Christ Church Grammar School

MATHEMATICS METHODS UNIT 1

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures ☐☐ ☐☐ ☐ ☐☐☐

In words _____

_____

Your name _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of CCGS assessments are detailed in the Reporting and Assessment policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free   **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1   **(7 marks)**

Solve the following equations for x.

(a) $(3x - 1)(x + 2) = 0$.   (2 marks)

$$x = \frac{1}{3} \qquad x = -2$$

✓ One solution correct

✓ both solutions correct.

(b) $x^2 - 6x - 7 = 0$.   (2 marks)

$$(x - 7)(x + 1) = 0$$

$$x = 7 \quad \text{and} \quad x = -1$$

✓ One solution correct

✓ both solutions correct.

(c) $(x - 11)^2 - 81 = 0$.   (3 marks)

$$(x - 11)^2 = 81$$
$$x - 11 = \pm 9$$
$$x = 20 \quad \text{\&} \quad x = 2$$

<u>OR</u>

$$(x - 11 - 9)(x - 11 + 9) = 0$$
$$(x - 20)(x - 2) = 0 \quad \therefore x = 20 \text{ \& }$$
$$x = 2$$

✓ Solves for x

✓ both solutions

✓ One solution

(Can also factorise using DOPS)

Question 2 **(8 marks)**

The straight line L has equation $4x + 2y = 1$.

(a) Write the equation of L in the form $y = mx + c$ to show that its gradient is -2. (2 marks)

$2y = 1 - 4x$

$y = \dfrac{1 - 4x}{2}$

$y = \dfrac{1}{2} - 2x$

✓ Rearrange to $y = mx + c$

✓ Simplify to show $M = -2$

Line L_1 is perpendicular to L and passes through the point $(2, 6)$.

Line L_2 is parallel to L and passes through the point $(1, -7)$.

(b) Determine the point of intersection of L_1 and L_2. (6 marks)

$L_1: \quad m = \dfrac{1}{2}$

$y = \dfrac{1}{2}x + c$

$6 = \dfrac{1}{2}(2) + c \quad c = 5$

$y = \dfrac{1}{2}x + 5$

$L_2: \quad y = -2x + c$

$-7 = -2(1) + c \quad c = -5$

$y = -2x - 5$

$-2x - 5 = \dfrac{1}{2}x + 5$

$-2\dfrac{1}{2}x = 10$

$\dfrac{-5}{2}x = 10$

$x = -4$

$y = -2(-4) - 5$
$\quad = 3$

$\therefore \quad (-4, 3) \quad$ is the intersection point.

✓ perpendicular gradient.

✓ equation for L_1

✓ L_2 equation.
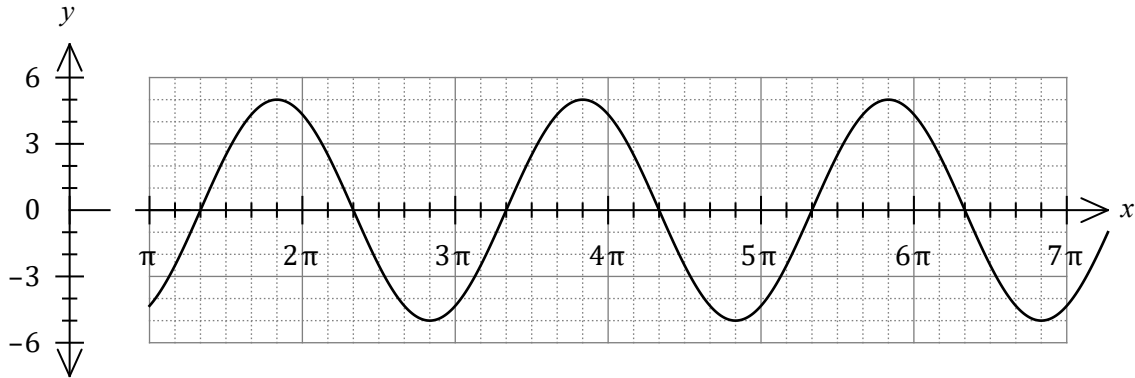
✓ Uses simultaneous equations to solve for x

✓ Calculates y value.

✓ Coordinate of intersection.

See next page

Question 3 **(4 marks)**

(a) The graph of $y = a\sin(x + b)$ is shown below, where a and b are positive constants.

Determine the value of a and the least value of b. (2 marks)

$a = 5$
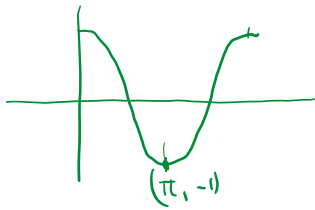
$b = \dfrac{\pi}{3}$

✓ Determines a value.

✓ Determines b value.

(b) Let $g(x) = 3 + \cos\left(\dfrac{x}{2}\right)$.

Determine the coordinates of the minimum of the graph of $y = g(x)$ for $0 \le x \le 4\pi$. (2 marks)

$(\pi, -1)$

$(\pi, -1) \Rightarrow (2\pi, 2)$

✓ x value correct

✓ y value correct.

Question 4 **(7 marks)**

Consider the function $f(x) = \dfrac{p}{x+q}$, where p and q are constants. The graph of $y = f(x)$ has an asymptote with equation $x = 2$ and passes through the point $(6, -1)$.

