Pricing the Term Structure with Linear Regressions

Adrian, Crump & Moench (2013)

Gustavo Amarante

The authors present a model for the term structure of interest rates

- It prices time-series and cross-section of the term structure using a **computationally fast** 3-step linear regression approach.
- Method allows the use of unspanned factors and estimation of a term structure **without observing a zero-coupon yield**.
- Derive the analytical asymptotic variance, allowing for inference.
- Authors suggest a 5-PCs model which outperforms Cochrane & Piazessi (2008).
- Identification of the price-of-risk parameters allows the **decompsition of yields between expected future yields and term premium**.

Contents

Outline

Outline: The Model

1. Pricing kernel is exponentially affine in the shocks that drive the economy
2. Prices of risk are affine in the state variables
3. Innovations on the state variables and return pricing errors are conditionally Gaussian

These assumptions give rise to yields that are affine in the state variables and whose coefficients on the state variables are subject to constraints across maturities (Duffie & Kan, 1996)

The Model

Structure

- State Variable Dynamics: $X_{t+1} = \mu + \Phi X_t + v_{t+1}$ where $v_{t+1} \sim N(0, \Sigma)$
- Princing Kernel: $M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \Sigma^{-\frac{1}{2}} v_{t+1}\right)$
- Prices of Risk: $\lambda_t = \Sigma^{-\frac{1}{2}}(\lambda_0 + \lambda_1 X_t)$

Then is possible to show that:

$$rx_{t+1}^{(n-1)} = \underbrace{\beta^{(n-1)\prime}(\lambda_0 + \lambda_1 X_t)}_{\textit{Expected Return}} \underbrace{- \frac{1}{2}\left(\beta^{(n-1)\prime}\Sigma\beta^{(n-1)} + \sigma^2\right)}_{\textit{Convexity Adjustment}} + \underbrace{\beta^{(n-1)\prime} v_{t+1}}_{\textit{Priced Return Innovation}} + \underbrace{e_{t+1}^{(n-1)}}_{\textit{Return Pricing Error}}$$

Stacking these equations across maturities and time periods gives us a way to estimate these parameters and it only requires a panel of returns and a corresponding set of spanning factors.

The affine term structure model is

$$y_t^{(n)} = -\frac{1}{n}\left(A_n + B_n' X_t + u_t^{(n)}\right)$$

where the cross-maturity coefficient restrictions are:

$$\begin{aligned} A_n &= A_{n-1} + B_{n-1}'\left(\mu - \lambda_0\right) + \frac{1}{2}\left(B_{n-1}' \Sigma B_{n-1} + \sigma^2\right) - \delta_0 \\ B_n' &= B_{n-1}'\left(\Phi - \lambda_1\right) - \delta_1' \end{aligned}$$

Notice that we can set λ_0 and λ_1 to zero and get a **risk-neutral yield curve**. The difference between the model implied yields (or observed yields, if available) and the risk-neutral yield is the **term premium**.

Outline: Empirical Results

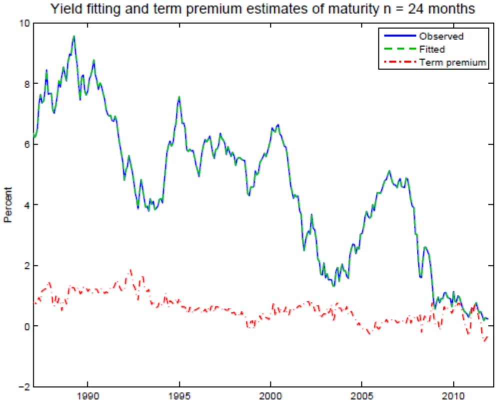
Empirical Results

Selecting the number of PC's

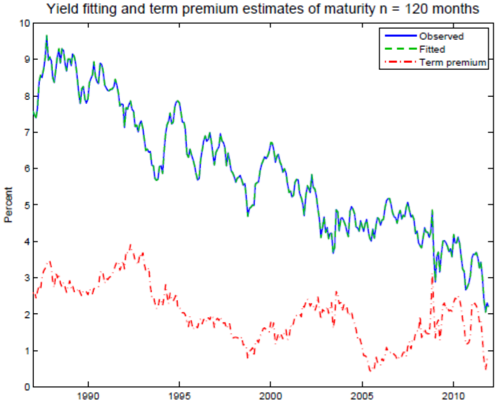
Number of Factors	GSW returns rk_{k-1}	GSW returns W_β	CRSP Fama returns rk_{k-1}	CRSP Fama returns W_β
Panel A: GSW yields				
$K = 3$	577.580	11216.622	333.092	2176.506
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 4$	629.077	4643.528	104.281	231.140
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 5$	885.261	21831.991	34.991	25.139
(p-value)	(0.000)	(0.000)	(0.000)	(0.009)
Panel B: Fama and Bliss yields + GSW six- and ten-year				
$K = 3$	228.202	9105.729	103.634	2090.322
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 4$	155.195	6194.862	86.344	889.096
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 5$	34.348	166.716	32.952	86.717
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
Panel C: H.15 yields				
$K = 3$	242.768	384.954	189.816	454.848
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 4$	61.473	279.181	101.817	44.675
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 5$	18.845	55.866	63.757	68.442
(p-value)	(0.009)	(0.000)	(0.000)	(0.000)
Panel D: CRSP constant-maturity yields				
$K = 3$	164.237	1154.031	68.376	1328.303
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 4$	152.737	462.968	61.836	728.340
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)
$K = 5$	22.550	72.438	23.945	188.009
(p-value)	(0.002)	(0.000)	(0.001)	(0.000)

Early research points out that 3 PCs are enough to explain cross-section variation in yields, but more recent papers emphasize the importance of more factors.

