An Institutional Theory of Momentum and Reversal

Vayanos and Woolley (2013, RFS)

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Setup of the theory/paper

- The efficient market hypothesis assumes that competition delivers asset prices in fair value, and there is **no need for intermediaries**.

- In the world where markets are inefficient, which is why one hires managers, the authors introduce intermediaries.

- With them, they introduce delegation that generates a cost of investing in the active fund (mgm't perk), which the manager partially appropriates, so the model has **principal-agent problem**.

Storyline of the model

- A positive shock hits an asset's fundamentals, causing good returns for the properly weighted active fund, or the costs decrease, in the asymmetric information settings.

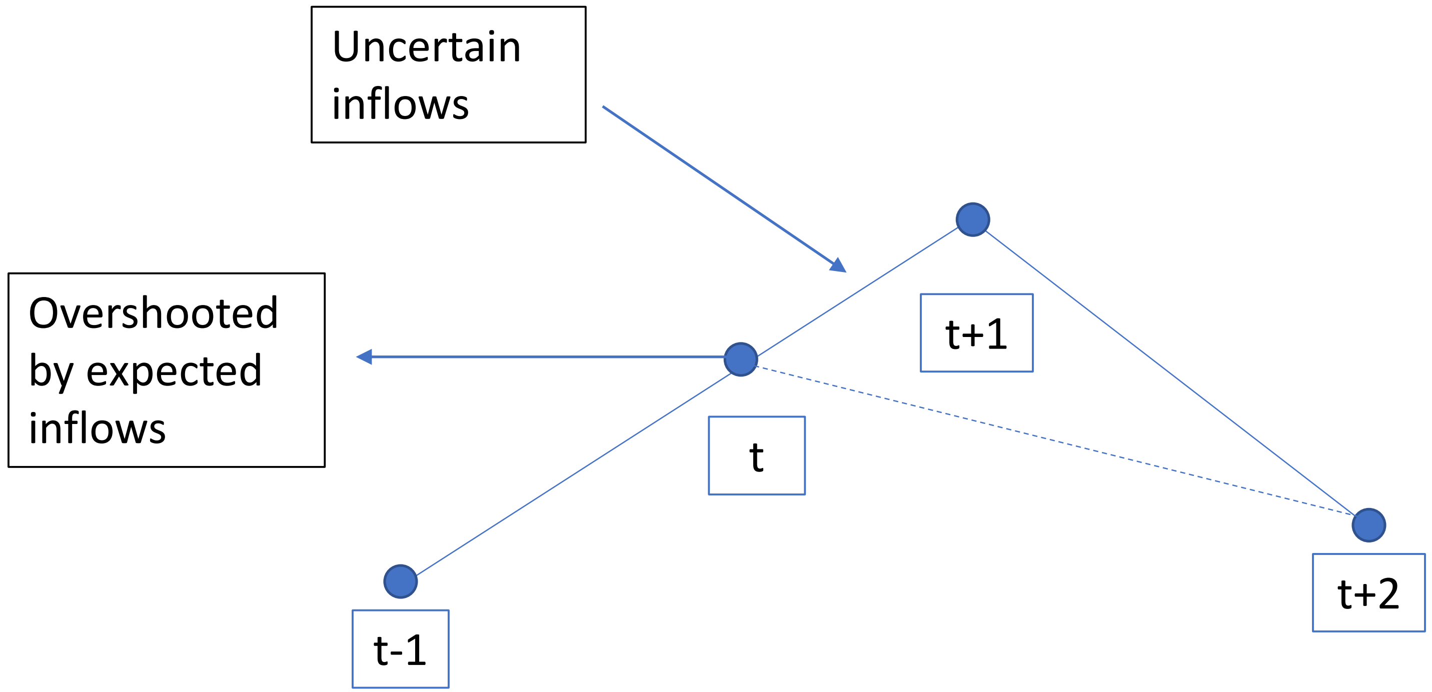
- Then the investors infer that the cost has decreased, and flows in to the active fund, from the index fund. These inflows are **uncertain (1)** in magnitude or timing.

- The inflows are **assumed to be gradual (2)** and if they trigger price increase and an increase in expected returns, assets that performed well (poorly) tend to continue to overperform (underperform).

- Momentum goes **further than fundamentals (3)**, later reversal occurs.