(a) Determine the value of p and the value of q. (3 marks)

$$f(x) = \frac{p}{x-2}$$

$$-1 = \frac{p}{6-2}$$

$$-4 = p$$

$$\therefore p = -4 \qquad q = -2$$

√ Correct q value

√ substitutes to find p

√ Correct p value.

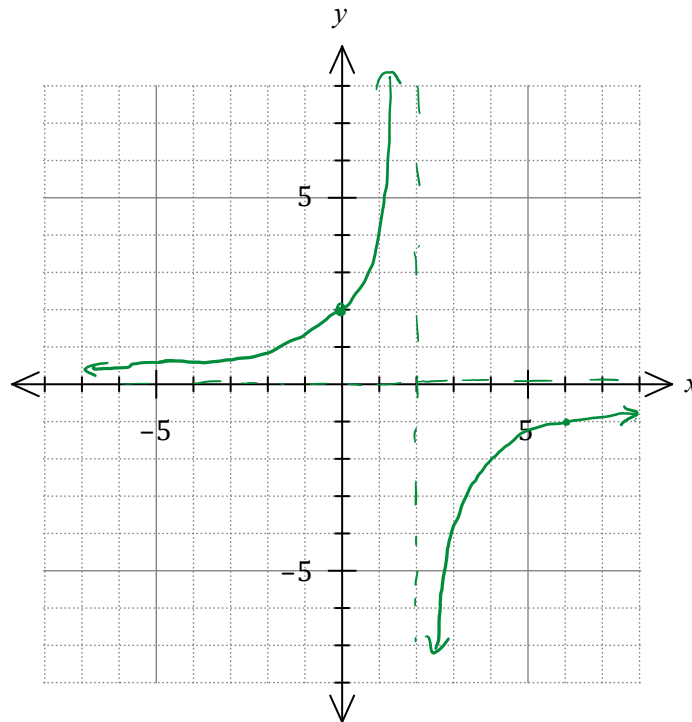(b) State the equation of the other asymptote of the graph of $y = f(x)$. (1 mark)

$$y = 0$$

√ Correct equation (must be y=)

(c) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)

√ passes through (6,-1) and (0,2)
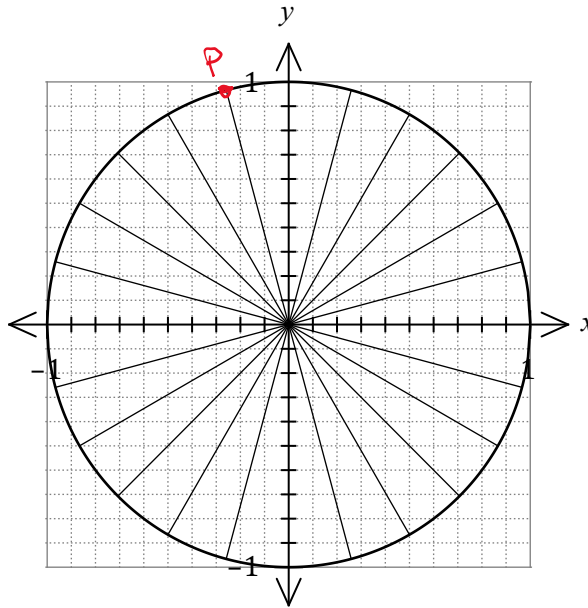
√ correct Vertical asymptote

√ Smooth hypebolic Shape.

Question 5 **(5 marks)**

(a)   A unit circle is shown.

Mark on the circumference of the
circle the point P so that a ray drawn from the origin to point P will make an anticlockwise
angle of $\dfrac{7\pi}{12}$ from the positive x-axis.

Hence estimate the value of $\cos\left(\dfrac{7\pi}{12}\right)$.

√ P marked correctly

√ estimates value.

$$\cos\left(\frac{7\pi}{12}\right) \approx -0.25$$

(2 marks)

(b)   Solve the equation $2\tan(3x-75°)+2=0$ for $0° \le x \le 90°$.   **(3 marks)**

$$2\tan(3x-75)=-2 \qquad -75° \le 3x-75 \le 195°$$
$$\tan(3x-75)=-1$$
$$3x-75=-45°, 135°$$
$$x=10°, 70°$$

√ Rearranges to isolate $3x-75$

√ One solution correct

√ Both solutions correct.

Question 6   **(7 marks)**

(a) Determine the number of possible combinations when four students must be chosen from a small class of six.   (2 marks)

$$^6C_4 = \frac{6!}{4! \cdot (6-4)!}$$

$$= \frac{6 \times 5 \times 4!}{4! \, (2)!}$$

$$= \frac{30}{2} = 15$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$\therefore$ 15 combinations

(green annotations) ✓ Uses formula or Pasals

✓ Correct value.

(b) Determine the coefficient of the x^2 term in the expansion of

(i) $(x+4)^3$.   (2 marks)

$$3(x)^2 (4)^1 = 12x^2$$

$$\therefore \quad 12$$

(green annotations) ✓ Shows method

✓ States coefficient.

