T-NA078-20 - Performance of spectral elements

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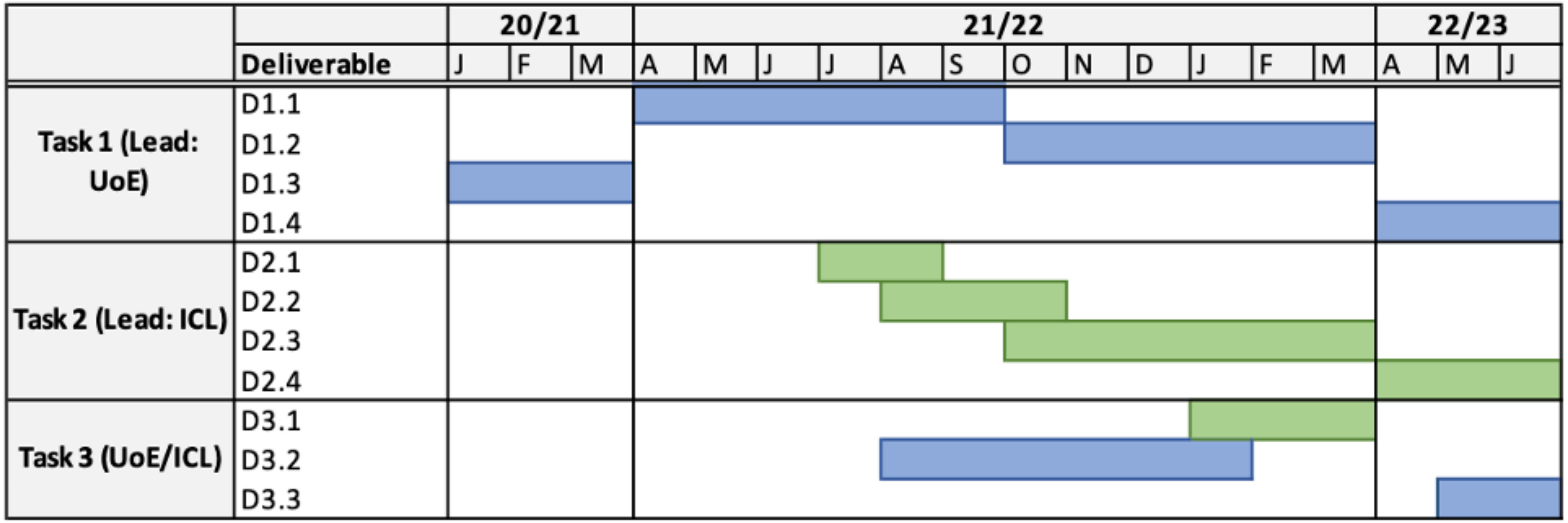
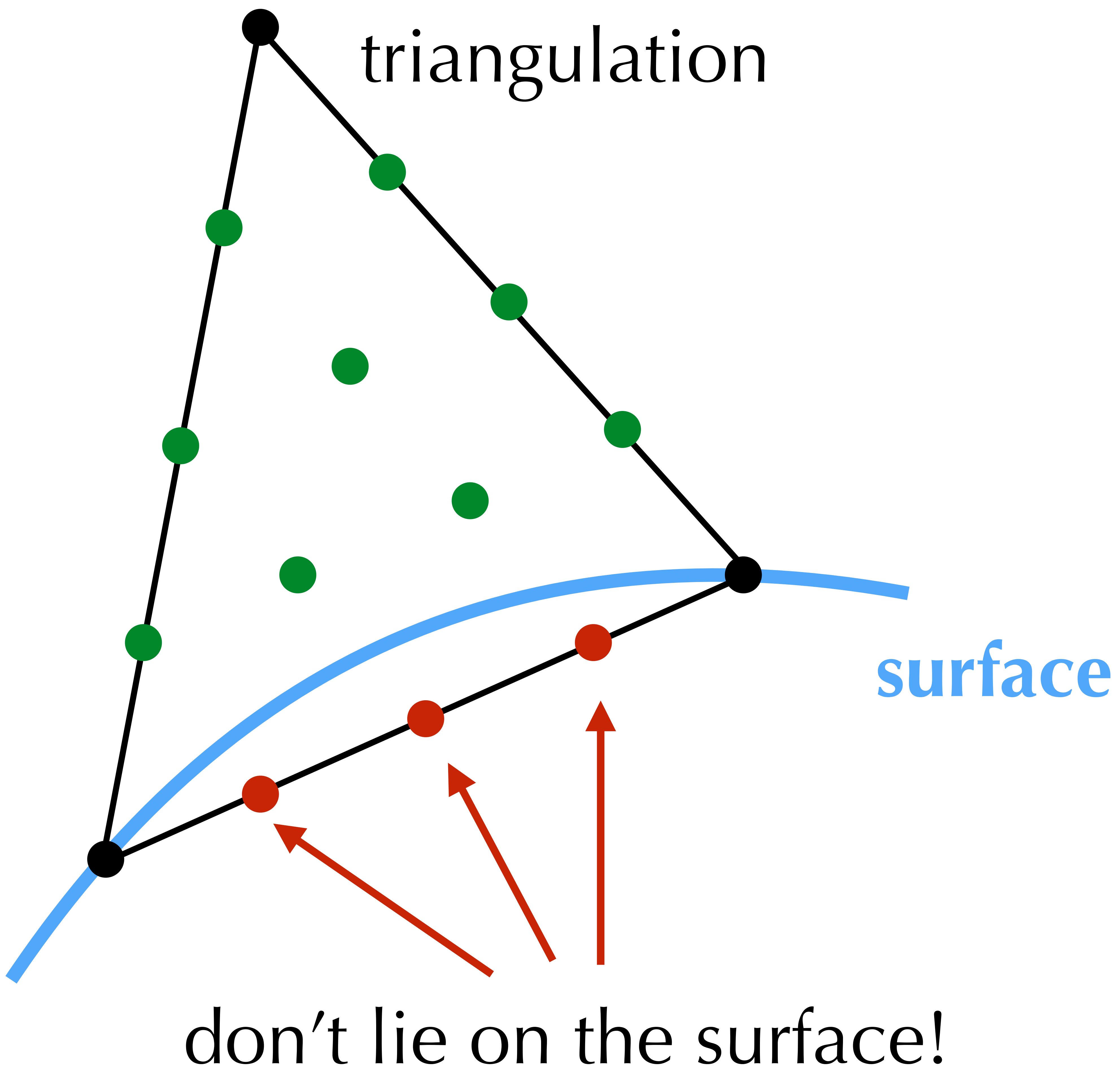
Overall project timeline