Yet, from whom they buy the positive expected returns assets, at period *t*?

- The manager acts as market maker, with the **bird-in-the-hand effect**:



Quick wrap-up

- The investor and the active fund manager are **rational**.
- There is delegation with uncertainty with management perk.
- Assets prices depend not only in cash flows discounted, as in a market efficient hypothesis, but **also due to fund flows**.

The (symmetric information) model – The assets

- The N risky assets' dividend follows:

$$dDt = \bar{F}dt + \sigma dB_t^D$$

where the covariance matrix Σ is defined by the Brownian part.

- The investor can invest in the risk free, the index fund and the active fund, **but not in individual stocks**.

The cost of investing in the active fund

- The manager extract a cost from investing in the **active fund**, or return gap of the mutual fund and a hypothetical portfolio, the C_t. It is proportional to the number of shares that the investor has, and assumes the process:

$$dC_t = \kappa(\bar{C} - \boldsymbol{\lambda C_t})dt + s dB_t^C$$

where, λ measures the efficiency of the perk extraction; so it is efficient, when equals one, and when zero, she only cares manager about fund performance only through her personal investment.

A friction and stock prices

- Finally assume **the adjustment in active fund have a friction** $\frac{\boldsymbol{\psi}\left(\frac{\boldsymbol{dy_t}}{\boldsymbol{dt}}\right)^{\mathbf{2}}}{\mathbf{2}}$, so that that would be gradual; y_t is the investor's number of shares in the active fund.

- The rate of adjustment is:

$$\frac{dy_t}{dt} \equiv v_t = b_0 - b_1 C_t - b_2 \mathrm{y_t}$$

- Note that the investor does see C_t, and the stock prices $S_t \equiv (S_{1t}, \dots, S_{Nt})'$ take the form:

$$S_t = \frac{\overline{F}}{r} - (a_0 + a_1 C_t + a_2 y_t),$$

The manager – Maximization problem

- The manager chooses her controls $(\overline{c_t}, \overline{y_t}, \boldsymbol{z_t})$ $-$ $\boldsymbol{z_t}$ is the number of shares in one share of the active fund –, to solve:

$$\max_{\overline{c_t}, \overline{y_t}, \boldsymbol{z_t}} [-\exp(-\bar{\alpha}\overline{c_t}) + D\bar{V} - \bar{\beta}\bar{V}]$$

subject to:

$$dW_t = rW_t dt + \overline{y_t}\boldsymbol{z_t}(\mathrm{dD_t} + \mathrm{dS_t} - \mathrm{rS_t dt}) + \boldsymbol{\lambda C_t} - \overline{c_t}dt$$

$$\boldsymbol{z_t} S_t = (\theta - x_t\eta)S_t$$

- They conjecture that the value function that satisfy the Bellman equation is:

$$\bar{V}(W_t, \overline{X_t}) \equiv -\exp[-(r\bar{\alpha}W_t + \overline{q_0} + (\overline{q_1}, \overline{q_2})\bar{X}_t + \frac{1}{2}\overline{X_t}'\bar{Q}\overline{X_t}]$$

where $\overline{X_t} \equiv (C_t, y_t)'$, and $\bar{Q}$ is a constant symmetric (2x2) matrix. By Proposition 1, they prove that is the case if $(\overline{q_0}, \overline{q_1}, \overline{q_2}, \bar{Q})$ satisfy a system of equations.

The manager – Cost on expected returns

- The manager optimization can be reduced over $(\bar{c}_t, \bar{y}_t, \boldsymbol{z_t})$ to $(\bar{c}_t, \hat{\boldsymbol{z}}_{\boldsymbol{t}})$, where $\hat{\boldsymbol{z}}_{\boldsymbol{t}} = \bar{y}_t * z_t$, that is, the number of asset shares in the active fund manager holds (what upper bar indicates).

- The FOC w.r.t. $\hat{\boldsymbol{z}}_{\boldsymbol{t}}$ is:

$$E_t(dR_t) = r\bar{\alpha}Cov_t(dR_t, \hat{\boldsymbol{z}}_{\boldsymbol{t}} dR_t) + (\overline{q_1} + \overline{q_{11}}C_t + \overline{q_{12}}y_t)Cov_t(dR_t, dC_t)$$

- The expected return required by the manager of an asset depends on her portfolio returns **and on the cost changes covariance with returns**.