(ii) $(5x-2)^6$.   (3 marks)

$$^6C_2 (5x)^2 (-2)^4$$

$$15 (25x^2)(16)$$

$$= 6000x^2$$

$$\therefore 6000$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$15 (5x)^2 (-4)^4$$

$$= 6000x^2$$

$$\therefore 6000$$

(green annotations) ✓ Shows use of Pascal's or combinations.

✓ Shows other components of term

✓ States coefficient.

Question 7 **(7 marks)**

Two polynomial functions are defined by $f(x) = (3x - 2)(x + 4)$ and $g(x) = x^3 - x^2 + 3x + 2$.

Determine the coordinates of the point(s) of intersection of $f(x)$ and $g(x)$.

$(3x-2)(x+4) = x^3 - x^2 + 3x + 2$

$3x^2 + 12x - 2x - 8 = x^3 - x^2 + 3x + 2$

$0 = x^3 - 4x^2 - 7x + 10$

When $x = 1$ $0 = (1)^3 - 4(1)^2 - 7(1) + 10$

$= 0$

$\therefore (x - 1)$ is a factor

$$
\begin{array}{r}
x^2 - 3x - 10 \\
(x-1){\overline{\smash{\big)}\,x^3 - 4x^2 - 7x + 10}} \\
-(x^3 - x^2) \\
\hline
-3x^2 - 7x \\
-(-3x^2 + 3x) \\
\hline
-10x + 10 \\
-(-10x + 10) \\
\hline
0
\end{array}
$$

allow by inspection

$(x-1)(x^2 - 3x - 10)$

$(x-1)(x-5)(x+2)$

$x = 1, 5, -2$

$(3(1)-2)(1+4) = 5$

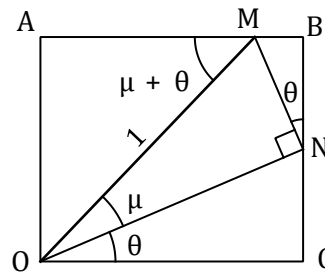$(3(5)-2)(5+4) = 13(9) = 117$

$(3(-2)-2)(-2+4) = -8(2) = -16$

$\therefore$ Intersects at $(1,5)$ $(5,117)$ & $(-2,-16)$

√ shows $f(x) = g(x)$

√ expands & rearranges to 0.

√ Finds first root (method must be clear)

√ fully factorises

√ x values

√√ co ordinates.

Question 8   **(7 marks)**

Consider rectangle $OABC$ that contains the right triangle OMN as shown.

Let the length of $OM = 1$,
$\angle NOC = \angle MNB = \theta$,
$\angle MON = \mu$ and
$\angle AMO = \mu + \theta$.

(a) Explain why $OC = \cos\mu\cos\theta$.   (2 marks)

As $ON = \cos\mu$

$OC = \cos\theta \times ON$

$\therefore OC = \cos\theta\cos\mu$

✓ Uses $\triangle OMN$ to find ON

✓ Uses $\triangle ONC$ to state OC.

(b) Determine expressions for the lengths of BM and AM and hence prove the angle sum identity $\cos(\mu + \theta) = \cos\mu\cos\theta - \sin\mu\sin\theta$.   (3 marks)

$\sin\theta = \dfrac{BM}{MN}$ $AM = \cos(\mu + \theta)$

$MN = \sin\mu$

$BM = \sin\theta\sin\mu$

$AM + BM = OC$

$\cos(\mu+\theta) + \sin\theta\sin\mu = \cos\theta\cos\mu$

$\therefore \cos(\mu+\theta) = \cos\theta\cos\mu - \sin\theta\sin\mu$

✓ Expression for BM

✓ Expression for AM

✓ Uses equal sides of $OABC$ to complete proof.

(c)   Use the identity from part (b) to show that $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$.   (2 marks)

$$\cos(x)\cos\left(\frac{\pi}{2}\right) - \sin(x)\sin\left(\frac{\pi}{2}\right)$$
$$\cos(x)(0) - \sin(x)(1)$$
$$= -\sin(x)$$

√ Substitutes into identity

√ Substitutes exact values & simplifies.

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Christ Church Grammar School

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures ☐☐ ☐☐☐ ☐☐☐

In words _____

_____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

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Section Two: Calculator-assumed   **65% (93 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
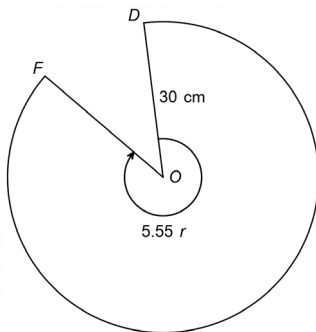
Working time: 100 minutes.

