Anisotropic Heat Transport

An initial exploration using the Nektar ++ Framework (D2.1 Baseline proxy-app)

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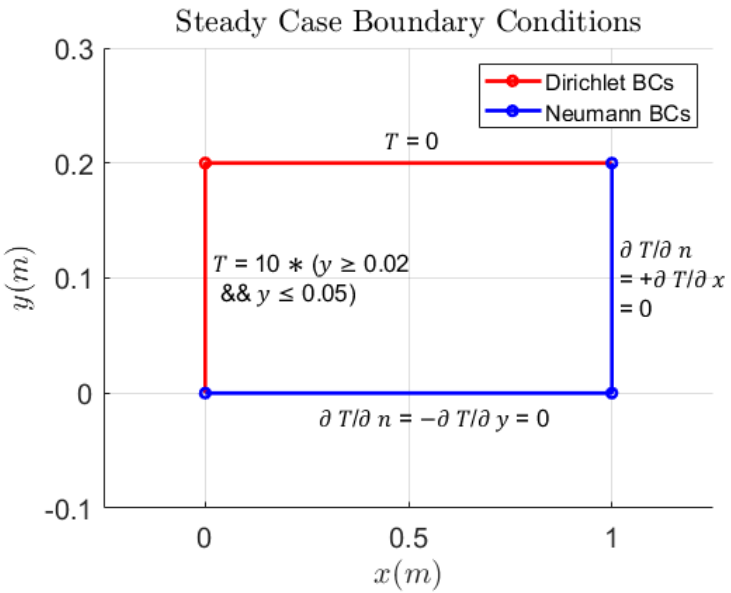
Steady State Case

$$0 = \nabla \cdot (\kappa_{\parallel} \boldsymbol{b}[\boldsymbol{b} \cdot \nabla T])$$

Initial Conditions:

$$T(x, y) = 0$$

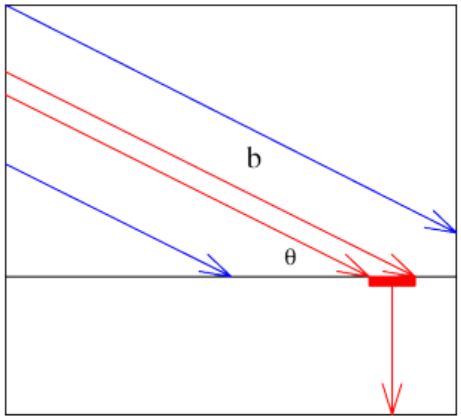


Thermal Parallel Diffusivity coefficient:

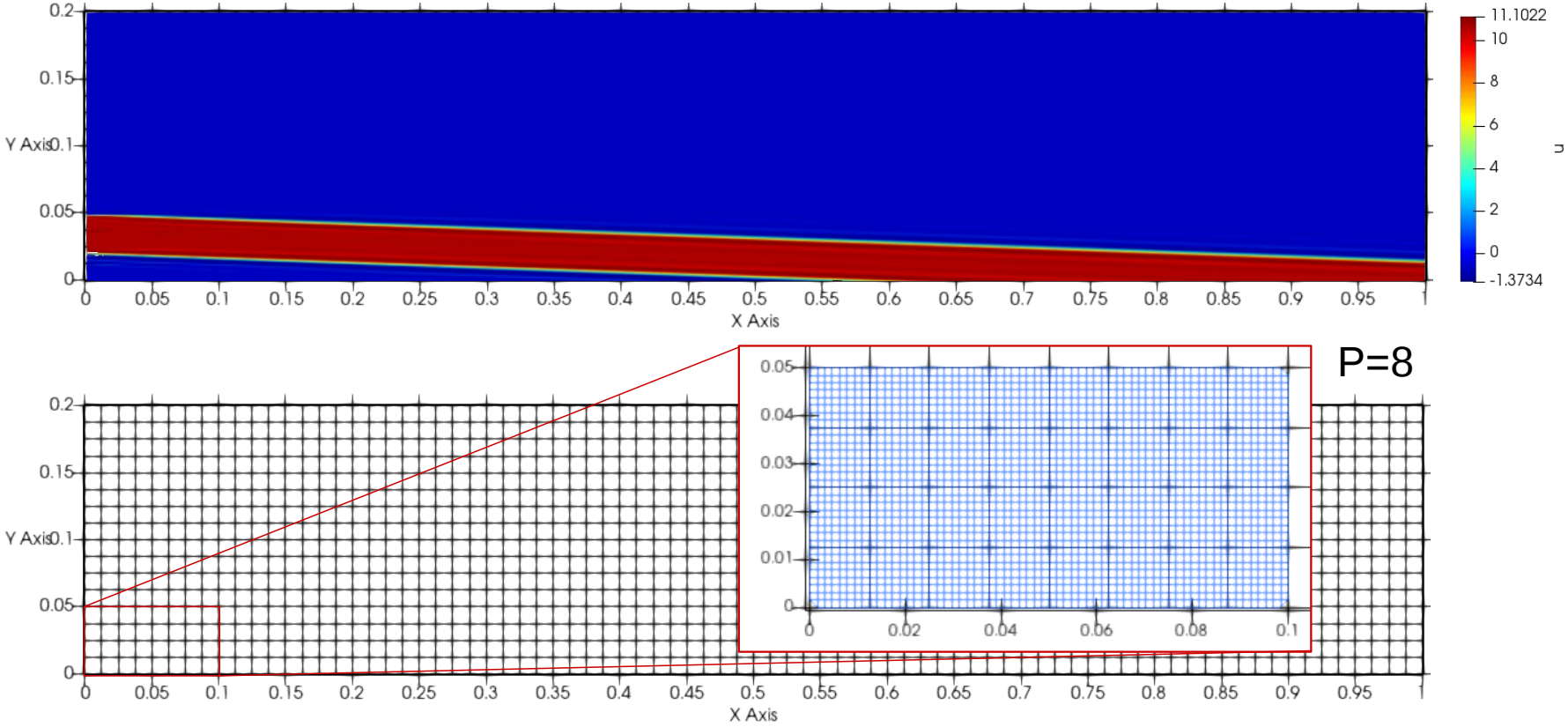
$$\kappa_{\parallel} = 12.8 \sqrt{2\pi^3} \frac{1}{\sqrt{m_e}} \frac{\epsilon_0^2}{e^4} \cdot \frac{(k_B T_e)^{\frac{5}{2}}}{Z^2 N \lambda}$$

$$= 3.1398\ e\ 6$$

$$\kappa_{\perp} = 0$$



Steady State Case



Unsteady Case

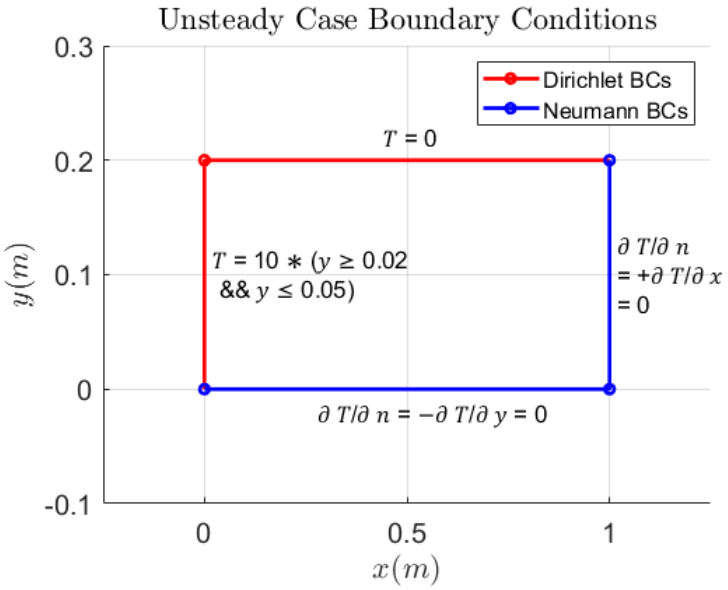
- $\frac{\partial T}{\partial t} = \frac{2}{3N} \nabla \cdot (D \cdot \nabla T)$

  where $D = \kappa_{\parallel} B^2 \begin{bmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{bmatrix}$

