Particle Systems and ODEs

Sai-Kit Yeung

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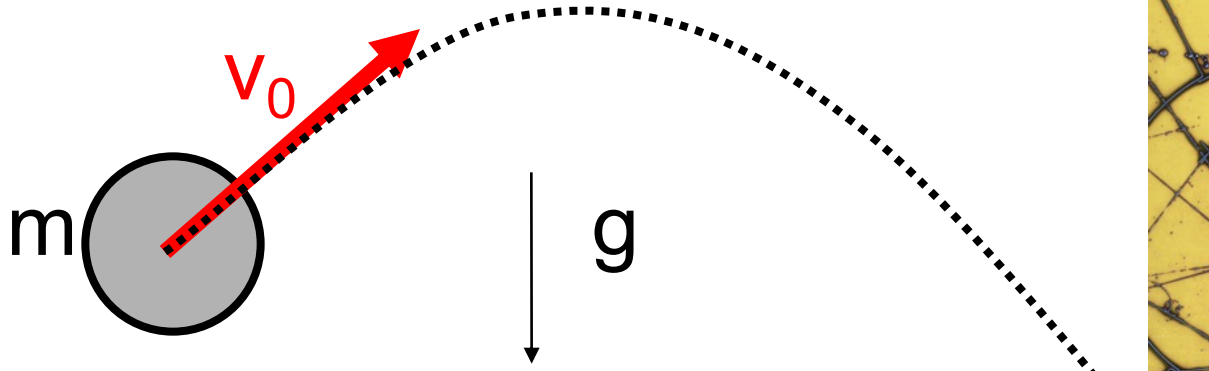
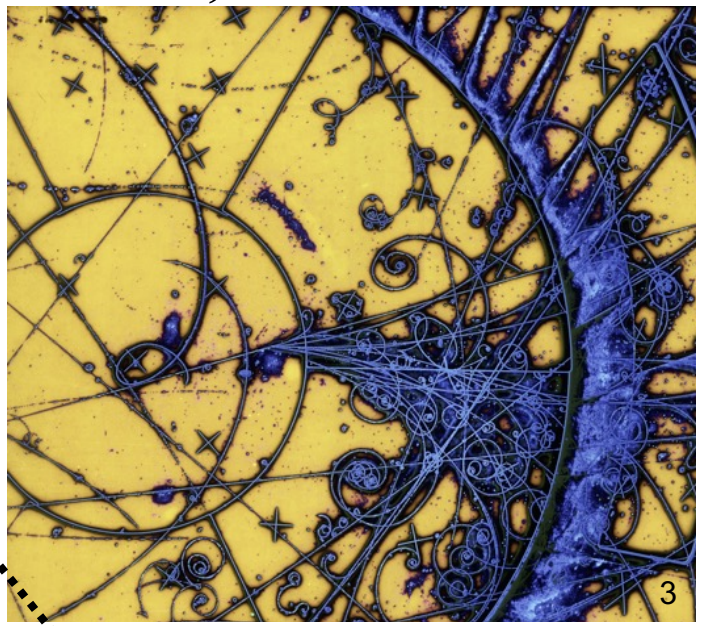
Types of Animation

- Keyframing
- Procedural
- Physically-based
  - Particle Systems: **TODAY**
    - Smoke, water, fire, sparks, etc.
    - Usually heuristic as opposed to simulation, but not always
    - Mass-Spring Models (Cloth) **NEXT CLASS**
  - Continuum Mechanics (fluids, etc.), finite elements
    - Not in this class
  - Rigid body simulation
    - Not in this class

Types of Animation: Physically-Based

- Assign physical properties to objects
  - Masses, forces, etc.
- Also procedural forces (like wind)
- Simulate physics by solving equations of motion
  - Rigid bodies, fluids, plastic deformation, etc.
- Realistic but difficult to control





Types of Dynamics

- Point

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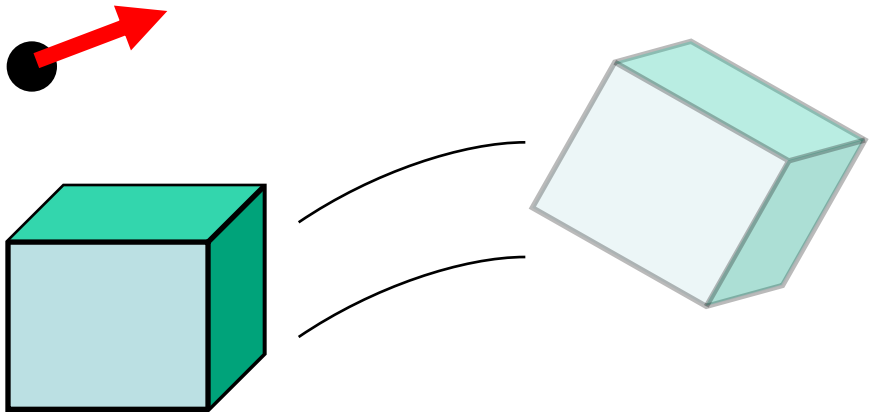
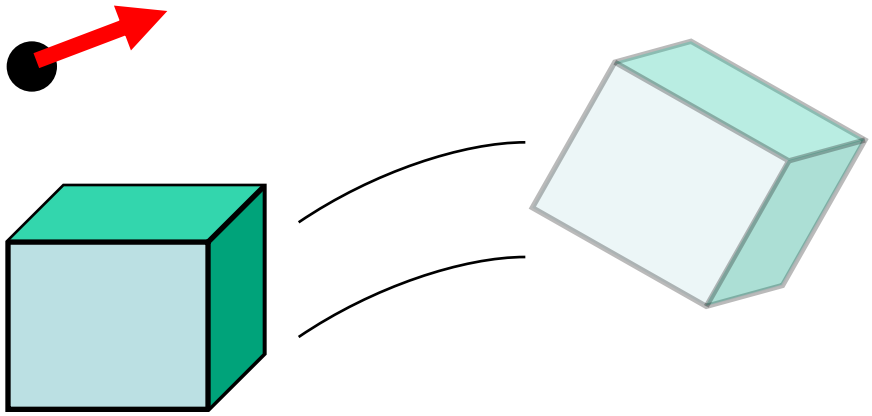
- Point
- Rigid body



Animation by Mark Carlson

Types of Dynamics

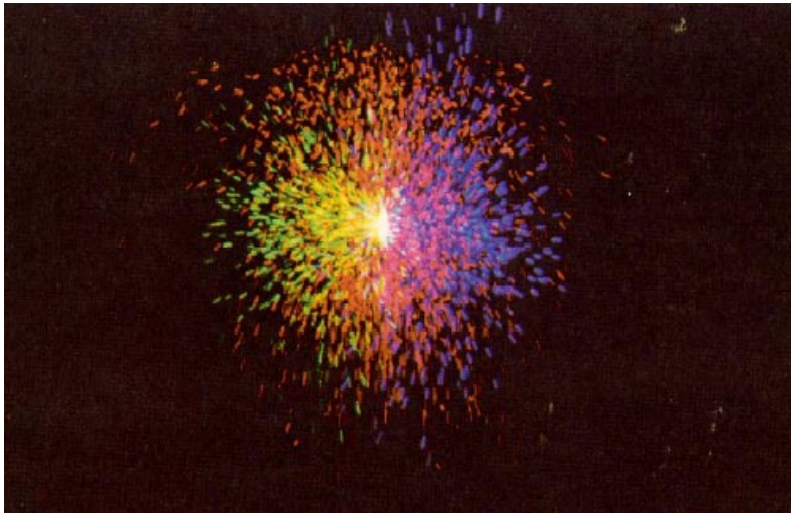
- Point
- Rigid body
- Deformable body
  (include clothes, fluids, smoke, etc.)



Mark Carlson

Today We Focus on Point Dynamics

- Lots of points!
- Particles systems
  - Borderline between procedural and physically-based



Outline

- Particle Systems Overview
- Simple Particle System
  - Algorithm
  - Math (Solving ODE)
- Other Particle Systems (Very Brief)

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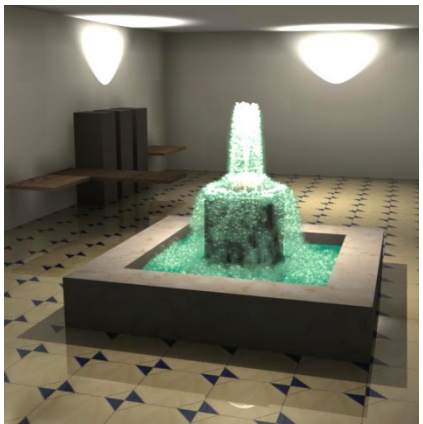
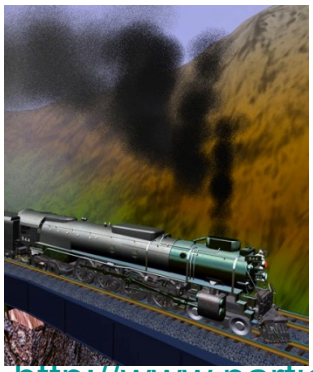
Particle Systems Overview

- Emitters generate tons of “particles”
  - Sprinkler, waterfall, chimney, gun muzzle, exhaust pipe, etc.

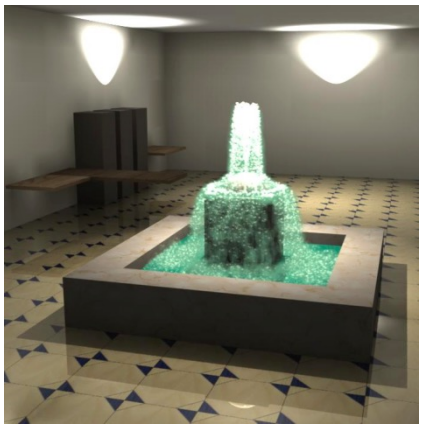


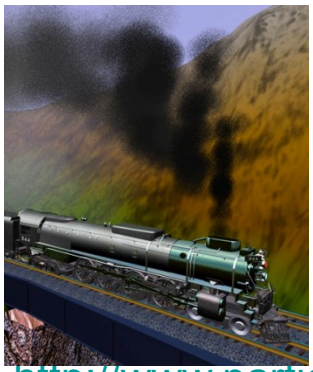


http://www.particlesystems.org/

Particle Systems Overview

- Emitters generate tons of “particles”
- Describe the external forces with a force field
  - E.g., gravity, wind

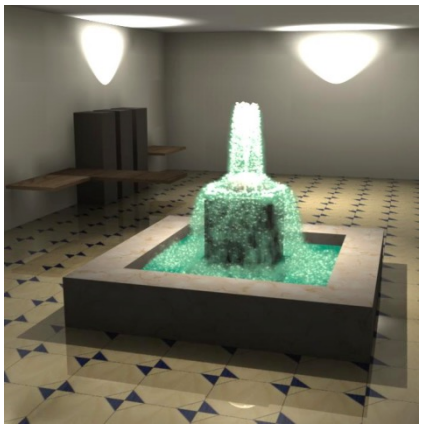


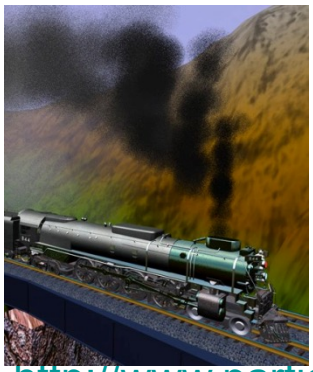


http://www.particlesystems.org/

Particle Systems Overview

- Emitters generate tons of “particles”
- Describe the external forces with a force field
- Integrate the laws of mechanics (ODEs)
  - Makes the particles move





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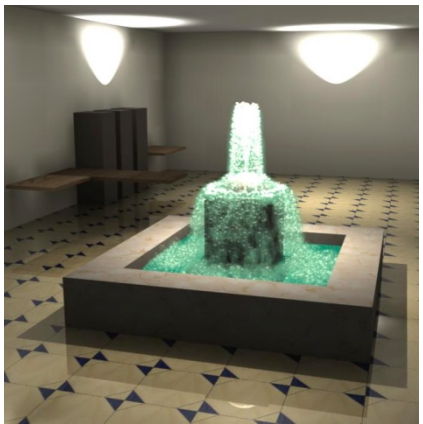
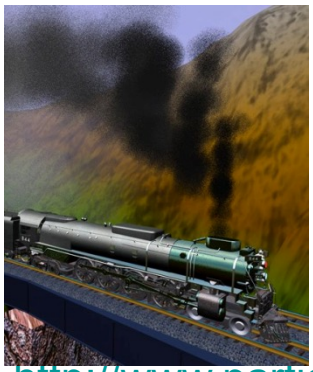
Particle Systems Overview

- Emitters generate tons of “particles”
- Describe the external forces with a force field
- Integrate the laws of mechanics (ODEs)
- In the simplest case, each particle is independent





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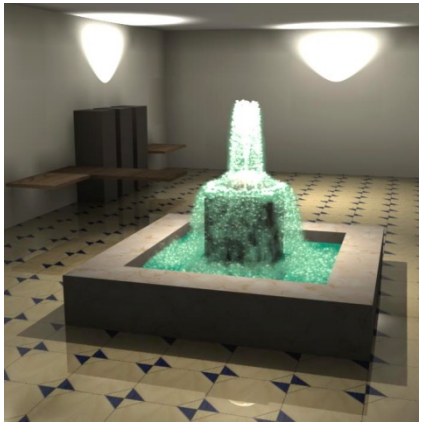
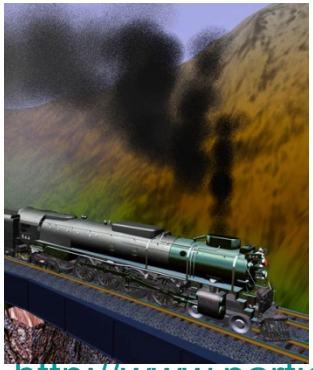
Particle Systems Overview

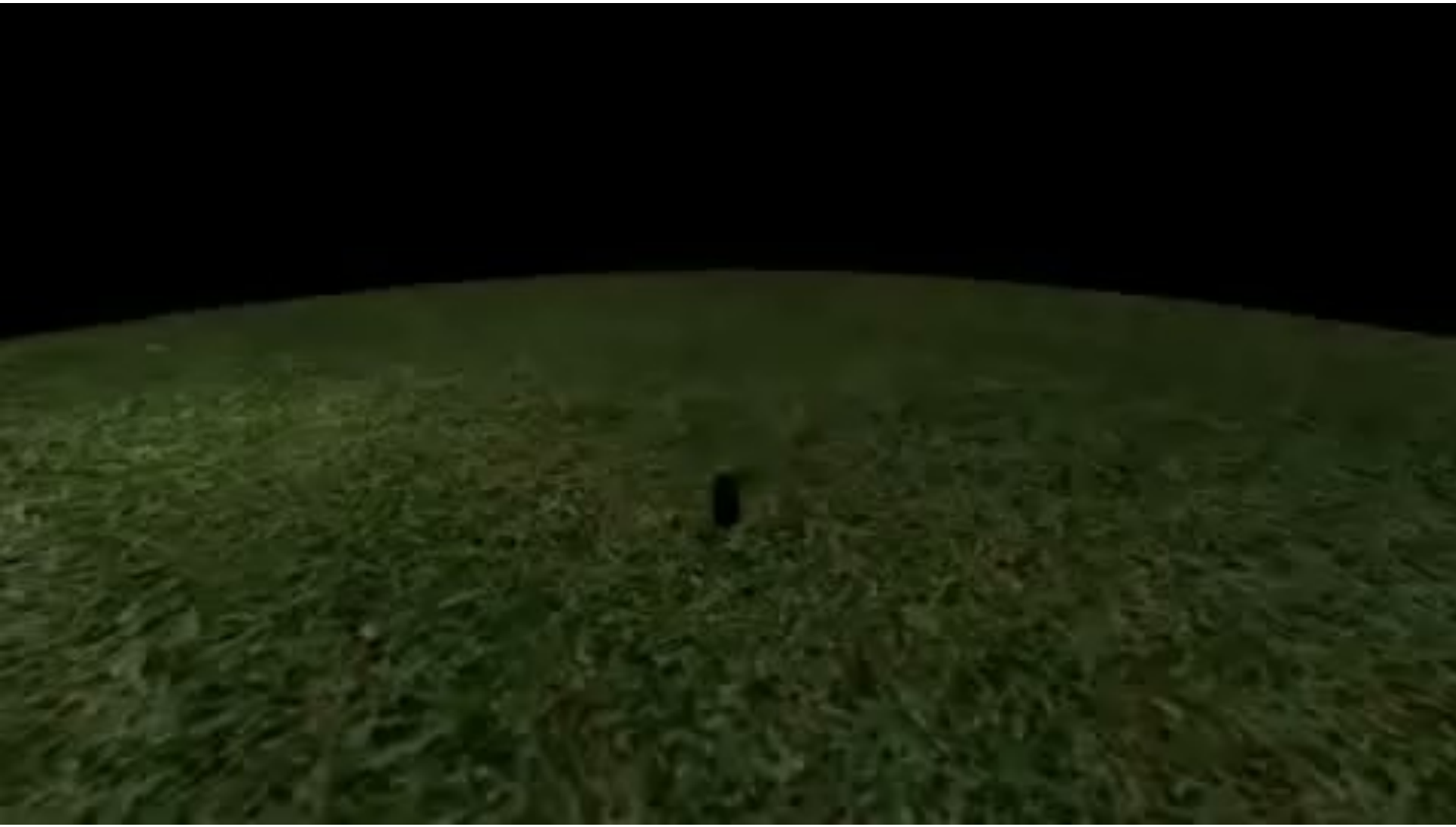
- Emitters generate tons of “particles”
- Describe the external forces with a force field
- Integrate the laws of mechanics (ODEs)
- In the simplest case, each particle is independent
- If there is enough randomness (in particular at the emitter) you get nice effects
  - sand, dust, smoke, sparks, flame, water, …





http://www.particlesystems.org/

Sprinkler



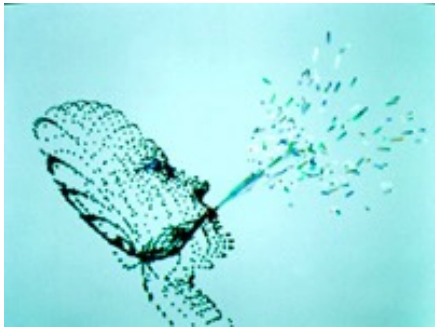
- http://www.youtube.com/watch?v=rhvH12nC6_Q

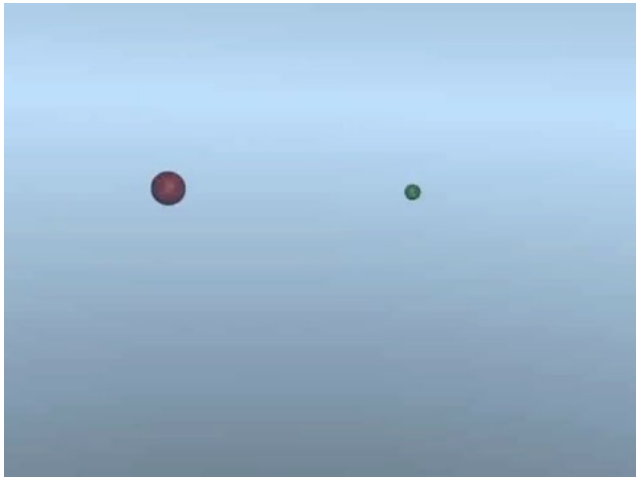
Fire

- http://www.youtube.com/watch?v=6hG00etwRBU

Generalizations

- More advanced versions of behavior
  - flocks, crowds
- Forces between particles
  - Not independent any more





See http://www.red3d.com/cwr/boids/ for discussion on how to do flocking.