- Notice that if $C_t = \bar{C} = s = 0$, then dC_t is zero always, and expected returns are only the first term in the RHS above (Corollary 1).

The investor’s maximization problem

- The investor chooses her controls $(c_t, \boldsymbol{x_t}, y_t)$ to solve – η is the shares in the index fund :

$$\max_{c_t, \boldsymbol{x_t} y_t} [-\exp(-\alpha c_t) + DV - \beta V]$$

subject to:

$$dW_t = rW_t dt + \boldsymbol{x_t} \eta dR_t + y_t(\boldsymbol{z_t} dR_t - \boldsymbol{C_t} dt) - \frac{\boldsymbol{\psi} \left(\frac{\boldsymbol{dy_t}}{\boldsymbol{dt}}\right)^{\boldsymbol{2}}}{\boldsymbol{2}} dt - c_t dt$$

- They conjecture that the value function that satisfy the Bellman equation is:

$$V(W_t, X_t) \equiv \exp\left[-\left(r\alpha W_t + q_0 + (q_1, q_2)X_t + \frac{1}{2} X_t' Q X_t\right)\right]$$

where $\mathrm{X_t} \equiv (C_t, y_t)'$. By Proposition 2, they prove that is the case if (q_0, q_1, q_2, Q) satisfy a system of equations.

Equilibrium

- Proposition 3: for small s, there is a unique linear equilibrium. The constants (b_1, b_2) are positive (as expected in the adjustment factor v_t), and a_1 and a_2 (from the stock pricing) are given by:

$$a_i = \gamma_i \Sigma p_f', where\ \gamma_1 > 0\ and\ \gamma_2 < 0, and$$

$$p_f \equiv \theta - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta,\ \text{ is the } \mathbf{flow\ portfolio}, \text{ where } \eta\Sigma p_f' = 0$$

- Long positions in p_f correspond to large components of the vector θ relative to η, and hence to stocks that the active fund overweights relative to the index fund.

- So, **flowing out** of the active to the index fund, the investor is selling a slice of the flow portfolio, thus **selling active o/w's, and buying u/w's**.

Corollaries – Costless delegation and fund flows

- Corollary 1 (Costless delegation): when $C_t = \bar{C} = s = 0$, there is no cost to invest in active fund, and, so, in the steady-state the investor will hold $\lim_{t\to\infty} y_t = \frac{\bar{\alpha}}{\alpha+\bar{\alpha}}$, and of $\lim_{t\to\infty} x_t = 0$. Now, $\theta_t = z_t$. Stocks' expected returns in the steady state are given by:

$$\lim_{t\to\infty} E_t(dR_t) = \frac{r\alpha\bar{\alpha}}{\alpha+\bar{\alpha}}\Sigma\theta' dt = \frac{r\alpha\bar{\alpha}}{\alpha+\bar{\alpha}} Cov_t(dR_t, \theta dR_t)$$

- Corollary 2 (Fund Flows): The change in the investor's holdings at time $t' > t$, caused by a change in C_t is negatively related to the flow portfolio p_f, that is, flows out of the active, where

$$\frac{\delta E_t(x_t'\eta + y_t' z_t')}{\delta C_t} = -\frac{b_1\left[e^{-\kappa(t'-t)} - e^{-b_2(t'-t)}\right]}{b_2-\kappa} p_f$$

Corollaries – Prices and comovement

- Corollary 3 (Prices): with an increase of C_t, the investor gradually sells flow portfolio; because managers are market-makers, they become averse to stocks that covaries with the flow portfolio, increasing expected returns, formally,

$$\frac{\delta S_t}{\delta C_t} = -\gamma_1 \Sigma p_f' = -\frac{\gamma_1}{1 + \frac{s^2 \gamma_1^2 \Delta}{\eta \Sigma \eta}} Cov_t(dR_t, \boldsymbol{p_f dR_t})$$

- Corollary 4 (Comovement): the covariance matrix, due to the change of C_t is:

$$Cov_t(dR_t, dR_t') = \left(\Sigma + \boldsymbol{s^2 \gamma_1^2 \Sigma p_f' p_f \Sigma}\right) dt$$

where the first term in the RHS is cashflow driven, and the second is **fund-flows driven**. It is positive for stock pairs whose covariance w/ the fund portfolio has the same sign, and is negative otherwise.

