NEPTUNE UQ FEAT. SLOT CONVECTION

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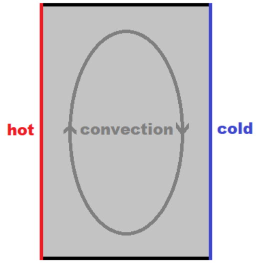
NEPTUNE Workshop (The Cosener's House, Abingdon)

6 September 2022

0.1 Nektar++ vertical natural convection (reprise)

- 2D heat transfer problem:

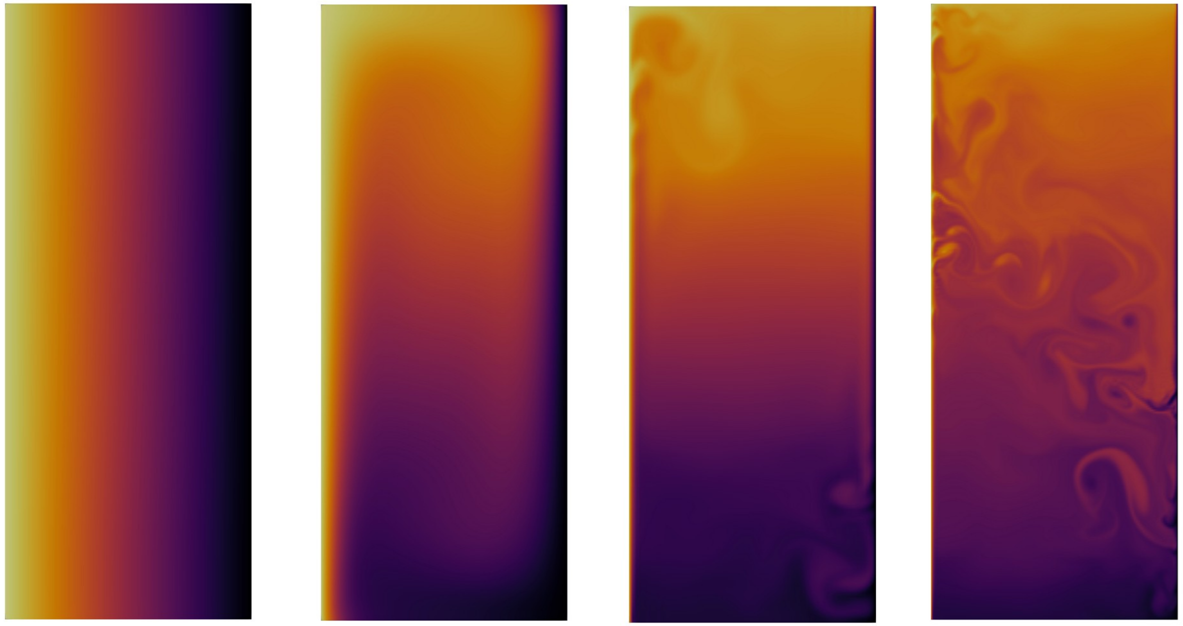


$$\begin{aligned} \frac{1}{Pr}\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) &= -\nabla p + Ra\, T\,\hat{\mathbf{y}} + \nabla^2\mathbf{u} \\ \left(\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T\right) &= \nabla^2 T \\ \nabla\cdot\mathbf{u} &= 0. \end{aligned}$$

- Parameter: Rayleigh number : strength of buoyancy force. (; air.)
- Phenomenology: increased gives increased heat transfer rate … diffusion, then steady convection, then 2D turbulence driven by large intermittent boundary fluctuations (cf. tokamak plasma edge).
- Goals: study of system as turbulent heat transport proxy; also investigate capability of Nektar++.

heat flux results vs. literature

Ra	Barletta *et al* (2005)	Lo *et al* (2005)	*Nektar++* (2021)
10^3	1.118	1.118	1.118
10^4	2.245	2.243	2.245
10^5	4.520/4.522	4.519	4.522
10^6	–	8.823	8.826
10^7	–	16.641	16.532

