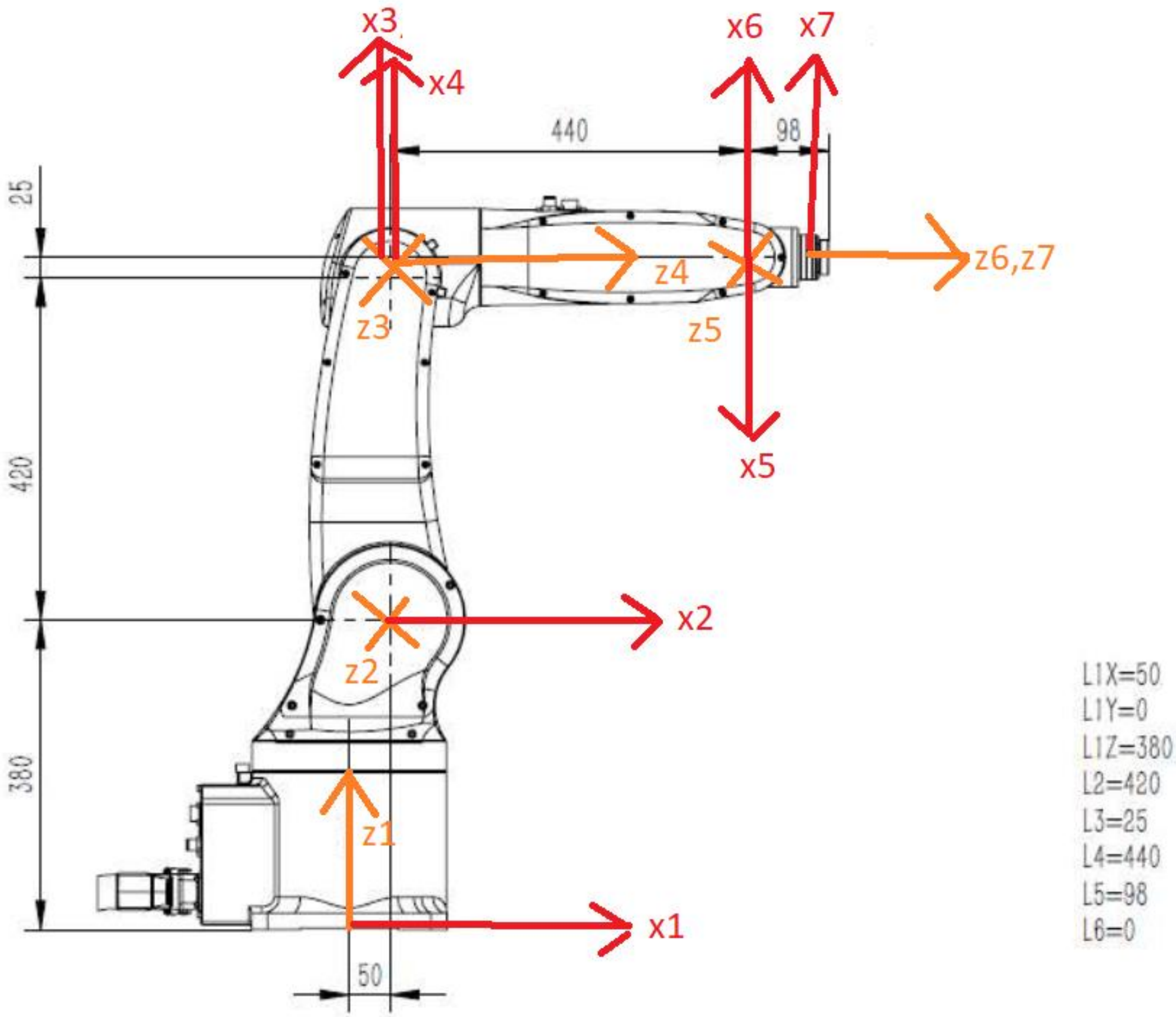
مینی پروژه دوم مکاترونیک   810198479   فاطمه نائینیان



	ai	bi	αi	θi
1	50	380	90	Θ1
2	420	0	0	Θ2
3	25	0	90	Θ3
4	0	440	90	Θ4
5	0	0	90	Θ5
6	0	98	0	Θ6

