4. Data Preprocessing

Knowledge Discovery in Databases with Exercises

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Outline

1. Overview
2. Data Cleaning
3. Data Integration
4. Data Reduction
5. Data Transformation and Data Discretization
6. Summary

Overview

Data Quality: Why Preprocess the Data?

- **Measures for data quality: A multidimensional view:**
  - **Accuracy:** correct or wrong, accurate or not.
  - **Completeness:** not recorded, unavailable.
  - **Consistency:** some modified but some not, dangling refs, etc.
  - **Timeliness:** timely updated?
  - **Believability:** how trustworthy is it, that the data is correct?
  - **Interpretability:** how easily can the data be understood?
  - And even many more!

Major Tasks in Data Preprocessing (I)

- **Data cleaning:**
  - Fill in missing values.
  - Smooth noisy data.
  - Identify or remove outliers.
  - Resolve inconsistencies.
- **Data integration:**
  - Integration of multiple databases.
  - Data cubes or files.

Major Tasks in Data Preprocessing (II)

- **Data reduction:**
  - Dimensionality reduction.
  - Numerosity reduction.
  - Data compression.
- **Data transformation and data discretization:**
  - Normalization.
  - Concept-hierarchy generation.

Data Cleaning

Dirty Data

- **Data in the real world is dirty.**
- **Lots of different kinds of dirty data:**
  - **Incomplete data:** lacking attributes, lacking values or containing aggregate data.
  - **Inconsistencies:** containing discrepancies in codes or names.
  - **Errors:** containing incorrect values.
  - **Noise:** containing small inaccuracies.
  - **Outliers:** containing extreme values.

Dirty Data: Incomplete Data

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- **Potential reasons:**
  - Data not yet available.
  - Technical malfunction.
  - Human error.
  - etc.
- **Potential solutions:**
  - Ignore the tuple.
  - Fill in the missing value manually.
    - Often infeasible.
  - Fill in automatically with:
    - A global constant.
    - The attribute mean.
    - The class mean.
    - The most probable value.

Mat. Nr.	Age
12345678	23
23061995	25
21241992	
	23
25052025	21
14912780	24

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Dirty Data: Inconsistencies

- **Potential reasons:**
  - Merging of data from different sources.
  - Missing conventions.
  - Human error.
  - etc.
- **Potential solutions:**
  - Manual data cleaning.
  - (Semi-)Automatic data cleaning.
    - Most often common inconsistencies can be detected and solved via rule based approaches.

Applicant	Grade
124	1.0
Michael	2.3
134	3.7
323	A-
174	2.0
123	1.6

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Dirty Data: Errors

- **Potential reasons:**
  - Malfunctions.
  - Transmission errors.
  - Human error.
  - etc.
- **Potential solutions:**
  - Ignore the tuple.
  - Manual data cleaning.
    - A subject matter expert (SME) is often needed to identify the errors.
  - (Semi-)Automatic data cleaning.
    - Errors are often highly case dependent and therefore there is no general solution.

Module	ECTS
EADEIS	5
MoL	5
DL	5
EDB	7.5
KDDmUe	6
POIS	5

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Dirty Data: Noise

- **Potential reasons:**
  - Small sensor inaccuracies.
  - Transmission errors.
  - etc.
- **Potential solutions:**
  - Data smoothing by:
    - Binning.
    - Regression.
    - Clustering.
    - etc.

Time	Temperature
08:01	14.123 ℃
08:02	14.153 ℃
08:03	14.163 ℃
08:04	14.723 ℃
08:05	14.126 ℃
08:06	14.463 ℃

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Errors $\Longleftrightarrow$ Noise

- Noise can be referred to as a special type of error.
- Not every error is noise!

Dirty Data: Outliers

- **Potential reasons:**
  - Errors.
  - Very rare events.
- **Potential solutions:**
  - If an error, treat them as one.
  - If a rare event, the outlier is interesting and can be used for further analysis.

Year	Max. Temp.
2026	32 ℃
2027	34 ℃
2028	33 ℃
2029	35 ℃
2030	61 ℃
2031	36 ℃

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2028	33 °C
2029	35 °C
2030	61 °C
2031	36 °C

Errors $\Longleftrightarrow$ Outliers

- Outliers might indicate errors.
- Not every outlier is an error!

Data Cleaning as a Process (I)

- **Data discrepancy detection:**
  - Use **metadata** (e.g. domain, range, dependency, distribution).
  - Check field overloading.
  - Check uniqueness rule, consecutive rule and null rule.
  - Use commercial tools:
    - **Data scrubbing:** use simple domain knowledge (e.g. postal code, spell-check) to detect errors and make corrections.
    - **Data auditing:** by analyzing data to discover rules and relationships to detect violators (e.g. correlation and clustering to find outliers).

Data Cleaning as a Process (II)

- **Data migration and integration:**
  - Data-migration tools: allow transformations to be specified.
  - ETL (Extraction/Transformation/Loading) tools: allow users to specify transformations through a graphical user interface.
- **Integration of the two processes.**
  - Iterative and interactive (e.g. the Potter's Wheel tool).

Data Integration

Data Integration

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- **Data integration:**
  - Combine data from multiple sources into a coherent store.
- **Schema integration:**
  - E.g. `A.cust-id` ≡ `B.cust-#`.
  - Integrate metadata from different sources.
- **Entity-identification problem:**
  - Identify the same real-world entities from multiple data sources.
  - E.g. Bill Clinton = William Clinton.
- **Detecting and resolving data-value conflicts:**
  - For the same real world entity, attribute values from different sources are different.
  - Possible reasons:
    - Different representations (coding).
    - Different scales, e.g. metric vs. British units.