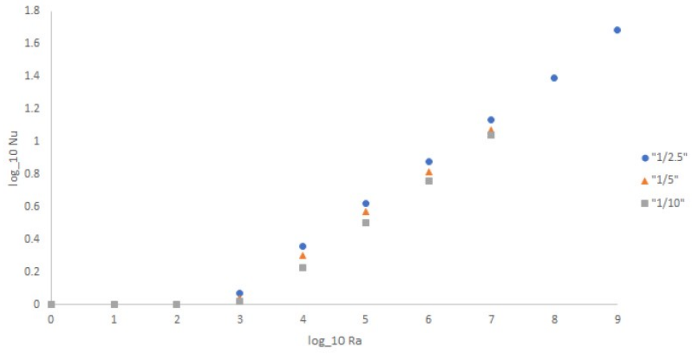


temperature fields for , ,

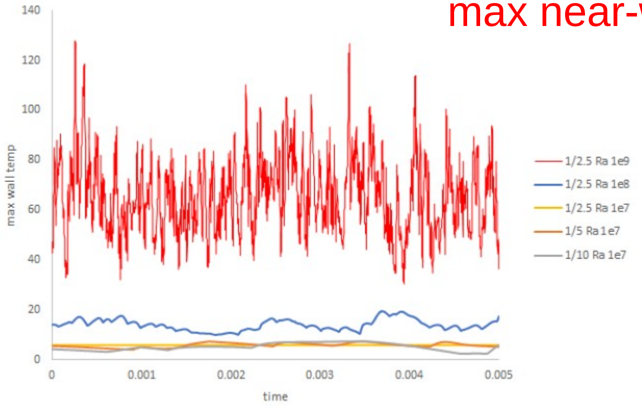
0.2 Nektar++ vertical natural convection (reprise)

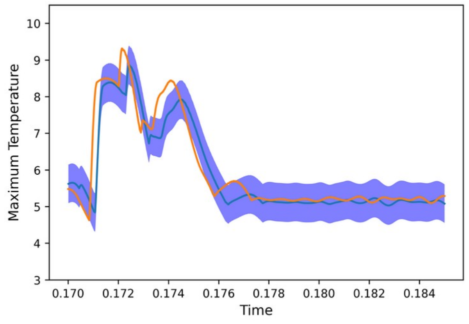
- Small modifications to Nektar++ to compute heat flux, and properties of wall-wave instability.
- Heat flux (Nusselt number): power laws (three different cavity aspect ratios)…

power laws



max near-wall temp fitted by GP (*EasyVVUQ*)





- Wall-wave instability: time series for maximum local temperature and position of hottest point (these were used with *EasyVVUQ* – constructed GP surrogate during hackathons).

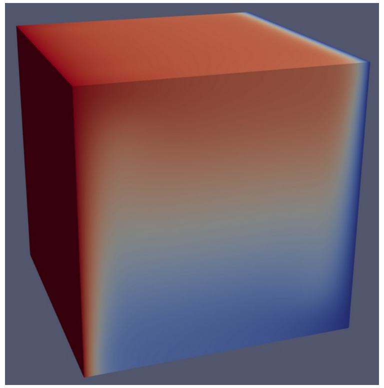
- Main conclusions: -refinement advised in preference to -refinement (more efficient). Large- dynamics mandates small time step in solver (also tried out different schemes for `SpectralVanishingViscosity` solver option – did not see a big improvement).

1.2 UKAEA in-house activity: Nektar++ V&V

- 3D version of global Nusselt number calculator implemented in Nektar++ (easy).
- Got preliminary results for Nusselt number for cubical cavity cf. *A first incursion into the 3D structure of natural convection of air in a differentially heated cubic cavity, from accurate numerical solutions* (E. Tric et al, 1999).

