

# Developing exascale spectral/hp element tools for fusion applications

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We are developing efficient high-order spectral/hp element solvers and proxyapps for the NEPTUNE project, examining the numerical and computational performance for testbed fusion applications. In particular we are:

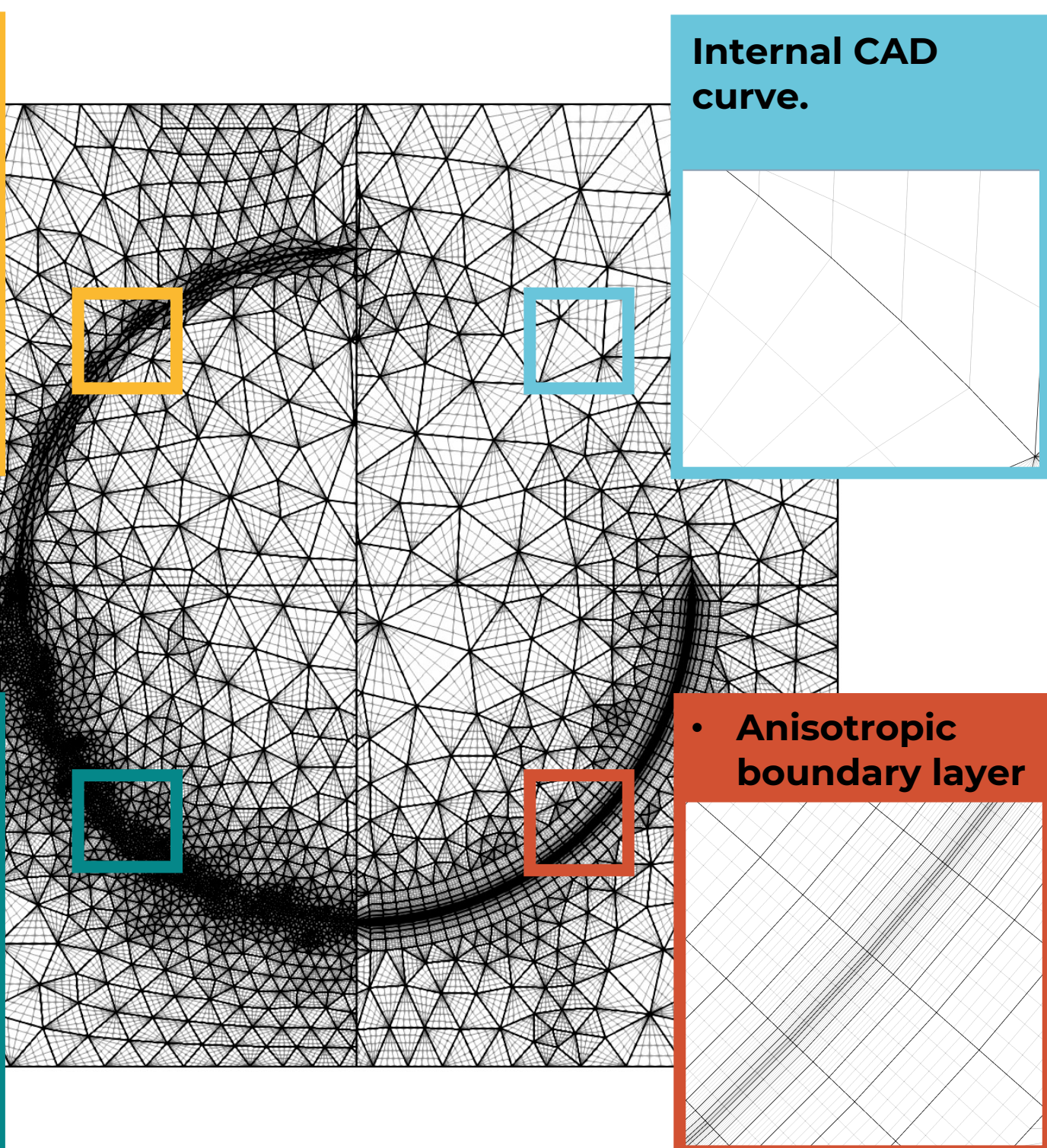
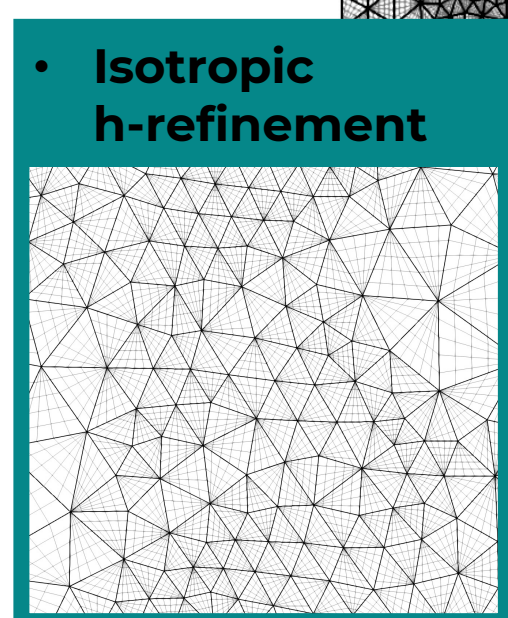
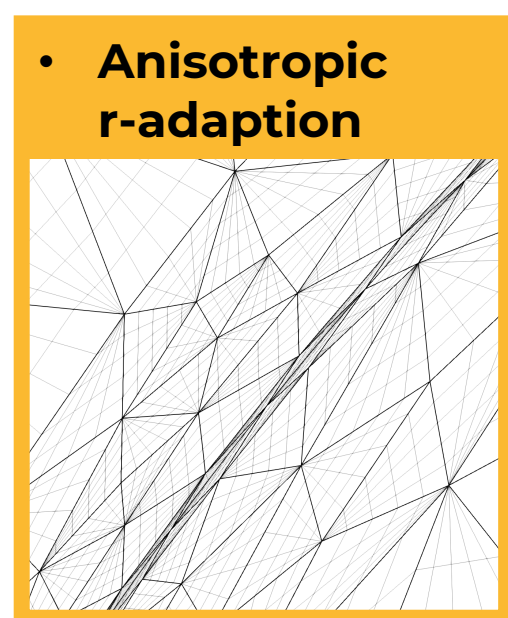