Even with different data sets, the tests unanimously support a five-factor specification, which will be used as a baseline specification.



Yield fitting and term premium estimates of maturity n = 24 months
Observed
Fitted
Term premium
Percent
10
8
6
4
2
0
−2
1990
1995
2000
2005
2010



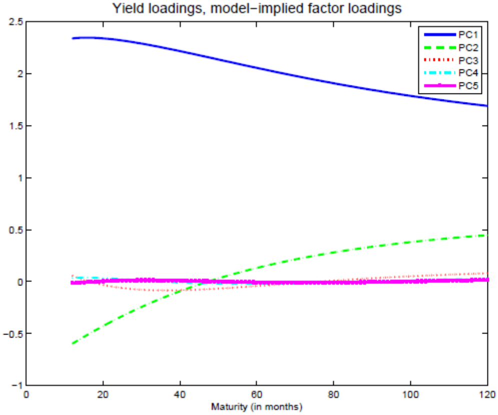
Yield fitting and term premium estimates of maturity n = 120 months
Observed
Fitted
Term premium
Percent
10
9
8
7
6
5
4
3
2
1
0
1990
1995
2000
2005
2010

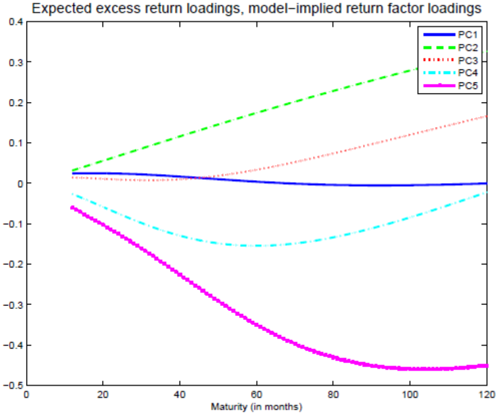
Empirical Results

Estimated Price of Risk Parameters

Factor	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	$\lambda_{1.5}$
PC1	**-0.019**	-0.003	**-0.016**	-0.005	0.012	**0.030**
(t-statistic)	(-2.566)	(-0.443)	(-2.160)	(-0.648)	(1.605)	(3.987)
PC2	0.013	0.027	-0.011	-0.003	-0.011	0.015
(t-statistic)	(0.951)	(1.914)	(-0.818)	(-0.213)	(-0.792)	(1.077)
PC3	-0.030	**-0.077**	-0.001	**-0.093**	**-0.132**	-0.056
(t-statistic)	(-0.951)	(-2.466)	(-0.029)	(-2.987)	(-4.244)	(-1.783)
PC4	0.042	0.064	-0.007	0.015	-0.058	**-0.086**
(t-statistic)	(1.062)	(1.594)	(-0.189)	(0.367)	(-1.461)	(-2.147)
PC5	0.005	**-0.105**	0.012	-0.004	-0.073	**-0.324**
(t-statistic)	(0.097)	(-2.028)	(0.243)	(-0.070)	(-1.431)	(-6.287)