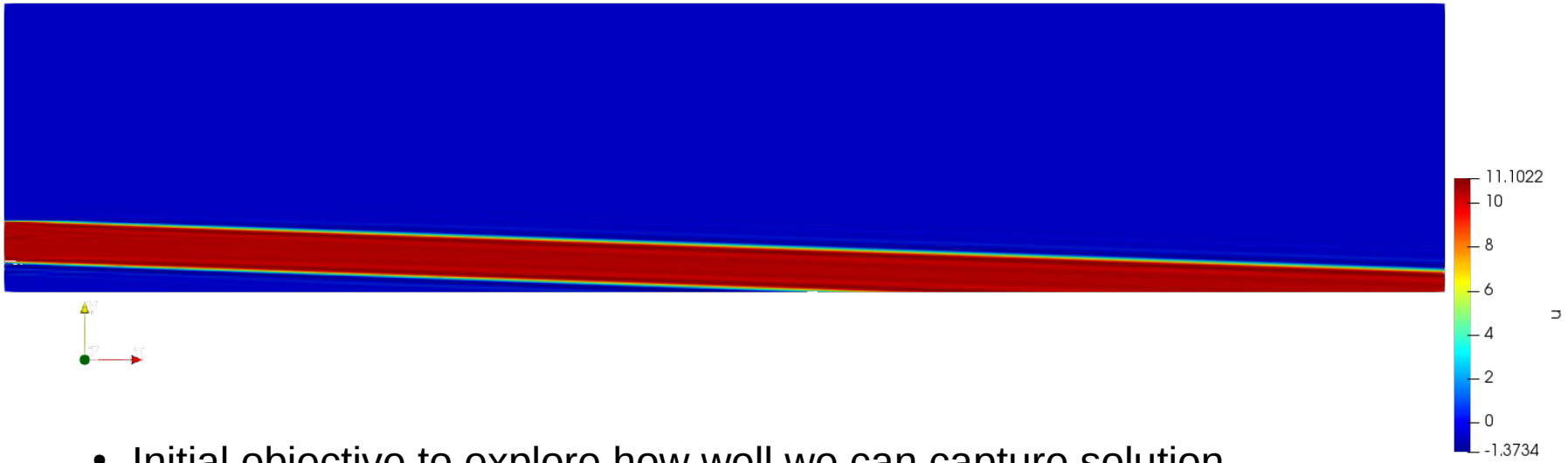
  Initial Conditions:

  $T(x, y;\ t = 0) = 0$

  - Extremely small diffusion coefficient
  - $\frac{2}{3N} \kappa_{\parallel} B^2 = 1.8839\ e\ -11$
  - Hence used N = 1 e 12
  - $\frac{2}{3N} \kappa_{\parallel} B^2 = 18.839$

