A reduced electron model for testing plasma dynamics for closed field lines

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1. Introduction

Calculation of the electrostatic potential is a challenge within the drift kinetic approximation. This is because, without further manipulation of the drift kinetic system of equations, there is no explicit equation to be solved for the potential: Instead, it appears as a parameter in the drift kinetic equations for both electrons and ions, which are then related to one another via quasineutrality. In report 2047357-TN-09-01 [1], we showed how the electrostatic potential could be calculated for a 2D drift kinetic model with a helical magnetic field and with periodic boundary conditions appropriate for closed field lines. Considerable care had to be taken to obtain an expression for the electrostatic potential. In particular, we showed that one has to retain higher order terms in the drift kinetic expansion parameter and thus solve a mixed, integral and partial differential equation for the potential at the domain's parallel boundary.

Thus far we have side-stepped the issue of calculating the potential in our ProxyApps by using a Boltzmann response for the electron dynamics so that no electron equations need to be solved at all. In this report, we outline a collisional model for electron dynamics. The purpose of this model is not to be physically accurate, but to bridge the complexity between the fully kinetic treatment and the relatively crude electron model employed thus far. Importantly, the collisional model for electrons will require the calculation of the potential as laid out in the aforementioned report, but will do so within a fluid framework that should be easier to test numerically.

To begin we will provide a brief overview of the physical system we aim to model, before presenting an overview of the recipe for obtaining the potential derived in earlier reports. We will then go on to provide a collisional model for electron dynamics, which follows from the original derivation of Braginskii [2].

2. Magnetic geometry and orderings

We consider a plasma consisting of a single ion species of charge e and mass m_i and electrons with charge $-e$ and mass m_e. Because we are focusing on the closed-field-line region of the plasma where the neutral density is low, we neglect neutrals here. The plasma is immersed in a helical magnetic field of the form

$$\mathbf{B}(r,\zeta) = B_z(r)\hat{\mathbf{z}} + B_\zeta(r)\hat{\boldsymbol{\zeta}}(\zeta), \tag{1}$$

where $\{r, z, \zeta\}$ are cylindrical coordinates, and $\hat{\mathbf{z}}$ and $\hat{\boldsymbol{\zeta}}$ are unit vectors in the direction of ∇z and $\nabla\zeta$, respectively. We assume that the plasma is electrostatic and does not vary in the symmetry direction $\hat{\boldsymbol{\zeta}}$.

The spatial domain we consider is $r \in [r_0, r_0 + L_r]$ and $z \in [0, L_z]$, with a periodic boundary condition in z. The characteristic length along a field line between two walls is

$$L_\parallel \sim \frac{B}{B_z} L_z. \tag{2}$$

The characteristic time it takes for ions to stream along the field line from one end of z to the other is $L_\parallel/v_{\mathrm{th,i}} \sim (B/B_z)(L_z/v_{\mathrm{th,i}})$, where $v_{\mathrm{th,i}} \doteq \sqrt{2T_i/m_i}$ is the ion thermal speed and T_i is the ion temperature.

The radial $\mathbf{E}\times\mathbf{B}$ drift speed is

$$v_{Er} \doteq -\frac{B_\zeta}{B}\frac{\partial\phi}{\partial z} \sim \frac{\rho_{\mathrm{i}}}{L_z} v_{\mathrm{th,i}}, \tag{3}$$

where $\rho_i \doteq v_{\mathrm{th,i}}/\Omega_i$ is the thermal ion gyroradius, $\Omega_i \doteq eB/m_i$ is the ion gyrofrequency, and we have ordered the electrostatic potential energy comparable to the thermal energy: $e\phi \sim T_i$. We order the times for ions to cross the domain in the radial and vertical directions to be comparable, giving

$$L_r \sim \frac{B}{B_z}\rho_{\mathrm{i}}. \tag{4}$$

Taking the drift kinetic limit, we then have

$$\frac{\rho_{\mathrm{i}}}{L_r} \sim \frac{B_z}{B} \sim \frac{B_z}{B_\zeta} \ll 1. \tag{5}$$