Handling Redundancy in Data Integration

- **Redundant data often occur when integrating multiple databases.**
  - **Object (entity) identification:**
    The same attribute or object may have different names in different databases.
  - **Derivable data:**
    One attribute may be a "derived" attribute in another table. E.g. annual revenue.
- **Redundant attributes:**
  - Can be detected by **correlation analysis** and **covariance analysis**.
- **Careful integration of the data from multiple sources:**
  - Helps to reduce/avoid redundancies and inconsistencies and improve mining speed and quality.

Correlation Analysis for Nominal Data (I)

- **Example:**
  We want to determine if the interests "Reads Books" and "Plays Chess" in the following table correlate with each other:

ID	Reads Books	Plays Chess
1	Y	Y
2	Y	Y
3	Y	N
...	...	...
1499	N	Y
1500	N	N

Correlation Analysis for Nominal Data (II)

- **General starting point:**
  - **The attributes A and B to be analyzed:**
    - A has n distinct values: $A := \{a_1, a_2, \dots, a_n\}$, where $n \in \mathbb{N}_{>1}$.
    - B has m distinct values: $B := \{b_1, b_2, \dots, b_m\}$, where $m \in \mathbb{N}_{>1}$.
  - **The set X of all distinct combinations:**
    - X is defined as follows: $X := \{(a, b) \mid a \in A \text{ and } b \in B\}$.
  - **The multi set Y of all tuples:**
    - The multiset Y over the set X is a mapping of X to the set of natural numbers $\mathbb{N}_0$. The number $Y(x)$, $x \in X$ tells how often x is contained in the multiset Y.

- **Starting point in the example:**
  - **The attributes A and B to be analyzed:**
    - A ("Reads Books") has 2 distinct values: $A := \{Y, N\}$
    - B ("Plays Chess") has 2 distinct values: $B := \{Y, N\}$
  - **The set X of all distinct combinations:**
    - X contains 4 distinct combinations: $X := \{(Y, Y), (Y, N), (N, Y), (N, N)\}$.
  - **The multi set Y of all tuples:**
    - Y contains 1500 tuples: $Y := \{(Y, Y), (Y, Y), \dots, (N, N)\}$.

Correlation Analysis for Nominal Data (III)

- **Actual quantity in Y:**

$$c_{ij} = \#\{(a, b) \in Y \mid a = a_i, b = b_j\} = Y((a_i, b_j))$$

- **Expected quantity (value of c_{ij}) in case of independence, i. e. no correlation:**

$$e_{ij} = \frac{\sum_{k=1}^{m} c_{ik}}{\#Y} \cdot \frac{\sum_{l=1}^{n} c_{lj}}{\#Y} \cdot \#Y = \frac{\sum_{k=1}^{m} c_{ik} \cdot \sum_{l=1}^{n} c_{lj}}{\#Y}$$

Please note that:

- The sum of all c_{ij} over an attribute a_i (or b_j) is identical to the sum of all e_{ij} over a_i (or b_j):

$$\sum_{k=1}^{m} e_{ik} = \sum_{k=1}^{m} c_{ik} \text{ and } \sum_{l=1}^{n} e_{lj} = \sum_{l=1}^{n} c_{lj}$$

Correlation Analysis for Nominal Data (IV)

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- **The values c_{ij} and e_{ij} are often presented in a contingency table:**

	a_1	$\dots$	a_n	
b_1	$c_{11}(e_{11})$	$\dots$	$c_{n1}(e_{n1})$	$\sum_{i=1}^{n} e_{i1}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
b_m	$c_{1m}(e_{1m})$	$\dots$	$c_{nm}(e_{nm})$	$\sum_{i=1}^{n} e_{im}$
	$\sum_{j=1}^{m} e_{1j}$	$\dots$	$\sum_{j=1}^{m} e_{nj}$	$\sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij}$

- **In our example it would look like this:**

	Plays Chess	Doesn't Play Chess	Sum (Row)
Reads Books	250 (e_{11})	200 (e_{21})	450
Doesn't Read Books	50 (e_{12})	1000 (e_{22})	1050
Sum (Column)	300	1200	1500

Expected Quantity for "Plays Chess" & "Reads Books"

$$e_{11} = \frac{\sum_{k=1}^{m} c_{1k} \cdot \sum_{l=1}^{n} c_{l1}}{\#Y} = \frac{\quad\cdot\quad}{\qquad} =$$

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Expected Quantity for "Plays Chess" & "Reads Books"

$$e_{11} = \frac{\sum_{k=1}^{m} c_{1k} \cdot \sum_{l=1}^{n} c_{l1}}{\#Y} = \frac{300 \cdot 450}{} =$$

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	a_1	...	a_n	
b_1	$c_{11}(e_{11})$	...	$c_{n1}(e_{n1})$	$\sum_{i=1}^{n} e_{i1}$
...	...	...	...	...
b_m	$c_{1m}(e_{1m})$	...	$c_{nm}(e_{nm})$	$\sum_{i=1}^{n} e_{im}$
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Sum (Column)	300	1200	1500

Expected Quantity for "Plays Chess" & "Reads Books"

$$e_{11} = \frac{\sum_{k=1}^{m} c_{1k} \cdot \sum_{l=1}^{n} c_{l1}}{\#Y} = \frac{300 \cdot 450}{1500} = 90$$

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- **In our example it would look like this:**

	Plays Chess	Doesn't Play Chess	Sum (Row)
Reads Books	250 (90)	200 (360)	450
Doesn't Read Books	50 (210)	1000 (840)	1050
Sum (Column)	300	1200	1500

Expected Quantity for "Plays Chess" & "Reads Books"

$$e_{11} = \frac{\sum_{k=1}^{m} c_{1k} \cdot \sum_{l=1}^{n} c_{l1}}{\#Y} = \frac{300 \cdot 450}{1500} = 90$$

Correlation Analysis for Nominal Data (V)

- **To determine the correlation the χ^2-test (Chi-squared test) is applied:**

$$\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(c_{ij} - e_{ij})^2}{e_{ij}}.$$

- **Calculation of χ^2 in our example:**

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93.$$

Correlation Analysis for Nominal Data (VI)

Null hypothesis of the χ^2-test

- The χ^2-test is used to test the null hypothesis H_0 of independence (i.e. no correlation).