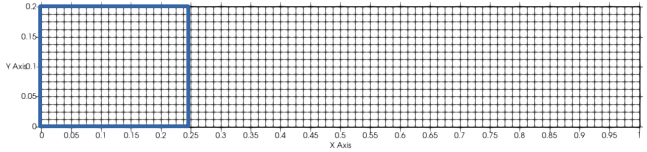


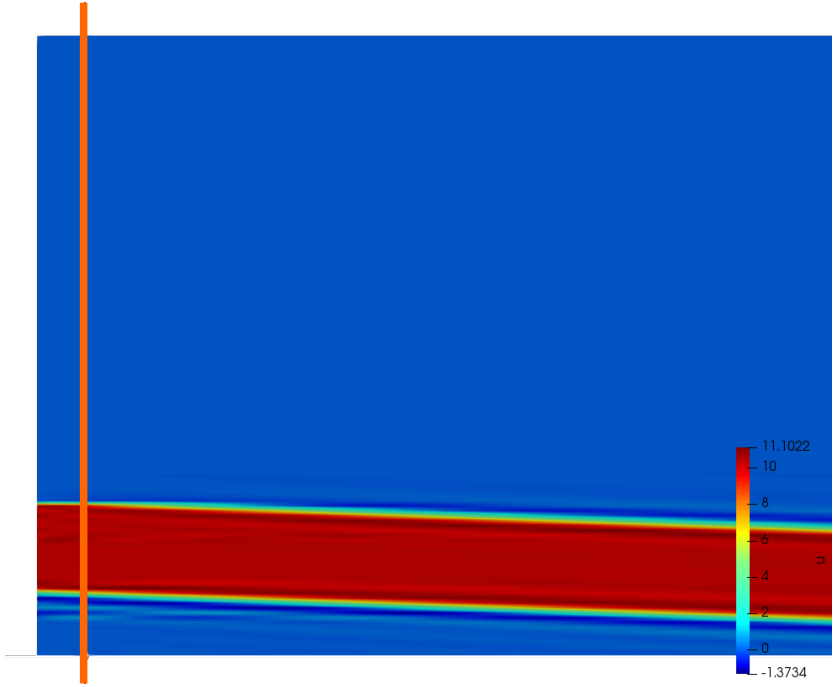
Unsteady Case

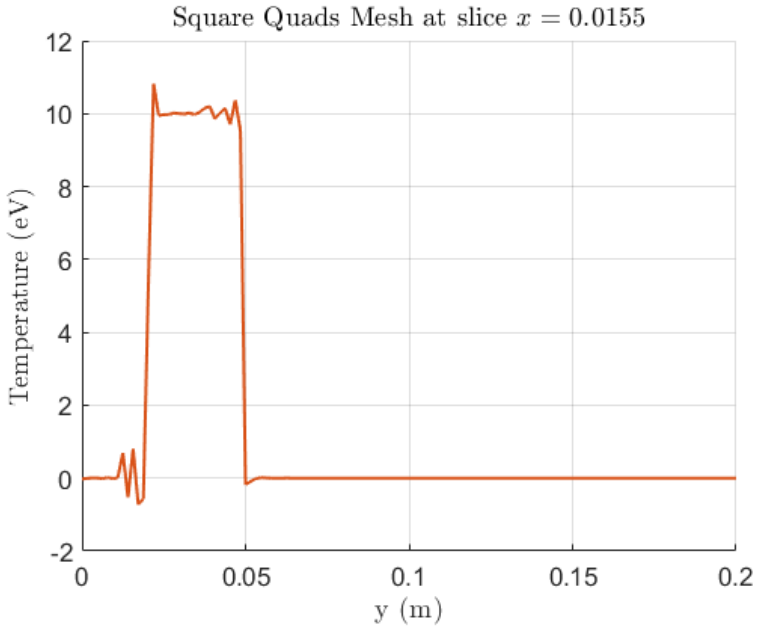


- Initial objective to explore how well we can capture solution with a tailored mesh

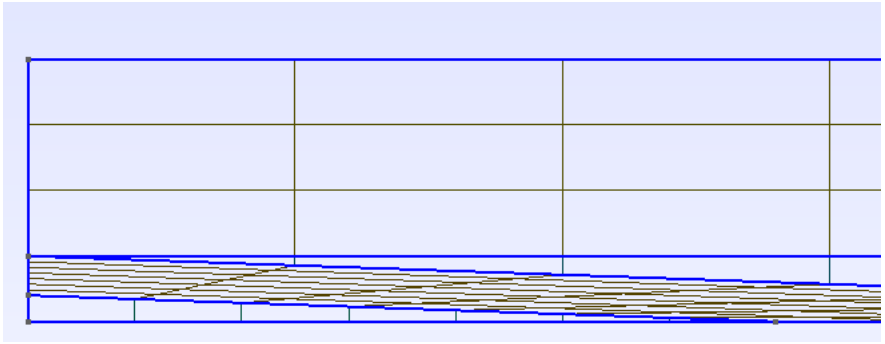
Spatial Discretisation Errors (P=8)