We'll come back to this a little later.

http://www.blendernation.com/2008/01/05/simulating-flocks-herds-and-swarms-using-experimental-blender-boids-particles/

Generalizations – Next Class

- Mass-spring and deformable surface dynamics
  - surface represented as a set of points
  - forces between neighbors keep the surface coherent

Image Witkin & Baraff

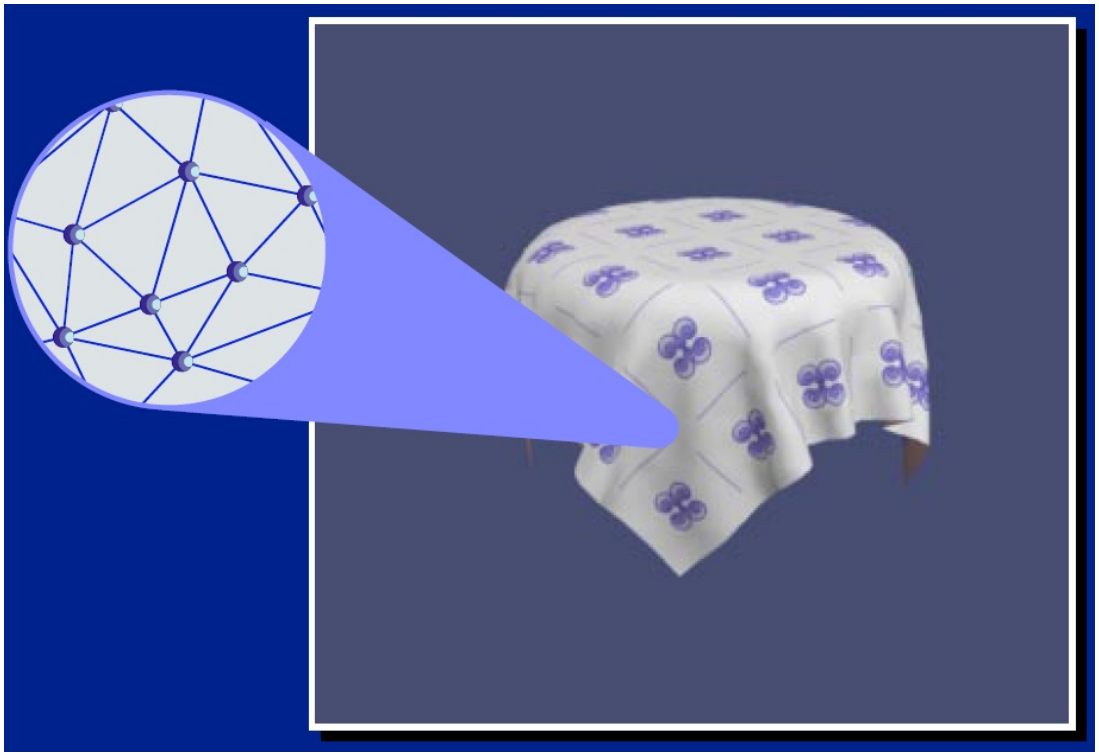
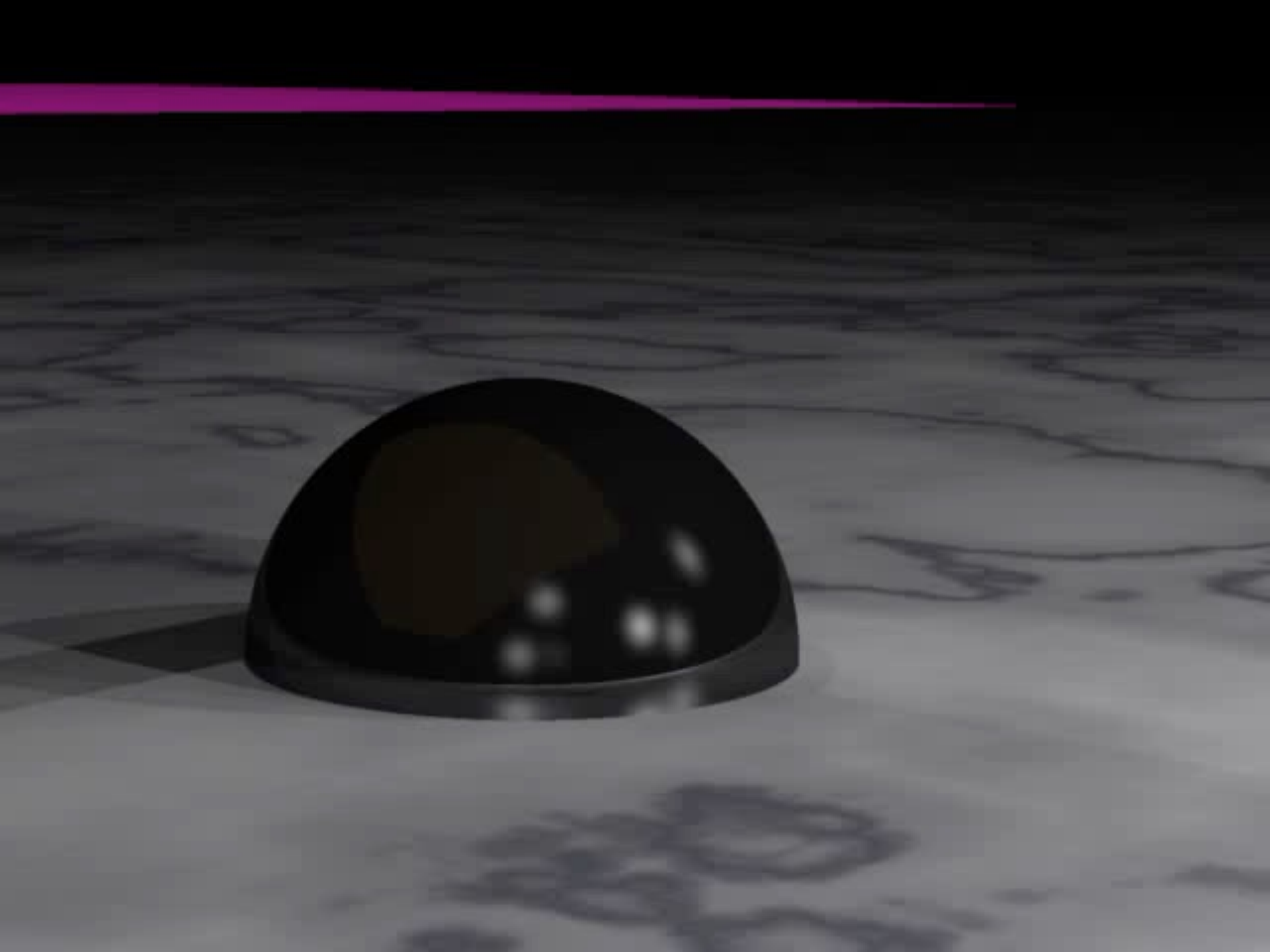


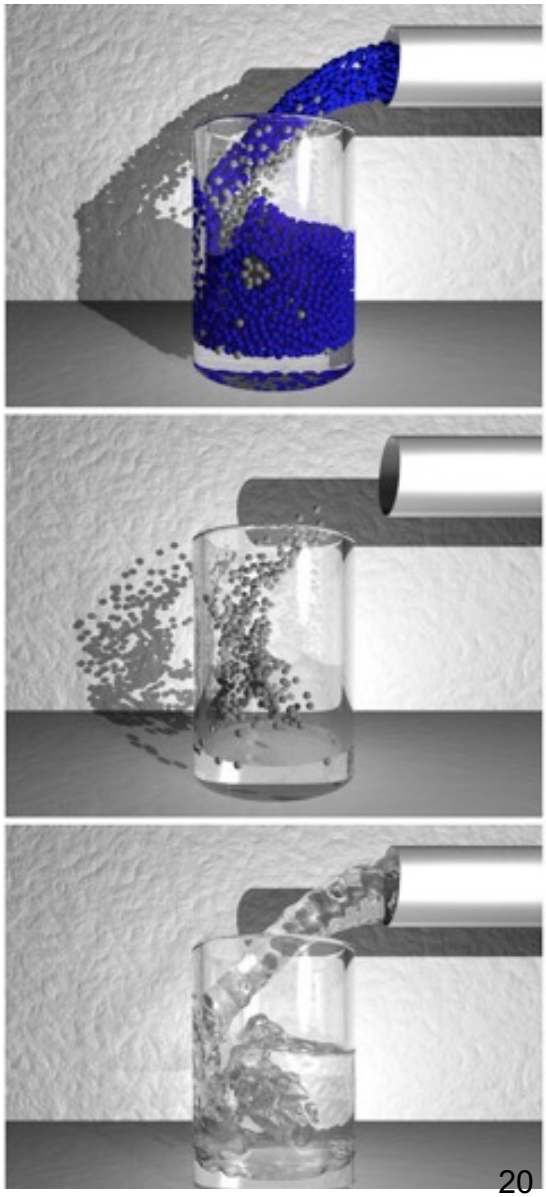
Image Michael Kass



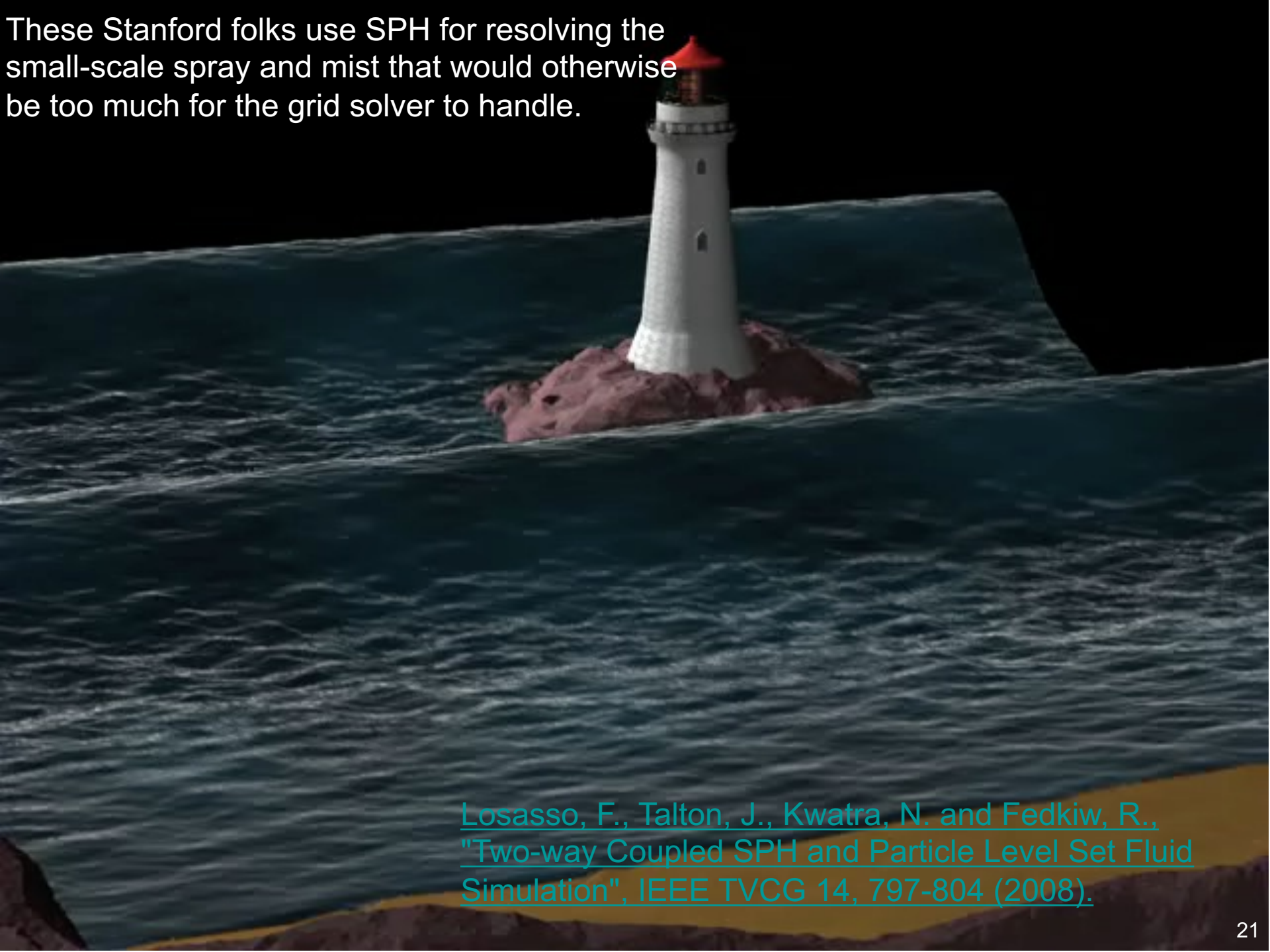
Generalizations

Müller et al. 2005

- It's not all hacks: Smoothed Particle Hydrodynamics (SPH)
  - A family of "real" particle-based fluid simulation techniques.
  - Fluid flow is described by the Navier-Stokes Equations, a nonlinear partial differential equation (PDE)
    - SPH discretizes the fluid as small packets (particles!), and evaluates pressures and forces based on them.



These Stanford folks use SPH for resolving the small-scale spray and mist that would otherwise be too much for the grid solver to handle.



Losasso, F., Talton, J., Kwatra, N. and Fedkiw, R., "Two-way Coupled SPH and Particle Level Set Fluid Simulation", IEEE TVCG 14, 797-804 (2008).

Real-time particles in games

- http://www.youtube.com/watch?v=6DicVajK2xQ

EA Fight Night 4 Physics Trailer

Take-Home Message

- Particle-based methods can range from pure heuristics (hacks that happen to look good) to "real" simulation

Andrew Selle et al.

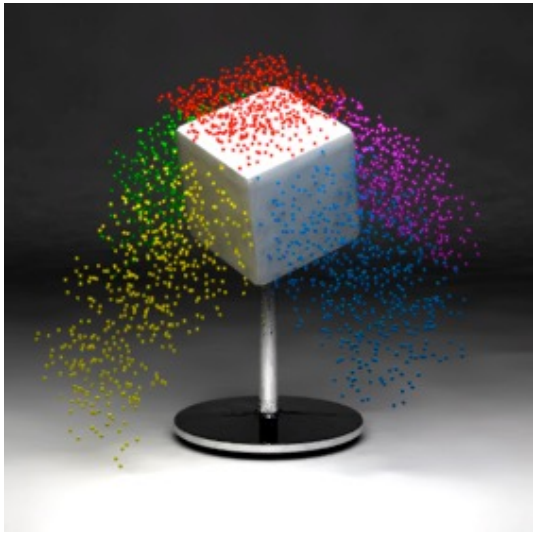
- Basics are the same:
  Things always boil
  down to integrating ODEs!
  - Also in the case of grids/computational meshes

Questions?

http://www.cs.columbia.edu/cg/ESIC/esic.html

What is a Particle System?