- Which χ^2 value indicates correlation?
  - The χ^2 value is compared with a critical value from the χ^2 distribution (see table on the next slide).
  - Before that is done the degrees of freedom (df) must be calculated:

$$\text{df} = (n-1) \cdot (m-1)$$

    Where n is the count of distinct values in A and m of distinct values in B.
  - And a significance level α must be defined (e.g. $\alpha = 0.005$).

Correlation Analysis for Nominal Data (VII)

- **In our example:**
  - The degrees of freedom (df) are:

$$\text{df} = (2-1)\cdot(2-1) = 1.$$

df/α	**0.025**	**0.010**	**0.005**
1	5.024	6.635	7.879
2	7.378	9.210	10.597
3	9.348	11.345	12.838
4	11.143	13.277	14.860
5	12.833	15.086	16.750
6	14.449	16.812	18.548
7	16.013	18.475	20.278
8	17.535	20.090	21.955
9	19.023	21.666	23.589

[1] Good link for a full table: `https://www.hawkeslearning.com/documents/statdatasets/stat_tables.pdf`

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  - The critical value from the χ^2 distribution[1] is:

$$\chi^2_{0.005,1} = 7.879.$$

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  - The critical value from the χ^2 distribution[1] is:

$$\chi^2_{0.005,1} = 7.879.$$

  - Our χ^2-value is bigger than the critical value:

$$\chi^2 = 507.93 > 7.879.$$

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$$\text{df} = (2-1) \cdot (2-1) = 1.$$

  - We set the significance level to $\alpha = 0.005$
  - The critical value from the χ^2 distribution[1] is:

$$\chi^2_{0.005,1} = 7.879.$$

  - Our χ^2-value is bigger than the critical value:

$$\chi^2 = 507.93 > 7.879.$$

  - Therefore we reject the null hypothesis H_0 and conclude that there is correlation between the two attributes.

df/α	**0.025**	**0.010**	**0.005**
1	5.024	6.635	7.879
2	7.378	9.210	10.597
3	9.348	11.345	12.838
4	11.143	13.277	14.860
5	12.833	15.086	16.750
6	14.449	16.812	18.548
7	16.013	18.475	20.278
8	17.535	20.090	21.955
9	19.023	21.666	23.589

[1] Good link for a full table: `https://www.hawkeslearning.com/documents/statdatasets/stat_tables.pdf`

Correlation Analysis of Numerical Data (I)

- **Numerical correlation can be determined with Pearson's product-moment coefficient:**

$$\mathrm{Cor}(A,B) = \frac{\sum_{i=1}^{n}(a_i-\mu_A)(b_i-\mu_B)}{n\cdot\sigma_A\sigma_B} = \frac{\sum_{i=1}^{n} a_i b_i - n\cdot\mu_A\mu_B}{n\cdot\sigma_A\sigma_B}.$$

where n is the number of tuples, a_i and b_i are the respective values of A and B in tuple i, μ_A and μ_B are the respective mean values of A and B, σ_A and σ_BB are the respective standard deviations of A and B

Properties of Pearson's product-moment coefficient

- If $\mathrm{Cor}(A,B) > 0$: A and B are positively correlated (the closer to 1, the stronger the correlation).
- If $\mathrm{Cor}(A,B) = 0$: A and B are independent.
- If $\mathrm{Cor}(A,B) < 0$: A and B are negatively correlated (the closer to -1, the stronger the correlation).

Correlation Analysis of Numerical Data (II)

- **It is also possible to visually detect numerical correlation:**

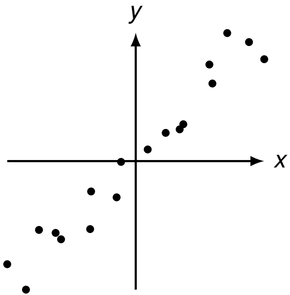


Figure: a) Positive correlation.

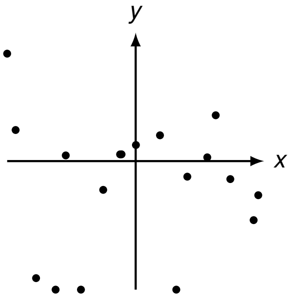


Figure: b) Uncorrelated/no correlation.

