Nektar++: High-dimensional spectral elements

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Outline

- Overview
- *TensorRegions* library
- Implementing core operations
- Projection proxy-app
- Vlasov-Poisson and advection proxy-apps

Overview

- Nektar++ is a spectral/hp element framework
- Existing support for solving PDE problems in domains in up to three dimensions:

$$\Omega \subset \mathbb{R}^d, \qquad d \leq 3$$

- 1D, 2D and 3D elements: segments, quads/tris, hex/tet/prism/pyramid
- Develop continuum modelling of plasma kinetics
- Vlasov-Poisson equation models the distribution of plasma for a particle species α, with position $\mathbf{x}$ and velocity $\mathbf{v}$:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot \frac{\partial f_\alpha}{\partial \mathbf{x}} + \lambda \frac{\partial \phi}{\partial \mathbf{x}} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0 \qquad \nabla^2 \phi = \int f_\alpha \, \mathrm{d}\mathbf{v}$$

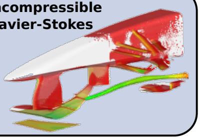
- In the 3D3V case, this leads to a 6D problem

This project aims to develop support higher-than-3D spaces in Nektar++.

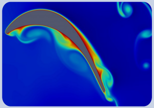
- Constructed as a tensor product of two (or potentially more) lower-dimensional spaces.
- Leverage implementation for lower-dimensional spaces

TensorRegions library

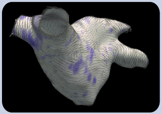
Incompressible Navier-Stokes



Compressible Navier-Stokes



Cardiac Electrophysiology



NekMesh

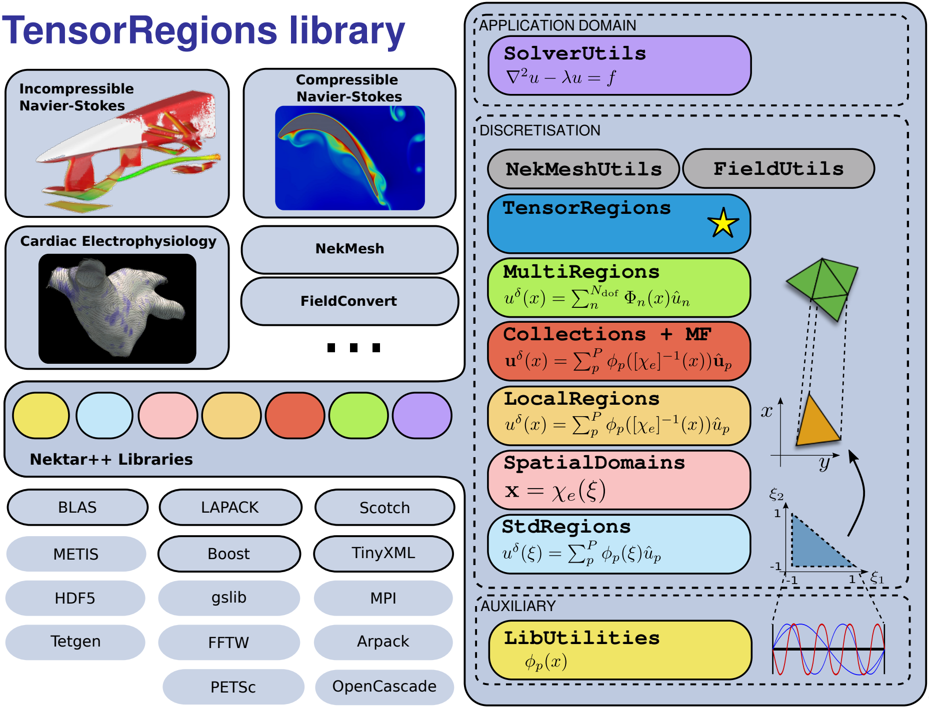
FieldConvert

...

