tape
  - Transitions (combination of state and symbol)
  - Propagation of final state

TDL Turing Machine

- Need to define:
  - Push and pop operations for tape
  - Transitions (combination of state and symbol)
  - Propagation of final state
- Turing-complete unification:
  - Data type (initial state and tape)
  - Computation type (with final state and tape)

Two-feature TDL Turing Machine

- Combine features:
  - Interleave left tape and right tape
(push and pop operations are a little fiddly...)
  - Use RESULT for both next state and final state
(need to split the machine into two nodes)

Demo time!

Grammar Engineering

- Make it easy to understand code

- Make it easy to invoke computation types

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  - Clear naming conventions
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- Make it easy to understand code
  - Clear naming conventions
- Make it easy to invoke computation types
  - Wrapper types

Naming conventions

- ```
  with-computation := *top* &
    [ RESULT *top* ].
  ```
  ```

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- ```
  with-computation := *top* &
     [ RESULT *top* ].
  ```

- ```
  input-with-func := input & with-computation &
     [ RESULT output ].
  ```

Unary function from `input` to `output`

- Define new types, matching `input` hierarchy
- Inherit from `with-computation`
- Specify output

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- Define new types, matching `input` hierarchy
- Inherit from `with-computation`
- Specify output
- (Pay attention to recursion, if present!)

Binary functions: Currying

- Curry binary function into pair of unary functions

```
input1-with-f1 := input1 & with-computation &
  [ RESULT input2-with-f2 ].

input2-with-f2 := input2 & with-computation &
  [ RESULT output ].
```

Binary functions: Currying

- Curry binary function into pair of unary functions

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input1-with-f1 := input1 & with-computation &
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input2-with-f2 := input2 & with-computation &
  [ RESULT output ].
```

- Will end up defining a type for each combination of input types (i.e. Cartesian product)

Binary functions: Currying

- Example in grammar: boolean logic
- Currying done once, to map to `bool-pair`
- Operations (and, or) defined on `bool-pair`

Wrapper Types

- A computation type can be defined once, and used in many rules

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- A computation type can be defined once, and used in many rules
- Wrapper types: a little boilerplate once, but more intuitive in each use

Wrapper Types

- $\begin{bmatrix} \textit{bool-with-neg} & \\ \text{RESULT} & \text{bool} \end{bmatrix} \begin{bmatrix} +\textit{-with-neg} & \\ \text{RESULT} & - \end{bmatrix} \begin{bmatrix} -\textit{-with-neg} & \\ \text{RESULT} & + \end{bmatrix}$

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- $\begin{bmatrix} \textit{bool-wrapper} & \\ \text{BOOL} & \text{bool} \end{bmatrix}$

Wrapper Types

- $\begin{bmatrix} \textit{bool-with-neg} & \\ \text{RESULT} & \text{bool} \end{bmatrix} \begin{bmatrix} \textit{+-with-neg} & \\ \text{RESULT} & - \end{bmatrix} \begin{bmatrix} \textit{−-with-neg} & \\ \text{RESULT} & + \end{bmatrix}$

- $\begin{bmatrix} \textit{neg-bool} & \\ \text{BOOL} & \boxed{1}\ \text{bool} \\ \text{NEG} & \begin{bmatrix} \text{BOOL} & \begin{bmatrix} \textit{bool-with-neg} \\ \text{RESULT}\ \boxed{1} \end{bmatrix} \end{bmatrix} \end{bmatrix}$

Wrapper Types

- Define a wrapper type for each data type, with a unique feature (e.g. BOOL)
- Define a subtype for each computation type, with a unique feature (e.g. NEG)

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- Define a wrapper type for each data type, with a unique feature (e.g. BOOL)
- Define a subtype for each computation type, with a unique feature (e.g. NEG)
- “Call” a type using its feature

Wrapper Types

```
...
FEAT.AND < #1, #2 >,
DTRS < [ FEAT #1 ], [ FEAT #2 ] >
...
```

Relational Constraints

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Relational Constraints

- Any deterministic relational constraint can be expressed as a computation type
- What about nondeterministic constraints?
(e.g. popping an arbitrary element from a list)
  - Delph-in unification is deterministic
  - Parsing is nondeterministic
(multiple parses for the same input)

Nondeterministic Constraints

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- Use unary rules to perform computation

Nondeterministic Constraints

- Idea: one edge per output
- Use unary rules to perform computation
- Need a feature to keep track of computation (fiddly recursion)
- Need to encapsulate from rest of grammar (fiddly protection for all other rules)

Example: head-comp

```
basic-head-1st-comp-phrase := basic-head-comp-phrase &
  [ SYNSEM.LOCAL.CAT.VAL.COMPS #comps,
    HEAD-DTR.SYNSEM.LOCAL.CAT.VAL.COMPS < #synsem . #comps >,
    NON-HEAD-DTR.SYNSEM #synsem ].

basic-head-2nd-comp-phrase := basic-head-comp-phrase &
  [ SYNSEM.LOCAL.CAT.VAL.COMPS < #firstcomp . #othercomps >,
    HEAD-DTR.SYNSEM.LOCAL.CAT.VAL.COMPS [ FIRST #firstcomp,
                                          REST < #synsem . #othercomps > ],
    NON-HEAD-DTR.SYNSEM #synsem ].
```

Example: head-comp

```
basic-head-any-comp-phrase := basic-head-comp-phrase &
  [ SYNSEM.LOCAL.CAT.VAL.COMPS #new-comps,
    HEAD-DTR.SYNSEM.LOCAL.CAT.VAL.COMPS #old-comps,
    NON-HEAD-DTR.SYNSEM #synsem,
    NONDETERMINISTIC [ POP-INPUT #old-comps,
                       POP-OUTPUT-LIST #new-comps,
                       POP-OUTPUT-ITEM #synsem ] ].
```

Summary

- Delph-in unification is Turing-complete
  (with two recursive features, lots of re-entrancies)
- Computation types allow relational constraints
- Wrapper types allow readable code
- Unary rules allow non-deterministic constraints