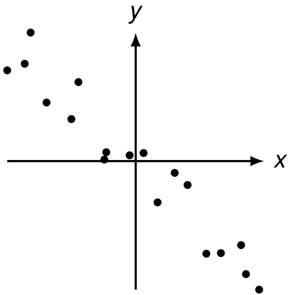
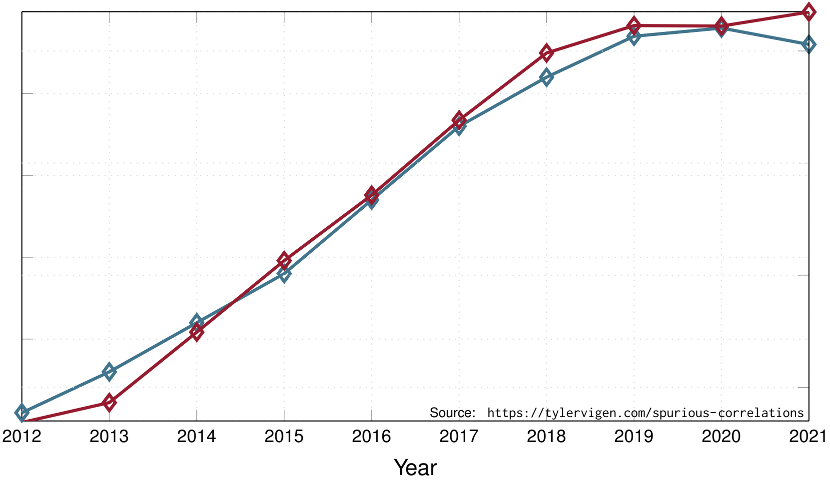
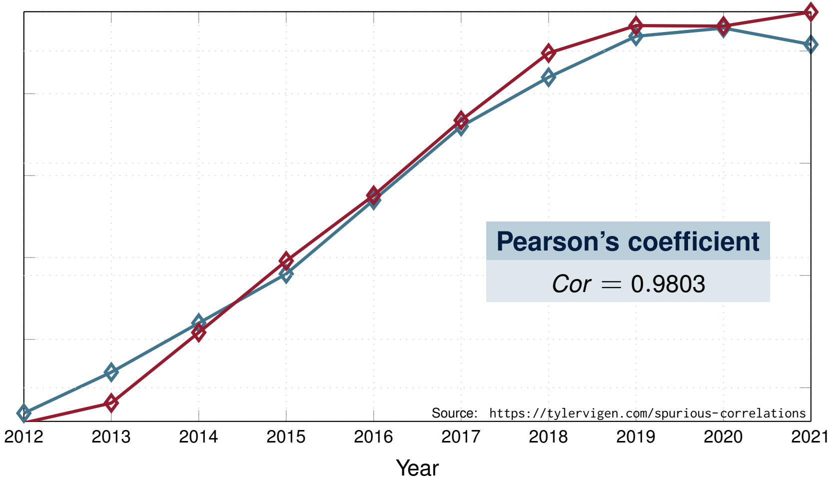


Figure: c) Negative correlation.