Question 9   **(5 marks)**

(a)   The diagram below shows a sector of a circle with centre O.
       The radius of the circle is 30 cm and $\angle DOF = 5.55$ radians.
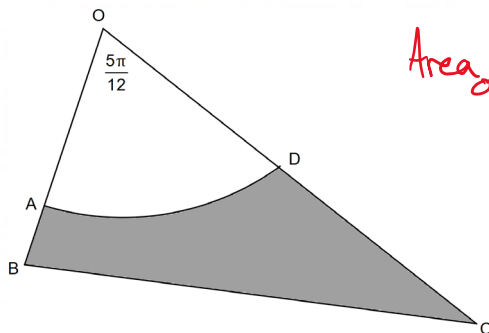   Calculate the length of the major arc DF   (2 marks)

$a = (30)(5.55)$

$= 166.5 \, cm$

✓ Substitutes into formula

✓ Calculates arc length

(b)   In the diagram below AOD is a sector of the circle with centre O. BOC is a triangle.
       In sector AOD, the radius is 30 cm and angle AOD is $\frac{5\pi}{12}$ radians.
       In triangle OBC, OB = 38 cm and OC = 55 cm.
       Calculate the shaded area of the shape with the vertices of ABCD rounded to 3 dp.
                                                                   (3 marks)

$Area_{OBC} = \frac{1}{2}(38)(55)\sin\left(\frac{5\pi}{12}\right)$

$= 1009.392$

$Area_{OAD} = \frac{1}{2}(30)^2\left(\frac{5\pi}{12}\right)$

$= 589.049$

$Shaded = 1009.39 - 589.049$

$= 420.344 \, cm^2$

✓ Calculates area of △OBC

✓ Calculates area of sector OAD

✓ Calculates shaded region

$(-1 \; if \; not \; 3 \, dp)$

Question 10   **(6 marks)**

(a) The parabolic graph of a relation is shown below.

(i) State the equation of its axis of symmetry.   (1 mark)

$$y = 0$$

✓ Correct equation

(ii) State the equation of the relationship between x and y.   (1 mark)

$$y^2 = x$$

✓ Correct equation.

(b) Points A and B have coordinates $(-7, 8)$ and $(5, 4)$ respectively. Determine the equation of the circle that has diameter AB.   (4 marks)

$$M = \frac{-7+5}{2}, \frac{8+4}{2}$$

$$= (-1, 6)$$

✓ Calculates midpoint

✓ Calculates radius

$$\text{radius} = \sqrt{(8-4)^2 + (-7-5)^2} \div 2 \quad \bigg| \quad \text{radius}^2 = (6-4)^2 + (-1-5)^2$$

OR

$$= 12.649 \div 2 \qquad\qquad = 40$$

$$= 6.325$$

$$\text{radius}^2 = 40$$

✓ Correct center in equation

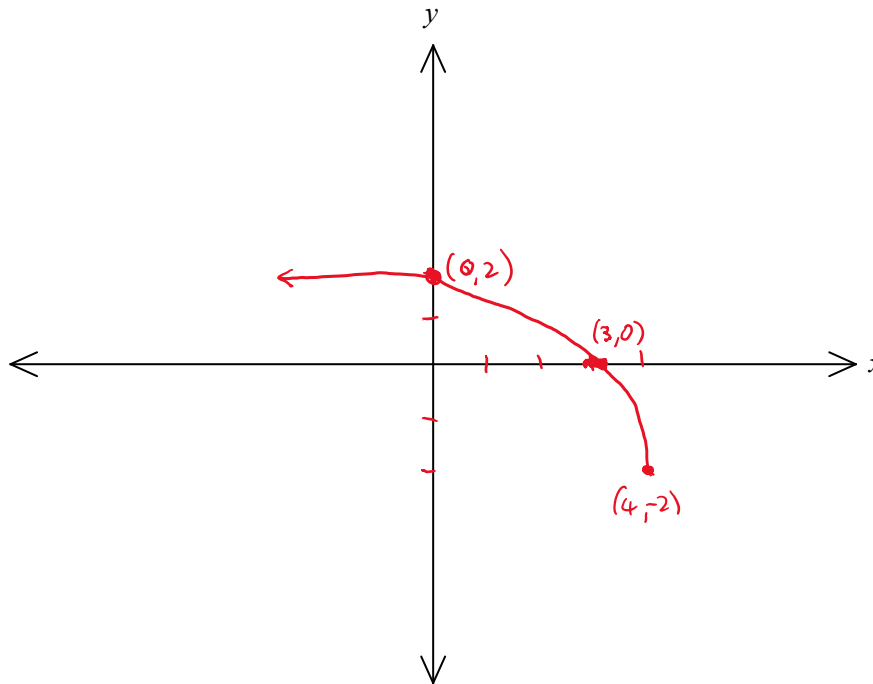$$(x+1)^2 + (y-6)^2 = 40$$

✓ Correct radius².

Question 11 (8 marks)

Let $f(x) = 2\sqrt{4-x} - 2$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)

√ x intercept
√ y intercept
√ shape.

Points marked on graph: (0,2), (3,0), (4,−2)

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = f(x)$:

(i) $y = 2\sqrt{1-x} - 2$. (2 marks)

Translation parallel to x axis left 3 units.

√ States translation with direction
√ Correct distance

(ii) $y = \sqrt{4-x} - 1$. (3 marks)

Dilation parallel to y axis, scale factor $= \frac{1}{2}$
Translation parallel to y axis up 1 unit.