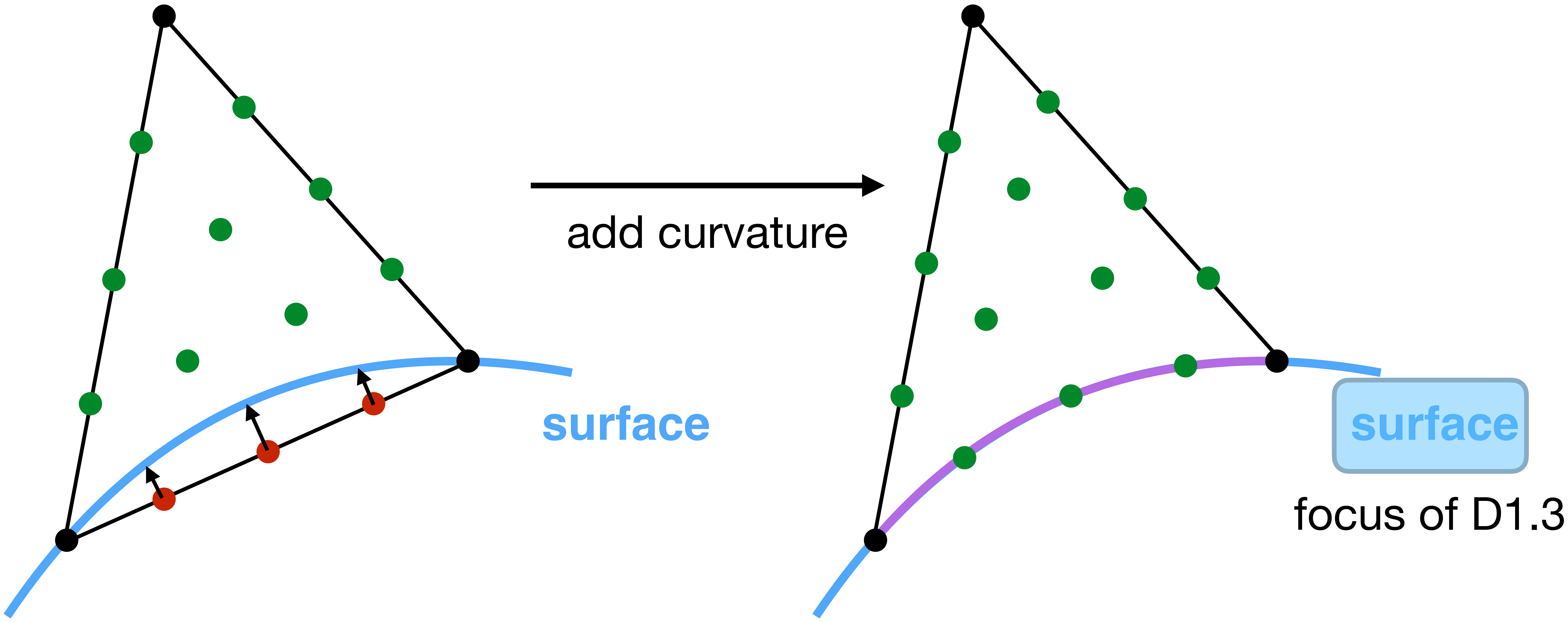
- **Task 1: high-order mesh generation for fusion applications.**
- Task 2: flexible & performance portable proxyapps.
- Task 3: dissemination & coordination activities.