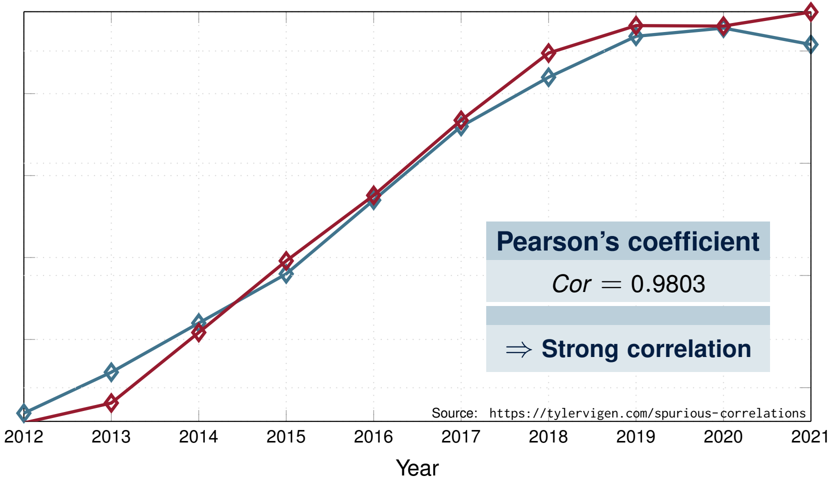
Correlation vs. Causality



Correlation vs. Causality



Correlation vs. Causality

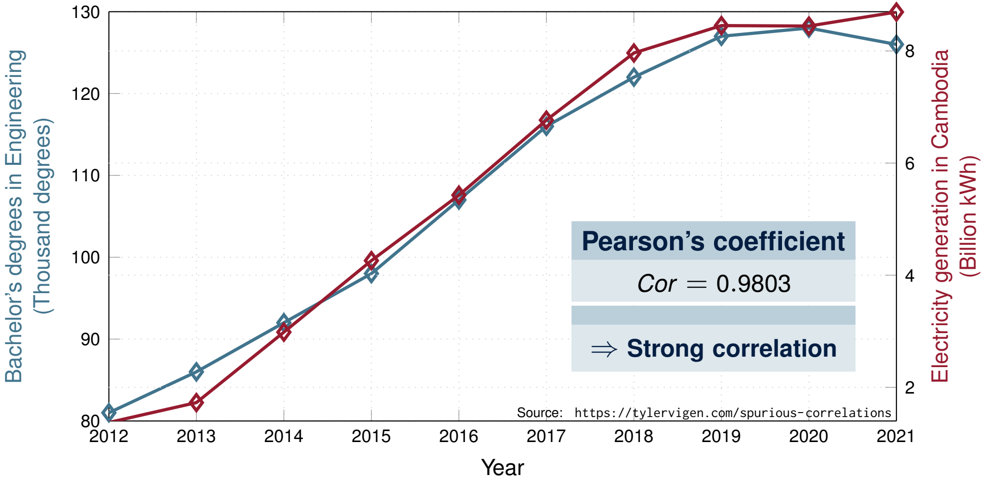


Pearson's coefficient

$Cor = 0.9803$

$\Rightarrow$ **Strong correlation**

Correlation vs. Causality

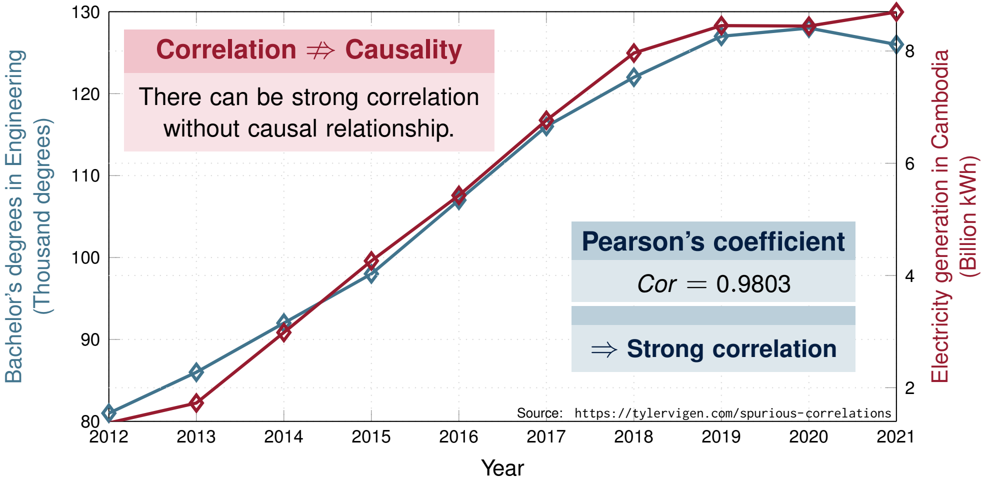


Pearson's coefficient

$Cor = 0.9803$

$\Rightarrow$ **Strong correlation**

Correlation vs. Causality



Correlation $\not\Rightarrow$ Causality

There can be strong correlation without causal relationship.

Pearson's coefficient

$Cor = 0.9803$

$\Rightarrow$ **Strong correlation**

Data Reduction

Data Reduction (I)

- **What is data reduction?**
  - Obtain a reduced representation of the data set that is much smaller in volume but yet produces the same (or almost the same) results.
- **Why data reduction?**
  - A database/data warehouse may store terabytes of data.
  - Complex data analysis may take a very long time to run on the complete data set.
- **Data reduction strategies:**
  - Dimensionality reduction, i.e. remove unimportant attributes.
    - Wavelet transforms.
    - Principal component analysis.
    - Attribute subset selection or attribute creation.

Data Reduction (II)

- **Data reduction strategies (continued):**
  - Numerosity reduction:
    - Regression and log-linear models.
    - Histograms, clustering and sampling.
    - Data cube aggregation.
  - Data compression.

Data Reduction (I): Dimensionality Reduction

- **Curse of dimensionality:**
  - When dimensionality increases data becomes increasingly sparse.
  - Density and distance between points become less meaningful.
  - The possible combinations of subspaces will grow exponentially.
- **Dimensionality reduction:**
  - Avoid the curse of dimensionality.
  - Help eliminate irrelevant features and reduce noise.
  - Reduce time and space required in data mining.
  - Allow easier visualization.
- **Dimensionality-reduction techniques:**
  - Wavelet transforms.
  - Principal component analysis.
  - Supervised and nonlinear techniques (e.g. feature selection).

Wavelet Transform

- **Discrete wavelet transform:**
  Transforms a vector X into a different vector X' of wavelet coefficients with the same length.
- **Compressed approximation:**
  Store only a small fraction of the strongest of the wavelet coefficients.
- **Similar to discrete fourier transform, but better lossy compression, localized in space.**
- **Method:**
  - The length of the vector must be an integer power of 2 (padding with 0's if necessary).
  - Each transform has two functions: smoothing and difference.
  - Applied to pairs of data, resulting in two sets of data with half the length.
  - The two functions are applied recursively until reaching the desired length.

Example: Wavelet Transform (I)

- **Initial vector:**
  - $X = (2, 2, 0, 2, 3, 5, 4, 4)$
- **First step:**
  - $(2, 2) \rightarrow$ Average: 2, Weighted difference: 0
  - $(0, 2) \rightarrow$ Average: 1, Weighted difference: -1
  - $(3, 5) \rightarrow$ Average: 4, Weighted difference: -1
  - $(4, 4) \rightarrow$ Average: 4, Weighted difference: 0
  - $A_1 = (2, 1, 4, 4), D_1 = (0, -1, -1, 0)$
- **Second step:**
  - $(2, 1) \rightarrow$ Average: 1.5, Weighted difference: 0.5
  - $(4, 4) \rightarrow$ Average: 4, Weighted difference: 0
  - $A_2 = (1.5, 4), D_2 = (0.5, 0)$

Example: Wavelet Transform (II)

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- **Third step:**
  - $(1.5, 4) \rightarrow$ Average: 2.75, Weighted difference: -1.25
  - $A_3 = (2.75)$, $D_3 = (-1.25)$
- **Resulting vector:**
  - $X' = (2.75, -1.25, 0.5, 0, 0, -1, -1, 0)$
- **Possible compression:**
  - Small detail coefficients ($D_{1,2,3}$) can be replaced by 0's, while retaining significant coefficients.

Resolution	Averages	Detail coefficients
8	$(2, 2, 0, 2, 3, 5, 4, 4)$	-
4	$(2, 1, 4, 4)$	$(0, -1, -1, 0)$
2	$(1.5, 4)$	$(0.5, 0)$
1	(2.75)	(-1.25)

Principal Component Analysis (PCA)

- **Main idea:**
  - Given a data set with n dimensions.
  - Find $k \leq n$ orthogonal vectors that capture the largest amount of data.
  - Works only for numeric data.
- **Example data set:**
  - Used on the next few slides to explain the steps of a PCA:

d_1	d_2	d_3
23	6	1
9	9	5
17	5	1
3	6	1

PCA - 1. Step: Standardization (I)

- **Procedure:**
  - Each value x within a dimension d_n is standardized with the help of the mean (μ_{d_n}) and standard deviation (σ_{d_n}) of d_n:

$$x' = \frac{x - \mu_{d_n}}{\sigma_{d_n}}$$

- **Reason:**
  - Each dimension should be considered equally in the analysis.
  - Dimensions with a wider range of values would dominate without this step.