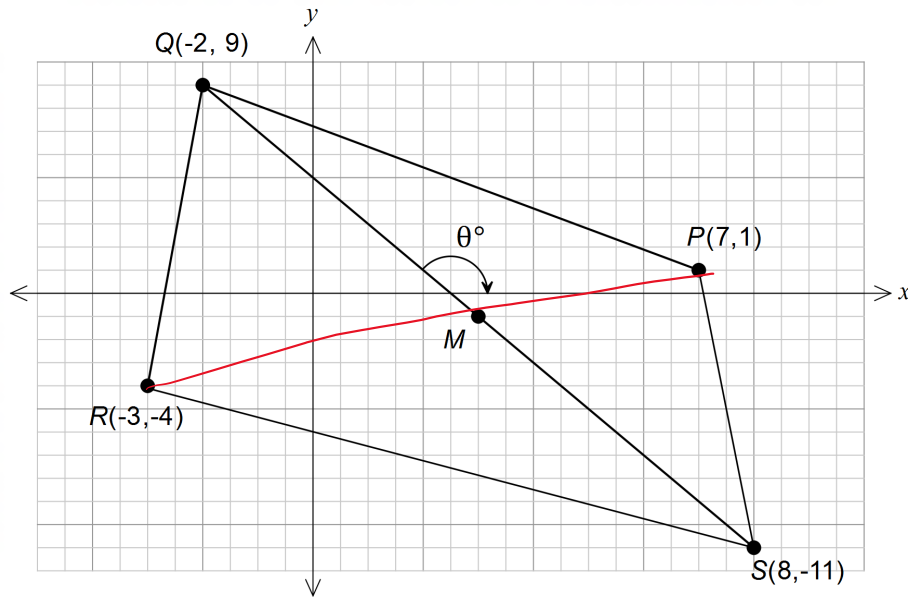√ States dilation parallel to y axis
√ Scale factor $\frac{1}{2}$
√ vertical translation with distance.

Question 12 **(9 marks)**

In the diagram *PQRS* is a quadrilateral having vertices $P(7,1), Q(-2,9), R(-3,-4)$ and $S(8,-11)$. *M* is the midpoint of *QS*.

(a) A line is drawn from *P* to *R*, determine the equation of the line *PR*. (2 marks)

$m = \dfrac{-4-1}{-3-7}$

$= \dfrac{1}{2}$

$y = \dfrac{1}{2}x + c$

$1 = \dfrac{1}{2}(7) + c \qquad c = -2.5$

$y = 0.5x - 2.5$

$y = \dfrac{1}{2}x - 2\frac{1}{2}$

$y = \dfrac{1}{2}x - \dfrac{5}{2}$

$2y = x - 5 \quad$ or $\quad 2y - x = -5$

✓ Calculates gradient

✓ Correct equation.
<u>or</u> Classpad.

allow any variation.

(b) Determine whether *M* lies on the line PR. (3 marks)

$M = (3, -1)$

$-1 = \dfrac{1}{2}(3) - \dfrac{5}{2}$

$-1 = -1$

∴ Yes M lies on the line.

✓ Subs M into PR

✓ Correct M coordinate.

✓ Yes or No

Question 12 continued

(c)   Show that QS is Perpendicular to PR.   (2 marks)

$$M_{QS} = \frac{-11 - 9}{8 + 2}$$

$$= -2$$

$$M_{PR} = \frac{1}{2}$$

$$\frac{1}{2} \times -2 = -1$$

$$\therefore \text{ perpendicular}$$

✓ Calculates gradients

✓ uses $m_1 \times m_2 = -1$ to compare.

(d)   Calculate θ, the angle of inclination of QS, as shown in the diagram.   (2 marks)

$$\tan \theta = -2$$

$$\theta = 116.57°$$

$$\underline{\underline{OR}} \quad 180 + \tan^{-1}(-2) = \theta$$

$$\theta = 116.57°$$

✓ Uses $\tan \theta = m$ to find angle

✓ correct angle.

Question 13 **(6 marks)**

A netball team collected money to go on a tour. They decided to sell cans of soft drinks at their home games. At the first game they sold 100 cans at $4 per can. They collected data and found that if they increased the price of the soft drinks, the sales would decrease. This is the data they collected:

Price per can	$4	$4.50	$5
Number of supporters willing to buy at the given price	100	88	79

One of the players claimed that the data suggested an inverse relation with the equation $n = \dfrac{296}{p}$ where n is the number of cans they can sell at p per can.

(a) Comment on the validity of the player's claim. (2 marks)

$$n = \frac{296}{4} = 74 \neq 100$$

∴ Not valid as the values do not fit the equation.

✓ Comments on validity

✓ Shows a value is not true.

(b) According to the given model, state the number of cans which would be sold if the price of a can was $5.50. (2 marks)
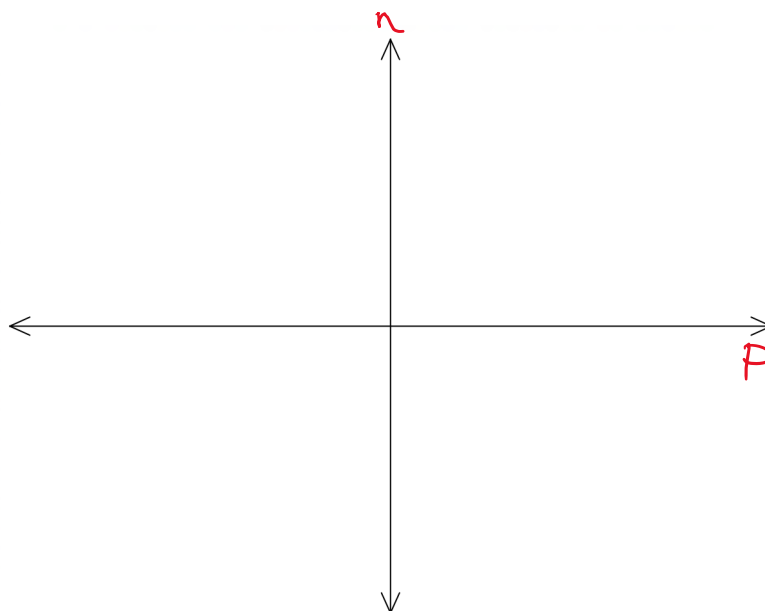
$$n = \frac{296}{5.5}$$