High-order mesh generation

- Good quality meshes are **essential** to finite element and finite volume simulations.
- You can have a very fancy solver, but if you can't mesh your geometry then you **can't run your simulation**!
- At high orders we have an additional headache, as we must **curve the elements** to fit the geometry.

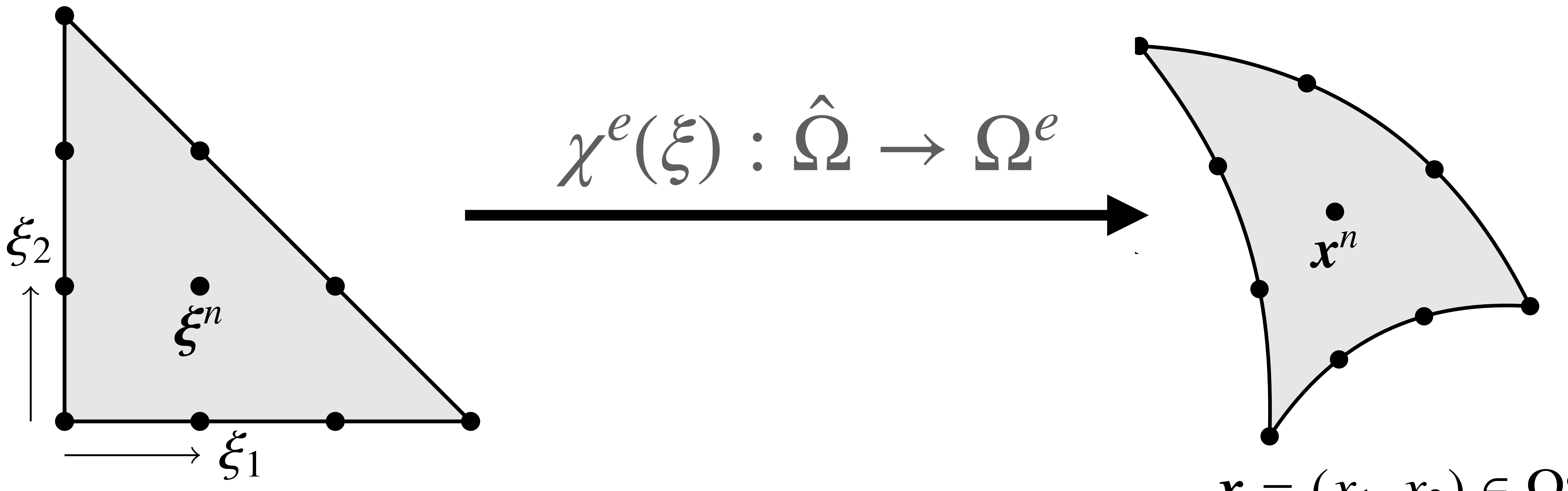


High-order mesh generation



D1.3: High-order surface mesh generation

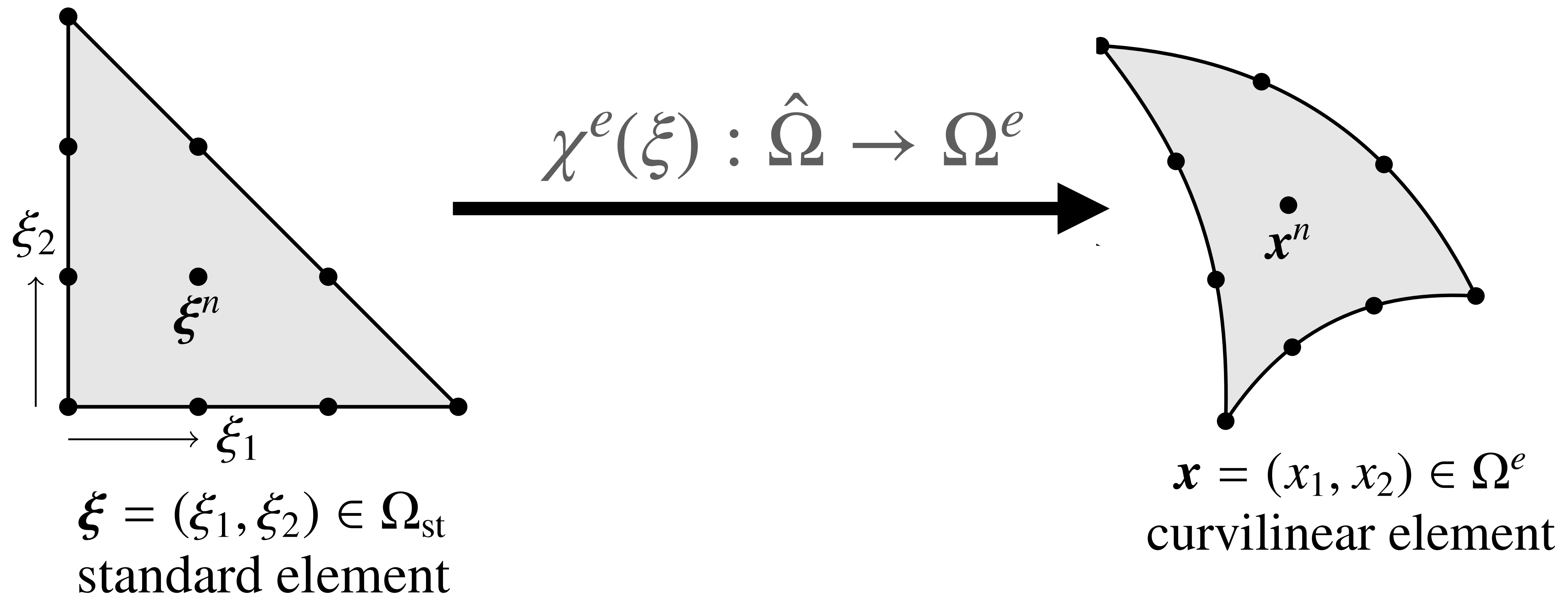
- Focus of this deliverable is around generation of **high-quality surface meshes** that accurately represent surfaces delivered from CAD.
- At high orders, the shape of a curved element is defined through a polynomial mapping, either at the same order of solution accuracy (isoparametric) or at lower/higher orders (sub/superparametric).



$\boldsymbol{\xi} = (\xi_1, \xi_2) \in \Omega_{\mathrm{st}}$
standard element

$\boldsymbol{x} = (x_1, x_2) \in \Omega^e$
curvilinear element

D1.3: High-order surface mesh generation



- Definition of the mapping depends on specifying a spatial position $\boldsymbol{x}^n$ for each nodal point ξ^n in the standard element.

