Cartesian grid q-space reconstruction

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For the purpose of the HARDI reconstruction workshop we used GQI2 [1], [2] a non-parametric method to find the ODFs [3], [4] for dMRI acquisitions on a keyhole cartesian grid in q-space [3]. From these ODFs we find the number of peaks (number of fiber compartments); if this number equals 1, we report the ODF from fitting the Single Tensor to the data, otherwise we report the original GQI2 ODF.

Following the approach of Wedeen et al. [5] the ODF (ψ_{DSI}) can be derived by obtaing the diffusion propagator $P(\mathbf{r})$ from the dMRI signal by a Fourier transform on the cartesian lattice and then integrating radially.

$$\psi_{DSI}(\hat{\mathbf{u}}) = \int_0^\infty P(r\hat{\mathbf{u}})r^2 dr \tag{1}$$

Yeh et al. [1] proposed a direct way to calculate a slightly different ODF using the Cosine transform. They estimated the spin density weighted propagator Q via from an unweighted truncated radial projection

$$\begin{aligned}\psi_{GQI}(\hat{\mathbf{u}}) &= \int_0^\lambda Q(r\hat{\mathbf{u}})dr \\ &= \lambda \int S(\mathbf{q})\mathtt{sinc}(2\pi r\mathbf{q}\cdot\hat{\mathbf{u}})d\mathbf{q} \end{aligned}\tag{2}$$

where λ is a smoothing factor called the diffusion sampling length.

We have instead developed an ODF like the one produced using DSI where we need to take into consideration the weighted truncated radial projection r^2. This will give us a different ODF which we symbolize with ψ_{GQI2}

$$\begin{aligned}\psi_{GQI2}(\hat{\mathbf{u}}) &= \int_0^\lambda Q(r\hat{\mathbf{u}})r^2 dr \\ &= \lambda^3 \int S(\mathbf{q})H(2\pi r\mathbf{q}\cdot\hat{\mathbf{u}})d\mathbf{q} \end{aligned}\tag{3}$$

where $H(x) = \begin{cases} \frac{2\cos(x)}{x^2} + \frac{(x^2-2)\sin(x)}{x^3}, & x \neq 0 \\ 1/3, & x = 0 \end{cases}$.

This equation can be implemented analytically in a simple matrix form

$$\boldsymbol{\psi}_{GQI2} = \mathbf{s}\cdot \mathtt{H}((6D\cdot G\circ\mathbf{b}\circ\mathbb{1})\cdot G)\lambda^3/\pi \tag{4}$$

where $\cdot$ denotes standard matrix or vector dot product, $\circ$ denotes the Hadamard product, $\mathbf{y}$ is ψ_{GQI} as a MD vector with components corresponding to the selected directions $\hat{\mathbf{u}}$ on the ODF sphere, $\mathbf{s}$ is a vector with all the signal values, $D = 0.0025$ where D is the free water diffusion coefficient, G is the $N\times 3$ matrix with the gradient vectors, $\mathbf{b}$ is the $N\times 1$ matrix with the b-values and $\mathbb{1}$ is the $N\times 3$ incidence matrix where all values are equal to 1.

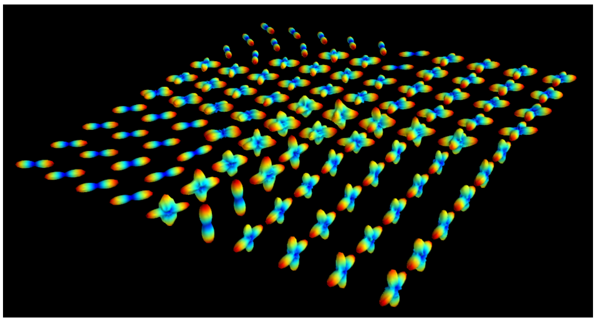

Figure 1. Result showing our reconstruction ODFs with the training data set (SNR 30) provided by the organizers of the HARDI Workshop 2012

We use $\lambda = 3.0, 3.3, 3.5$ with the provided phantom data with SNRs $10, 20, 30$ respectively. In the case where a single peak is found the Single Tensor model is fitted using Weighted Least Squares and the standard ODF for that is generated.

The source code for the methods described here can be found in DIPY (dipy.org). An example of our method can be seen in Fig. 1. GQI2 is a method theoretically identical with the framework of Equatorial Inversion Transform (EIT) [2] with Laplacian weighting. This builds on the formulation in eq. 4.

Model-based methods for ODF estimation, like the Single Tensor or Multi Tensor, require a number of parameters to be fitted. By contrast for model-free methods fitting is not necessary and the directionality of the underlying tissue can be approximated by a re-parametrization or re-transformation of the signal. The latter is usually more efficient than fitting models with many parameters which typically call for iterative methods.

References

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