$$= 53.8 \text{ cans}$$

$$\approx 54 \text{ or } 53 \text{ cans.}$$

✓ Calculates number of cans

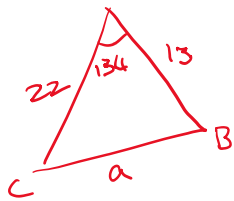(c) Draw a sketch of the graph of this model labelling the axes appropriately. (2 marks)

✓ Only one quadrant.

✓ axes labelled

Question 14 (9 marks)

(a) Triangle ABC is such that $b = 22$ cm, $c = 13$ cm and $\angle A = 134°$. Determine, with justification, the length of side a. (3 marks)

$$a^2 = 22^2 + 13^2 - 2(22)(13)\cos 134$$
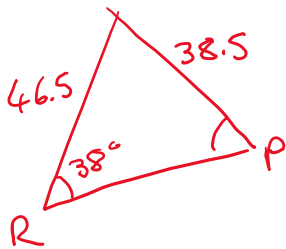$$= \sqrt{1050.345}$$
$$a = 32.41 \text{ cm}$$

✓ Uses cosine rule.

✓ Calculates a value.

✓ Units

(b) Triangle PQR is such that $p = 46.5$ cm, $r = 38.5$ cm and $\angle R = 38°$. Determine all possible areas of this triangle. (6 marks)

$$\frac{\sin P}{46.5} = \frac{\sin 38}{38.5}$$

$$P = \sin^{-1}\left(\frac{46.5 \times \sin 38}{38.5}\right)$$

$$= 48.04 \qquad OR \quad P = 180 - 48.04$$
$$= 131.96$$

$$R = 180 - 48.04 - 38 \qquad R = 10.04°$$
$$= 93.96°$$

$$\text{Area} = \frac{1}{2}(46.5)(38.5)\sin(93.96) \qquad \text{Area} = \frac{1}{2}(46.5)(38.5)\sin(10.04)$$
$$= 892.99 \text{ cm}^2 \qquad = 156.02 \text{ cm}^2$$

✓ Uses sine Rule

✓ Calculates ∠P

✓ Calculates ∠R

✓ Calculates one area correctly

✓ Calculates 2nd angles for P & R

✓ Calculates 2nd area correctly.

allow any dp
Must be rounded correctly.

Question 15   **(10 marks)**

The loudness L of sound, in decibels (dB), emitted by a machine t minutes after it is switched on can be modelled by
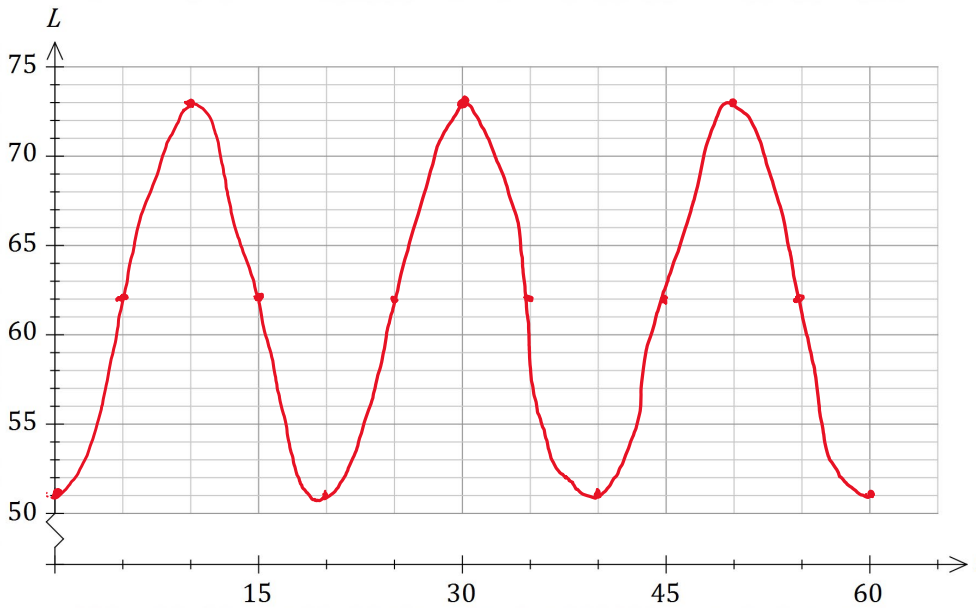
$$L = 62 - 11\cos\left(\frac{\pi t}{10}\right)$$

(a)   Determine the initial loudness emitted by the machine.   (1 mark)

$t = 0$ $L = 51$ dB   ✓ Calculates Loudness

(b)   Draw the graph of L against t on the axes below for the first 60 minutes.   (4 marks)

✓ Correct y intercept

✓ correct amplitude

✓ Correct period.