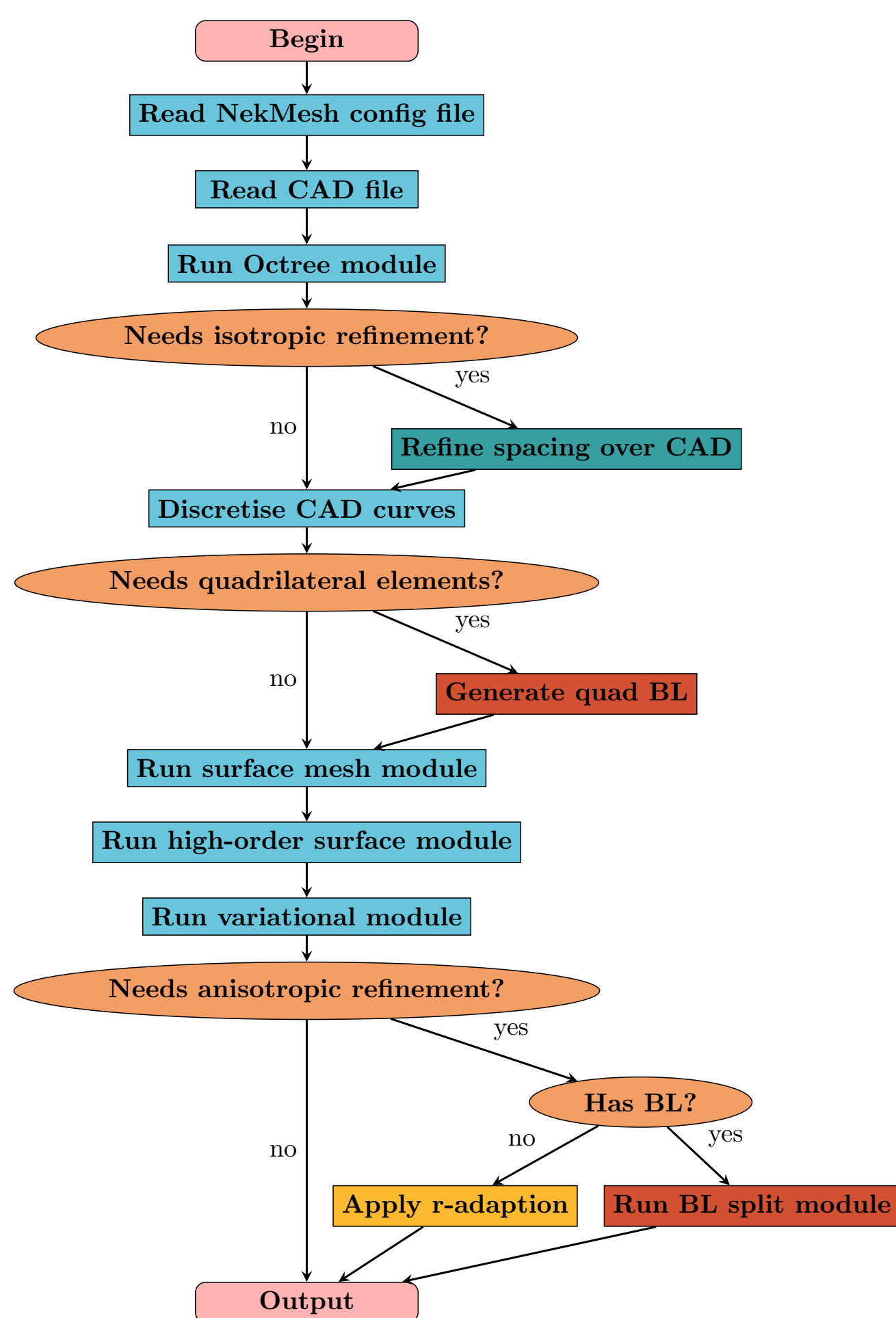
- generating high-quality meshes that are suited to the highly anisotropic physics found in these simulations;
- developing high-performance execution kernels for the heterogeneous hardware that powers exascale computing.