- Assuming the *linear mesh* has been optimised, vertices of each triangle are fixed. But how do we select the positions of nodal points, so that the high-order element is a 'good' approximation of the surface?

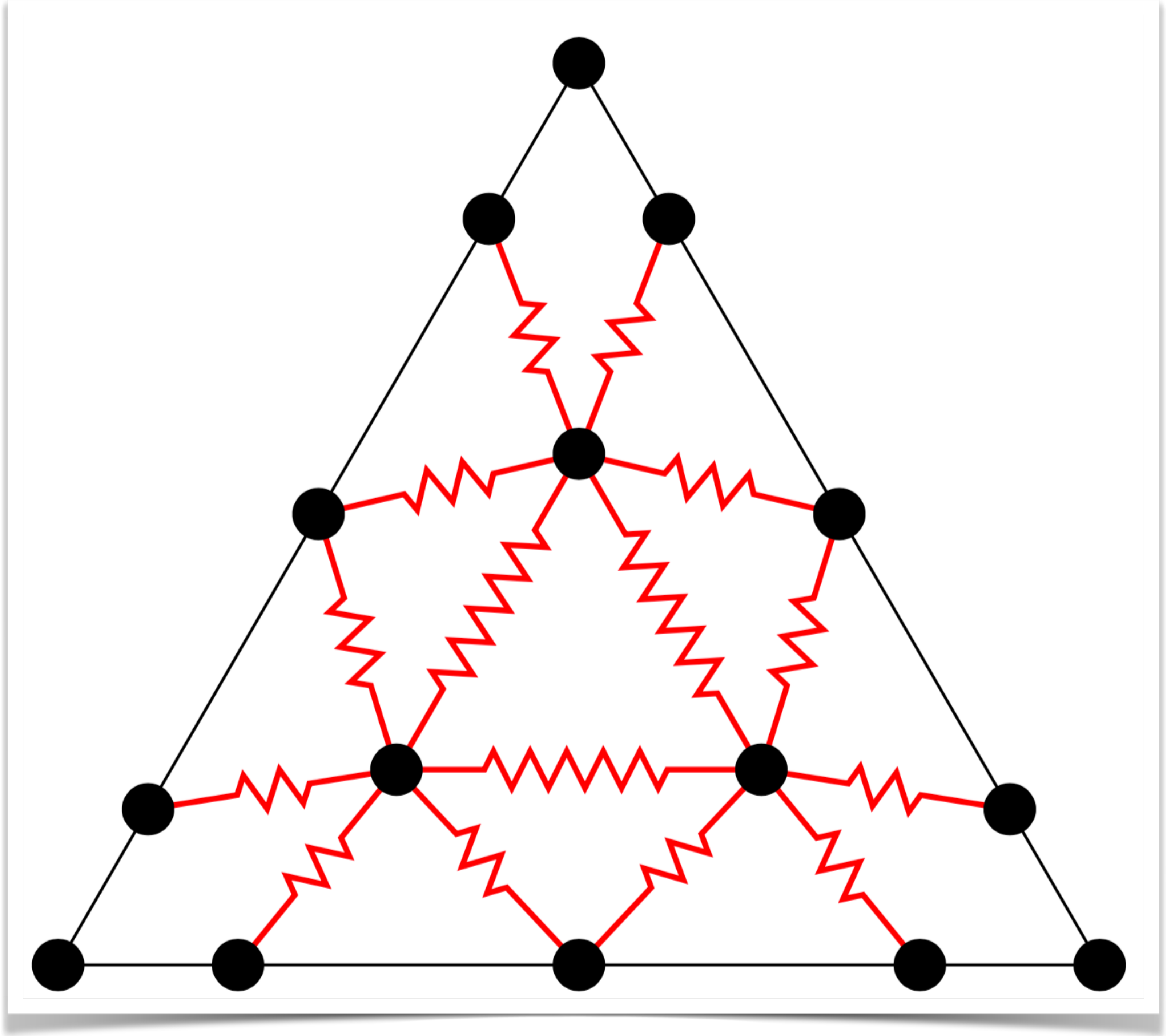
- The CAD engine gives us the ability to query a given surface/curve depending on its own parametrisation (splines, NURBS, etc), but doesn't solve this problem.