Forward kinematic

$$ai = \begin{bmatrix} a_i \cos(\theta i) \\ a_i \sin(\theta i) \\ b_i \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 50\cos(\theta 1) \\ 50\sin(\theta 1) \\ 380 \end{bmatrix} \quad a_2 = \begin{bmatrix} 420\cos(\theta 2) \\ 420\sin(\theta 2) \\ 0 \end{bmatrix} \quad a_3 = \begin{bmatrix} 25\cos(\theta 3) \\ 25\sin(\theta 3) \\ 0 \end{bmatrix}$$

$$a_4 = \begin{bmatrix} 0 \\ 0 \\ 440 \end{bmatrix} \quad a_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad a_6 = \begin{bmatrix} 0 \\ 0 \\ 98 \end{bmatrix}$$

$$Q_i = \begin{bmatrix} \cos(\theta i) & -\cos(\alpha i)\sin(\theta i) & \sin(\alpha i)\sin(\theta i) \\ \sin(\theta i) & \cos(\alpha i)\cos(\theta i) & -\sin(\alpha i)\cos(\theta i) \\ 0 & \sin(\alpha i) & \cos(\alpha i) \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} \cos(\theta 1) & 0 & \sin(\theta 1) \\ \sin(\theta 1) & 0 & -\cos(\theta 1) \\ 0 & 1 & 0 \end{bmatrix} \quad Q_2 = \begin{bmatrix} \cos(\theta 2) & -\sin(\theta 2) & 0 \\ \sin(\theta 2) & \cos(\theta 2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos(\theta 3) & 0 & \sin(\theta 3) \\ \sin(\theta 3) & 0 & -\cos(\theta 3) \\ 0 & 1 & 0 \end{bmatrix} \quad Q_4 = \begin{bmatrix} \cos(\theta 4) & 0 & \sin(\theta 4) \\ \sin(\theta 4) & 0 & -\cos(\theta 4) \\ 0 & 1 & 0 \end{bmatrix}$$

$$Q_5 = \begin{bmatrix} \cos(\theta 5) & 0 & \sin(\theta 5) \\ \sin(\theta 5) & 0 & -\cos(\theta 5) \\ 0 & 1 & 0 \end{bmatrix} \quad Q_6 = \begin{bmatrix} \cos(\theta 6) & -\sin(\theta 6) & 0 \\ \sin(\theta 6) & \cos(\theta 6) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$p = a_1 + Q_1 a_2 + Q_1 Q_2 a_3 + Q_1 Q_2 Q_3 a_4 + Q_1 Q_2 Q_3 Q_4 a_5 + Q_1 Q_2 Q_3 Q_4 Q_5 a_6$$

$$Q_1 a_2 = \begin{bmatrix} \cos(\theta 1) & 0 & \sin(\theta 1) \\ \sin(\theta 1) & 0 & -\cos(\theta 1) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 420\cos(\theta 2) \\ 420\sin(\theta 2) \\ 0 \end{bmatrix} = \begin{bmatrix} 420\, c(\theta 1)\, c(\theta 2) \\ 420\, s(\theta 1)\, c(\theta 2) \\ 420\, s(\theta 2) \end{bmatrix}$$