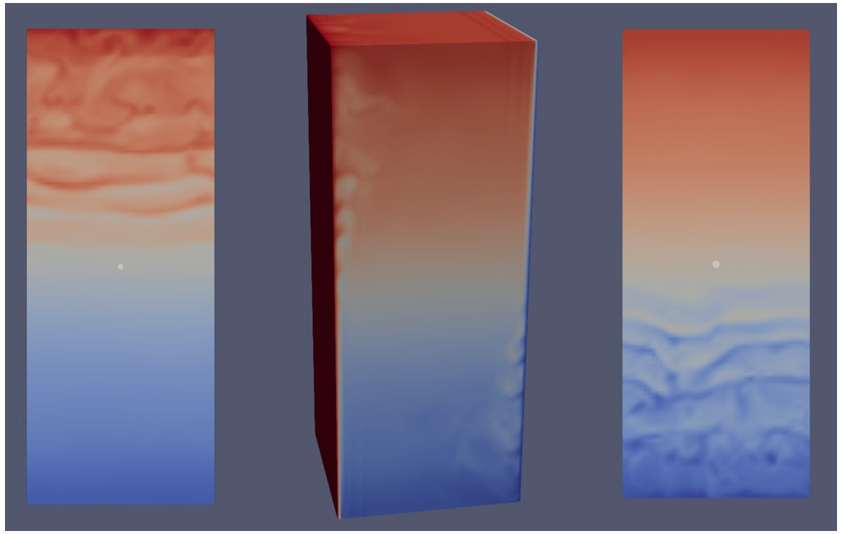
p	Nu (literature: 4.3370)
5	4.3002
6	4.3293
7	4.3378
8	4.3373

p	Nu (literature: 8.6407)
5	8.6600
6	8.4898
7	8.5763
8	8.6339



- Aspect ratio 2.5 cavity results:

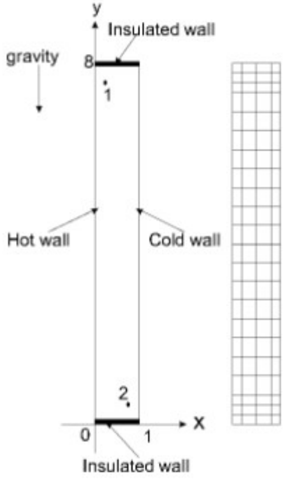
, elements, .



2.1 Slot convection: multi-level GP-ROM for Smallab

- Interesting problem to attack with multi-level GPs (build GP-ROM then use discrepancy with model to build another GP-ROM, etc …).
- Nektar++ convection problems are relevant (interesting features e.g. transition from time-periodic behaviour to turbulent chaos).
- Current plan (subject to meeting to happen next few weeks) is to treat MIT benchmark problem (2D 8:1 ratio cavity, Rayleigh numbers close to turbulent transition), then move on to 3D examples based on Smallab experiments (Dom Buta, Wayne Arter). 2D case easy as accessible to single PC – turbulent 3D not. FabNEPTUNE. Dom Buta presentation recorded.

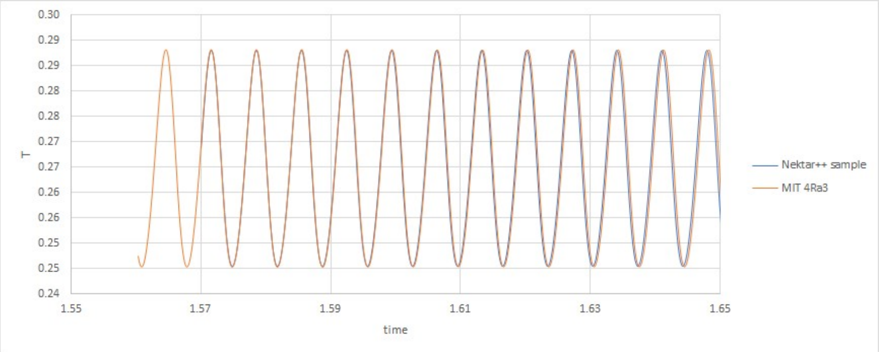
2D MIT benchmark



(QoIs)

1. The x-velocity at point 1
2. The temperature at point 1
3. The Nusselt number along both sides of the wall in an integral valued function,

$$Nu(t) = \frac{1}{H}\int_0^H \left|\frac{\partial\Theta}{\partial x}\right|_{x=0,W} dx$$



Smallab apparatus (Buta)

From Elder, *Laminar free convection in a vertical slot*, J. Fluid Mech (1965) vol.23, pt.1

2.2 FabNEPTUNE plans

- Investigate 3D structure of 3D problem (189k elts, inaccessible to single desktop), compare experiments.
- Investigate numerically "phase diagram" of various scaling regimes for Nusselt number. Compare to known Rayleigh-Benard case. Can be done in 2D first.