## Anisotropic high-order mesh generation for internal flow features using embedded CAD curves

High-order simulations require **high-order meshes** which conform to the underlying geometry, including curving elements to align with this geometry. In this project we have been extending our capabilities to resolve **geometry-interior features** such as the plasma separatrix found in a tokamak's interior.

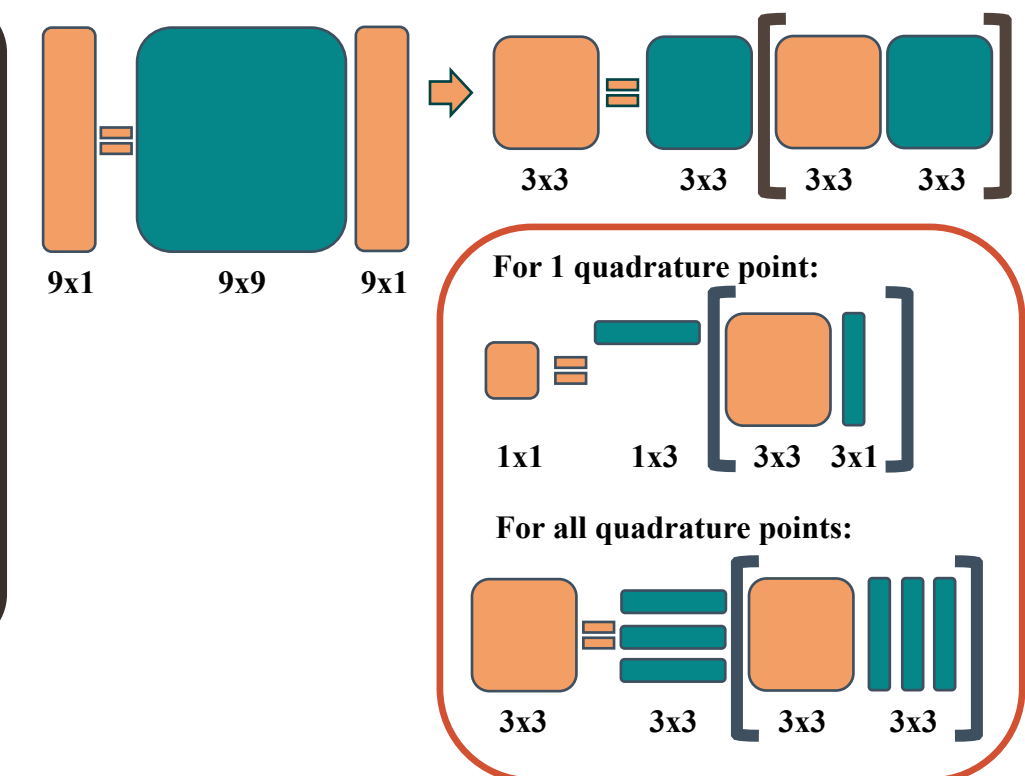
The flowchart demonstrates the bottom-up approach taken in our mesh generator **NekMesh** to generate high-order meshes. These have been extended to resolve these features, by using interior CAD geometries to conform and refine the mesh with the flow structures. This includes:

- Isotropic  $h$ -refinement;
- Anisotropic mesh refinement using  $r$ -adaptation;
- Quadrilateral anisotropic boundary layer (BL) meshes generated on both sides of the CAD geometry.