Corollaries – Expected returns

- Corollary 5 (Expected returns): the cross-section of stocks' expected returns depends on the mkt index, and the flow portfolio as two risk factors, given by

$$E_t(dR_t) = \frac{r\alpha\bar{\alpha}}{\alpha+\bar{\alpha}} - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}Cov_t(dR_t,\eta dR_t) + \Lambda_t Cov_t\left(dR_t,\boldsymbol{p_f dR_t}\right)$$

where $\Lambda_t = r\bar{\alpha} + \frac{1}{1+\frac{s^2\gamma_1^2\Delta}{\eta\Sigma\eta'}}\left(\boldsymbol{\gamma_1^R C_t} + \gamma_2^R y_t - \gamma_1 s^{2\overline{q_1}}\right)$

- For small s, $\boldsymbol{\gamma_1^R < 0, if\ \lambda < \lambda^R}$, and is positive otherwise, and the constant $\gamma_2^R < 0$.

Corollaries – Expected returns and bird-in-the-hand effect

- An increase in C_t, of investing in the active fund, creates an anticipation of future outflows, so the prices of stocks that covary positively with the flow portfolio **decrease** at t, according Corollary 3.

- Corollary 5 implies that when $\gamma_1^R < 0$, the expected returns of those stocks **decrease** at time t.

- Bird-in-the-hand: since there is an anticipation of future outflows, the stocks are underpriced, and **in the long-run offers an attractive return**.

- The manager could earn more returns if she waits for the outflows to happen, but it might not occur. So, she guarantees the long run return by buying the stocks with decreased expected returns.

Δt

Corollaries – Return predictability and career concerns

- Corollary 6 (Return predictability): C_t effects on expected returns causes return to be predictable based on past returns. The covariance between stock returns at time t and $t' > t$ is:

$$Cov_t\left(dR_t, dR_{t'}\right) = \left[\chi_1 e^{-\kappa\left(t'-t\right)} + \chi_2 e^{-b_2\left(t'-t\right)}\right]\Sigma p_f' p_f \Sigma dt dt'$$

- The term in square brackets is positive if $t' - t < \hat{u}$, and negative if $t' - t > \hat{u}$. And $\hat{u} > 0 \; if \; \boldsymbol{\lambda} < \boldsymbol{\lambda^R}, \text{zero otherwise}$ (no momentum part). **The first part is momentum, and the second, reversal**.

- Corollary 7 (Career concerns): when λ is not too large $(\lambda < \lambda^R)$, an increase in λ, capturing manager's career concerns, and against bird-in-the-hand effect,

  - Raises γ_1, and this increases the non-fundamental volume and covariance of stocks
  - Lowers $\chi_1 + \chi_2$, reducing the size of momentum and its horizon

Pinpoints of asymmetric model

- The investor does not see C_t, so she estimate it, with $C_t \sim N(\widehat{C}_t, K)$.

- The dividends now have a stochastic mean-reverse expected dividend return, dF_t. Then, stock prices also reflects these two uncertain variables.

- To infer the C_t, the investor uses Kalman filter.

- The manager optimization continue to be the same, as she still observes C_t, but the investor will take into account $\widehat{C}_t$.

Δt

The (asymmetric information) model – The assets

- The N risky assets' dividend follows:

$$dDt = F_t dt + \sigma dB_t^D$$
$$dF_t = \kappa(\bar{F} - F_t)dt + \phi\sigma dB_t^F$$

- The investor does not see C_t (the only asymmetry) nor F_t (so that she can't infer C_t entirely through prices), but $C_t \sim N\left(\widehat{C}_t, K_t\right)$. Now, stock prices are:

$$S_t = \frac{\bar{F}}{r} + \frac{F_t - \bar{F}}{r + \kappa} - \left(a_0 + \boldsymbol{a_1 \widehat{C_t}} + a_2 C_t + a_3 y_t\right)$$