Surface node placement

- Better surface representations can be obtained via optimisation of the nodal positions.
- Simple spring analogy, each of the interior/edge nodes are connected via springs of strength proportional to their length. For example, on a CAD curve connected to a surface we find

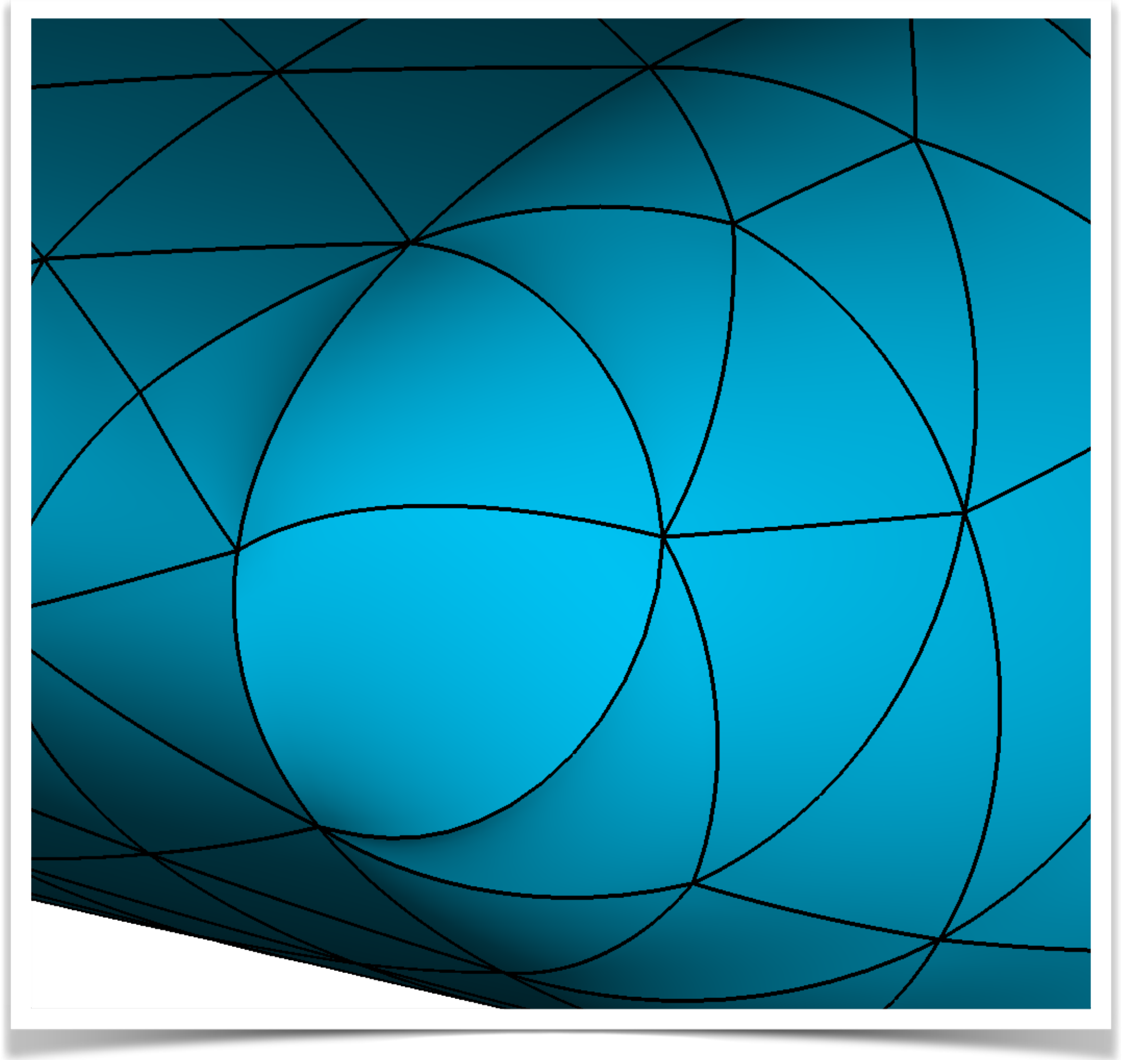
$$\min_{\mathbf{u},\mathbf{v}} f(\mathbf{u},\mathbf{v}) = \sum_{s} \frac{\mathbf{x}(u_1^s, v_1^s) - \mathbf{x}(u_2^s, v_2^s)}{w_s}$$