3. Drift kinetic system of equations

The drift kinetic equations describing the evolution of the particle distribution f_s for the ions and electrons are

$$\begin{aligned} &\frac{\partial f_i}{\partial t} - \frac{1}{B}\frac{\partial\phi}{\partial z}\frac{\partial f_i}{\partial r} + \left(v_\parallel\frac{B_z}{B} + \frac{1}{B}\frac{\partial\phi}{\partial r}\right)\frac{\partial f_i}{\partial z} - \frac{eB_z}{m_i B}\frac{\partial\phi}{\partial z}\frac{\partial f_i}{\partial v_\parallel} \\ &\quad = C_{ii}[f_i] + C_{ie}[f_i, f_e] + S_i, \end{aligned} \tag{6}$$

and

$$\begin{aligned}\frac{\partial f_e}{\partial t} - \frac{1}{B}\frac{\partial \phi}{\partial z}\frac{\partial f_e}{\partial r} + \left(v_\parallel \frac{B_z}{B} + \frac{1}{B}\frac{\partial \phi}{\partial r}\right)\frac{\partial f_e}{\partial z} + \frac{eB_z}{m_e B}\frac{\partial \phi}{\partial z}\frac{\partial f_e}{\partial v_\parallel} \\ = C_{ee}[f_e] + C_{ei}[f_e, f_i] + S_e,\end{aligned} \tag{7}$$

where $v_\parallel$ is the parallel component of the particle velocity $\mathbf{v}$, $C_{ss'}$ accounts for the effect on species s of collisions with species s', S_s is a source accounting for, e.g., heating and fueling, and angle brackets denote an average over particle gyroangle.

The distribution functions f_i and f_e are related via quasineutrality,

$$\sum_s n_s = \sum_s \int d^3v f_s = 0. \tag{8}$$

One of the main challenges in solving this system of equations is that there is no explicit equation for the electrostatic potential. If one were to try, e.g., to solve Eqs. (6) and (7) using an explicit time advance algorithm with ϕ at the previous time level as an input, then in general the solutions for f_i and f_e so obtained would not satisfy Eq. (8). This procedure could be iterated, with ϕ varied until quasineutrality were satisfied, or one could develop an approach that guarantees satisfaction of quasineutrality from the outset. We have presented two such approaches in previous reports: The first assumes a Boltzmann electron response and thus avoids entirely the need to solve the electron dynamics, while the second requires the inclusion of higher order corrections in the drift kinetic expansion parameter. The former may not be sufficiently accurate, while the latter introduces significant complication in numerical solution. In the following sections we provide a means by which the accuracy of the Boltzmann response approximation can be estimated (and somewhat improved) and a fluid model that should make implementation and testing of the higher-order approach easier.

We will assume in this report that f_i can be obtained, provided ϕ, either via direct solution of the above drift kinetic equation or via the moment-kinetic approach derived in report 2047357-TN-09-01 [1]. Our focus here will be on obtaining simplified models for electron dynamics.

4. Extended Boltzmann response model

Thus far we have modelled the electrons as having a Boltzmann response in the closed-field-line region of the plasma. This means their density is given by

$$n_e = N_0 \exp\left(\frac{e\phi}{T_e}\right) = n_i, \tag{9}$$

where the latter equality is a consequence of quasineutrality. If a solution of this form is expected, one could expand the electron distribution function about it and solve for the difference:

$$f_e = f_{MB} + \delta f_e, \tag{10}$$

with

$$f_{MB} \doteq N_0 \left(\frac{m_e}{2\pi T_e}\right)^{3/2} \exp\left(-\frac{m_e v^2}{2T_e}\right) \exp\left(\frac{e\phi}{T_e}\right). \tag{11}$$

Taking the density moment of Eq (10) then gives

$$n_e = N_0 \exp\left(\frac{e\phi}{T_e}\right) + \delta n_e, \tag{12}$$

with

$$\delta n_e \doteq \int d^3v \delta f_e. \tag{13}$$

We can rearrange Eq. (12) to solve for the electrostatic potential:

$$\phi = \phi_0 + \phi_1, \tag{14}$$

with

$$\frac{e\phi_0}{T_e} \doteq \ln \frac{n_i}{N_0} \tag{15}$$

and

$$\frac{e\phi_1}{T_e} \doteq \ln\left(1 - \frac{\delta n_e}{n_i}\right), \tag{16}$$

where we have used quasineutrality to replace n_e with n_i.