- Collection of many small simple pointlike things
  - Described by their current state: position, velocity, age, color, etc.
- Particle motion influenced by external force fields and internal forces between particles
- Particles created by generators or emitters
  - With some randomness
- Particles often have lifetimes
- Particles are often independent
- Treat as points for dynamics, but rendered as anything you want



http://en.wikipedia.org/wiki/Image:Particle_Emitter.jpg

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Simple Particle System: Sprinkler

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PL: linked list of particle = empty;
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Image Jeff Lander

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For each time step
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spread=0.1;//how random the initial velocity is
colorSpread=0.1; //how random the colors are
For each time step
    Generate k particles
        p=new particle();
        p->position=(0,0,0);
        p->velocity=(0,0,1)+spread*(rnd(), rnd(), rnd());
        p.color=(0,0,1)+colorSpread*(rnd(), rnd(),rnd());
        PL->add(p);
```

Image Jeff Lander

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        PL->add(p);
    For each particle p in PL
        p->position+=p->velocity*dt; //dt: time step
        p->velocity-=g*dt; //g: gravitation constant
        glColor(p.color);
        glVertex(p.position);
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Image Jeff Lander

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Image Jeff Lander

Questions?

Path forward

- Basic particle systems are simple hacks
- Extend to physical simulations, e.g., clothes
- For this, we need to understand numerical integration
- This lecture: point particles
- Next lecture: mass-spring and clothes

Ordinary Differential Equations

$$\frac{d\,\mathbf{X}(t)}{dt} = f(\mathbf{X}(t), t)$$

Derivative of function over time relates (f) to the function at the current time

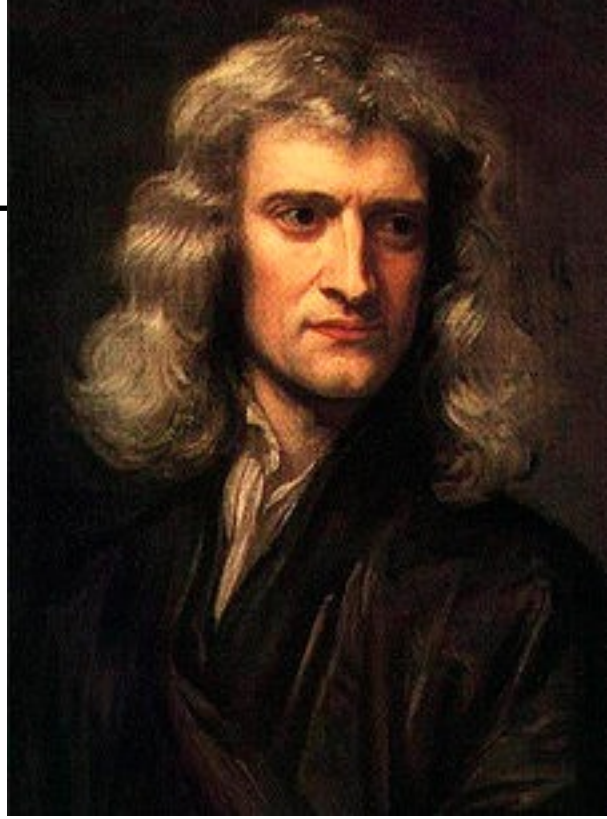
- Given a function f(X,t) compute X(t)
- Typically, initial value problems:
  - Given values $X(t_0)=X_0$
  - Find values X(t) for $t > t_0$

X - state

Did we just see it?

- We can use lots of standard tools

Newtonian Mechanics



- Point mass: 2nd order ODE

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{F} = m\frac{d^2\vec{x}}{dt^2}$$

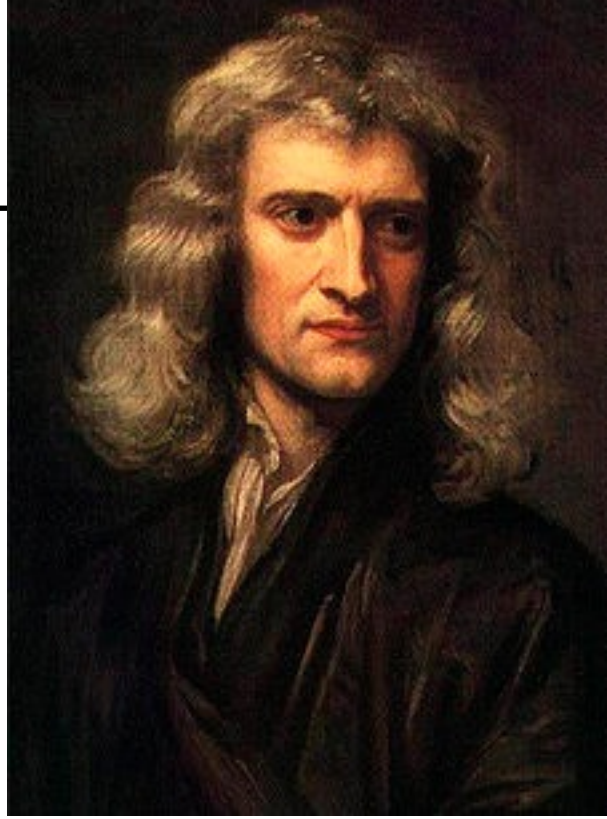
- Position $\boldsymbol{x}$ and force $\boldsymbol{F}$ are vector quantities
  - We know $\boldsymbol{F}$ and m, want to solve for $\boldsymbol{x}$

- You have all seen this a million times before

F – constant or depend on X

e.g., particle in the middle of the wind

Reduction to 1st Order



- Point mass: 2nd order ODE