$$Q_1 Q_2 a_3 = \begin{bmatrix} \cos(\theta 1) & 0 & \sin(\theta 1) \\ \sin(\theta 1) & 0 & -\cos(\theta 1) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta 2) & -\sin(\theta 2) & 0 \\ \sin(\theta 2) & \cos(\theta 2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25\cos(\theta 3) \\ 25\sin(\theta 3) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta 1)c(\theta 2) & -c(\theta 1)s(\theta 2) & s(\theta 1) \\ s(\theta 1)c(\theta 2) & -s(\theta 1)s(\theta 2) & -c(\theta 1) \\ s(\theta 2) & c(\theta 2) & 0 \end{bmatrix} \begin{bmatrix} 25\, c(\theta 3) \\ 25\, s(\theta 3) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 25\, c(\theta 1)c(\theta 2)c(\theta 3) - 25\, c(\theta 1)s(\theta 2)s(\theta 3) \\ 25\, s(\theta 1)c(\theta 2)c(\theta 3) - 25\, s(\theta 1)s(\theta 2)s(\theta 3) \\ 25\, s(\theta 2)c(\theta 3) + 25\, c(\theta 2)s(\theta 3) \end{bmatrix} = \begin{bmatrix} 25\, c(\theta 1)c(\theta 2 + \theta 3) \\ 25\, s(\theta 1)c(\theta 2 + \theta 3) \\ 25\, s(\theta 2 + \theta 3) \end{bmatrix}$$

$$Q_1Q_2Q_3a_4 = \begin{bmatrix} \cos(\theta1) & 0 & \sin(\theta1) \\ \sin(\theta1) & 0 & -\cos(\theta1) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta2) & -\sin(\theta2) & 0 \\ \sin(\theta2) & \cos(\theta2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta3) & 0 & \sin(\theta3) \\ \sin(\theta3) & 0 & -\cos(\theta3) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 440 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta1)c(\theta2) & -c(\theta1)s(\theta2) & s(\theta1) \\ s(\theta1)c(\theta2) & -s(\theta1)s(\theta2) & -c(\theta1) \\ s(\theta2) & c(\theta2) & 0 \end{bmatrix} \begin{bmatrix} 440\ s(\theta3) \\ -440\ c(\theta3) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 440\ c(\theta1)c(\theta2)\ s(\theta3) + 440\ c(\theta1)s(\theta2)\ c(\theta3) \\ 440\ s(\theta1)c(\theta2)\ s(\theta3) + 440\ s(\theta1)s(\theta2)\ c(\theta3) \\ 440\ s(\theta2)\ s(\theta3) - 440\ c(\theta2)\ c(\theta3) \end{bmatrix} = \begin{bmatrix} 440\ c(\theta1)s(\theta2+\theta3) \\ 440\ s(\theta1)s(\theta2+\theta3) \\ -440\ c(\theta2+\theta3) \end{bmatrix}$$

$$Q_1Q_2Q_3Q_4a_5 = Q_1Q_2Q_3Q_4 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_1Q_2Q_3Q_4Q_5a_6 = \begin{bmatrix} \cos(\theta1) & 0 & \sin(\theta1) \\ \sin(\theta1) & 0 & -\cos(\theta1) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta2) & -\sin(\theta2) & 0 \\ \sin(\theta2) & \cos(\theta2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta3) & 0 & \sin(\theta3) \\ \sin(\theta3) & 0 & -\cos(\theta3) \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta4) & 0 & \sin(\theta4) \\ \sin(\theta4) & 0 & -\cos(\theta4) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta5) & 0 & \sin(\theta5) \\ \sin(\theta5) & 0 & -\cos(\theta5) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 98 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta1)c(\theta2) & -c(\theta1)s(\theta2) & s(\theta1) \\ s(\theta1)c(\theta2) & -s(\theta1)s(\theta2) & -c(\theta1) \\ s(\theta2) & c(\theta2) & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta3) & 0 & \sin(\theta3) \\ \sin(\theta3) & 0 & -\cos(\theta3) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta4) & 0 & \sin(\theta4) \\ \sin(\theta4) & 0 & -\cos(\theta4) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 98\sin(\theta5) \\ -98\cos(\theta5) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta1)c(\theta2)c(\theta3) - c(\theta1)s(\theta2)s(\theta3) & s(\theta1) & c(\theta1)c(\theta2)s(\theta3) + c(\theta1)s(\theta2)c(\theta3) \\ s(\theta1)c(\theta2)c(\theta3) - s(\theta1)s(\theta2)s(\theta3) & -c(\theta1) & s(\theta1)c(\theta2)s(\theta3) + s(\theta1)s(\theta2)c(\theta3) \\ s(\theta2)c(\theta3) + c(\theta2)s(\theta3) & 0 & s(\theta2)s(\theta3) - c(\theta2)c(\theta3) \end{bmatrix} \begin{bmatrix} 98\cos(\theta4)\sin(\theta5) \\ 98\ \sin(\theta4)\cos(\theta5) \\ -98\cos(\theta5) \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta1)c(\theta2+\theta3) & s(\theta1) & c(\theta1)c(\theta2+\theta3) \\ s(\theta1)c(\theta2+\theta3) & -c(\theta1) & s(\theta1)c(\theta2+\theta3) \\ s(\theta2+\theta3) & 0 & -c(\theta2+\theta3) \end{bmatrix} \begin{bmatrix} 98\cos(\theta4)\sin(\theta5) \\ 98\ \sin(\theta4)\cos(\theta5) \\ -98\cos(\theta5) \end{bmatrix}$$