- We use a bounded BFGS optimisation strategy with analytic derivative (using CAD to query gradients) to perform optimisation.

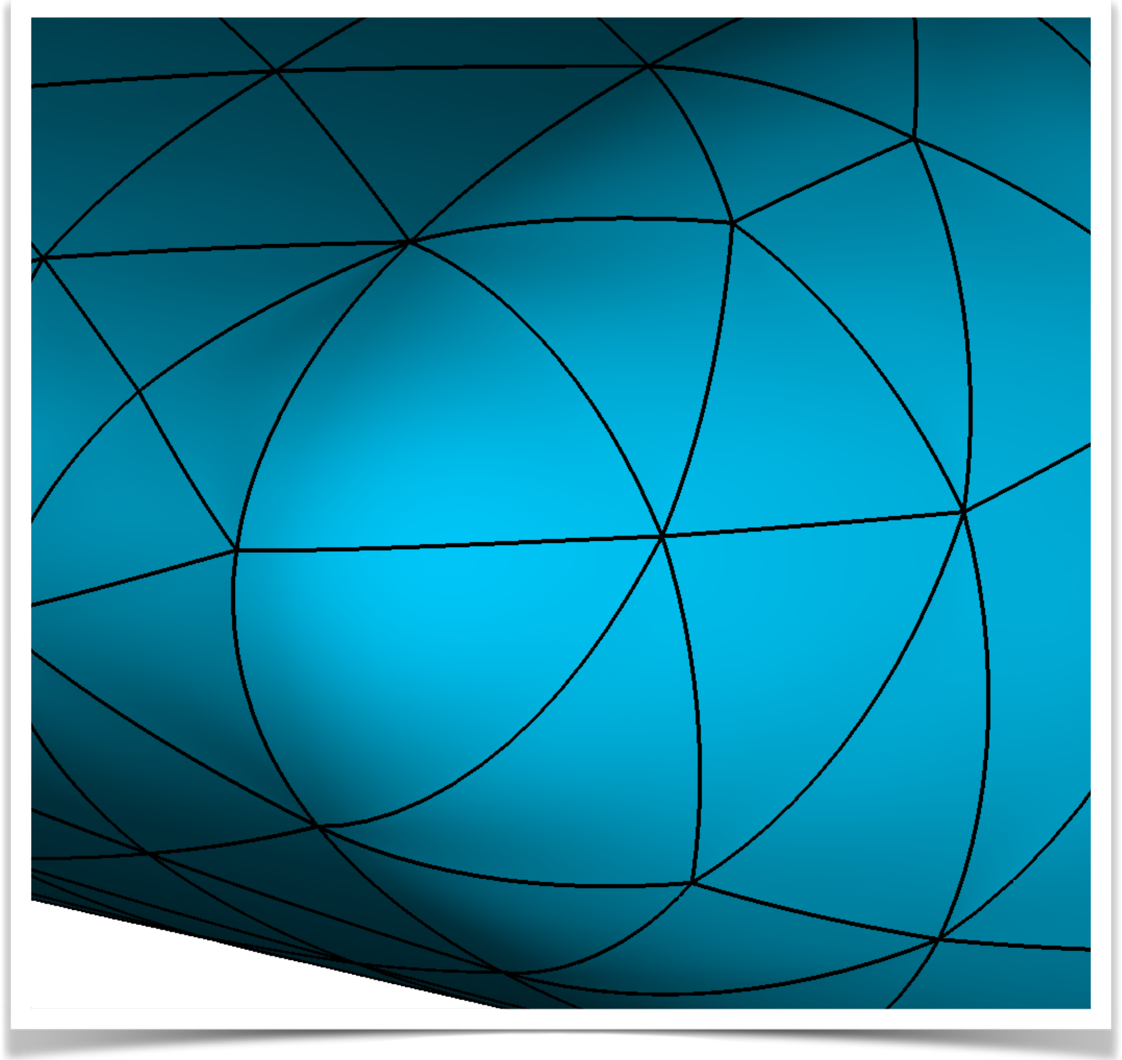


Surface node placement

- Applying optimisation greatly increases surface quality representation. Similar techniques (variational optimisation) can further improve element quality.
- In report quantifies attainable surface mesh quality in two metrics as listed in call:
    - accuracy of normal approximation vs. CAD surface;
    - L^2 distance from CAD to FE surface.
- Using sample geometries from UKAEA & other known-good complex geometries to illustrate the methodology.



before optimisation