$$\vec{F} = m\vec{a} \quad \text{or} \quad \vec{\boldsymbol{F}} = m\frac{d^2\vec{\boldsymbol{x}}}{dt^2}$$

- Corresponds to system of first order ODEs

$$\begin{cases} \frac{d}{dt}\vec{\boldsymbol{x}} = \vec{\boldsymbol{v}} \\ \frac{d}{dt}\vec{\boldsymbol{v}} = \vec{\boldsymbol{F}}/m \end{cases}$$

2 unknowns (x, v) instead of just x

Reduction to 1st Order

$$\begin{cases} \frac{d}{dt}\vec{x} = \vec{v} \\ \frac{d}{dt}\vec{v} = \vec{F}/m \end{cases}$$

2 variables (x, v)
instead of just one

- Why reduce?

Reduction to 1st Order

$$\begin{cases} \frac{d}{dt}\vec{x} = \vec{v} \\ \frac{d}{dt}\vec{v} = \vec{F}/m \end{cases}$$

2 variables (x, v)
instead of just one

- Why reduce?
  - Numerical solvers grow more complicated with increasing order, can just write one 1st order solver and use it
  - Note that this doesn't mean it would always be easy :-)

Notation

- Let's stack the pair (x, v) into a bigger state vector X

$$\boldsymbol{X} = \begin{pmatrix} \vec{\boldsymbol{x}} \\ \vec{\boldsymbol{v}} \end{pmatrix}$$

For a particle in 3D, state vector X has 6 numbers

$$\frac{d}{dt}\boldsymbol{X} = f(\boldsymbol{X}, t) = \begin{pmatrix} \vec{\boldsymbol{v}} \\ \vec{\boldsymbol{F}}(x, v)/m \end{pmatrix}$$

Now, Many Particles

- We have N point masses
  - Let's just stack all xs and vs in a big vector of length 6N

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{x}_N \\ \boldsymbol{v}_N \end{pmatrix} \qquad f(\boldsymbol{X}, t) = \begin{pmatrix} \boldsymbol{v}_1 \\ \boldsymbol{F}^1(\boldsymbol{X}, t) \\ \vdots \\ \boldsymbol{v}_N \\ \boldsymbol{F}^N(\boldsymbol{X}, t) \end{pmatrix}$$

Now, Many Particles

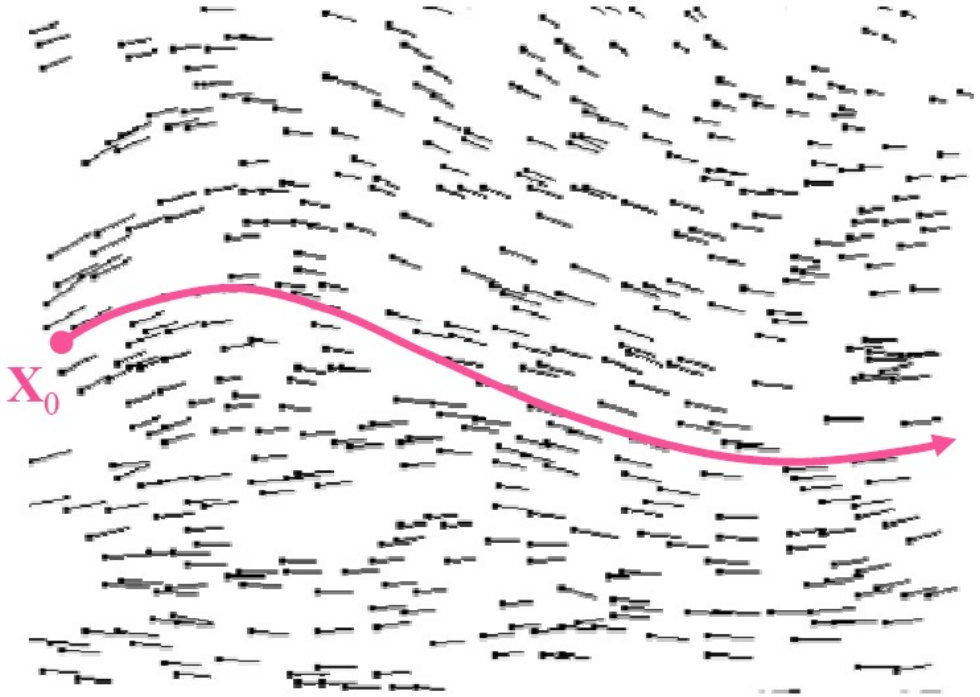
- We have N point masses
  - Let's just stack all xs and vs in a big vector of length 6N
  - F^i denotes the force on particle i
    - When particles don't interact, F^i only depends on x_i and v_i.

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{x}_N \\ \boldsymbol{v}_N \end{pmatrix} \qquad f(\boldsymbol{X}, t) = \begin{pmatrix} \boldsymbol{v}_1 \\ \boldsymbol{F}^1(\boldsymbol{X}, t) \\ \vdots \\ \boldsymbol{v}_N \\ \boldsymbol{F}^N(\boldsymbol{X}, t) \end{pmatrix}$$

f gives d/dt X, remember!

Path through a Vector Field

- ***X(t)***: path in multidimensional phase space

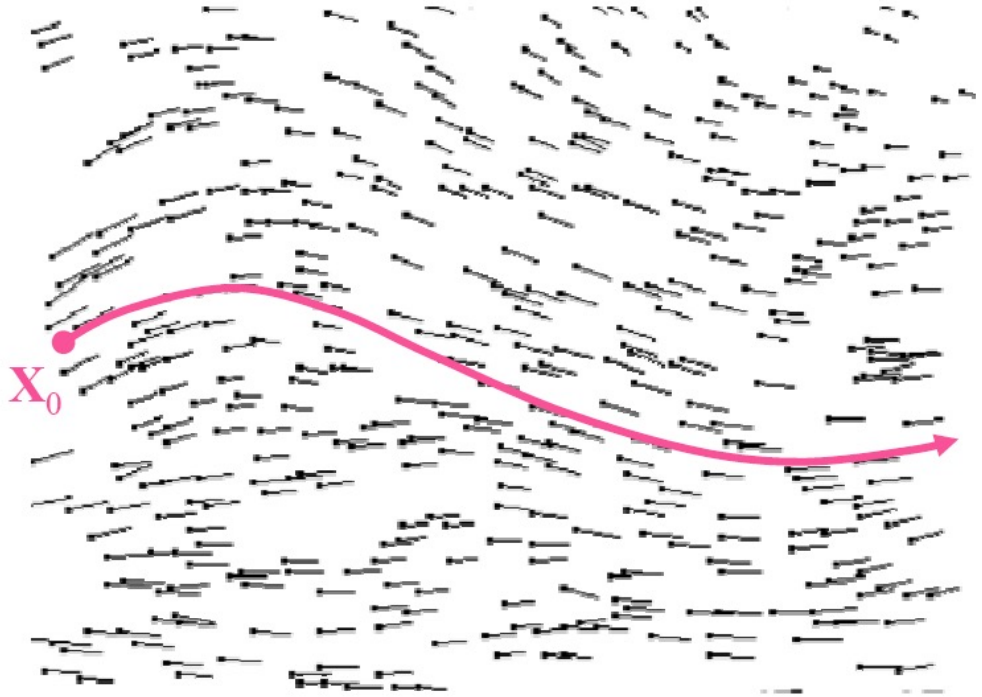


$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state X at time t, where will X be after an infinitely small time interval dt ?"

Path through a Vector Field

- ***X(t)***: path in multidimensional phase space



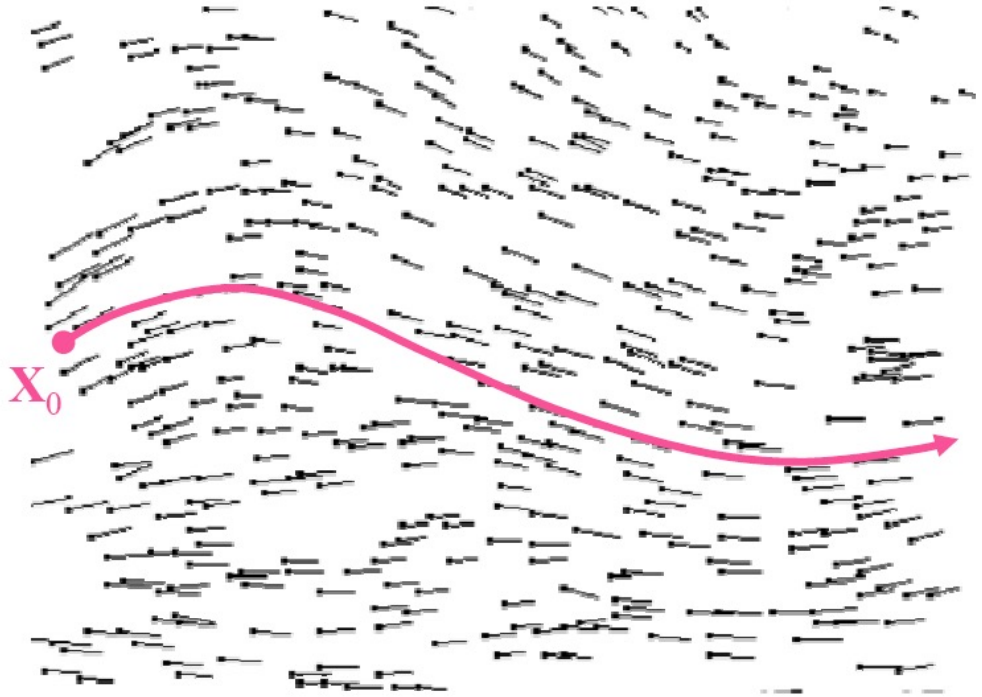
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state X at time t, where will X be after an infinitely small time interval dt ?"

- $f = \mathrm{d}/\mathrm{d}t\ \boldsymbol{X}$ is a vector that sits at each point in phase space, pointing the direction.(Given!)