$$= \begin{bmatrix} 98\ c(\theta1)c(\theta2+\theta3)\ c(\theta4)\ s(\theta5) + 98\ s(\theta1)\ s(\theta4)\ c(\theta5) - 98\ c(\theta1)c(\theta2+\theta3)\ c(\theta5) \\ 98s(\theta1)c(\theta2+\theta3)\ c(\theta4)\ s(\theta5) - 98c(\theta1)\ s(\theta4)\ c(\theta5) - 98\ s(\theta1)c(\theta2+\theta3)\ c(\theta5) \\ 98\ s(\theta2+\theta3)\cos(\theta4)\sin(\theta5) + 98c(\theta2+\theta3)\cos(\theta5) \end{bmatrix}$$

$$p = a_1 + Q_1a_2 + Q_1Q_2a_3 + Q_1Q_2Q_3a_4 + Q_1Q_2Q_3Q_4a_5 + Q_1Q_2Q_3Q_4Q_5a_6$$

$$p = \begin{bmatrix} 50\ c(\theta1) \\ 50\ s(\theta1) \\ 380 \end{bmatrix} + \begin{bmatrix} 420\ c(\theta1)c(\theta2) \\ 420\ s(\theta1)c(\theta2) \\ 420\ s(\theta2) \end{bmatrix} + \begin{bmatrix} 25\ c(\theta1)c(\theta2+\theta3) \\ 25\ s(\theta1)c(\theta2+\theta3) \\ 25\ s(\theta2+\theta3) \end{bmatrix} + \begin{bmatrix} 440\ c(\theta1)s(\theta2+\theta3) \\ 440\ s(\theta1)s(\theta2+\theta3) \\ -440\ c(\theta2+\theta3) \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 98\ c(\theta1)c(\theta2+\theta3)\ c(\theta4)\ s(\theta5) + 98\ s(\theta1)\ s(\theta4)\ c(\theta5) - 98\ c(\theta1)c(\theta2+\theta3)\ c(\theta5) \\ 98s(\theta1)c(\theta2+\theta3)\ c(\theta4)\ s(\theta5) - 98c(\theta1)\ s(\theta4)\ c(\theta5) - 98\ s(\theta1)c(\theta2+\theta3)\ c(\theta5) \\ 98\ s(\theta2+\theta3)\cos(\theta4)\sin(\theta5) + 98c(\theta2+\theta3)\cos(\theta5) \end{bmatrix} =$$

$$= \begin{bmatrix} 50c(\theta1) + 420c(\theta1)c(\theta2) + 25c(\theta1)c(\theta2+\theta3) + 440c(\theta1)s(\theta2+\theta3) + 98c(\theta1)c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) + 98s(\theta1)\,s(\theta4)\,c(\theta5) - 98c(\theta1)c(\theta2+\theta3)c(\theta5) \\ 50s(\theta1) + 420s(\theta1)c(\theta2) + 25s(\theta1)c(\theta2+\theta3) + 440s(\theta1)s(\theta2+\theta3) + 98s(\theta1)c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta1)\,s(\theta4)\,c(\theta5) - 98s(\theta1)c(\theta2+\theta3)c(\theta5) \\ 380 + 420s(\theta2) + 25s(\theta2+\theta3) - 440c(\theta2+\theta3) + 98\,s(\theta2+\theta3)\,c(\theta4)\,s(\theta5) + 98c(\theta2+\theta3)c(\theta5) \end{bmatrix}$$