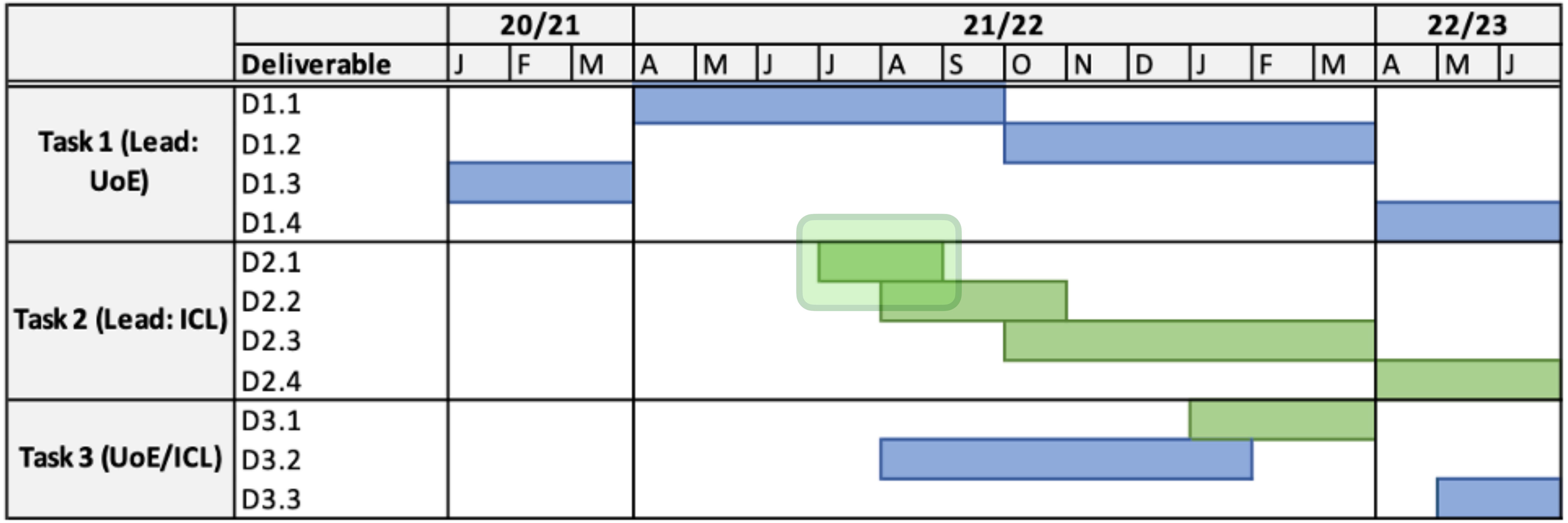
after optimisation

Looking ahead

- **D2.1:** A number of developments around the anisotropic heat transport problem to be presented later:

$$\nabla \cdot (\kappa_{\parallel}\mathbf{b}[\mathbf{b} \cdot \nabla T] + \kappa_{\perp}(\nabla T - \mathbf{b}[\mathbf{b} \cdot \nabla T])) = 0$$

- Have recruited students at ICL to help investigate aspects of this further.



Looking ahead

- **D1.2:** Working on improving software quality of quad-based cross-field solver in preparation for deliverable.

- **D3.2:** Have solicited input into what aspects of Nektar++ various parties are interested in. Will follow up on that this week.