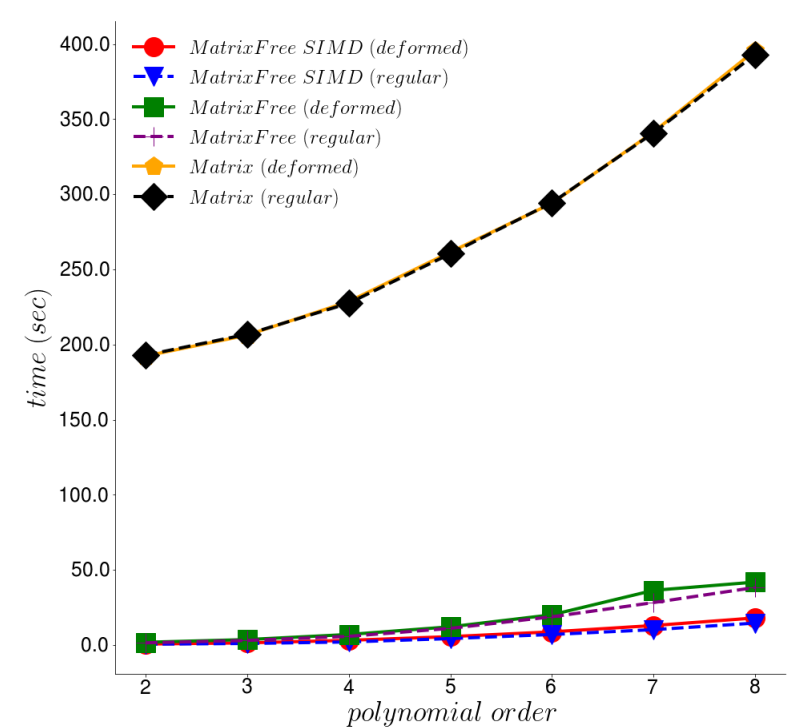


## Helmholtz matrix-free operators in Nektar++

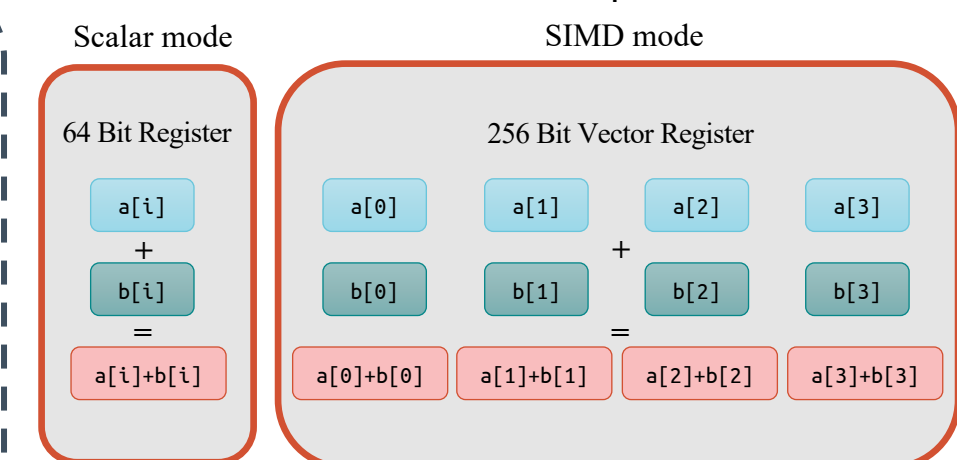
We have developed efficient finite element operators using **matrix-free evaluations** of the block inner-product, derivative and interpolation kernels from which they are constructed. These target **x86, ARM and GPU** architectures, ensuring use of SIMD vectorization through templating and data interlacing of elemental data. We leverage **sum factorization** for reduced operator complexity & improved throughput.



Sum-factorization (backward transformation) improves operator complexity and efficiency



Performance of CPU Helmholtz matrix-free operator



Single-instruction, multiple-data (SIMD) intrinsics are used to ensure optimal performance of kernel evaluations

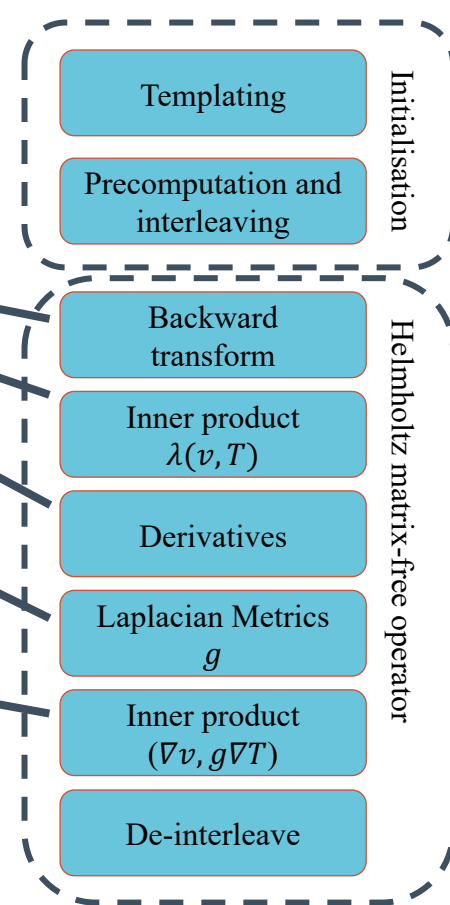
## Nektar++ matrix free libraries and their organization