$$orientation = Q_{\varphi\theta\psi} = Q_1Q_2Q_3Q_4Q_5Q_6$$

Inverse kinematic

X = $50c(\theta1) + 420c(\theta1)c(\theta2) + 25c(\theta1)c(\theta2+\theta3) + 440c(\theta1)s(\theta2+\theta3) + 98c(\theta1)c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) + 98s(\theta1)\,s(\theta4)\,c(\theta5) - 98c(\theta1)c(\theta2+\theta3)c(\theta5)$

y= $50s(\theta1) + 420s(\theta1)c(\theta2) + 25s(\theta1)c(\theta2+\theta3) + 440s(\theta1)s(\theta2+\theta3) + 98s(\theta1)c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta1)\,s(\theta4)\,c(\theta5) - 98s(\theta1)c(\theta2+\theta3)c(\theta5)$

$$x^2 + y^2 = (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) + 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5))^2 + (98\,s(\theta4)\,c(\theta5))^2$$

$$\begin{aligned} x^2 + y^2 = 50 * (&50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) + 420c(\theta2) \\ &* (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) + 25c(\theta2+\theta3) \\ &* (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) + 440s(\theta2+\theta3) \\ &* (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) + 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) \\ &* (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) - 98c(\theta2+\theta3)c(\theta5) \\ &* (50 + 420c(\theta2) + 25c(\theta2+\theta3) + 440s(\theta2+\theta3) \\ &+ 98c(\theta2+\theta3)\,c(\theta4)\,s(\theta5) - 98c(\theta2+\theta3)c(\theta5)) \end{aligned}$$

برای ساده تر شدن محاسبات زین پس برای مثال c(θ2) را با c2 نشان میدهیم

$$\begin{aligned} x^2 + y^2 = 2500 &+ 42000c_2 - 86240c_5s_{2+3}^2 + 9604c_4^2s_5^2c_{2+3}^2 - 19208c_4c_5s_5c_{2+3}s_{2+3} + 9604c_5^2s_{2+3}^2 \\ &+ 9604s_4^2s_5^2 + 2500c_{2+3} + 44000s_{2+3} + 9800c_4s_5c_{2+3} + 22000c_{2+3}s_{2+3} \\ &+ 4900c_4s_5c_{2+3}^2 - 4900c_5c_{2+3}s_{2+3} + 193600s_{2+3}^2 - 9800c_5s_{2+3} + 176400c_2^2 \\ &+ 21000c_2c_{2+3} + 369600c_2s_{2+3} + 82320c_2c_4s_5c_{2+3} - 82320c_2c_5s_{2+3} + 625c_{2+3}^2 \\ &+ 86240c_4s_5c_{2+3}s_{2+3} \end{aligned}$$

حال به سراغ روش تحلیلی کتاب میرویم چون از روش بالا به جواب نمیرسیم.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = e = c + Q_{\varphi\theta\psi}\begin{bmatrix} b_6 \\ 0 \\ 0 \end{bmatrix} = c + Q_{\varphi\theta\psi}\begin{bmatrix} 98 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_{\varphi\theta\psi} = \begin{bmatrix} c_\theta c_\varphi & c_\theta s_\varphi & -s_\varphi \\ c_\varphi s_\theta s_\psi - c_\psi s_\varphi & c_\psi c_\varphi + s_\theta s_\psi s_\varphi & c_\theta s_\psi \\ s_\psi s_\varphi + c_\psi c_\varphi s_\theta & c_\psi s_\theta s_\varphi - c_\varphi s_\psi & c_\theta c_\psi \end{bmatrix}$$

$$c = e - Q_{\varphi\theta\psi}\begin{bmatrix}98\\0\\0\end{bmatrix} = \begin{bmatrix}x - 98c_\theta c_\varphi\\ y - 98c_\varphi s_\theta s_\psi + 98c_\psi s_\varphi\\ z - 98s_\psi s_\varphi - 98c_\psi c_\varphi s_\theta\end{bmatrix} = \begin{bmatrix}x_c\\y_c\\z_c\end{bmatrix}$$