Path through a Vector Field

- ***X(t)***: path in multidimensional phase space



$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

"When we are at state X at time t, where will X be after an infinitely small time interval dt ?"

- *f* is a function of X and t, but not necessary depends on t. If it does, the vector field will keep changing

Think an example?

Recap: ODE

$$\frac{d\,\mathbf{X}(t)}{dt} = f(\mathbf{X}(t), t)$$

- *** Given a function f(X,t) compute X(t) ***
- Typically, initial value problems:
  - *** Given values $X(t_0)=X_0$ ***
  - Find values X(t) for $t > t_0$
- f may or may not depends on t
- Think about a point on the ocean, f is the ocean current, X should be the path of the point driven by f

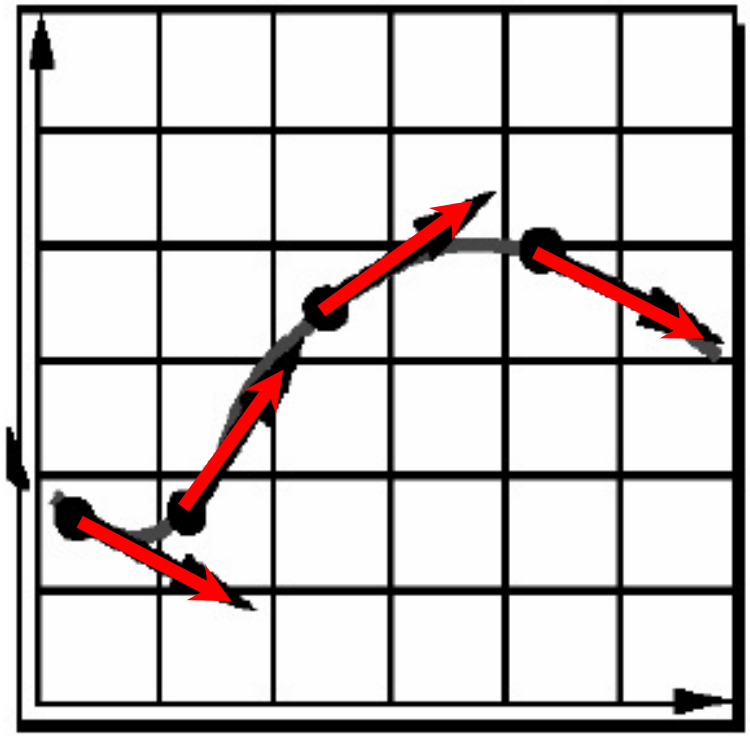
Questions?

Numerics of ODEs

- Numerical solution is called “integration of the ODE”
- Many techniques
  - Today, the simplest one
  - Next class, we’ll look at some more advanced techniques

Intuitive Solution: Take Steps

- Current state X
- Examine f(X,t) at (or near) current state
- Take a step to new value of X



$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$

$$\Rightarrow \text{“}\mathrm{d}\boldsymbol{X} = \mathrm{d}t\, f(\boldsymbol{X}, t)\text{”}$$

f = d/dt X is a vector that sits at each point in phase space, pointing the direction.

Euler's Method

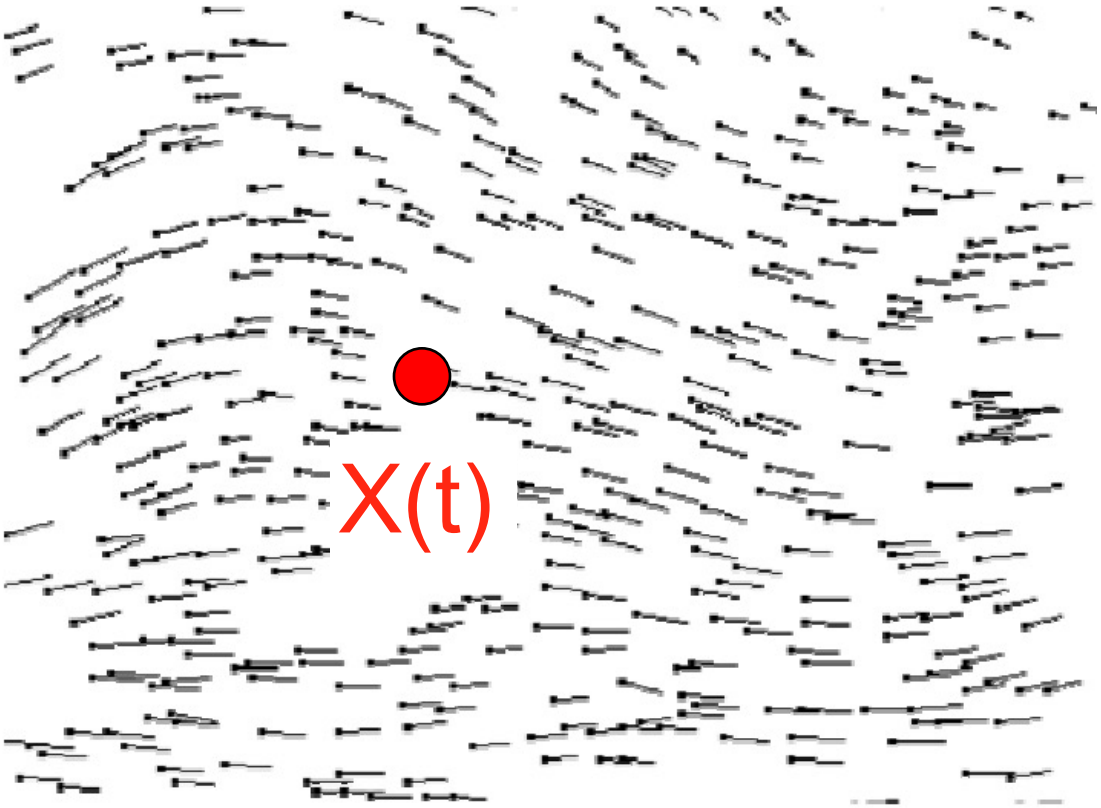
- Simplest and most intuitive
- Pick a step size h
- Given $X_0=X(t_0)$, take step:

$$t_1 = t_0 + h$$
$$\mathbf{X}_1 = \mathbf{X}_0 + h\, f(\mathbf{X}_0, t_0)$$

- Piecewise-linear approximation to the path
- Basically, just replace dt by a small but finite number

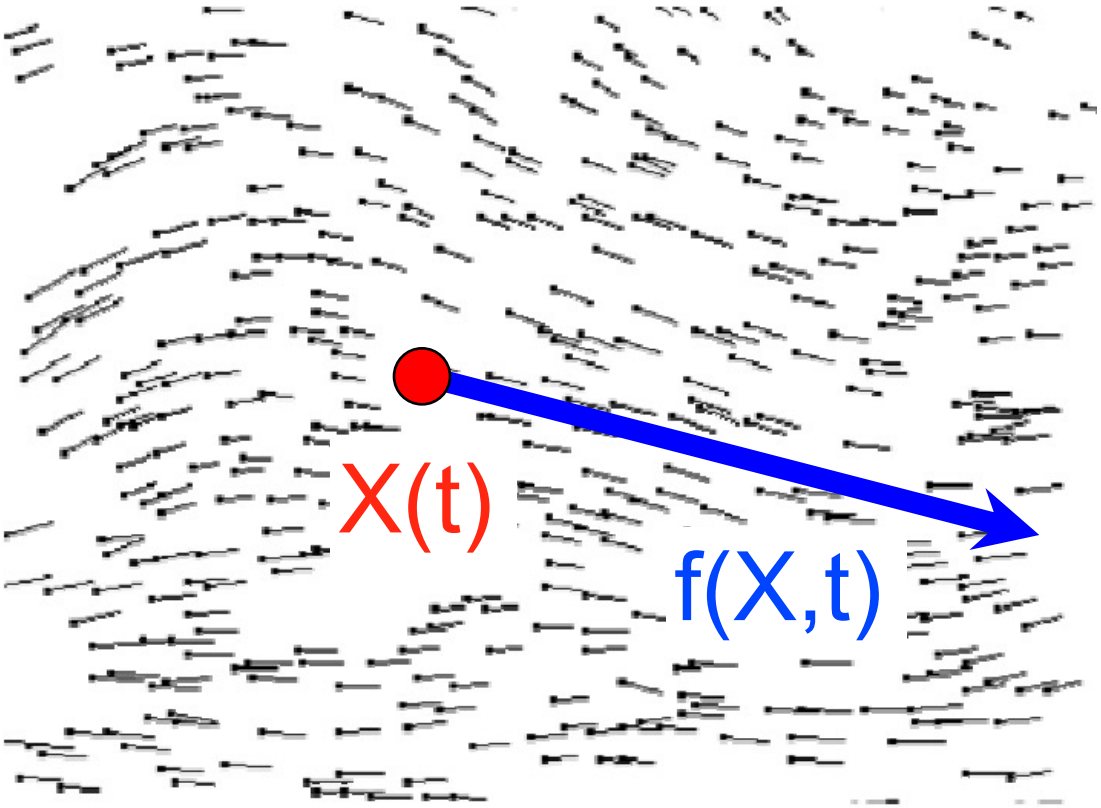
Euler, Visually

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$



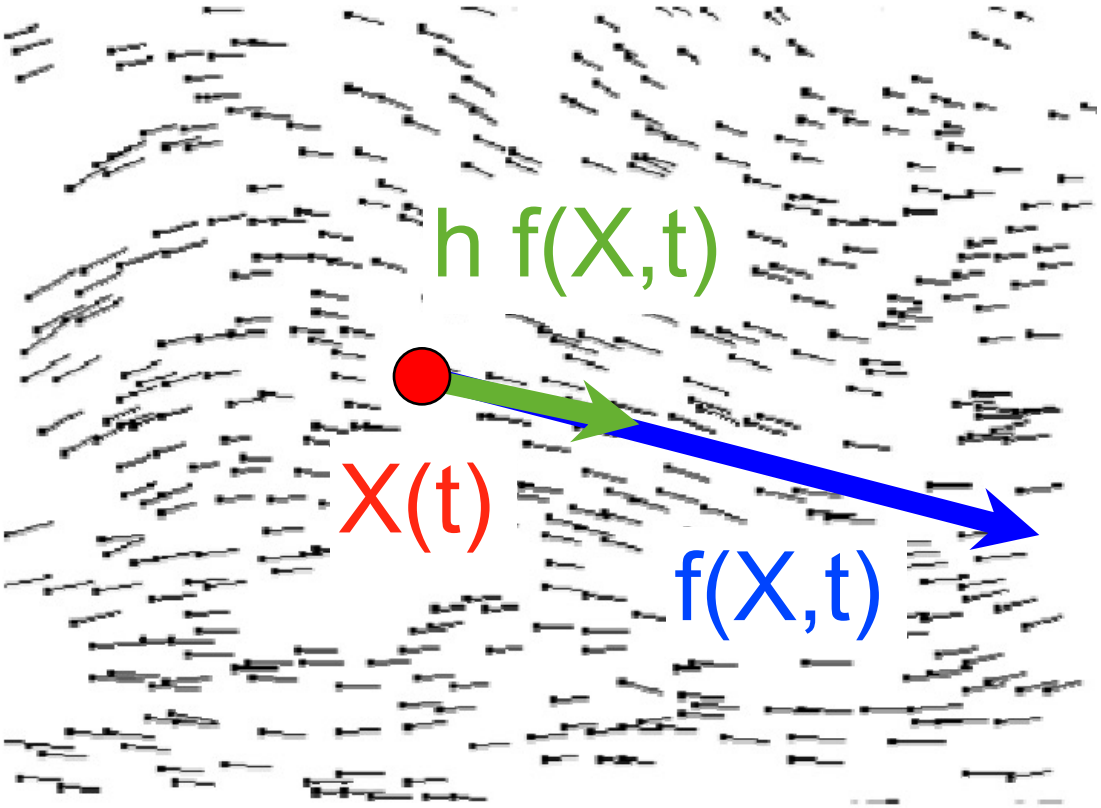
Euler, Visually

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$



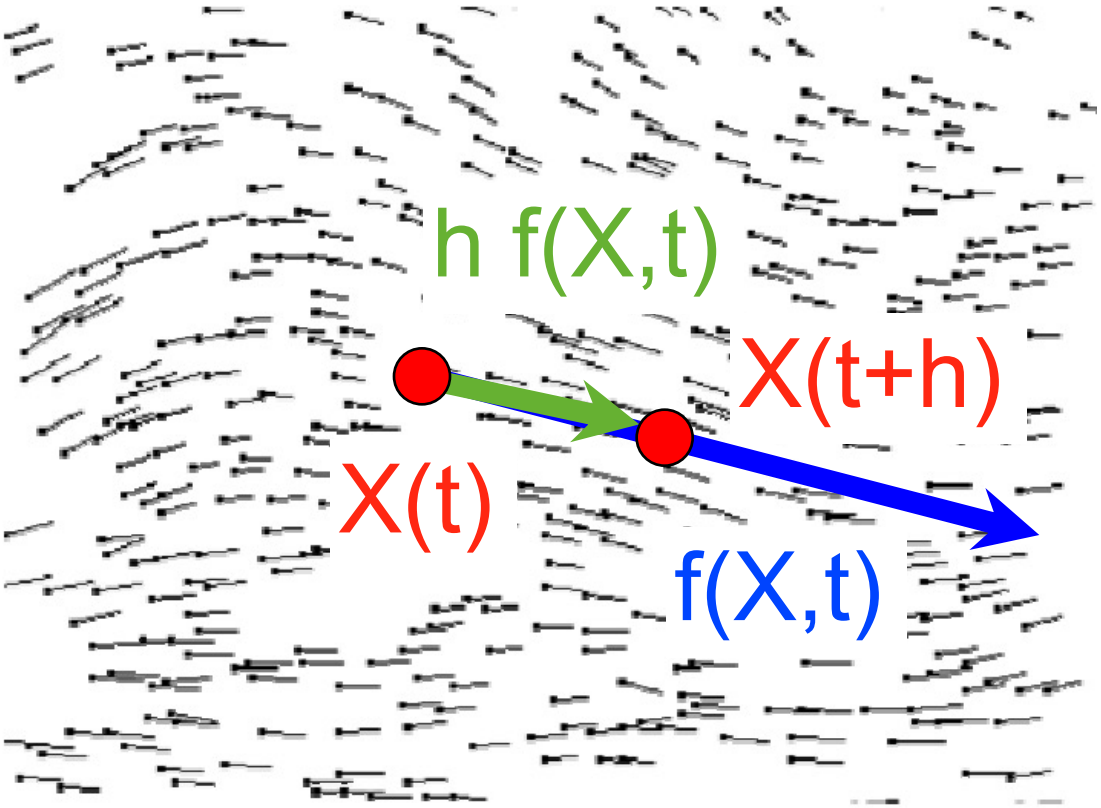
Euler, Visually

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$



Euler, Visually

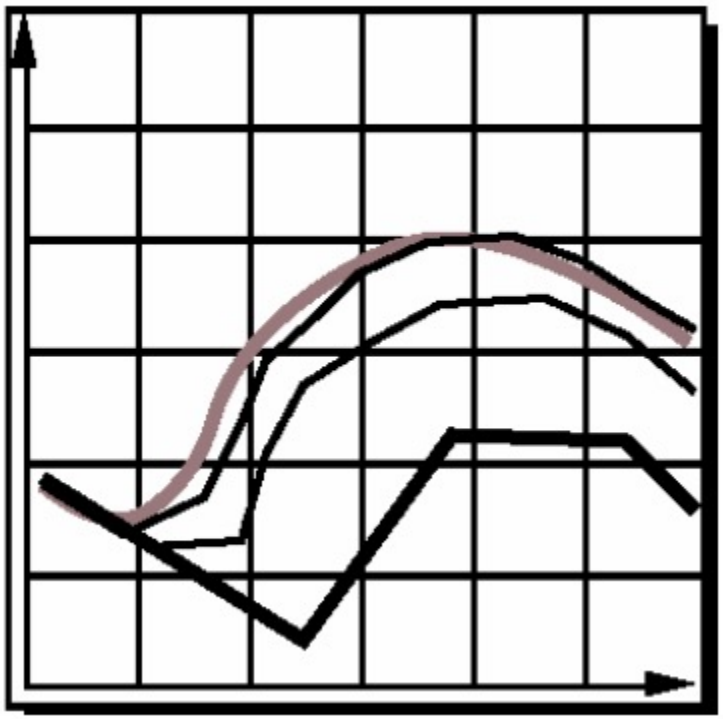
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{X} = f(\boldsymbol{X}, t)$$



Questions?

Effect of Step Size

- Step size controls accuracy
- Smaller steps more closely follow curve
  - May need to take many small steps per frame
  - Properties of f(X, t) determine this (more later)



Euler's method: Inaccurate

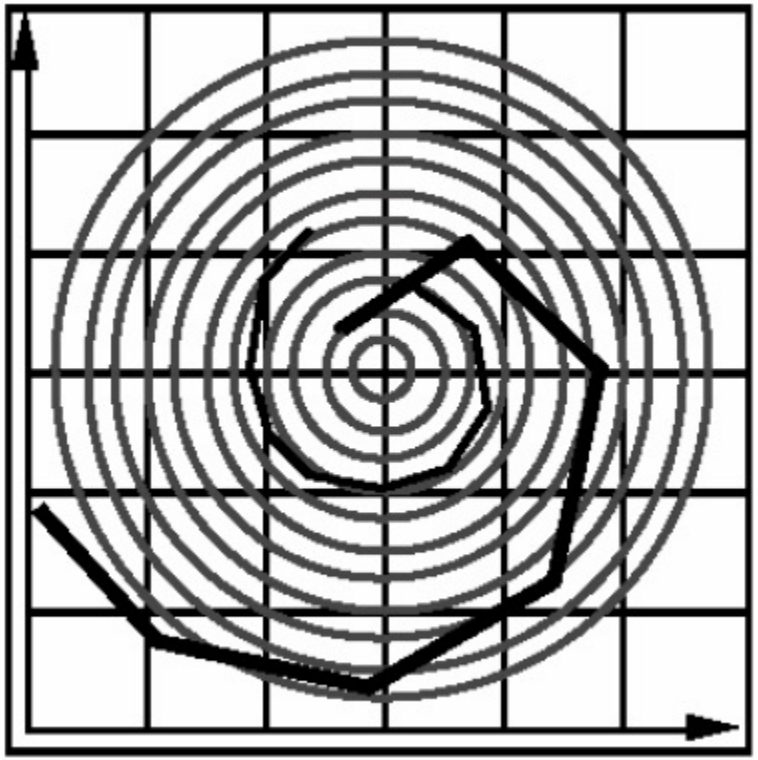
- Moves along tangent; can leave solution curve, e.g.:

$$f(\mathbf{X},t) = \begin{pmatrix} -y \\ x \end{pmatrix}$$

- Exact solution is circle:

$$\mathbf{X}(t) = \begin{pmatrix} r\cos(t+k) \\ r\sin(t+k) \end{pmatrix}$$

The true form that you want to recover



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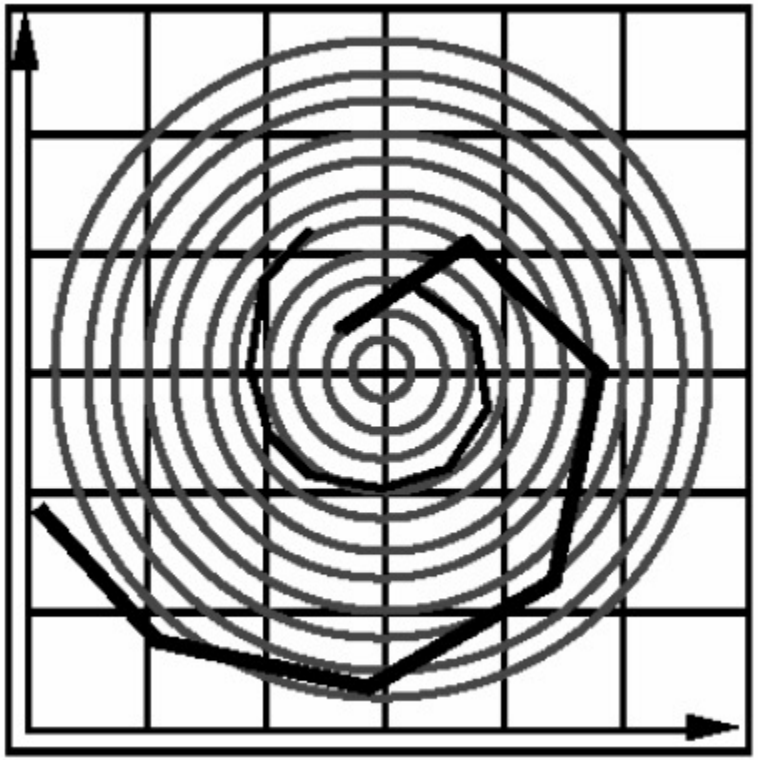
- Exact solution is circle:

$$\mathbf{X}(t) = \begin{pmatrix} r\cos(t+k) \\ r\sin(t+k) \end{pmatrix}$$

- Euler spirals outward no matter how small h is
  - will just diverge more slowly



More Accurate Alternatives

- Midpoint, Trapezoid, Runge-Kutta

More on this during next class

- Extremely valuable resource: SIGGRAPH 2001 course notes on physically based modeling

Outline

- Particle Systems Overview
- Simple Particle System
  - Algorithm
  - Math (Solving ODE)
- Other Particle Systems (Very Brief)

Other Particle Systems

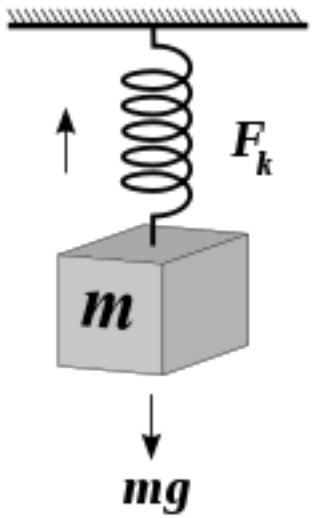
- Particles are not independent
  - Force among them!

What is a Force?

- A force changes the motion of the system
  - Newton says: When there are no forces, motion continues uniformly in a straight line (good enough for us)

- Forces can depend on location, time, velocity
  - Gravity, spring, viscosity, wind, etc.

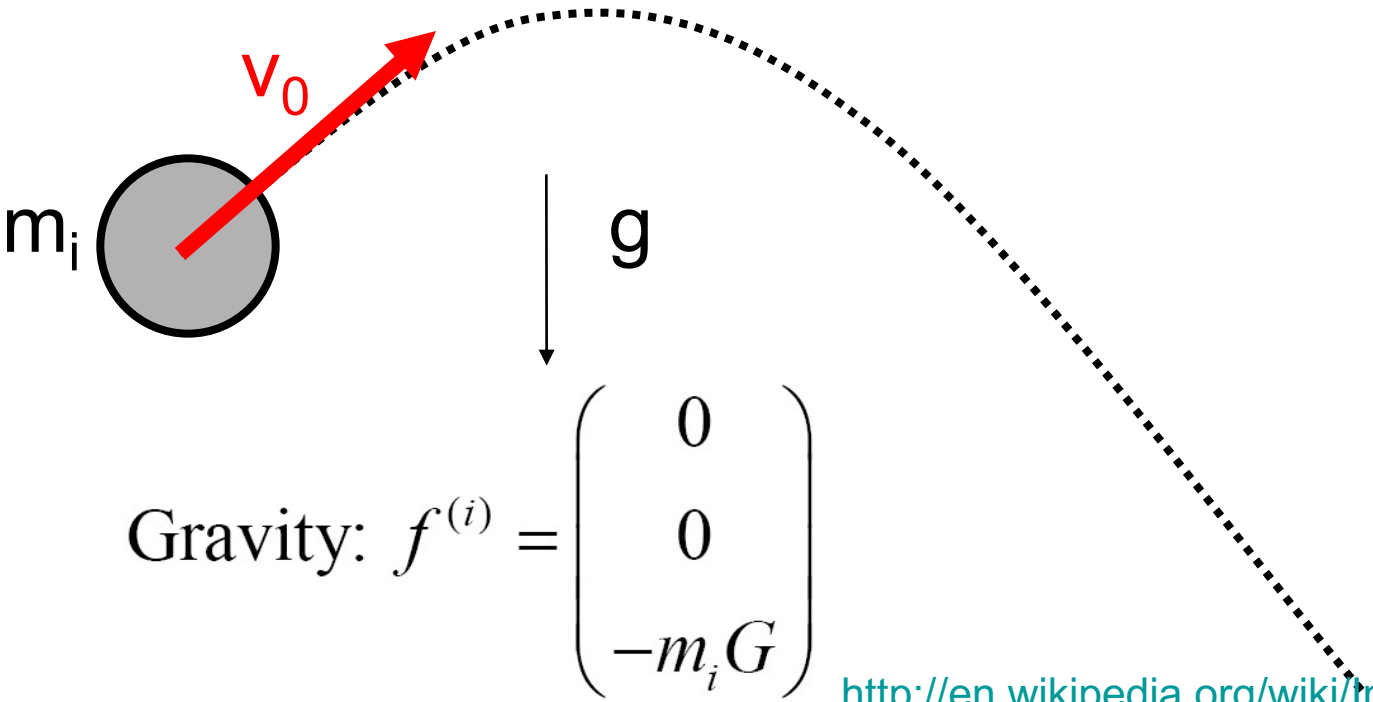
- For point masses, forces are vectors
  - Ie., to get total force, take vector sum of everything



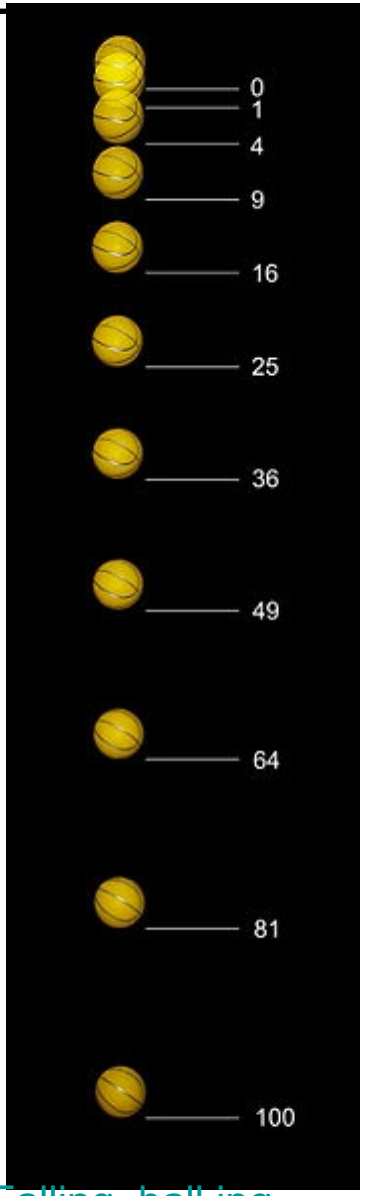
Wikipedia

Forces: Gravity on Earth

- Depends only on particle mass
- f(X,t) = constant
- Hack for smoke, etc: make gravity point up!
  - Well, you can call this buoyancy, too.



$$\text{Gravity: } f^{(i)} = \begin{pmatrix} 0 \\ 0 \\ -m_i G \end{pmatrix}$$



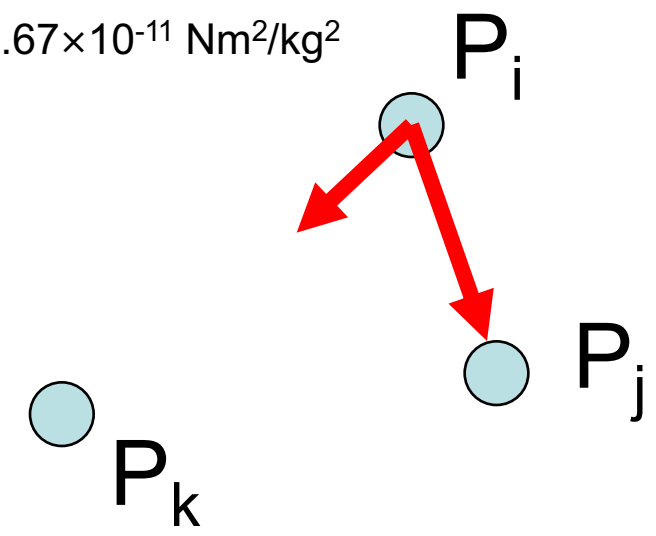
http://en.wikipedia.org/wiki/Image:Falling_ball.jpg

Forces: Gravity (N-body problem)

- Gravity depends on all other particles
- Opposite for pairs of particles
- Force in the direction of p_i-p_j with magnitude inversely proportional to square distance

$$\|\boldsymbol{F}_{ij}\| = \frac{G\, m_i\, m_j}{r^2}$$

where $G=6.67\times10^{-11}$ Nm^2/kg^2



- Testing all pairs is $O(n^2)$!

Particles are not independent!

Real-Time Gravity Demo

NVIDIA

http://www.youtube.com/watch?v=uhTuJZiAG64

An Aside on Gravity

- That was Brute Force
  - Meaning all $O(n^2)$ pairs of particles were considered when computing forces
  - Yes, computers are fast these days, but this gets prohibitively expensive soon. (The square in n^2 wins.)

- Hierarchical techniques approximate forces caused by many distant attractors by one force, yields O(n)!
  - Fast Multipole Method", Greengard and Rokhlin, J Comput Phys 73, p. 325 (1987)
  - This inspired very cool hierarchical illumination rendering algorithms in graphics (hierarchical radiosity, etc.)

Forces: Viscous Damping

$$f^{(i)} = -dv^{(i)}$$

- Damping force on particle i determined its velocity
  - Opposes motion
  - E.g. wind resistance
- Removes energy, so system can settle
- Small amount of damping can stabilize solver
- Too much damping makes motion like in glue

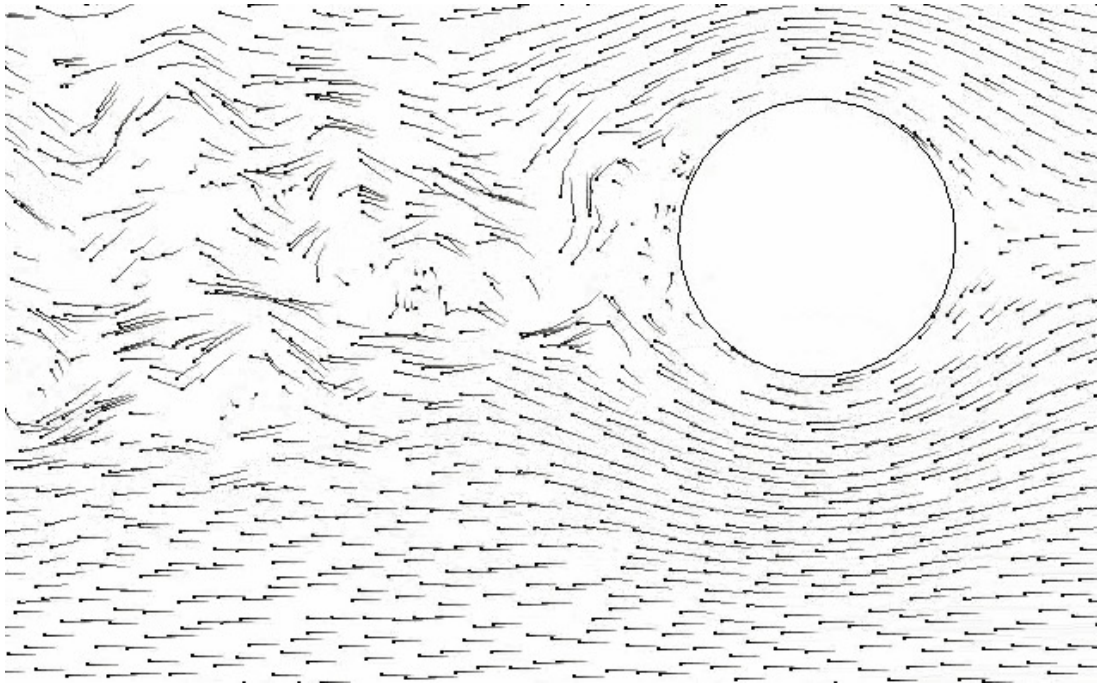
Forces: Spatial Fields

- Externally specified force (or velocity) fields in space