✓ Shape

(c)   State the maximum loudness emitted by the machine and the time this maximum was first reached.   (2 marks)

73 dB , 10 min   ✓ Max L

✓ t value.

(d)   A health and safety inspector can deem a machine unsafe if the loudness it emits exceeds 70 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unsafe.   (3 marks)

$L \geqslant 70$ when $7.59 \leq t \leq 12.41$, 3 times   ✓ identifies end points

$12.41 - 7.59 = 4.82$ min

$3(4.82) = 14.46$ min   ✓ total < 70 dB

∴ Not unsafe as less than 15 mins.   ✓ Yes or No.

See next page

Question 16 **(9 marks)**

(a) Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = 4$ and an axis intercept at $(0, 6)$.

(i) State the value of the constant c. (1 mark)

$$C = 6$$

✓ States y intercept for c.

(ii) Determine the value of the constant b. (2 marks)

$$x = -\frac{b}{2a}$$
$$4 = -\frac{b}{2(1)}$$
$$b = -8$$

✓ Shows method clearly

✓ States b value.

(b) Let $g(x) = -(x + 3)^2 + 5$. Determine

(i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

$$(-3, 5)$$

✓ correct turning point.

(ii) the domain and range of $g(x)$. (3 marks)

$$\text{Domain} : \{x \in R\}$$ ✓ Value of domain

$$\text{Range} : \{g(x) \in R : g(x) \leq 5\}$$ ✓ Value of range

✓ notation.

(iii) the coordinates of the turning point of the graph of $y = g(x + 2) - 3$. (2 marks)

$$\begin{matrix} (-3, & 5) \\ -2 & -3 \end{matrix}$$
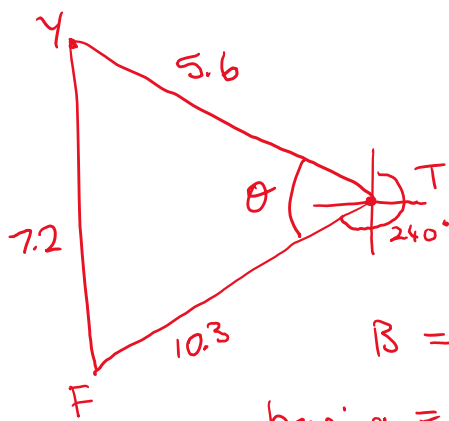$$= (-5, 2)$$

✓ Correct x

✓ correct y

-1 if not a coordinate.

Question 17   **(8 marks)**

(a)   A ferry and a trawler receive a request for help from a stranded yacht. The ferry is 7.2 km from the yacht. The trawler is 5.6 km from the yacht. The ferry is 10.3 km from the trawler and is on a bearing of 240° from the trawler. Draw a diagram and hence calculate the bearing of the yacht from the trawler.   (4 marks)

$$\theta = \cos^{-1}\left(\frac{7.2^2 - 5.6^2 - 10.3^2}{-2(5.6)(10.3)}\right)$$

$$= 42.09$$

√ Correct diagram with sides & bearing.

√ Use cosine rule

$$B = 240 + 42.09$$

$$\text{bearing} = 282.09°T$$
$$\text{or } 282°T$$

√ Calculates θ

√ Calculates bearing.

OR

$$\text{Bearing} = 240 - 42.09$$
$$= 197.91°T$$
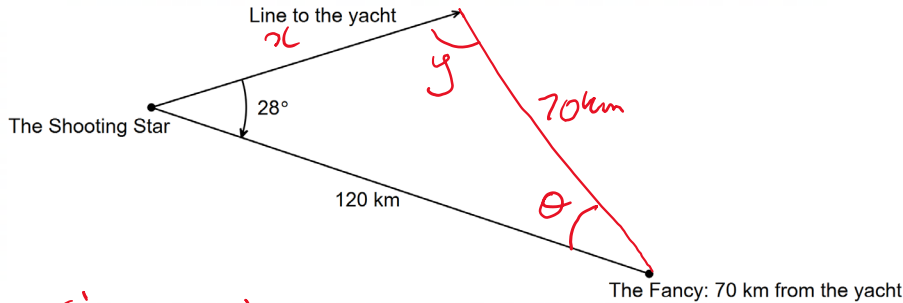$$\text{or } 197.9°T$$
$$\text{or } 198°T$$

Question 17 continued

(b)   Two cargo ships, The Shooting Star and The Fancy, are 120 km apart when they pick up the distress call from the stranded yacht from (a). The captain of The Fancy estimates that the yacht is 70 km away and that the angle between the line from The Fancy to The Shooting Star and the line from The Shooting Star to the yacht is 28°. What are two possible distances, to the nearest tenth of a km, from The Shooting Star to the yacht?