همانند روش کتاب ضرایب را تعریف میکنیم.

$$\lambda_i = \cos\alpha_i\,, \qquad \mu_i = \sin\alpha_i$$

$$t_i = \tan\frac{\theta_i}{2} \quad s_i = \frac{2t_i}{1+t_i^2}, \qquad c_i = \frac{1-t_i^2}{1+t_i^2}$$

$$Ac_1 + Bs_1 + Cc_3 + Ds_3 + E = 0$$
$$Fc_1 + Gs_1 + Hc_3 + Is_3 + J = 0$$

$$A = 2a_1x_c$$

$$B = 2a_1y_c$$

$$C = 2a_2a_3 - 2b_2b_4\mu_2\mu_3$$

$$D = 2a_3b_2\mu_2 + 2a_2b_4\mu_3$$

$$E = a_2^2 + a_3^2 + b_2^2 + b_3^2 + b_4^2 - a_1^2 - x_c^2 - y_c^2 - (z_c - b_1)^2 + 2b_2b_3\lambda_2 + 2b_2b_4\lambda_2\lambda_3 + 2b_3b_4\lambda_3$$

$$F = y_c\mu_1$$

$$G = -x_c\mu_1$$

$$H = -b_4\mu_2\mu_3$$

$$I = a_3\mu_2$$

$$J = b_2 + b_3\lambda_2 + b_4\lambda_2\lambda_3 - (z_c - b_1)\lambda_1$$

با جایگذاری ، به ضرایب زیر میرسیم.

$$A = 100x_c$$

$$B = 100y_c$$

$$C = 21000$$

$$D = 369600$$

$$E = 223725 - x_c^2 - y_c^2 - z_c^2 + 760z_c$$

$$F = y_c$$

$$G = -x_c$$

$$H = 0$$

$$I = 0$$

$$J = 0$$

$$\begin{matrix} Ac_1 + Bs_1 + Cc_3 + Ds_3 + E = 0 \\ Fc_1 + Gs_1 + Hc_3 + Is_3 + J = 0 \end{matrix} \quad => \quad \begin{bmatrix} c_1 \\ s_1 \end{bmatrix} = \begin{bmatrix} A & B \\ F & G \end{bmatrix}^{-1} \begin{bmatrix} -Cc_3 - Ds_3 - E \\ -Hc_3 - Is_3 - J \end{bmatrix}$$

$$A_{11} = a_2 + a_3 c_3 + b_4 \mu_3 s_3$$

$$A_{12} = -a_3 \lambda_2 s_3 + b_3 \mu_2 + b_4 \lambda_2 \mu_3 c_3 + b_4 \mu_2 \lambda_3$$

$$A_{11} = 420 + 25c_3 + 440s_3$$

$$A_{12} = -25s_3 + 440c_3$$

$$\begin{bmatrix} c_2 \\ s_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ -A_{12} & A_{11} \end{bmatrix}^{-1} \begin{bmatrix} x_c c_1 + +y_c s_1 - 50 \\ z_c - 380 \end{bmatrix}$$

$$Rt_3^4 + St_3^3 + Tt_3^2 + Ut_3 + V = 0$$

$$R = 4a_1^2(J - H)^2 + \mu_1^2(E - C)^2 - 4(x_c^2 + y_c^2)a_1^2\mu_1^2$$

$$S = 4[4a_1^2 I(J - H) + \mu_1^2 D(E - C)]$$

$$T = 2[4a_1^2(J^2 - H^2 + 2I^2) + \mu_1^2(E^2 - C^2 + 2D^2) - 4(x_c^2 + y_c^2)a_1^2\mu_1^2]$$

$$U = 4[4a_1^2 I(H + J) + \mu_1^2 D(C + E)]V = 4a_1^2(J + H)^2 + \mu_1^2(E + C)^2 - 4(x_c^2 + y_c^2)a_1^2\mu_1^2$$