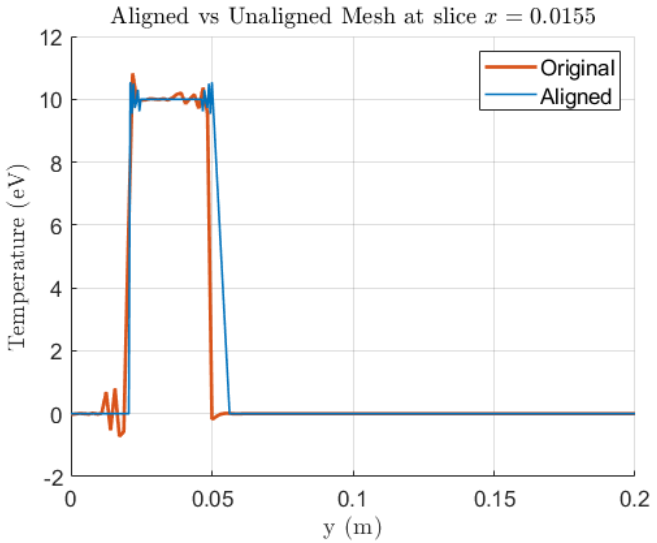






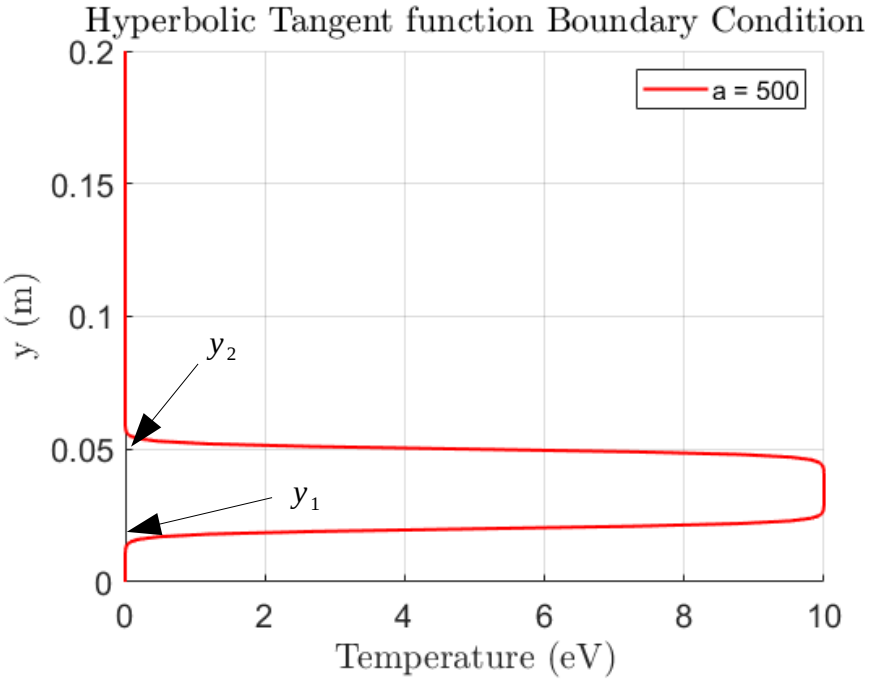
Mesh alignment with B

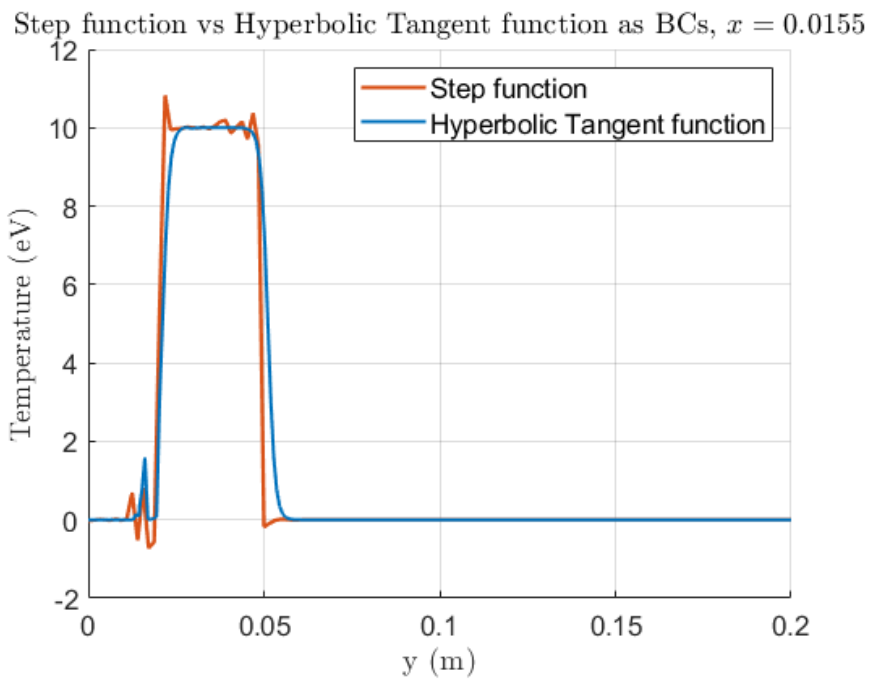