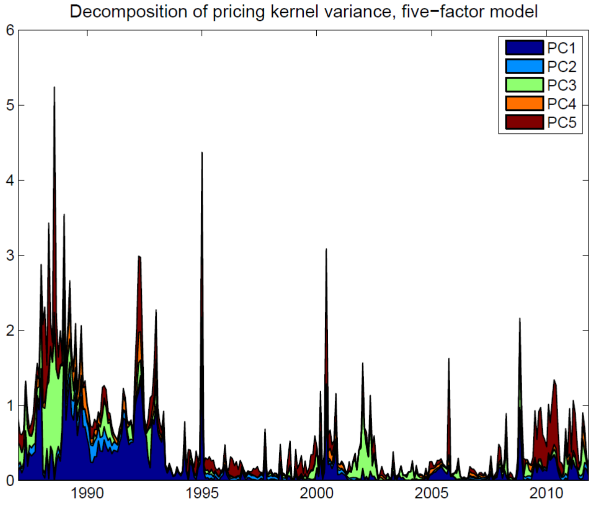
Empirical Results
Loadings





Empirical Results

Pricing Kernel Variance Decomposition

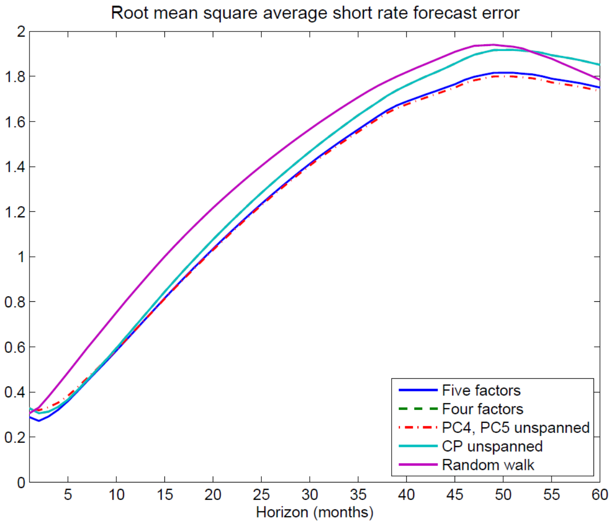


- **First 3 PCs + Cochrane-Piazessi Factor:**
  - CP factor is important to explain treasury return dynamics, but tight parametric restrictions on risk price dynamics are not supported by the author's estimates.
  - Captures similar term premium dynamics but with larger in sample yield fitting errors.
  - Inferior out-of-sample performance in forecasting short-term interest rates.

The authors also give extensions to the model for handling unspanned factors, imposing restrictions on the market price of risk, computing maximum sharpe ratios and working with daily data.

Empirical Results

Other Specifications



Outline: Conclusion

Conclusions

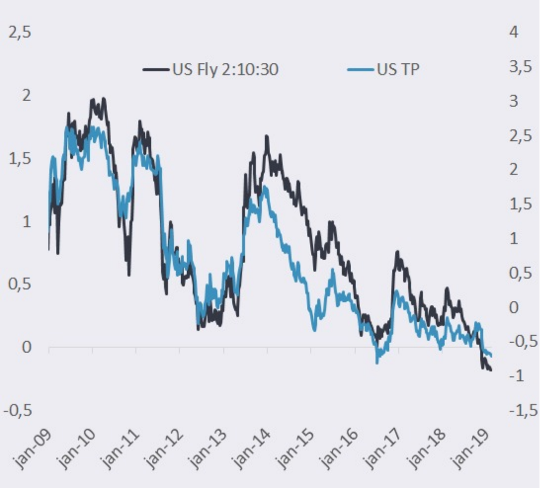
- **Benefits of the Model**
  - Computationally fast
  - Small pricing errors
  - Provides asymptotic standard errors for the model parameters
  - Only requeires a panel of returns and their factors, either spanned or not.

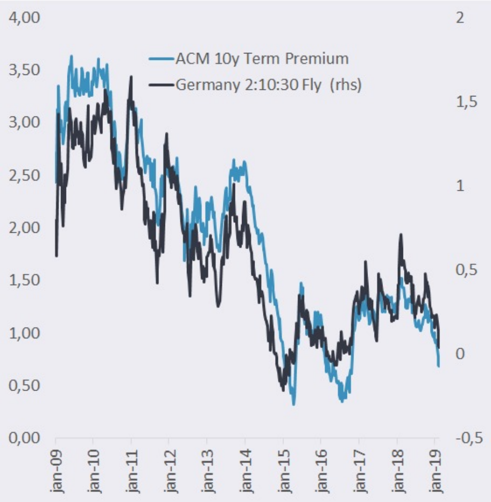
- **Empirical Analysis**
  - The first three principal components are not sufficient to span the cross-section of returns.
  - The 5-PC model gives rise to similar term premium dynamics as the Cochrane-Piazessi (2008)
  - Practically all of the time variation in risk premium is associated with the level risk, but it's dynamics are mainly explained by the second and fifth principal component.

One More Thing...

We find a strong correlarion between the ACM term premium and the 2y-10y-30y butterfly portfolios

One More Thing...





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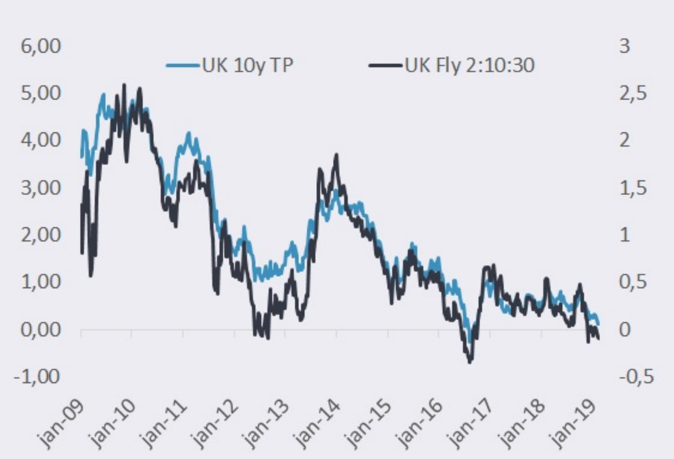

One More Thing...

5y Correlation between Term Premium and FLY 2:10:30	
US	0,92
Germany	0,95
UK	0,95
Japan	0,80

5y Correlation between Term Premium and FLY 2:10:30 (diffs)	
US	0,81
Germany	0,69
UK	0,76
Japan	0,48