- Force on particle i depends only on its position
- Arbitrary functions
  - wind
  - attractors, repulsors
  - vortices
- Can depend on time
- Note: these add energy, may need damping

Bridson et al.

Example: Procedural Spatial Field

- Curl noise for procedural fluid flow, R. Bridson, J. Hourihan, and M. Nordenstam, Proc. ACM SIGGRAPH 2007.



Plausible, conrollable force fields – just advecting particles along the flow gives cool results!

And it's simple, too!

Curl-Noise for Procedural Fluid Flow

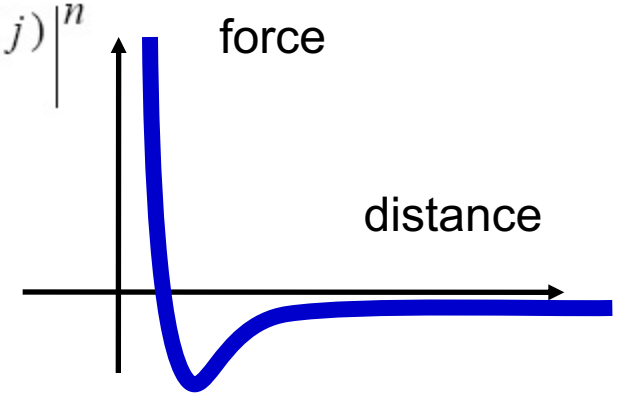
Robert Bridson
Jim Hourihan
Markus Nordenstam

Forces: Other Spatial Interaction

$$\text{Spatial interaction: } f^{(i)} = \sum_j f(x^{(i)}, x^{(j)})$$

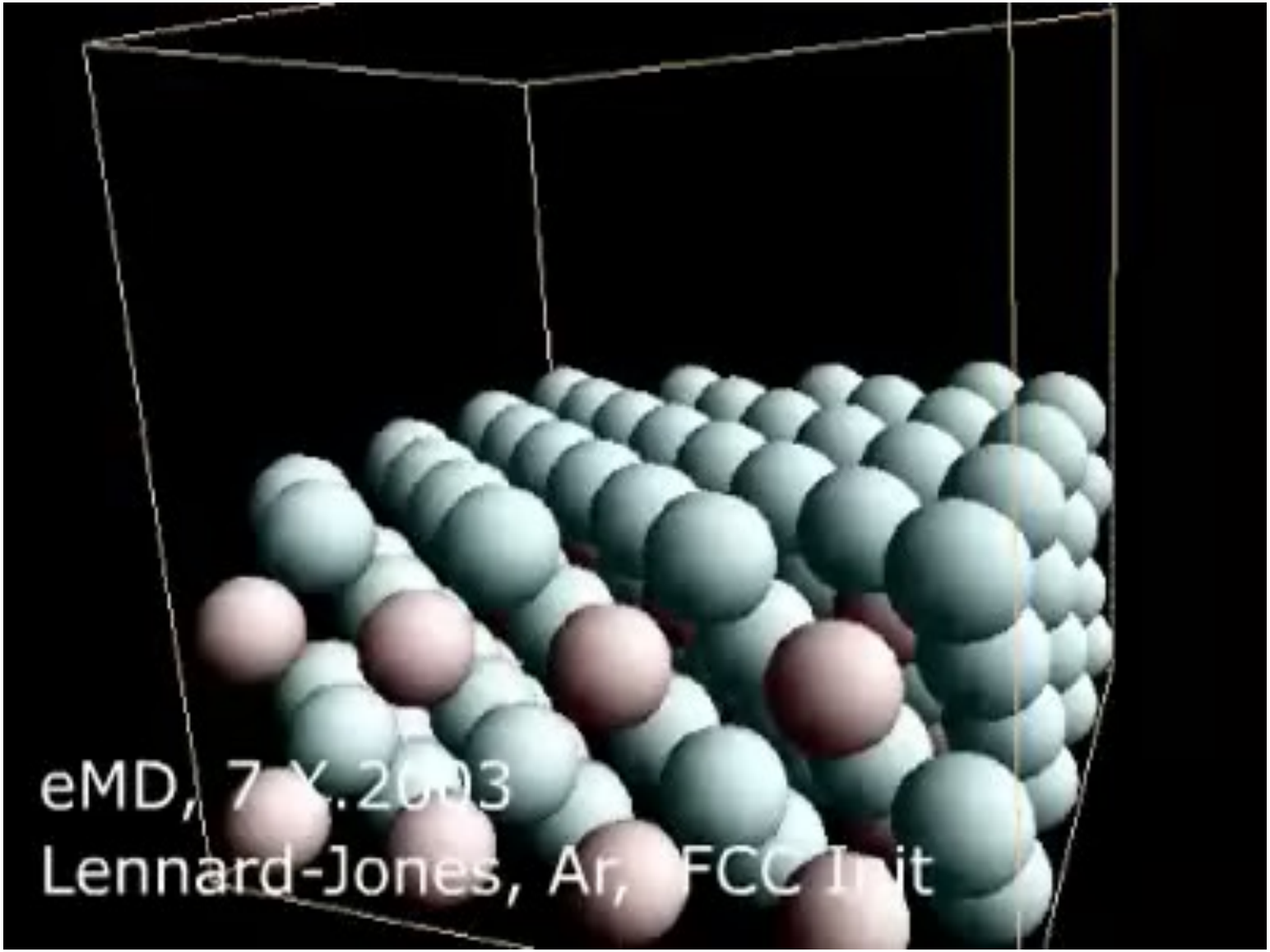
- E.g., approximate fluid using Lennard-Jones force:

$$f(x^{(i)}, x^{(j)}) = \frac{k_1}{\left|x^{(i)} - x^{(j)}\right|^m} - \frac{k_2}{\left|x^{(i)} - x^{(j)}\right|^n}$$



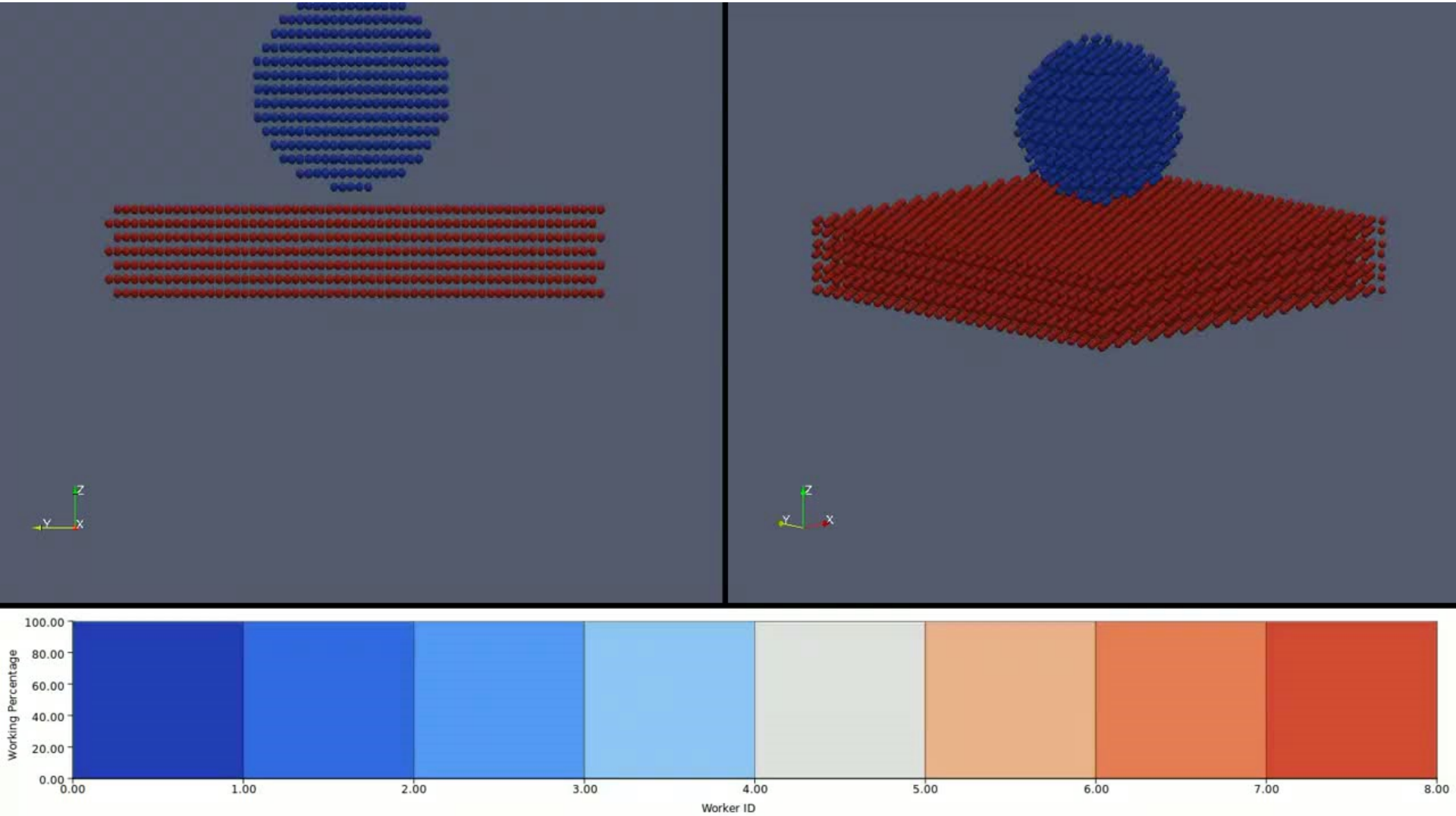
- Repulsive + attractive force
- Again, $O(N^2)$ to test all pairs
  - usually only local
  - Use buckets to optimize

Particles are not independent!



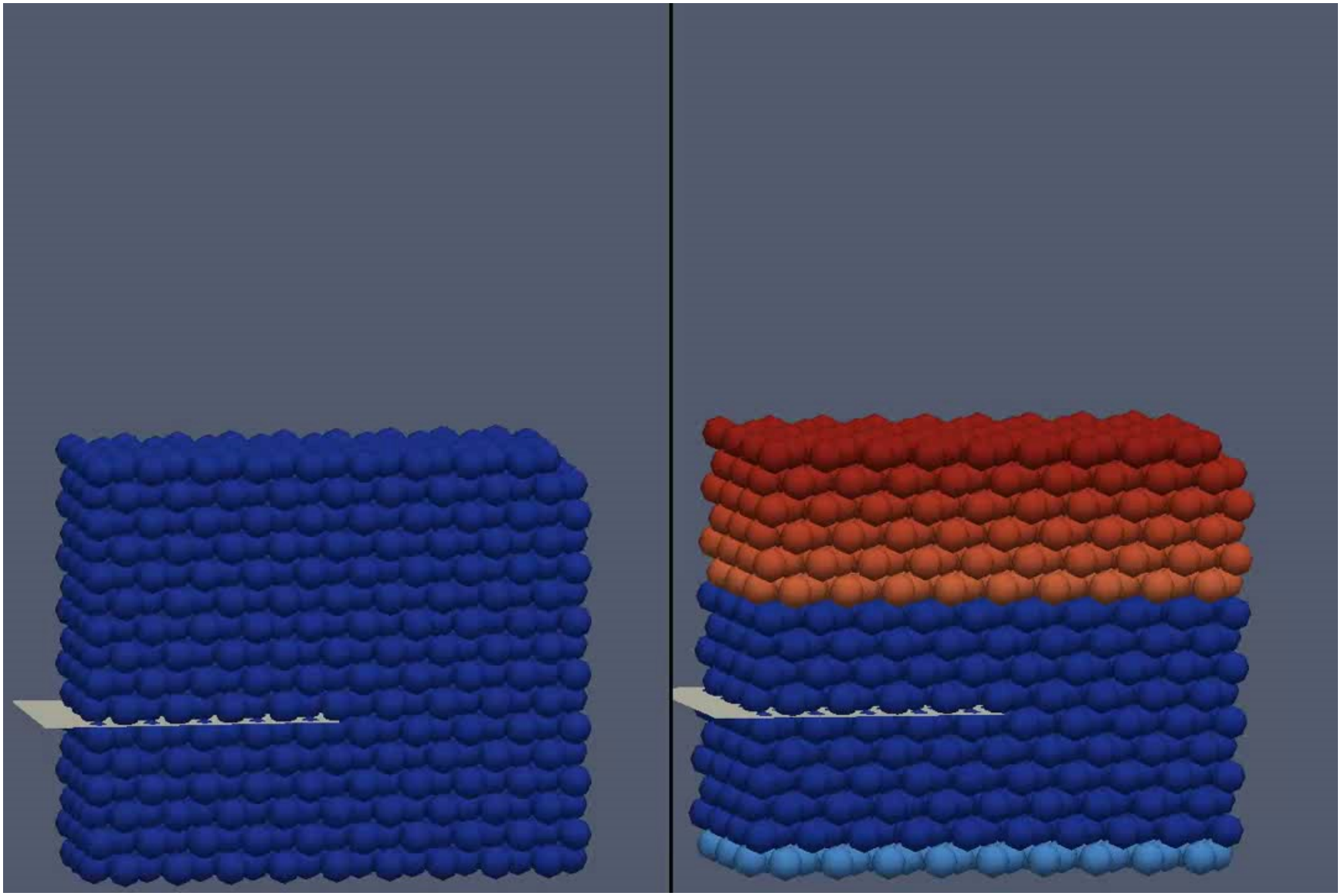
http://www.youtube.com/watch?v=nl7maklgYnI&feature=related

Lennard-Jones forces



http://www.youtube.com/watch?v=XfjYlKxKIWQ&feature=autoplay&list=PL0605C44C6E8D5EDB&lf=autoplay&playnext=2

Questions?



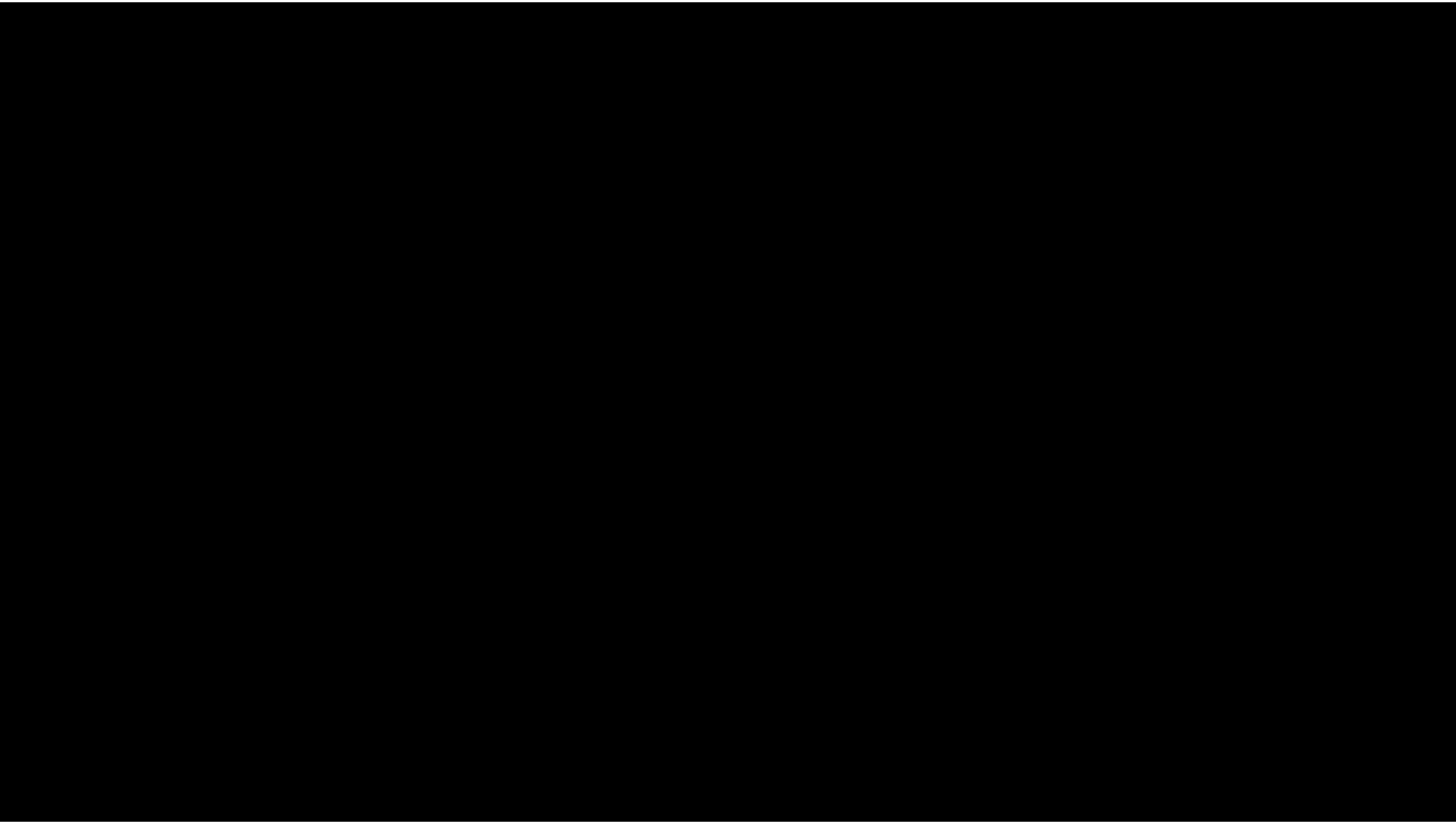
http://www.youtube.com/watch?v=dHWCT7RPjPo

More Eyecandy from NVIDIA

- Fluid flow solved using a regular grid solver
  - This gives a velocity field
- 0.5M smoke particles advected using the field
  - That means particle velocity is given by field
- Particles are for rendering, motion solved using other methods

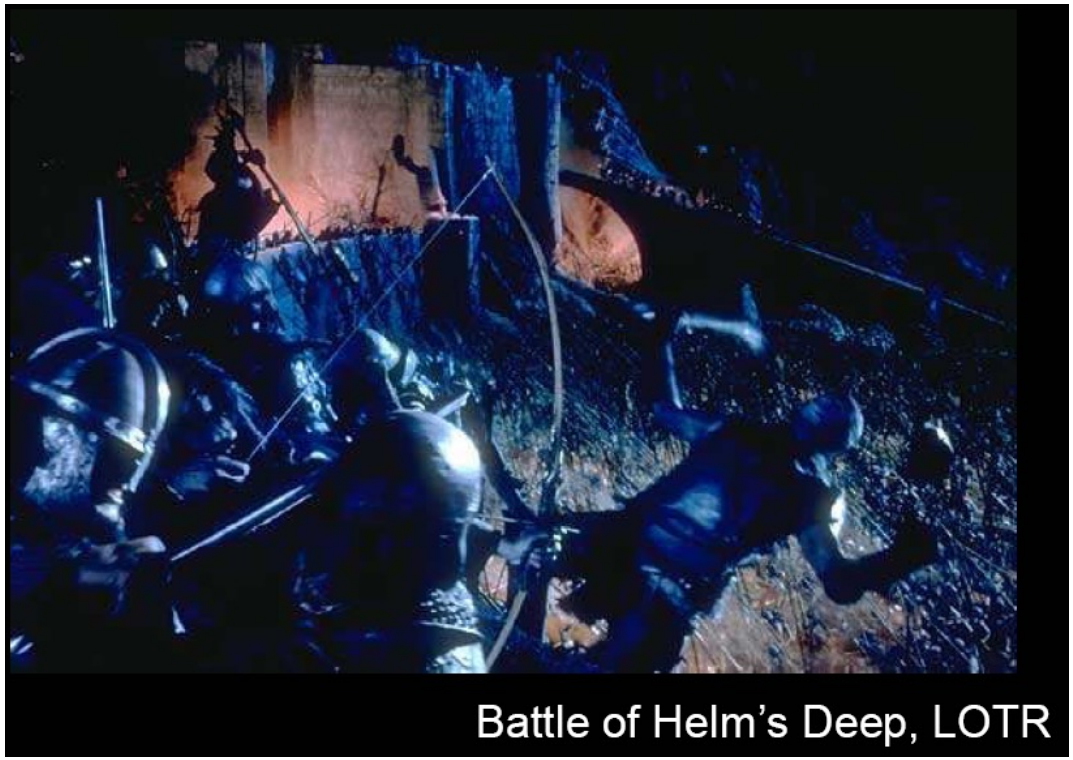
NVIDIA

More Eyecandy from NVIDIA



More Advanced "Forces"

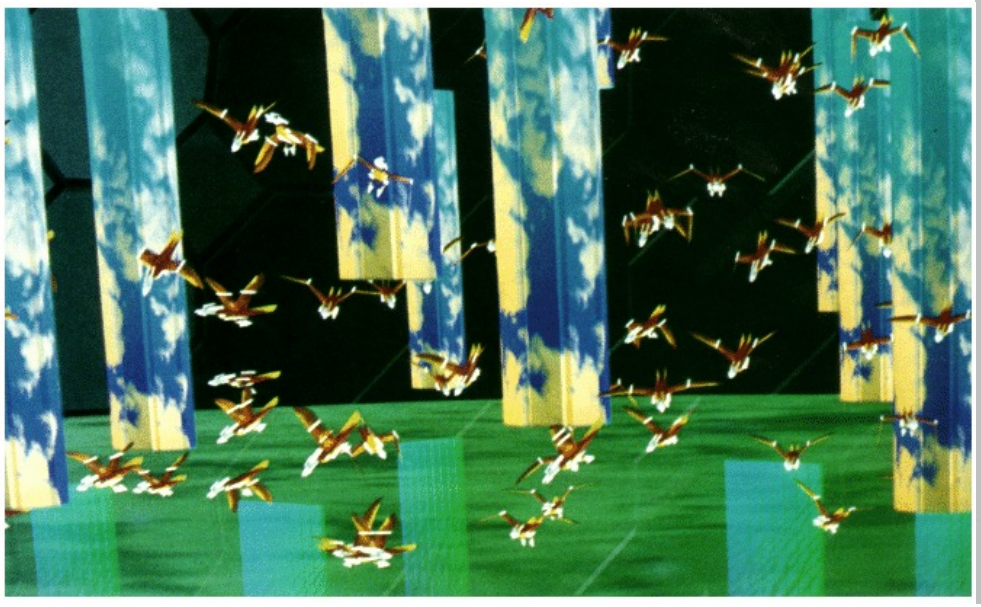
- Flocking birds, fish shoals
  - http://www.red3d.com/cwr/boids/
- Crowds (www.massivesoftware.com)

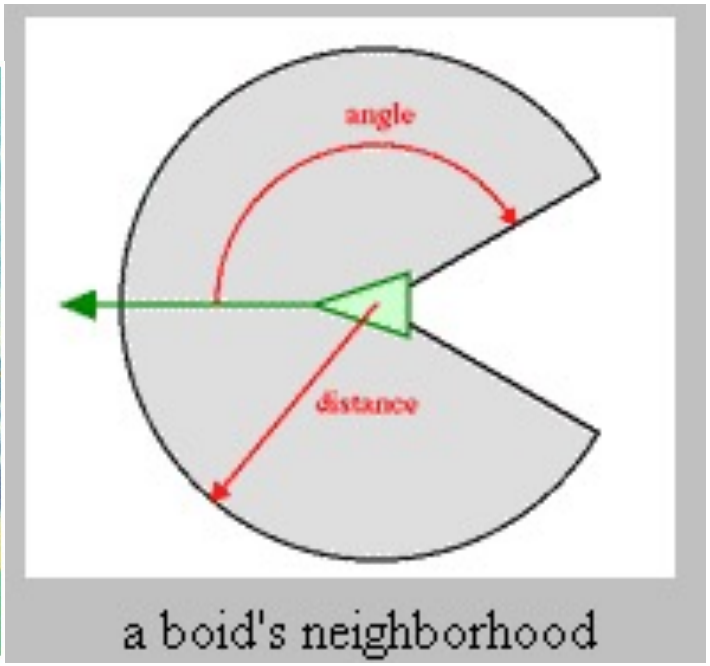


Battle of Helm's Deep, LOTR

Flocks (“Boids”)

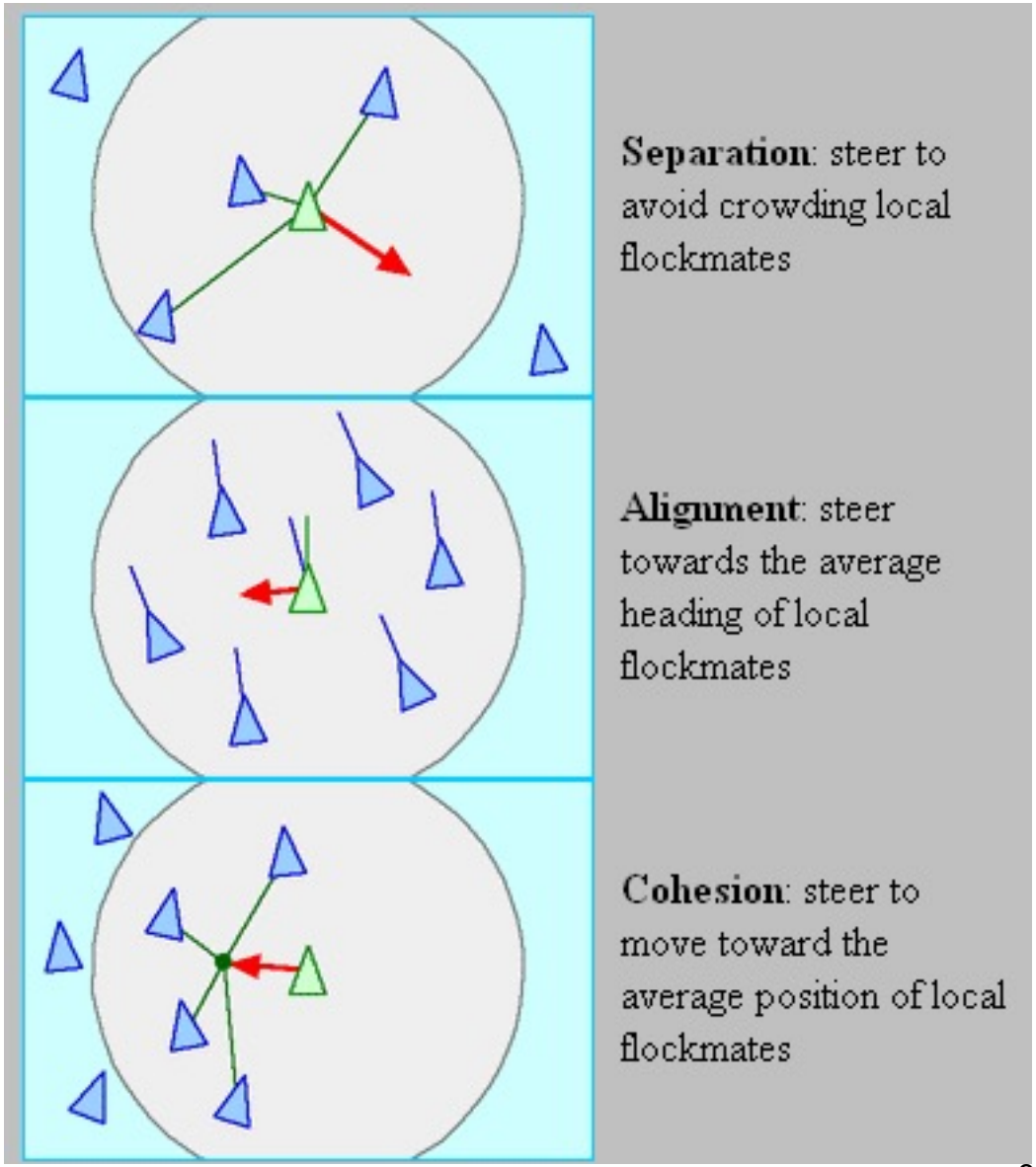
- From Craig Reynolds
- Each bird modeled as a complex particle (“boid”)
- A set of forces control its behavior
- Based on location of other birds and control forces





Flocks (“Boids”)

- (“Boid" was an abbreviation of "birdoid". His rules applied equally to simulated flocking birds, and shoaling fish.)

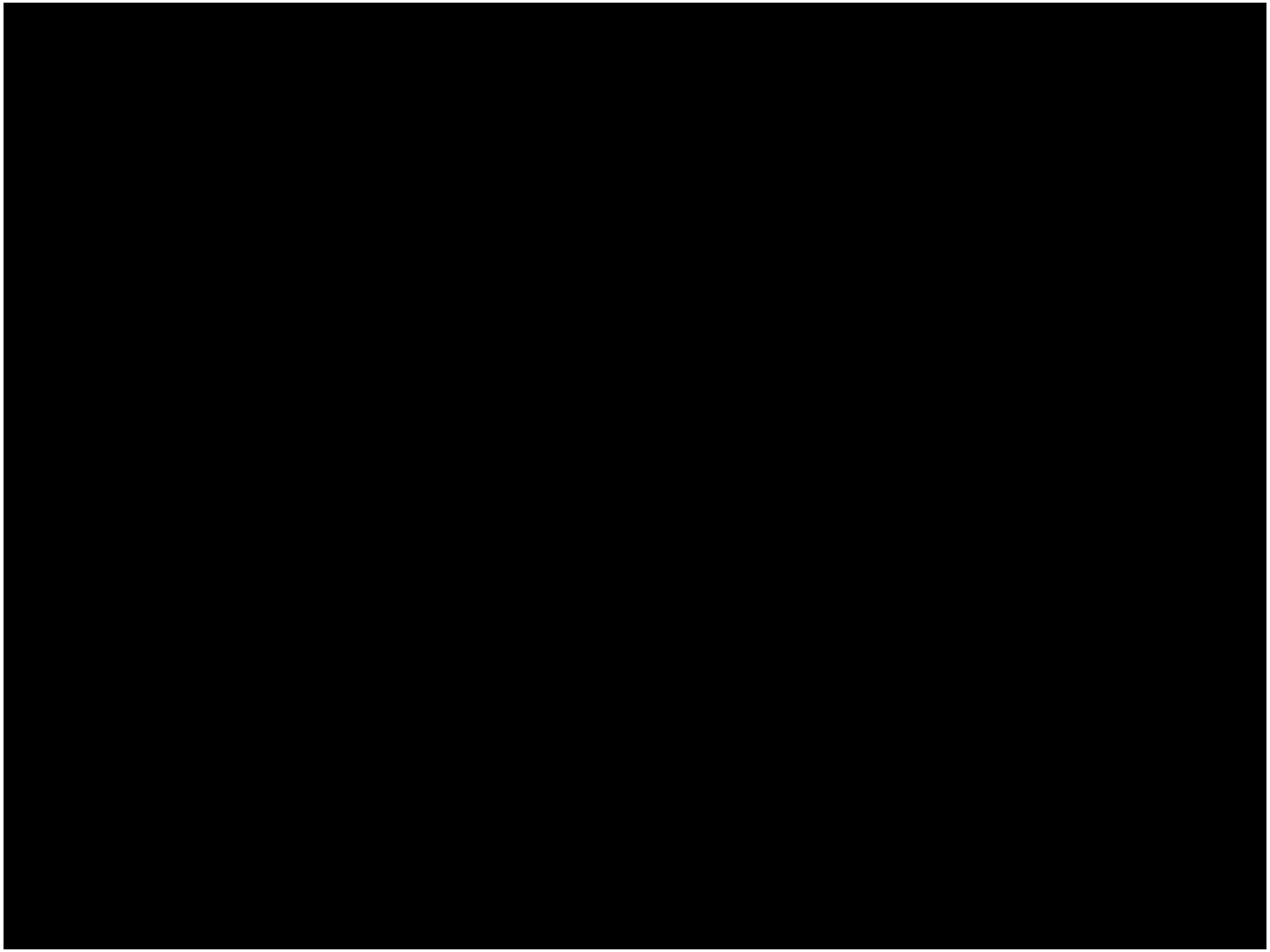


Flocks (“Boids”)

Craig Reynolds

Predator-Prey

- http://www.youtube.com/watch?v=rN8DzlgMt3M

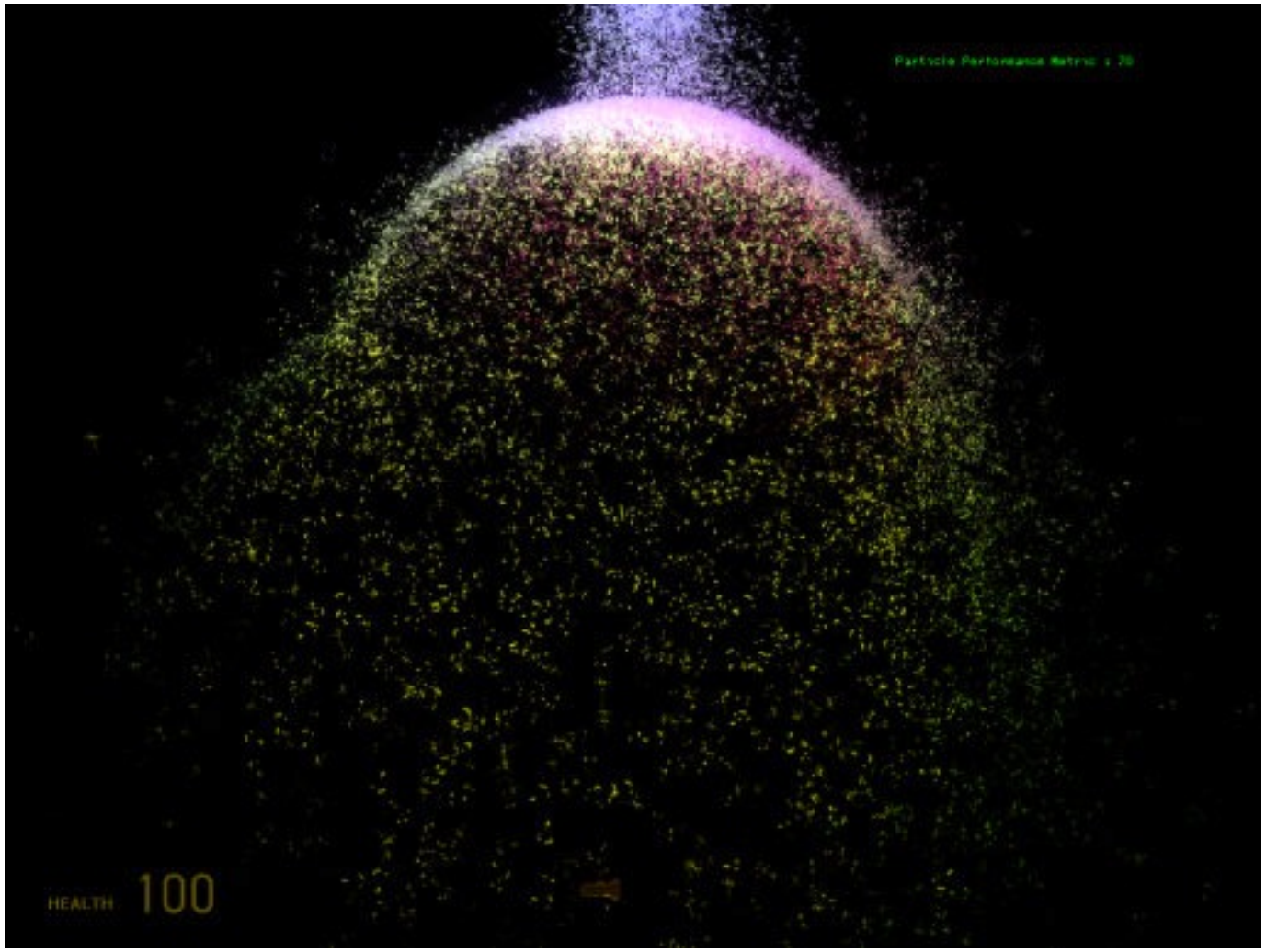


Massive software

- http://www.massivesoftware.com/
- Used for battle scenes in the Lord of The Rings

https://www.fxguide.com/featured/massive-gets-bigger/

Questions?



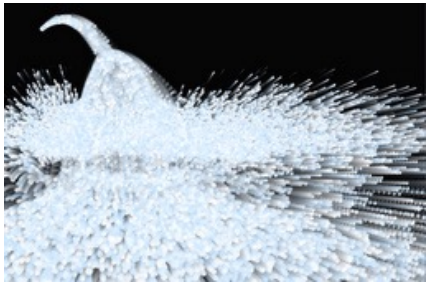
Valve Software