Smoothing of boundary condition

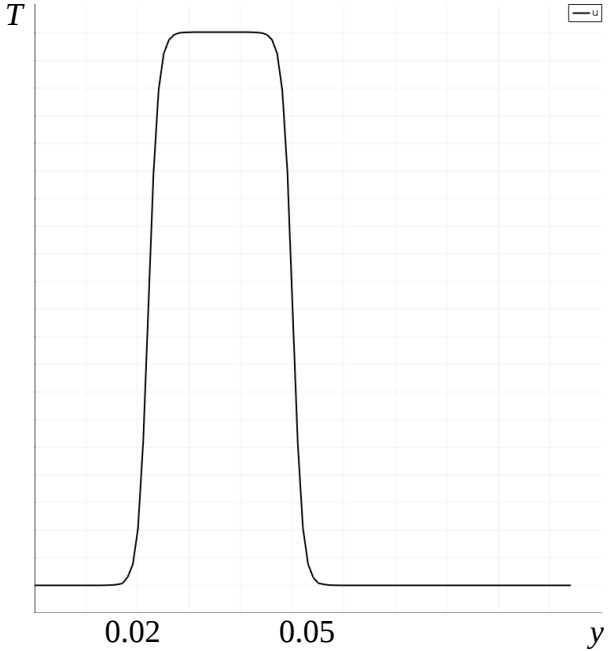
$$T(y)=1+\tfrac{1}{2}\tanh(a(y-y_1))\tanh(a(y_2-y))$$