Substituting the assumed form of f_e from Eq. (10) into the electron drift kinetic equation (7) gives

$$\begin{aligned} &\frac{\partial \delta f_e}{\partial t} - \frac{1}{B}\frac{\partial \phi}{\partial z}\frac{\partial \delta f_e}{\partial r} + \left(v_\parallel \frac{B_z}{B} + \frac{1}{B}\frac{\partial \phi}{\partial r}\right)\frac{\partial \delta f_e}{\partial z} + \frac{eB_z}{m_e B}\frac{\partial \phi}{\partial z}\frac{\partial \delta f_e}{\partial v_\parallel} \\ &= (C_{ee}[f_e] - C_{ee}[f_{MB}]) + C_{ei}[\delta f_e, f_i] + S_e[f_e] - \frac{\partial f_{MB}}{\partial t}, \end{aligned} \tag{17}$$

where

$$\frac{\partial f_{MB}}{\partial t} = f_{MB}\left(\frac{\partial \ln T_e}{\partial t}\left(\frac{mv^2}{2T_e} - \frac{3}{2}\right) + \frac{\partial}{\partial t}\ln\frac{n_i}{N_0} + \frac{\partial}{\partial t}\ln\left(1 - \frac{\delta n_e}{n_i}\right)\right). \tag{18}$$

The time derivatives of T_e and n_i above can be eliminated by using the ion and electron continuity equations, but there is no equivalent, closed-form equation for δn_e. This makes it difficult to use an explicit time advance scheme to evolve δf_e.

To overcome this, we can assume that $\delta n_e \ll n_e$; i.e., we assume that the electron response is close to Boltzmann. With this assumption, we have

$$\frac{e\phi_1}{T_e} \approx -\frac{\delta n_e}{n_i} \ll \frac{e\phi_0}{T_e}, \tag{19}$$

and

$$\begin{aligned} v_\parallel \frac{B_z}{B}\frac{\partial \delta f_e}{\partial z} + \frac{eB_z}{m_e B}\frac{\partial \phi_0}{\partial z}\frac{\partial \delta f_e}{\partial v_\parallel} &= C_{ee}^{(\ell)}[\delta f_e] + C_{ei}[\delta f_e, f_i] + S_e[f_{MB}] \\ &- f_{MB}\left(\frac{\partial \ln T_e}{\partial t}\left(\frac{mv^2}{2T_e} - \frac{3}{2}\right) + \frac{\partial}{\partial t}\ln\frac{n_i}{N_0}\right), \end{aligned} \tag{20}$$

where $C_{ee}^{(\ell)}$ is the linearized electron-electron collision operator, we have used the maximal ordering for collisions ($\nu_{ee} \sim v_{\mathrm{th,e}}/L_\parallel$), and we have kept only the lowest order terms in $\sqrt{m_e/m_i}$. Upon solving this equation for δf_e, the corresponding density δn_e could be compared to n_i to check if the Boltzmann approximation is valid. If not, one could use the alternative approach for computing ϕ that was proposed in report 2047357-TN-09-01 [1]. We outline this approach below and suggest a simple fluid model (first derived by Braginskii [2]) that could be used to test it.

5. Electron fluid equations

Electron fluid equations are obtained by taking the appropriate velocity moments of Eq. (7). The current conservation equation, obtained by combining the electron and ion continuity equations and enforcing quasineutrality, is

$$\frac{B_z}{B}\frac{\partial}{\partial z}\left(n_e\left(u_i - u_e\right)\right) = 0, \tag{21}$$

where u_s is the parallel flow of species s. We have assumed $\int d^3v\,(S_i - S_e)$ is small to obtain the above result. The electron parallel momentum equation is

$$-\frac{B_z}{B}\frac{\partial p_{\parallel,e}}{\partial z} + en_e\frac{B_z}{B}\frac{\partial \phi}{\partial z} + F_\parallel[f_e, f_i] = 0, \tag{22}$$

where $p_{\parallel,e}$ is the electron parallel pressure, and

$$F_\parallel[f_e, f_i](z,t) \doteq \int d^3v\; m_e v_\parallel C_{ei}[f_e, f_i] \tag{23}$$

is the parallel friction force between electrons and ions. The term proportional to S_e has been neglected in (22) as small in m_e/m_i because we order $S_e \sim f_e v_{\mathrm{th,i}}/L_\parallel$.