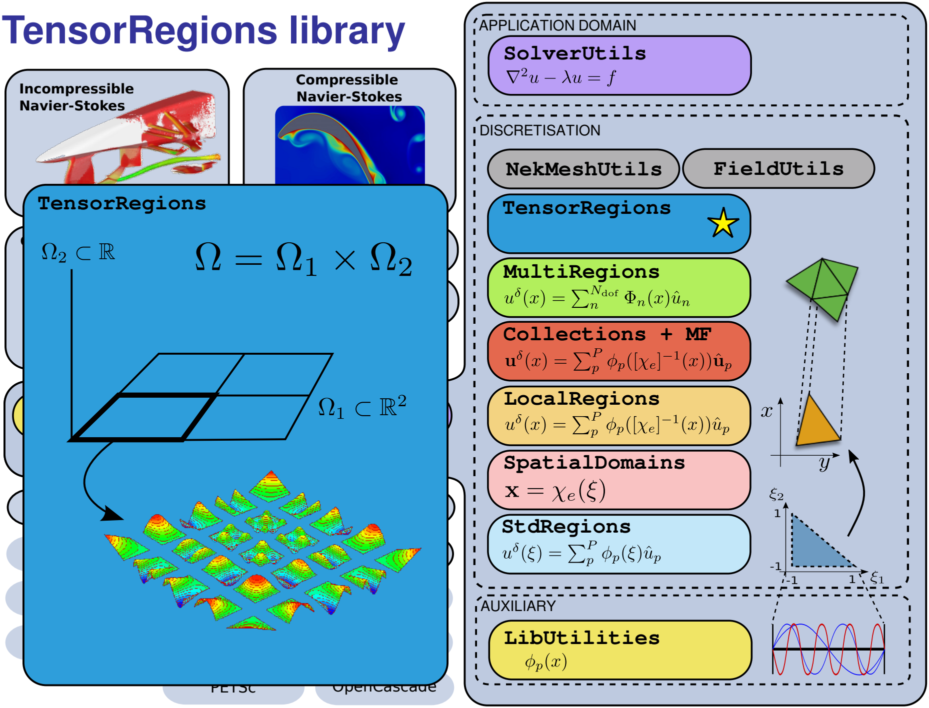
Nektar++ Libraries

BLAS
LAPACK
Scotch
METIS
Boost
TinyXML
HDF5
gslib
MPI
Tetgen
FFTW
Arpack
PETSc
OpenCascade

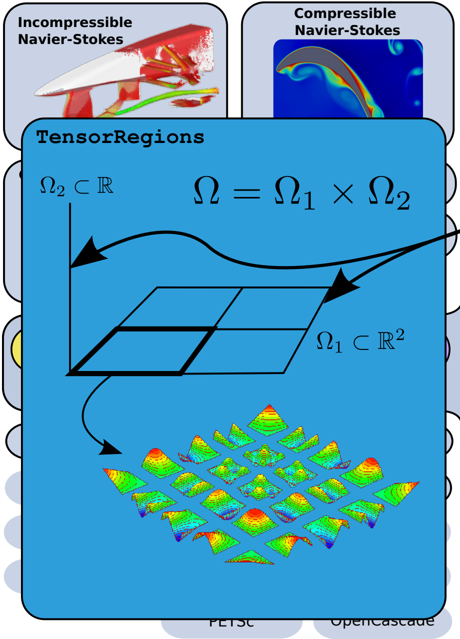
TensorRegions library

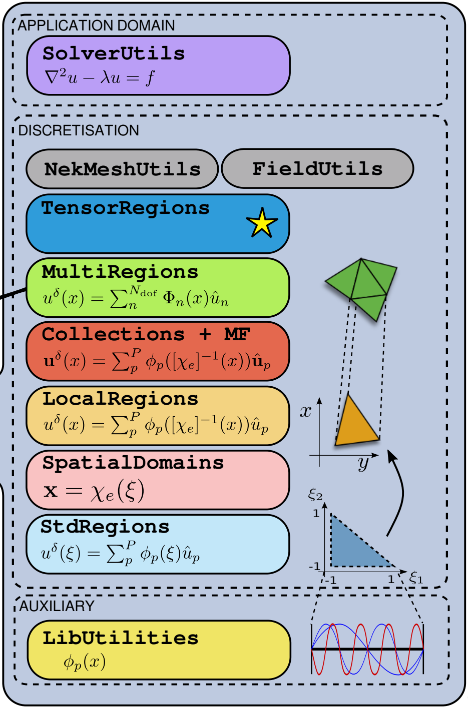


TensorRegions library



TensorRegions library





TensorRegions library

Two key classes:

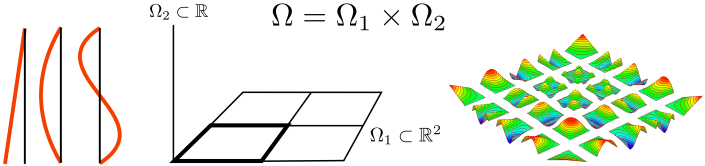
`TensorRegion`

- Describes a tensor-product domain
- Encapsulates two `MultiRegions` objects: DG or CG
- Provides interfaces for performing core operations

`TensorStorage`

- Data container for solutions on a TensorRegion
- Can hold Physical or Spectral/hp coefficient representation
- Exposes a `View` class which provides access to the data along one component of the tensor.
  $\rightarrow$ enables use of underlying MultiRegions operators.

Implementing Core Operators



- Tensor-product construction

$$\phi_\Omega(x, y, z) = \phi_{\Omega_1}(x, y)\phi_{\Omega 2}(z)$$

- Mirrors tensor-product construction of 2D and 3D elements
- Application of operators leverages existing lower dimensional operators e.g. Backward transformation:

$$\mathbf{B}_\Omega = \mathbf{B}_{\Omega_1}^\top \mathbf{B}_{\Omega_2}$$

Projection proxy-app

- Initial proxy-app to test core operators
- Transforms a solution represented by physical values to the spectral/hp element modal coefficient representation
- Solves

$$\mathbf{M}\hat{\mathbf{u}} = \mathbf{B}^{\top}\mathbf{W}\mathbf{u}$$

  elementally with a discontinuous Galerkin scheme for Ω_1 and Ω_2.
- Leverages tensor-product formulation to decouple operations in the two constituent spaces.

Advection and Vlasov-Poisson proxy-apps

- Currently under development to demonstrate `TensorRegion` library
- 6D advection proxy-app as groundwork for Vlasov-Poisson
- Vlasov-Poisson initially supporting 1D1V
- Will support 1D3V by the end of project.

Thank you for listening!