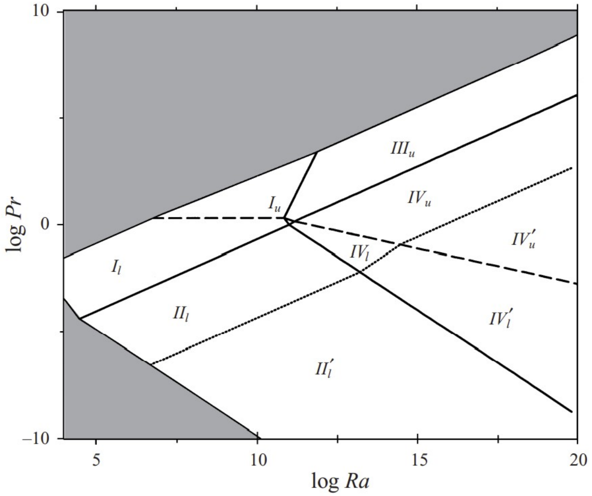


FIGURE 2. Phase diagram in the (Ra, Pr)-plane. The power laws and the corresponding prefactors (to be determined in §4) in the respective regimes are summarized in table 2. The tiny regime to the right of regime I_l is regime III_l. The dashed line is $\lambda_u = \lambda_\theta$. The shaded regime for large Pr is where $Re \leqslant 50$, and in the shaded regime for low Pr we have $Nu = 1$. The dotted line indicates the non-normal-nonlinear onset of turbulence in the BL shear flow discussed in §2.6. The scaling in regime II'_l is therefore as in the bulk-dominated regime IV_l. The power laws for the boundaries between the different regimes are given in table 3.

Regime	Dominance of	BL	Nu	Re
I_l	$\epsilon_{u,BL}$, $\epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.27Ra^{1/4}Pr^{1/8}$	$0.037Ra^{1/2}Pr^{-3/4}$
I_u		$\lambda_u > \lambda_\theta$	$0.33Ra^{1/4}Pr^{-1/12}$	$0.039Ra^{1/2}Pr^{-5/6}$
II_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,BL}$	$\lambda_u < \lambda_\theta$	$0.97Ra^{1/5}Pr^{1/5}$	$0.47Ra^{2/5}Pr^{-3/5}$
(II_u)		$\lambda_u > \lambda_\theta$	$(\sim Ra^{1/5})$	$(\sim Ra^{2/5}Pr^{-2/3})$
III_l	$\epsilon_{u,BL}$, $\epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$6.43 \times 10^{-6}Ra^{2/3}Pr^{1/3}$	$5.24 \times 10^{-4}Ra^{2/3}Pr^{-2/3}$
III_u		$\lambda_u > \lambda_\theta$	$3.43 \times 10^{-3}Ra^{3/7}Pr^{-1/7}$	$6.46 \times 10^{-3}Ra^{4/7}Pr^{-6/7}$
IV_l	$\epsilon_{u,bulk}$, $\epsilon_{\theta,bulk}$	$\lambda_u < \lambda_\theta$	$4.43 \times 10^{-4}Ra^{1/2}Pr^{1/2}$	$0.036Ra^{1/2}Pr^{-1/2}$
IV_u		$\lambda_u > \lambda_\theta$	$0.038Ra^{1/3}$	$0.16Ra^{4/9}Pr^{-2/3}$

TABLE 2. The power laws for Nu and Re of the theory presented, including the prefactors which are adopted from four pieces of experimental information in §4. The exact values of the prefactors depend also on how the Reynolds number is defined, see the first paragraph of §4. Regime II_u is in brackets as it turns out that it does not exist for this choice of prefactors.

S. Grossmann and D. Lohse, *Scaling in thermal convection: a unifying theory*, J. Fluid Mech. (2000) vol.407 pp. 27-56.

2.2 FabNEPTUNE plans

- Coupled models, Nektar++ CWIPI to couple convective cell and diffusion in solid.
- Interesting time-dependent behaviour due to quasi-steady state.
- Effect of solid on the fluid model.