PCA - 1. Step: Standardization (II)

- **Example:**
  - Mean and standard deviation per dimension:

	d_1	d_2	d_3
μ	13.000000	6.500000	2.0
σ	8.793937	1.732051	2.0

  - Standardized data set:

d_1	d_2	d_3
$+1.137147$	-0.288675	-0.5
-0.454859	$+1.443376$	$+1.5$
$+0.454859$	-0.866025	-0.5
-1.137147	-0.288675	-0.5

PCA - 2. Step: Covariance Matrix (I)

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- **Procedure:**
  - A n x n covariance matrix is generated that contains the covariance between each possible attribute pairing. When the dimensions are compared with themselves, the variance always replaces the covariance:

$$\begin{bmatrix} Var(d_1) & \dots & Cov(d_1, d_n) \\ \dots & \dots & \dots \\ Cov(d_n, d_1) & \dots & Var(d_n) \end{bmatrix}$$

- **Reason:**
  - Dimensions that are highly correlated contain redundant information.
  - This step helps to identify these correlations.

- **Example:**
  - The 3 x 3 covariance matrix of our example:

	d_1	d_2	d_3
d_1	$+1.000000$	-0.350150	-0.303239
d_2	-0.350150	$+1.000000$	$+0.962250$
d_3	-0.303239	$+0.962250$	$+1.000000$

PCA - 3. Step: Eigenvalues (I)

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- **Procedure:**
  - The eigenvectors and eigenvalues of the covariance matrix (C) are computed by solving the following equation:

$$C\nu = \lambda\nu$$

  - If an n digit vector ν satisfies this equation for a $\lambda \in \mathbb{R}$, then ν is called an eigenvector with associated eigenvalue λ
- **Reason:**
  - The determined eigenvectors are called **principal components** of the dataset. The eigenvalues indicate which of these principal components has which importance for the significance of the dataset.
  - By sorting the eigenvectors in descending order according to their eigenvalues, the principal components that contain the most information can be identified.

PCA - 3. Step: Eigenvalues (II)

- **Example:**
  - Eigenvalues and eigenvectors in our example:

$$\lambda_1 = +2.14823654, \nu_1 = \begin{bmatrix} +0.37342507 \\ -0.92684562 \\ -0.03887043 \end{bmatrix}$$

$$\lambda_2 = +0.81530433, \nu_2 = \begin{bmatrix} -0.66009198 \\ -0.23604255 \\ -0.71313568 \end{bmatrix}$$

$$\lambda_3 = +0.03645914, \nu_3 = \begin{bmatrix} -0.6517916 \\ -0.2919608 \\ +0.69994757 \end{bmatrix}$$

  - Sorting these three eigenvectors by their significance, we arrive at the order ν_1, ν_2, ν_3

PCA - 4. Step: Feature matrix (I)

- **Procedure:**
  - The top N eigenvectors are selected to create a feature matrix from them.
  - There is no fixed rule exactly how many eigenvectors should be selected.
  - The dimensionality reduction is larger the fewer eigenvectors are chosen.
  - The information loss increases with each eigenvector that is discarded.
- **Reason:**
  - It must be considered carefully how much information can be given up in favor of dimensionality reduction.

PCA - 4. Step: Feature matrix (II)

- **Example:**
  - In our example ν_1 carries approx. 72% of the information:

$$\frac{2.14823654}{2,14823654 + 0,81530433 + 0,03645914} = 0.71607885$$

  - It might be interesting to keep only the eigenvector ν_1 and discard the other two eigenvectors. Our feature matrix therefore looks as follows:

$$\begin{bmatrix} +0.37342507 \\ -0.92684562 \\ -0.03887043 \end{bmatrix}$$

PCA - 5. Step: Transformation (I)

- **Procedure:**
  - The original data set (D) gets multiplied with the feature matrix (F), to create a new data set (N) with lower dimensionality:

$$N = D \cdot F$$

- **Reason:**
  - This step applies the dimensionality reduction to each tuple.
  - The PCA is completed with this step.

PCA - 5. Step: Transformation (II)

- **Example:**
  - Our dataset after the transformation and with the PCA completed looks like this:

$$\begin{bmatrix} +0.711632 \\ -1.565948 \\ +0.991963 \\ -0.137647 \end{bmatrix}$$

  - It is to be expected that this dataset still contains about 72% of its original information, which can be further used for data mining, while having to deal with a lot less dimensions.

Attribute-Subset Selection

- **Another way to reduce dimensionality of data.**
- **Redundant attributes:**
  - Duplicate much or all of the information contained in other attributes.
    - E.g. purchase price of a product and the amount of sales tax paid.
- **Irrelevant attributes:**
  - contain no information that is useful for the data-mining task at hand.
    - E.g. students' ID is often irrelevant to the task of predicting students' GPA.

Heuristic Search in Attribute Selection

- **There are 2^d possible attribute combinations of d attributes.**
- **Typical heuristic attribute-selection methods:**
  - Best single attribute under the attribute-independence assumption: choose by significance tests (e.g. t-test, see Chapter 7 "Classification").
  - Best step-wise feature selection:
    - The best single attribute is picked first.
    - Then next best attribute condition to the first ...
- **Step-wise attribute elimination:**
  - Repeatedly eliminate the worst attribute.
- Best combined attribute selection and elimination.
- Optimal branch and bound:
  - Use attribute elimination and backtracking.