- Finally, the adjustment in the active fund investment is:

$$v_t \equiv \left(\frac{dy_t}{dt}\right) = b_0 - b_1 \widehat{C}_t - b_2 y_t$$

The investor – Inference of C_t

- Proposition 4: The mean $\widehat{C}_t$ of the investor's conditional distribution of C_t evolves according to the process

$$
\begin{aligned}
&d\widehat{C}_t \\
&= \kappa\left(\bar{C} - \widehat{C}_t\right)dt - \beta_1\{\theta dD_t - C_t dt - \boldsymbol{E_t}(\boldsymbol{\theta dD_t} - \boldsymbol{C_t dt}) - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}[\eta dD_t - E_t(\eta dD_t)]\} \\
&- \beta_2 p_f\left[dS_t + a_1 d\widehat{C}_t + a_3 dy_t - E_t\left(dS_t + a_1 d\widehat{C}_t + a_3 dy_t\right)\right]
\end{aligned}
$$

- The second term in RHS is the learning through dividends, and the third the learning through prices.
  - The investor lowers her estimate of C_t if net dividends $\boldsymbol{\theta dD_t} - \boldsymbol{C_t dt}$ (in curly brackets above) are above expectations, based on regressions of net dividends on ηD_t, the performance of the index fund.
  - If the price of the true market portfolio is above expectations, the investor lowers C_t.
  - As for dividends, the investor knows that the price could be high due to low C_t, and since the manager could be expecting future dividends to be high (F_t small), she does it comparing with the price of the index fund.

Optimizations and Equilibrium

- The manager solves her problem in the same way as before.
- The investor now optimizes the same problem, but instead of using C_t, she uses the inference $\widehat{C_t}$.
- By Propositions 5 and 6, the proposed value functions satisfy the Bellman equations of these optimizations, where the constants satisfy their respective system of equations.
- Proposition 7 (Equilibrium existence): For small s, there exists a unique linear equilibrium. The constants $(b_1, b_2, \gamma_1, \gamma_2)$ are positive and the constant γ_3 is negative.

Corollaries – Amplification and Comovement

- Amplification: let a negative cashflow shock hit an o/w asset in the active fund, the fund return loweres, and investors infer that C_t has increased, and flows out to the index fund. This **pressures all o/w prices down again**.

- Comovement: in symmetric information, expected returns comove due to an C_t increase (ER/ER comovement). Now, one of two o/w stocks by active fund has a negative cashflow shock, it generates a CF/ER comovement.

- Corollary 8 (Comovement and amplification): the covariance matrix of stock returns is:

$$Cov_t(dR_t, dR_t') = \left(f\Sigma + k\Sigma p_f' p_f \Sigma\right)dt$$

- Corollary 9 (Expected returns): the expected returns is equal to the prior model, except that,

$$\Lambda_t = r\bar{\alpha} + \frac{1}{f + \frac{k\Delta}{\eta\Sigma\eta'}}\left(\gamma_1^R \widehat{C_t} + \gamma_2^R C_t + \gamma_3^R y_t - k_1\overline{q_1} - k_2\overline{q_2}\right)$$

Corollaries – Return predictability (based on cashflows)

- Corollary 10 (Return Predictability based on cashflows): the covariance between cashflows shocks (dD_t, dF_t) at time t and $t' > t$ is given by:

$$Cov_t(dD_t, dR_t') = \frac{\beta_1(r+\kappa)Cov_t(dF_t, dR_{t'})}{\beta_2 \phi^2} = \left[\chi_1^D e^{-(\kappa+\rho)(t'-t)} + \chi_2^D e^{-b_2(t'-t)}\right] \Sigma p_f' p_f \Sigma dt dt'$$

This would have a $\widehat{\boldsymbol{u}^{\boldsymbol{D}}}$ **threshold** so for small s, the term in the sq. brackets is positive if $t' - t < \hat{u}^D$, and negative otherwise.

- Corollary 11 (Return Predictability): is the same as the corollary for the prior model, with $\hat{u}$ as a threshold.

Momentum and value strategies

- They construct the Sharpe ratio of momentum and value strategies, with proper weightings.

- And simulated the model to find a maximum Sharpe ratio with both strategies with **61%**, while value and momentum (4m holding period) alone **render 26% and 40%,** respectively.

Conclusion

- Vayanos and Woolley (2013) built a empirically rejectful model explaining why prices **overreact and reversal**.
- Not only that, they explained several asset pricing phenomena through the fund flows story affecting expected returns.
- Finally, if one has the model of asset mispricing, she should combine the strategies of momentum and reversal.