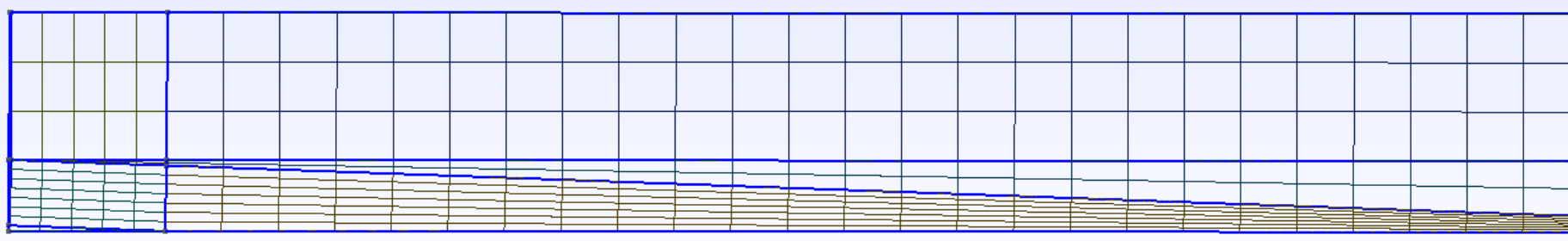




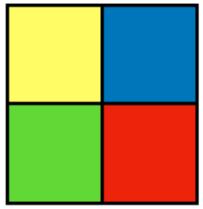
Smooth BC with Improved mesh

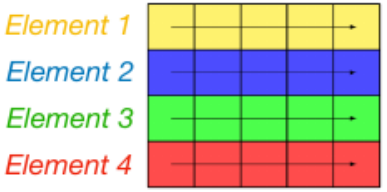
- Improved mesh element distribution
- Captures smoothed BC



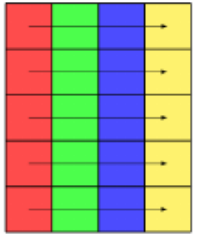


Vectorisation of matrix-free operations (D2.2)

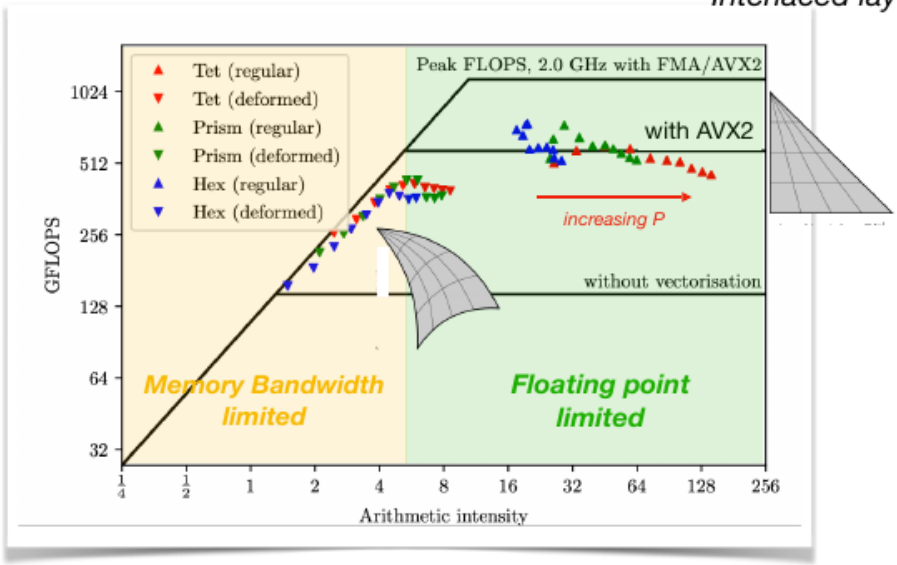




Standard contiguous layout



Interlaced layout



Imperial College London

Efficient Matrix-Free High-Order Finite Element Evaluation for Simplicial Elements., D. Moxey, R. Amici & R.M. Kirby

Next steps & Discussion

- Establish best-case result for numerical approximation.
- Is a smoothed boundary condition acceptable?
- What value of ‘a’ is acceptable?
- If we apply smoothers (e.g. SVV), what level of smoothing is acceptable?
- Investigate ability of unstructured meshes to capture solution.
- Mixed structured mesh (in transition boundary) with unstructured triangles elsewhere.