Attribute Creation (Feature Generation)

- **Create new attributes (features) that can capture the important information in a data set more effectively than the original ones.**
- **Three general methodologies:**
  - Attribute extraction.
    - Domain-specific.
  - Mapping data to new space (see: data reduction).
    - E.g. Fourier transformation, wavelet transformation, manifold approaches (not covered).
  - Attribute construction:
    - Combining features (see: discriminative frequent patterns in Chapter 5).
    - Data discretization.

Data Reduction (II): Numerosity Reduction

- **Reduce data volume by choosing alternative, smaller forms of data representation.**
- **Parametric methods (e.g., regression):**
  - Assume the data fits some **model** (e.g. a function).
  - Estimate model parameters.
  - Store only the parameters.
  - Discard the data (except possible outliers):
    - Ex. log-linear models obtain value at a point in *m*-dimensional space as the product of appropriate marginal subspaces.
- **Non-parametric methods:**
  - Do not assume models.
  - Major families: histograms, clustering, sampling, . . .

Histogram Analysis

- **Divide data into buckets and store aggregate (e.g. average) of each bucket.**
- **Two different partitioning rules:**
  - **Equal-width:** equal width of each bucket.
  - **Equal-frequency (or equal-depth)**: equal number of tuples in each bucket.

Histogram Analysis

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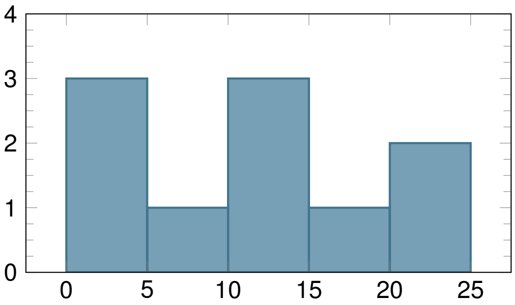

Histogram Analysis

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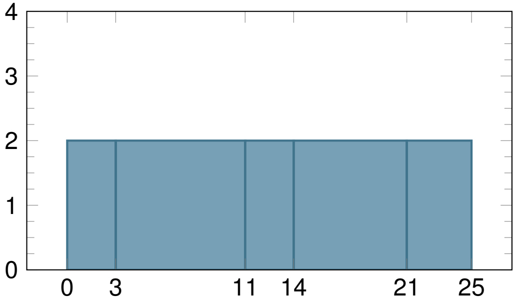

Clustering

- **Partition data set into clusters based on similarity and store cluster representation (e.g., centroid and diameter) only.**
  - Can be very effective if data points are close to each other under a certain norm and choice of space.
  - Can have hierarchical clustering and be stored in multidimensional index-tree structures.
  - There are many choices of clustering algorithms.
  - Cluster analysis will be studied in depth in Chapter 7.

Sampling

- **Obtain a small sample x to represent the whole data set X.**
- **Allow a mining algorithm to run in complexity that is potentially sub-linear to the size of the data.**
- **Key principle: Choose a representative subset of the data.**
  - Simple random sampling may have very poor performance in the presence of skew.
  - Develop adaptive sampling methods, e.g. stratified sampling.
- **Note: Sampling may not reduce database I/Os.**
  - One page at a time.

Types of Sampling

- **Simple random sampling.**
  - There is an equal probability of selecting any particular item.
- **Sampling without replacement.**
  - Once an object is selected, it is removed from the population.
- **Sampling with replacement.**
  - A selected object is not removed from the population.
- **Stratified sampling:**
  - Partition the data set and draw samples from each partition: Proportionally, i.e. approximately the same percentage of the data.
  - Used in conjunction with skewed data.

Data Reduction (III): Data Compression

- **String compression.**
  - There are extensive theories and well-tuned algorithms.
  - Typically lossless, but only limited manipulation is possible without expansion.
- **Audio/video compression.**
  - Typically lossy compression, with progressive refinement.
  - Sometimes small fragments of signal can be reconstructed without reconstructing the whole.
- **Time sequence is not audio.**
  - Typically short and varies slowly with time.
- **Dimensionality and numerosity reduction may also be considered as forms of data compression.**

Data Transformation and Data Discretization

Data Transformations

- Functions applied to a finite set of samples.
- **Methods:**
  - Smoothing: Remove noise from data.
  - Attribute/feature construction: New attributes constructed from the given ones.
  - Aggregation: Summarization, data-cube construction.
  - Normalization: Scaled to fall within a smaller, specified range.
    - Min-max normalization
    - Z-score normalization.
    - Normalization by decimal scaling.
  - Discretization: concept-hierarchy climbing.

Normalization

- **Min-max normalization (to some interval $[\min, \max]$):**

$$a_{\text{new}} = \frac{a - \min_A}{\max_A - \min_A}(\max - \min) + \min .$$

Example: let income range from \$12,000 to \$98,000 normalized to $[0, 1]$.
Then \$73,600 is mapped to $\frac{73{,}600 - 12{,}000}{98{,}000 - 12{,}000}(1 - 0) + 0 = 0.716$.

- **Z-score normalization:**

$$a_{\text{new}} := z(a) = \frac{a - \mu_A}{\sigma_A}, \text{ with } \mu \text{ being the mean and } \sigma \text{ the standard deviation.}$$

Example: let $\mu = 54{,}000$ and $\sigma = 16{,}000$. Then $\frac{73{,}000 - 54{,}000}{16{,}000} = 1.188$.