Where do particles come from?

- Often created by generators or emitters
  - Can be attached to objects in the model
- Given rate of creation: particles/second
  - record t_{last} of last particle created

$$n = \lfloor (t - t_{last})\, rate \rfloor$$

  - create n particles.
    update t_{last} if $n > 0$



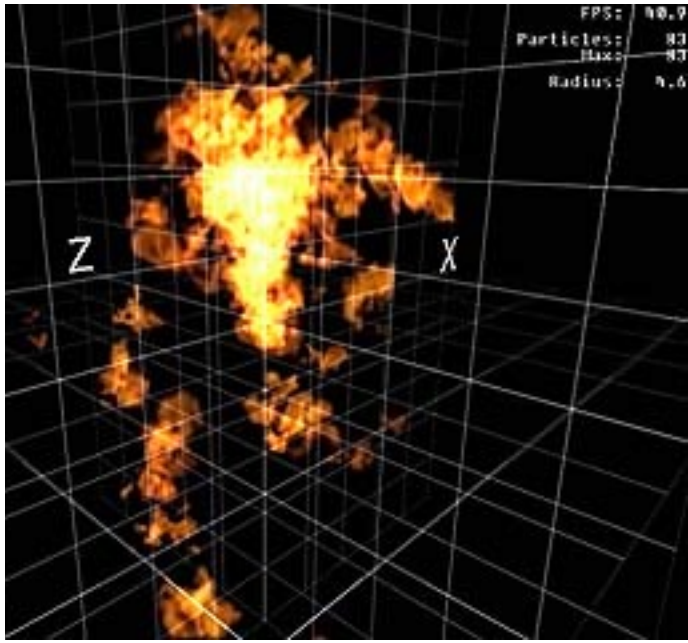
http://www.particlesystems.org/

- Create with (random) distribution of initial x and v
  - if creating $n > 1$ particles at once, spread out on path

Particle Controls

MAX PAYNE 2

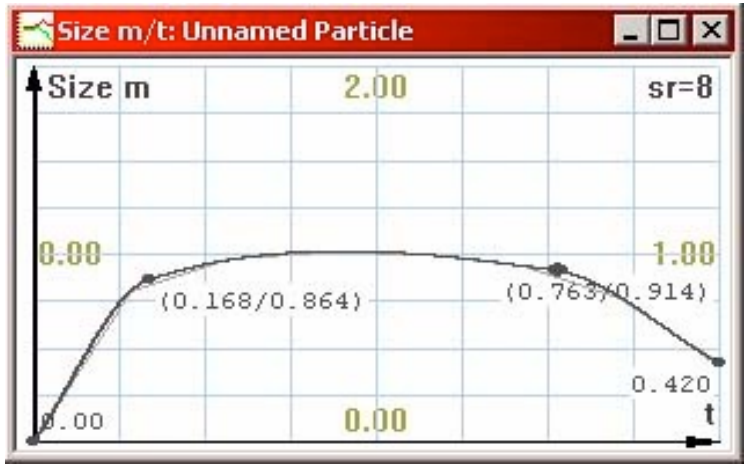
- In production tools, all these variables are time-varying and controllable by the user (artist)
  - Emission rate, color, velocity distribution, direction spread, textures, etc. etc.
    - All as a function of time!
  - Example: ParticleFX (Max Payne Particle Editor)
    - Custom editor software
    - You can download it (for Windows) and easily create your own particle systems. Comes with examples!
    - This is what we used for all the particles in the game!

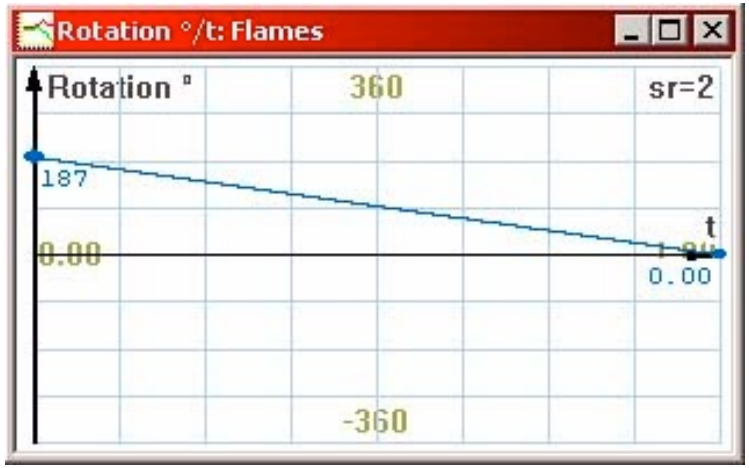


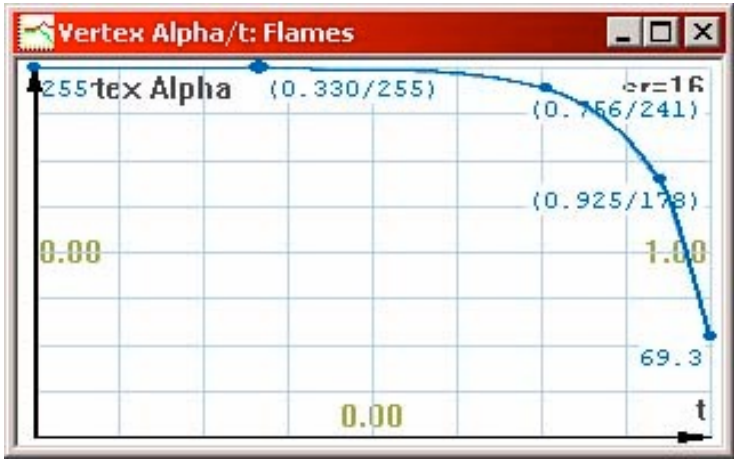
Emitter Controls

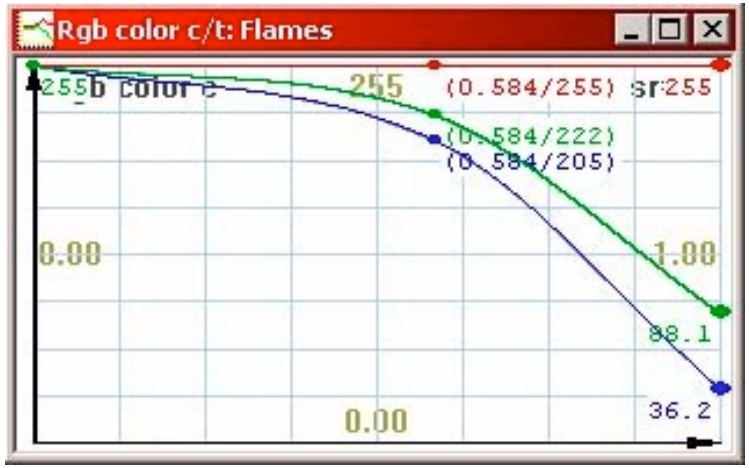
MAX PAYNE 2

- Again, reuse splines!









Emitter Controls

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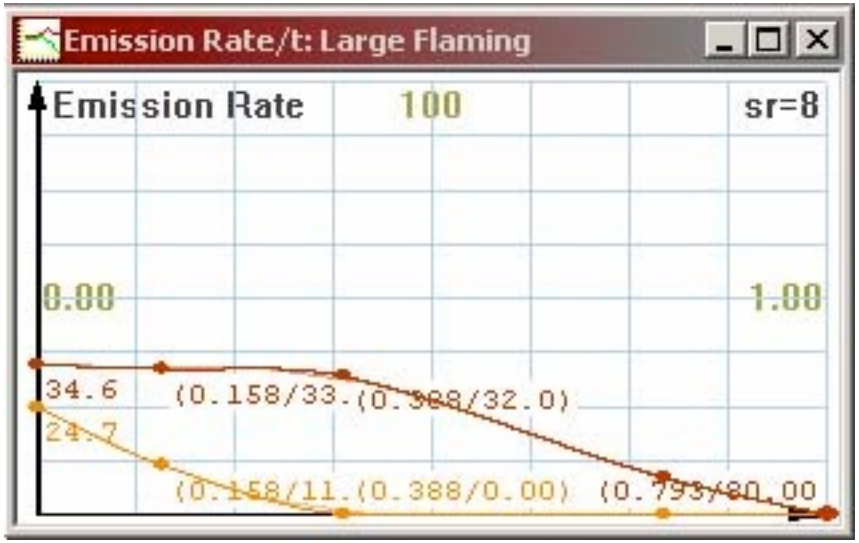
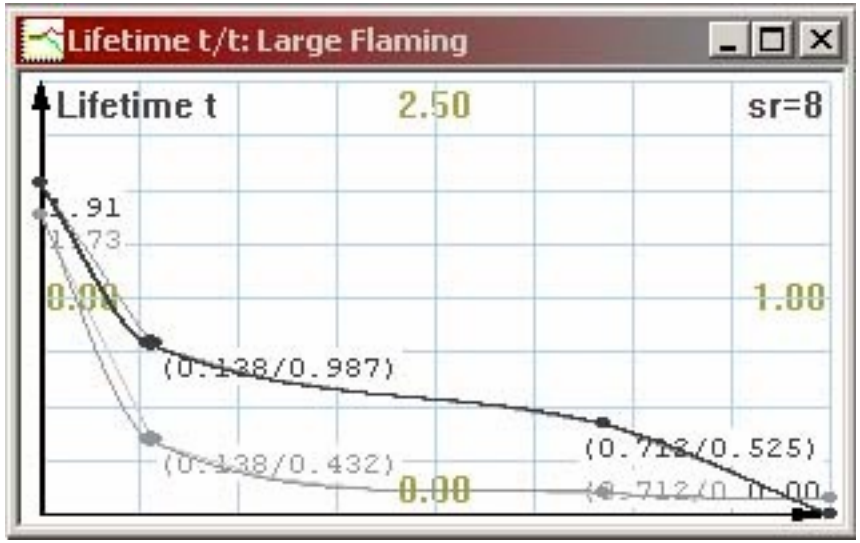
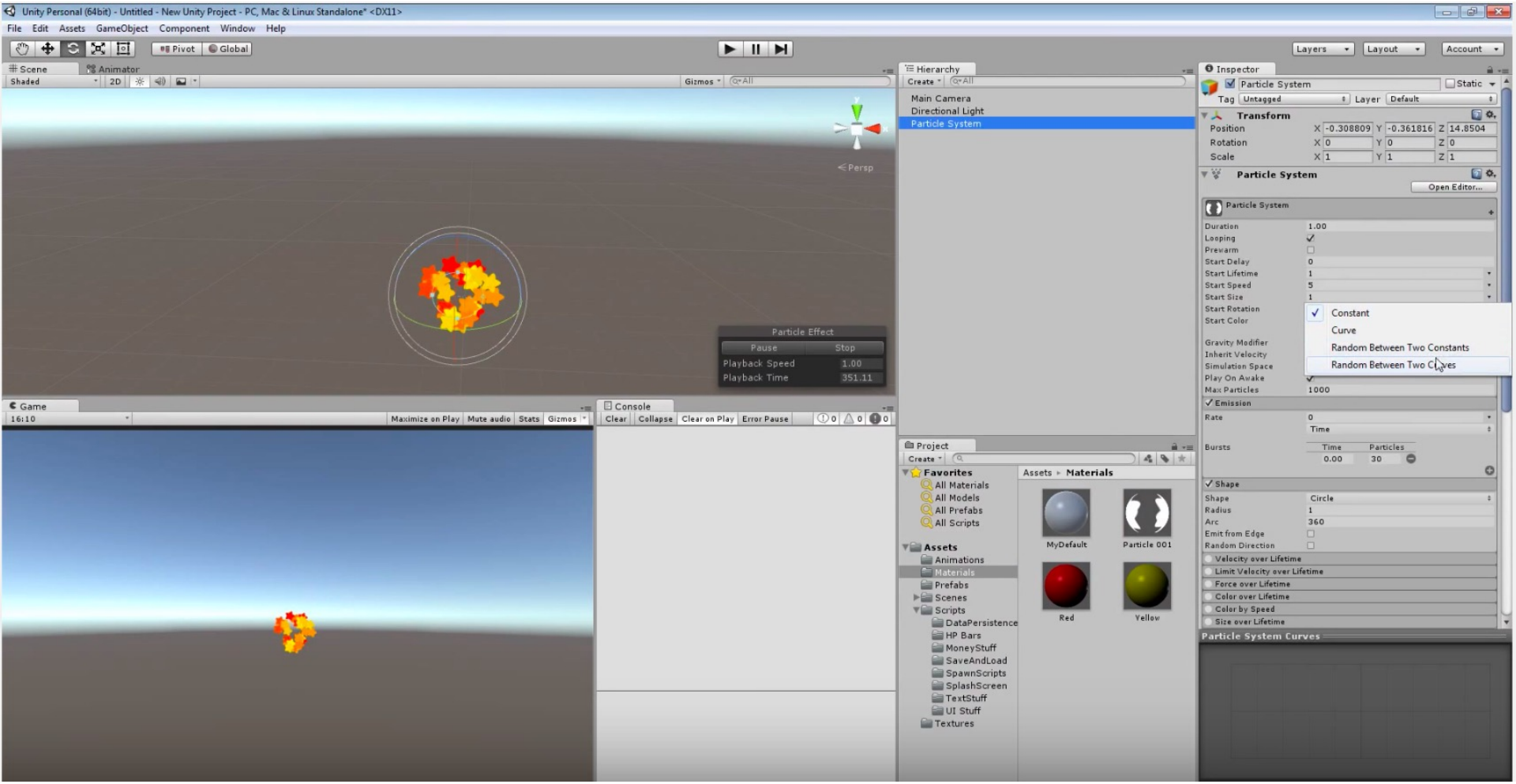




Controls from ParticleFX by Remedy Entertainment

Unity Engine



Rendering and Motion Blur

- Often not shaded (just emission, think sparks)
  - But realistic non-emissive particles needs shadows, etc.
- Most often, particles don't contribute to the z-buffer, i.e., they do not fully occlude stuff that's behind
  - Rendered with z testing on
    (particles get occluded by solid stuff)
- Draw a line for motion blur
  - (x, x+v dt)
  - Or an elongated quad with texture

Metal Gear Solid by Konami

Rendering and Motion Blur

Metal Gear Solid by Konami

Rendering and Motion Blur

- Often use texture maps (fire, clouds, smoke puffs)
  - Called “billboards” or “sprites”
  - Always parallel to image plane

Image from Sameboat

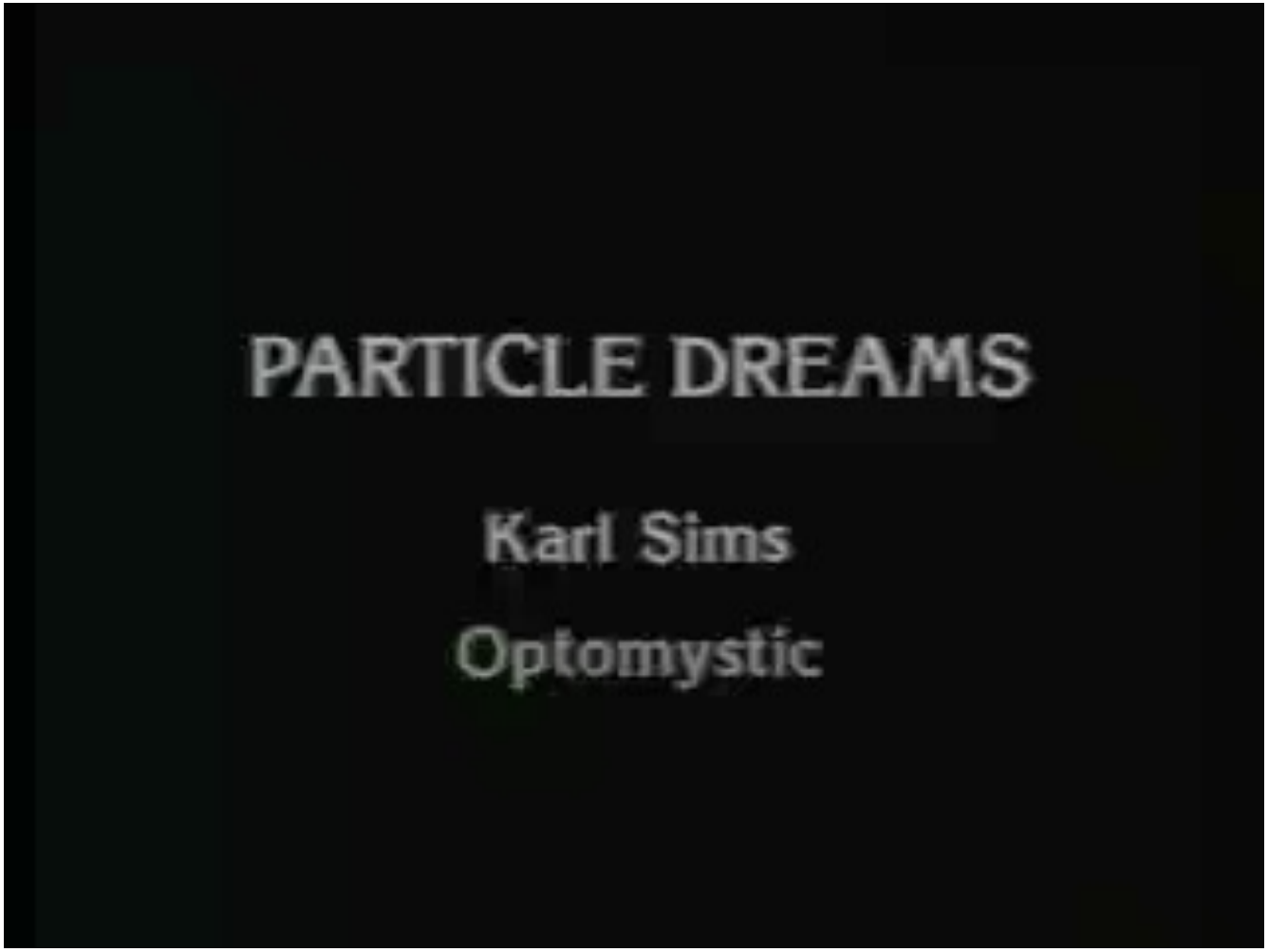
Star Trek 2 – The Wrath of Khan

- One of the earliest particle systems (from 1982)
- Also, fractal landscapes

- Described in [Reeves, 1983]

Paramount Pictures

Questions?



Early particle fun by Karl Sims
http://archive.org/details/sims_particle_dreams_1988

Additional materials

- Further reading
  - Witkin, Baraff, Kass: Physically-based Modeling Course Notes, SIGGRAPH 2001
    - Extremely good, easy-to-read resource. Highly recommended!
  - William Reeves: Particle systems—a technique for modeling a class of fuzzy objects, Proc. SIGGRAPH 1983
    - The original paper on particle systems