Finally, the electron energy equation is

$$\begin{aligned}\frac{3}{2}n_e\left(\frac{\partial T_e}{\partial t} - \frac{1}{B}\frac{\partial \phi}{\partial z}\frac{\partial T_e}{\partial r} + \left(u_e\frac{B_z}{B} + \frac{1}{B}\frac{\partial \phi}{\partial r}\right)\frac{\partial T_e}{\partial z}\right) = -\frac{B_z}{B}\left(\frac{\partial q_{\parallel,e}}{\partial z} + p_{\parallel,e}\frac{\partial u_e}{\partial z}\right)\\ + \frac{3n_e m_e \nu_{ei}}{m_i}\left(T_i - T_e\right) + F_\parallel[f_e, f_i]\left(u_i - u_e\right) + \int d^3v\left(\frac{m_e}{2}\left|\mathbf{v} - u_e\hat{\mathbf{z}}\right|^2 - \frac{3}{2}T_e\right)S_e,\end{aligned} \tag{24}$$

where $q_{\parallel,e}$ is the electron parallel heat flux,

$$q_{\parallel,e} \doteq \int d^3w m_s w_\parallel^3 f_e. \tag{25}$$

Eqs. (21) and (22) can be solved for $u_e(r,z,t)$ and $\phi(r,z,t)$, respectively, provided the boundary values $u_e(r,0,t)$ and $\phi(r,0,t)$ are given. The boundary value for the parallel flow is obtained by dividing Eq. (22) by n_e and integrating in z:

$$\int_0^{L_z} dz\left(-\frac{1}{n_e}\frac{B_z}{B}\frac{\partial p_{\parallel,e}}{\partial z} + \frac{F_\parallel[f_e, f_i]}{n_e}\right) = 0, \tag{26}$$

where it should be noted that $F_\parallel[f_e, f_i]$ will depend on u_e. An equation for $\phi(r,0,t)$ is obtained by going to higher order in the current conservation equation (for details, see [1]:

$$\frac{\partial}{\partial r}\Bigg(-\frac{\partial}{\partial t}\int_0^{L_z} dz\ F_r + \frac{1}{B}\frac{\partial}{\partial r}\int_0^{L_z} dz\ \frac{\partial \phi}{\partial z}F_r + \frac{1}{B}\int_0^{L_z} dz\ \mathbf{F}_{\rm ext}\cdot\hat{\mathbf{z}}\Bigg) = 0, \tag{27}$$

where $\mathbf{F}_{\rm ext}$ is the external force applied to the plasma and

$$F_r \doteq \frac{m_i n_i}{B^2}\left(\frac{\partial \phi}{\partial r} + \frac{1}{e n_i}\frac{\partial p_{\perp,i}}{\partial r}\right). \tag{28}$$

Provided the distribution f_i, we can close this set of equations if $q_{\parallel,e}$ can be expressed in terms of the other low-order fluid moments of f_e. This is made possible when the plasma is sufficiently collisional by performing a Chapman-Enskog expansion in short collisional mean-free-path. The resulting set of fluid equations are the drift-reduced Braginskii equations [2]: We provide a brief outline of their derivation next.

6. Drift-reduced Braginskii equations

Our starting point is the electron drift kinetic equation (7). We first change variables from particle velocity to peculiar velocity $\mathbf{w} \doteq \mathbf{v} - u_e\hat{\mathbf{b}}$, with $\hat{\mathbf{b}}$ the unit vector in the direction of $\mathbf{B}$:

$$\begin{aligned}&\frac{\partial f_e}{\partial t} - \frac{1}{B}\frac{\partial \phi}{\partial z}\frac{\partial f_e}{\partial r} + \left(\left(w_\parallel + u_e\right)\frac{B_z}{B} + \frac{1}{B}\frac{\partial \phi}{\partial r}\right)\frac{\partial f_e}{\partial z} + \frac{eB_z}{m_e B}\frac{\partial \phi}{\partial z}\frac{\partial f_e}{\partial w_\parallel}\\ &\quad - \left(\frac{\partial u_e}{\partial t} - \frac{1}{B}\frac{\partial \phi}{\partial r}\frac{\partial u_e}{\partial r} + \left(\left(w_\parallel + u_e\right)\frac{B_z}{B} + \frac{1}{B}\frac{\partial \phi}{\partial r}\right)\frac{\partial u_e}{\partial z}\right)\frac{\partial f_e}{\partial w_\parallel}\\ &\quad = C_{ee}[f_e] + C_{ei}[f_e, f_i] + S_e,\end{aligned} \tag{29}$$