Caveats – Consistency of the rationality feature

- They use a rational agents model with inefficient information assessment of the investor, and assume gradual information flow (the parameter ψ in the model).

  “Gradual outflows can be the consequence of **investor inertia** or institutional constraints, and we simply assume them.”

- The investors cannot invest in individuals stocks.

  “This could be reflect the **cost of learning** about individual stocks and trading them”.

Caveats – Specificity of the stocks in empirical evidences

- Their model states that the very stock that had a positive (negative) shock will be bought (sold), due to inflows (outflows). To test it one should show that the asset is or not being bought (sold). There is **limited space for liquidating, or renewing positions in the short-run**.

- Now, Lou (2012) shows evidences in the contrary, as he states: “Together, these results suggest that managers on average invest part of their inflows in their existing holdings, **and that they use more of their new capital to initiate new positions if the portfolio-average holding size or liquidity cost is larger**.”

Table 2
Fund responses to capital flows

	The Outflow Sample				The Inflow Sample			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	**−0.059**	−0.029	−0.022	−0.022	**−0.032**	0.000	0.020	0.020
	(−6.62)	(−1.32)	(−0.85)	(−0.88)	(−3.42)	(0.02)	(1.22)	(1.21)
$flow_{i,t}$	**0.970**	**1.028**	**1.107**	**1.107**	**0.618**	**0.737**	**0.858**	**0.855**
	(16.82)	(17.64)	(10.97)	(11.27)	(15.78)	(14.64)	(10.57)	(10.57)
$own_{i,j,t-1}$		0.429		**−1.196**		−0.766		−0.471
		(1.35)		(−2.35)		(−1.50)		(−0.65)
$flow_{i,t} \times own_{i,j,t-1}$		−2.355		**−20.588**		**−12.431**		−1.669
		(−0.58)		(−3.25)		(−3.74)		(−0.51)
$liqcost_{j,t-1}$		**−7.455**		**−5.755**		**−7.529**		**−3.416**
		(−2.97)		(−5.38)		(−3.95)		(−4.77)
$flow_{i,t} \times liqcost_{j,t-1}$		**−28.559**		**−13.999**		**−25.748**		**−8.433**
		(−2.48)		(−2.18)		(−3.71)		(−2.39)
$own_{i,t-1}$			**2.171**	**3.924**			−0.364	0.212
			(3.58)	(4.06)			(−0.44)	(0.18)
$flow_{i,t} \times own_{i,t-1}$			11.265	**41.242**			**−21.337**	**−19.235**
			(1.32)	(3.10)			(−3.20)	(−2.58)
$liqcost_{i,t-1}$			−11.127	−6.084			**−18.461**	**−15.505**
			(−1.89)	(−1.24)			(−3.08)	(−2.79)
$flow_{i,t} \times liqcost_{i,t-1}$			−57.295	−44.609			**−51.076**	**−42.332**
			(−1.90)	(−1.43)			(−3.01)	(−2.49)
Adjusted R^2	4.68%	6.31%	6.21%	6.43%	9.53%	10.07%	11.36%	11.46%
No. observations	1,207,060	1,044,623	1,207,060	1,044,623	2,462,355	2,215,898	2,462,355	2,215,898

This table reports regression analyses of mutual fund trading in response to capital flows. The dependent variable in all specifications is the percentage change in shares held by fund i in stock j from quarters $t-1$ to t with split adjustments. The main independent variable of interest is $flow_{i,t}$, which is the net capital flow to fund i in quarter t divided by the fund's total net assets at the end of the previous quarter. Other control variables include $own_{i,j,t-1}$, the percentage of all shares outstanding of stock j that is held by fund i at the end of quarter $t-1$; and $liqcost_{j,t-1}$, the effective half bid-ask spread estimated from the Basic Market-Adjusted model as described in Hasbrouck (2006, 2009). $own_{i,t-1}$ and $liqcost_{i,t-1}$ are the portfolio-weighted average ownership share and effective bid-ask spread, respectively. The coefficients are estimated using a panel OLS approach with quarter fixed effects. t-statistics, shown in parentheses, are computed based on standard errors clustered at the fund level. Coefficient estimates significant at the 5% level are indicated in bold.