- **Normalization by decimal scaling:**

$$a_{\text{new}} = \frac{a}{10^k}, \text{ where } k \text{ is the smallest integer such that } \max(|a_{\text{new}}|) < 1.$$

Discretization

- **Three types of attributes:**
  - Nominal – values from an unordered set, e.g. color, profession.
  - Ordinal – values from an ordered set, e.g. military or academic rank.
  - Numerical – numbers, e.g. integer or real numbers.
- **Divide the value range of a continuous attribute into intervals:**
  - **Interval labels** can then be used to replace actual data values.
  - Reduce data size by discretization.
  - Supervised vs. unsupervised.
  - Split (top-down) vs. merge (bottom-up).
  - Discretization can be performed recursively on an attribute.
  - Prepare for further analysis, e.g. classification.

Data-Discretization Methods

- **Typical methods:**
  - All the methods can be applied recursively.
  - **Binning:**
    - Unsupervised, top-down split.
  - **Histogram analysis:**
    - Unsupervised, top-down split.
  - **Clustering analysis:**
    - Unsupervised, top-down split or bottom-up merge.
  - **Decision-tree analysis:**
    - Supervised, top-down split.
  - **Correlation (e.g. χ^2) analysis:**
    - Unsupervised, bottom-up merge.

Simple Discretization: Binning

- **Equal-width (distance) partitioning:**
  - Divides the range into N intervals of equal size: uniform grid.
  - If A and B are the lowest and highest values of the attribute, the width of intervals will be: $W = \frac{(B-A)}{N}$.
  - The most straightforward, but outliers may dominate presentation.
  - Skewed data is not handled well.
- **Equal-depth (frequency) partitioning:**
  - Divides the range into N intervals, each containing approximately the same number of samples.
  - Good data scaling.
  - Managing categorical attributes can be tricky.

Binning Methods for Data Smoothing

- **Sorted data for price (in dollars):**
  4, 8, 9, 15, 21, 21, 24, 25, 26, 28, 29, 34.
- **Partition into equal-frequency (equal-depth) bins:**
  Bin 1: 4, 8, 9, 15,
  Bin 2: 21, 21, 24, 25,
  Bin 3: 26, 28, 29, 34.
- **Smoothing by bin means:**
  Bin 1: 9, 9, 9, 9,
  Bin 2: 23, 23, 23, 23,
  Bin 3: 29, 29, 29, 29.
- **Smoothing by bin boundaries:**
  Bin 1: 4, 4, 4, 15,
  Bin 2: 21, 21, 25, 25,
  Bin 3: 26, 26, 26, 34.

Classification & Correlation Analysis

- **Classification:**
  - E.g. decision-tree analysis.
  - Supervised: Class labels given for training set e.g. cancerous vs. benign.
  - Using **entropy** to determine split point (discretization point).
  - Top-down, recursive split.
  - Details will be covered in Chapter 6.
- **Correlation analysis:**
  - E.g. χ^2-merge: χ^2-based discretization.
  - Supervised: use class information.
  - Bottom-up merge: find the best neighboring intervals (those having similar distributions of classes, i.e., low χ^2 values) to merge.
  - Merge performed recursively, until a predefined stopping condition.

Concept-hierarchy Generation

- **Concept hierarchy:**
  - Organizes concepts (i.e. attribute values) hierarchically.
  - Usually associated with each dimension in a data warehouse.
  - Facilitates **drilling and rolling** in data warehouses to view data at multiple granularity.
- **Concept-hierarchy formation:**
  - Recursively reduce the data by collecting and replacing **low-level concepts** (such as numerical values for age) by **higher-level concepts** (such as youth, adult, or senior).
  - Can be explicitly specified by domain experts and/or data-warehouse designers.
  - Can be automatically formed for both numerical and nominal data.
  - For numerical data, use discretization methods shown.

Concept-hierarchy Generation for Nominal Data

- **Specification of a partial/total ordering of attributes explicitly at the schema level by users or experts.**
  - $\#(\text{streets}) \prec \#(\text{city}) \prec \#(\text{state}) \prec \#(\text{country})$.
- **Specification of a hierarchy for a set of values by explicit data grouping.**
  - $\#(\{''Urbana'', ''Champaign'', ''Chicago''\}) \prec \#(\text{Illinois})$.
- **Specification of only a partial set of attributes.**
  - Only $\#(\text{street}) \prec \#(\text{city})$, not others.
- **Automatic generation of hierarchies (or attribute levels) by the analysis of the number of distinct values.**
  - E.g. for a set of attributes: $\{\text{street}, \text{city}, \text{state}, \text{country}\}$.
  - See on the next slides.

Automatic Concept-hierarchy Generation

- **Some hierarchies can be automatically generated based on the analysis of the number of distinct values per attribute.**
  - The attribute with the most distinct values is placed at the lowest level of the hierarchy.
  - Exceptions, e.g. weekday, month, quarter, year.
- Example:

$$
\begin{aligned}
\#(\text{streets}) = 674.339 > \#(\text{city}) &= 3567, \\
\#(\text{city}) = 3567 > \#(\text{province or state}) &= 356, \\
\#(\text{province or state}) = 356 > \#(\text{country}) &= 15.
\end{aligned}
$$

Summary

Summary

- **Data quality:** Accuracy, completeness, consistency, timeliness, believability, interpretability.
- **Data cleaning:** E.g. missing/noisy values, outliers.
- **Data integration from multiple sources:**
  - Entity identification problem.
  - Remove redundancies.
  - Detect inconsistencies.
- **Data reduction:**
  - Dimensionality reduction.
  - Numerosity reduction.
  - Data compression.
- **Data transformation and data discretization:**
  - Normalization.
  - Concept-hierarchy generation.

Any questions about this chapter?

Ask them now or ask them later in our forum:

https://www.studon.fau.de/studon/goto.php?target=lcode_OLYeD79h