We make the subsidiary expansion $\lambda_{ee}/L_\parallel \sim \lambda_{ei}/L_\parallel \sim \sqrt{m_e/m_i}$ so that collisions between electrons are faster than any other process in Eq. (7). Defining $\epsilon = \sqrt{m_e/m_i}$ and expanding $f_e = f_{e0} + f_{e1} + \ldots$, with $f_{e1}/f_{e0} \sim \epsilon$, the lowest order drift kinetic equation for electrons is

$$C_{ee}[f_{e0}] + C_{ei}[f_{e0}, f_i] = 0. \tag{30}$$

Multiplying Eq. (30) by $\ln f_{e0}$ and integrating over velocity space, Boltzmann's H-Theorem indicates that f_{e0} is an isotropic Maxwellian; i.e.,

$$f_{e0} = \frac{n_e}{\pi^{3/2} v_{\rm th,e}^3}\exp\left(-\frac{w^2}{v_{\rm th,e}^2}\right). \tag{31}$$

It's worth noting a couple of things about this solution. First, the density, temperature and parallel flow appearing inside it are defined to be the exact values. This means that the corresponding moments of f_{e1} must vanish. Second, the isotropy of f_{e0} constrains $p_{\parallel,e} = p_e$.

At next order, the electron drift kinetic equation becomes

$$\begin{aligned} C_{ee}^{(\ell)}[f_{e1}] + \mathcal{L}[f_{e1} - \frac{m_e w_\parallel \left(u_i - u_e\right)}{T_e} f_{e0}] \\ = w_\parallel \frac{B_z}{B} \frac{\partial f_{e0}}{\partial z} + \frac{e B_z}{m_e B} \frac{\partial \phi}{\partial z} \frac{\partial f_{e0}}{\partial w_\parallel}, \end{aligned} \tag{32}$$

with $C_{ee}^{(\ell)}$ the linearized electron-electron collision operator and $\mathcal{L}_{ei}$ the Lorentz operator. We use the electron momentum equation (22) to eliminate ϕ and substitute

$$\frac{\partial f_{e0}}{\partial z} = \left(\frac{\partial \ln p_e}{\partial z} + \left(\frac{m_e w^2}{2T_e} - \frac{5}{2} \right) \frac{\partial \ln T_e}{\partial z} \right) f_{e0}, \tag{33}$$

$$\frac{\partial f_{e0}}{\partial w_\parallel} = -\frac{2 w_\parallel}{v_{\mathrm{th,e}}^2} f_{e0}, \tag{34}$$

and

$$\mathcal{L} \left[\frac{m_e w_\parallel \left(u_i - u_e\right)}{T_e} f_{e0} \right] = -\frac{3\sqrt{\pi}}{4} \left(\frac{2T_e}{m_e w^2} \right)^{3/2} \frac{m_e \nu_{ei} \left(u_i - u_e\right) w_\parallel}{T_e} f_{e0} \tag{35}$$

into Eq. (32) to obtain

$$\begin{aligned} & C_{ee}^{(\ell)}[f_{e1}] + \mathcal{L}_{ei}[f_{e1}] \\ & = \Bigg(- \frac{3\sqrt{\pi}}{4} \left(\frac{2T_e}{m_e w^2} \right)^{3/2} \frac{m_e \nu_{ei} \left(u_i - u_e\right) w_\parallel}{T_e} \\ & - \frac{2 w_\parallel}{v_{\mathrm{th,e}}^2} \frac{1}{m_e n_e} \left(\frac{B_z}{B} \frac{\partial p_{\parallel,e}}{\partial z} - F_\parallel[f_{e0}, f_i] \right) \\ & + w_\parallel \frac{B_z}{B} \left(\frac{\partial \ln n_e}{\partial z} + \frac{\partial \ln T_e}{\partial z} \left(\frac{m_e w^2}{2T_e} - \frac{3}{2} \right) \right) \Bigg) f_{e0}, \end{aligned} \tag{36}$$

with the parallel friction force

$$F_\parallel[f_{e0}, f_i] = -n_e \nu_{ei} \left(u_e - u_i\right). \tag{37}$$

Substituting Eq. (37) into Eq. (36):

$$\begin{aligned} & C_{ee}^{(\ell)}[f_{e1}] + \mathcal{L}_{ei}[f_{e1}] + \frac{2\gamma_{ei} n_i f_{0e}}{m_e p_e} w_\parallel \int d^3 w \frac{w_\parallel}{(w')^3} f_{e1} \\ & = \left(\left(1 - \frac{3\sqrt{\pi}}{4} \left(\frac{2T_e}{m_e w^2} \right)^{3/2} \right) \frac{m_e \nu_{ei} \left(u_i - u_e\right)}{T_e} \right. \\ & \left. + \frac{B_z}{B} \left(\frac{\partial \ln T_e}{\partial z} \left(\frac{m_e w^2}{2T_e} - \frac{5}{2} \right) \right) \right) w_\parallel f_{e0} \end{aligned} \tag{38}$$