**Algorithm 1** Overview of matrix-free evaluation of Helmholtz operator

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1: Helmholtz( $\bar{T}, \omega, B, \nabla B, J, \kappa_c$ )
2: for each element group do
3:    $T^h \leftarrow \text{BndTrans}(\bar{T}, B)$ 
4:    $out \leftarrow \lambda \cdot \text{IPProduct}(T, B, \omega, |J|)$ 
5:    $DT^h \leftarrow \text{PhysDeriv}(T^h, D, (J)^{-1})$ 
6:   if element is deformed then
7:     for each quadrature point  $\eta$  do
8:        $g \leftarrow (J(\eta))^{-1} \kappa_c(\eta) (J(\eta))^{-T}$ 
9:        $DT^h(\eta) \leftarrow g DT^h(\eta)$ 
10:    end for
11:   else
12:      $g \leftarrow (J)^{-1} \kappa_c(J)^{-T}$ 
13:     for each quadrature point  $\eta$  do
14:        $DT^h(\eta) \leftarrow g DT^h(\eta)$ 
15:     end for
16:   end if
17: end for
18: for each dimension  $d$  do
19:    $out \leftarrow out + \text{IPProduct}(DT^h, \partial_d B, \omega, |J|)$ 
20: end for
21: return  $out$ 
```

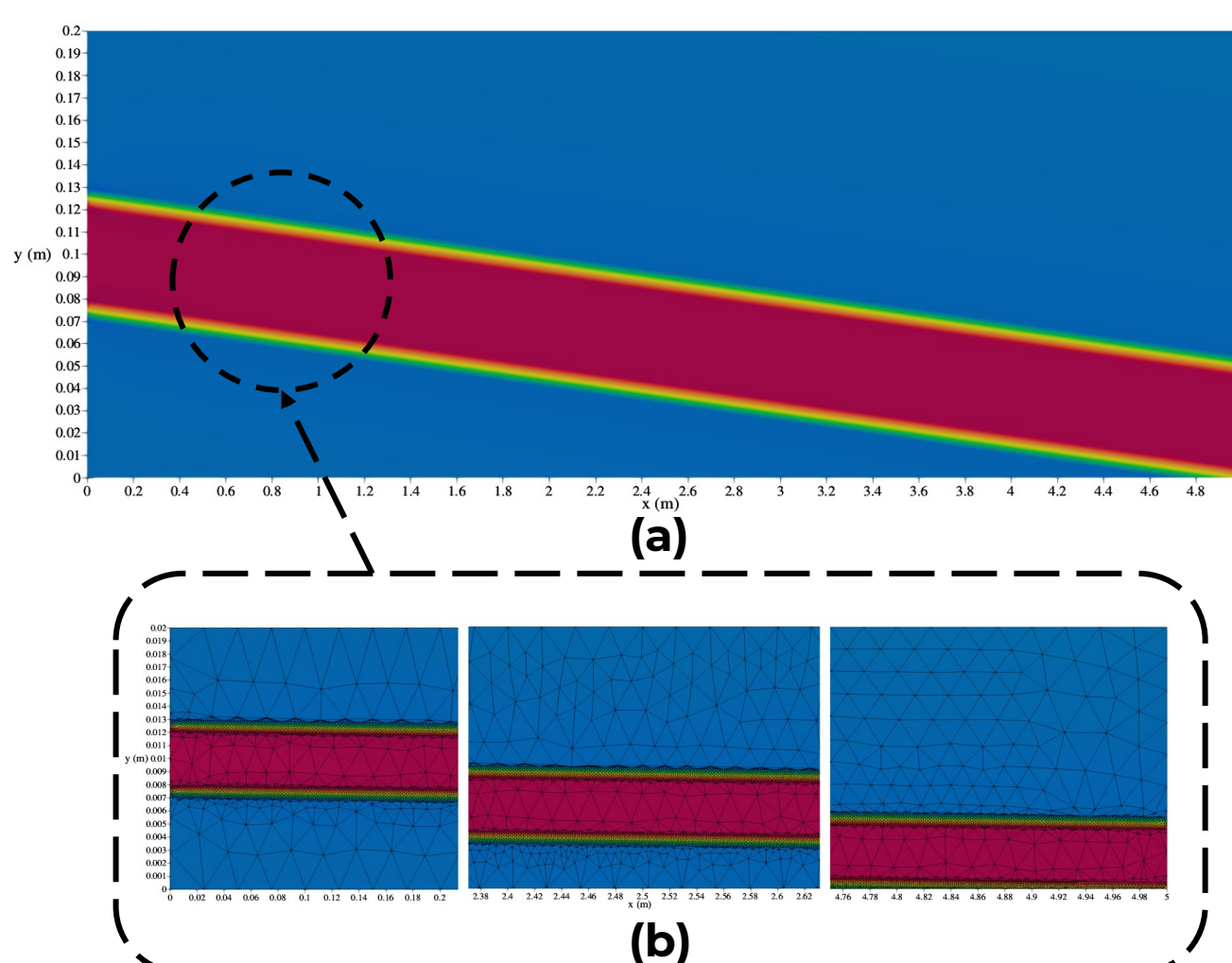
Coordination of kernels to construct a matrix-free Helmholtz operator ( $\nabla^2 + \lambda$ )



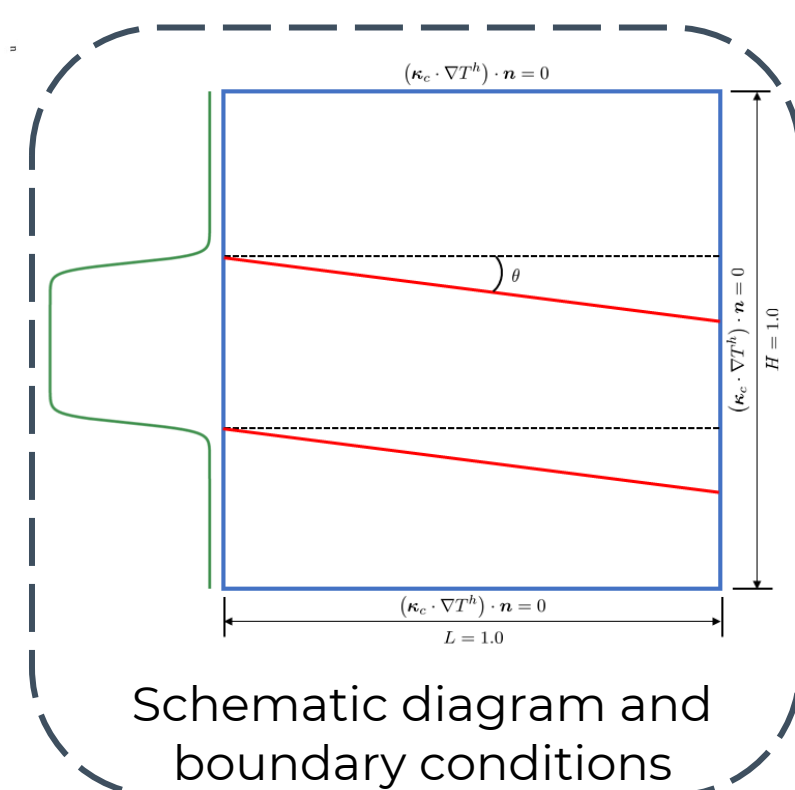
## Thermal conduction of anisotropic magnetised plasma simulation

We have used this infrastructure to develop proxyapps for NEPTUNE use cases, such as a highly anisotropic heat transport problem:

$$\frac{3}{2} n \frac{\partial T}{\partial t} = \nabla \cdot \left[ \left( (\kappa_{\parallel} - \kappa_{\perp}) \cos^2 \theta + \kappa_{\perp} \right) \partial_x T + \left( (\kappa_{\parallel} - \kappa_{\perp}) \cos \theta \sin \theta \right) \partial_y T \right] + Q$$



Snapshot of the thermal conduction of magnetized plasma: **(a)** strong thermal anisotropy along the magnetic field in plasma and **(b)** the refined mesh along scrape-off layer



Schematic diagram and boundary conditions