This form of the drift kinetic equation can be solved by expanding f_{e1} in terms of generalized Laguerre polynomials:

$$f_{e1} = \sum_{k=1}^{\infty} \hat{f}_k L_k^{(3/2)} \left(\frac{w^2}{v_{\mathrm{th,e}}^2} \right). \tag{39}$$

The sum starts at $k = 1$ because there is no flow contained in f_{e1} by construction. In practice we keep only the first two terms in the above expansion. Using the above expansion and taking the generalized Laguerre transform of the lefthand side of Eq. (38) gives

$$\mathrm{LHS} = -n_e \nu_{ee} \begin{pmatrix} \sqrt{2} + \frac{13}{4} & \frac{3\sqrt{2}}{4} + \frac{69}{16} \\ \frac{3\sqrt{2}}{4} + \frac{69}{16} & \frac{45\sqrt{2}}{16} + \frac{433}{64} \end{pmatrix} \begin{pmatrix} \hat{f}_1 \\ \hat{f}_2 \end{pmatrix}. \tag{40}$$

Transforming the righthand side yields

$$\int d^3w \; w_\parallel^2 f_{e0} L_k^{(3/2)}(x) \left(\frac{B_z}{B} \left(\frac{w^2}{v_{\mathrm{th,e}}^2} - \frac{5}{2} \right) \frac{\partial \ln T_e}{\partial z} + \left(1 - \frac{3\sqrt{\pi}}{4} \left(\frac{v_{\mathrm{th,e}}}{w} \right)^3 \right) \frac{m_e \nu_{ei} \left(u_i - u_e \right)}{T_e} \right)$$
$$= -\frac{5}{2} \frac{p_e}{m_e} \frac{B_z}{B} \frac{\partial T_e}{\partial z} \delta_{k1} - n_e \nu_{ei} \left(u_i - u_e \right) \left(\frac{3}{2} \delta_{k1} + \frac{15}{8} \delta_{k2} \right). \tag{41}$$

Equating the two sides of the equation, we can solve for $\hat{f}_1$ and $\hat{f}_2$. The result is

$$f_{e1} = w_\parallel f_{e0} \left(\frac{1}{\nu_{ei}} \frac{B_z}{B} \frac{\partial \ln T_e}{\partial z} \left(c_1 L_1^{(3/2)} + c_2 L_2^{(3/2)} \right) + \frac{m_e \left(u_i - u_e \right)}{T_e} \left(c_3 L_1^{(3/2)} + c_4 L_2^{(3/2)} \right) \right) \tag{42}$$

with $c_1 = 1.265$, $c_2 = -0.6325$, $c_3 = 0.284$ and $c_4 = 0.032$.

Using the solution for f_{e1} in Eq. (25) gives the classic Braginskii result for the parallel electron heat flux,

$$q_{\parallel,e} = -3.16 \frac{p_e}{m_e \nu_{ei}} \frac{B_z}{B} \frac{\partial T_e}{\partial z} - 0.71 p_e \left(u_i - u_e \right). \tag{43}$$

7. Discussion

We have proposed two models for electron dynamics to enable the calculation of the electrostatic potential. The first model assumes the electron response is almost Boltzmann and calculates the correction as a means of testing the assumption. If the obtained correction is large enough to invalidate the Boltzmann assumption, there are (at least) two options left: use an iterative method to find self-consistent solutions for f_i, f_e and ϕ (either by varying ϕ in the original drift kinetic equations or by varying, e.g., $\partial \delta n_e / \partial t$ in Eq. (17); or use current conservation at higher order via Eq. (27) to obtain ϕ. For the latter approach, it will be useful to use a simple fluid model for electron dynamics to facilitate implementation and testing. We proposed here the Braginskii fluid model composed of Eqs. (21), (22), (24), (26), (27), (37) and (43).

[1] F. I. Parra, M. Barnes, and M. R. Hardman. 2d drift kinetic model with periodic boundary conditions. *Excalibur/Neptune Report*, 9:2047357–TN–09–01 M1.6, 2021.

[2] S. I. Braginskii. Transport phenomena in a completely ionized two-temperature plasma. *Sov. Phys. JETP*, 6:358, 1958.