(4 marks)

$$\frac{\sin y}{120} = \frac{\sin 28}{70}$$

$$y = 53.59° \qquad\qquad \theta = 98.41°$$

$$\text{\&} \ y = 180 - 53.59$$

$$\approx 126.41° \qquad \theta = 25.59°$$

$$\frac{x}{\sin 98.41} = \frac{70}{\sin(28)} \qquad \frac{x}{\sin(25.59)} = \frac{70}{\sin(28)}$$

$$x = 147.5 \,km \qquad\qquad x = 64.4 \,km$$

OR $\quad solve\left(70^2 = x^2 + 120^2 - 2x(120)\cos(28)\right)$

(handwritten margin notes:)

✓ Uses sine rule to find y. or cosine

✓ Ambiguous case for θ.

✓ Shows method for one distance

✓ both distances given.

−1 if not to nearest tenth of km.

Question 18   **(8 marks)**

(a)   A row of Pascal's Triangle is shown below in combination notation.

       $^5C_0$ $^5C_1$ $^5C_2$ $^5C_3$ $^5C_4$ $^5C_5$

      (i)   State the natural numbers in the row above this row.   (1 mark)

$$1, 5, 10, 10, 5, 1$$

(handwritten, right margin)
```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```
✓ Correct values (all)

      (ii)   Expand and simplify $(p+q)^4$.   (2 marks)

$$p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$$

✓ Shows expansion

✓

(b)   (i)   State the degree of the polynomial $(3-2x)^5$.   (1 mark)

$$5$$

✓ Correct degree.

      (ii)   Without expanding, show clearly that the co-efficient of x^3 in the binomial expansion of $(3-2x)^5$ is -720.   (3 marks)

$$10(3)^2(-2x)^3 = 90(-8)x^3$$
$$= -720x^3$$

$$\therefore \text{coefficient is } -720$$

✓ Shows all factors

✓ Shows only 2 factors

✓ Simplifies

(c)   From a squad of 9 basketball players, a coach needs to select a team of 5 to play during a match.

      How many different teams are possible?   (1 mark)

$$C_5 = \frac{9!}{5!(9-5)!}$$
$$= 126$$

$$\therefore 56 \text{ teams}$$

✓ Calculates value.

Question 19 (7 marks)

The equation $f(x) = ax^3 + bx^2 - 12x + 8$ has two solutions, where $f(x) = k$ and a, b and k are constants.

The graph of $y = f(x)$ cuts the x-axis at $x = 2$, $x = -2$, and at one other point.

Determine the value(s) of the constant k, rounded to 2 decimal places. Explain your reasoning.

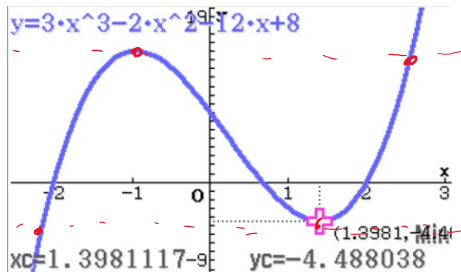$f(2) = 0$ $a(2)^3 + b(2)^2 - 12(2) + 8 = 0$

$8a + 4b - 24 + 8 = 0$

$f(-2) = 0$ $a(-2)^3 + b(-2)^2 - 12(-2) + 8 = 0$

$-8a + 4b + 24 + 8 = 0$

Solve $\begin{cases} 8a + 4b = 16 \\ -8a + 4b = 32 \end{cases} \Big|_{a,b}$ in classpad

$a = 3$ $b = -2$

$\therefore f(x) = 3x^3 - 2x^2 - 12x + 8$

For two solutions to exist k must be at local maximum or local minimum.

Minimum $y = -4.488$

Maximum $y = 15.023$

$\therefore k = -4.49$

or $k = 15.02$

Right margin annotations:

✓ Shows substitution of $x = -2$ & $x = 2$ or finds $f(x)$ by inspection **clearly**.

✓ Shows method to solve for a

✓ Shows method to solve for b.

✓ Describes case for solution

✓ States value of local max or min
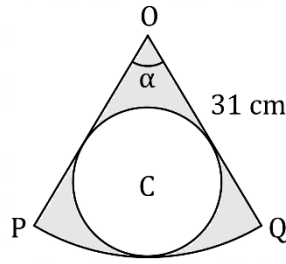
✓ States k value.

(-1 if not to 2 dp).

Graph annotations:

y=3·x^3−2·x^2−12·x+8

xc=1.3981117−9   yc=−4.488038

(1.3981, Min)

Question 20 **(8 marks)**

The diagram shows sector OPQ of a circle centre O of radius 31 cm and $\alpha = 40°$.

Circle C is inside the sector and just touches OP, OQ and arc PQ.
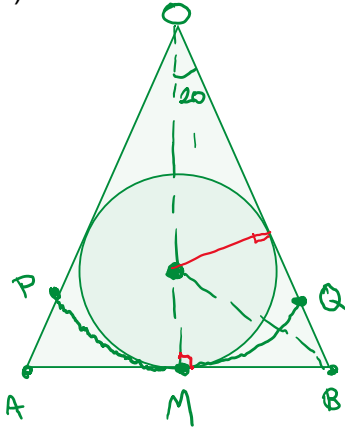
(a) Determine the area of sector OPQ. (2 marks)

$$\text{Area} = \frac{40}{360}\pi (31)^2$$

$$= 335.45 \text{ cm}^2$$

✓ Substitutes into formula

✓ Calculates area.

(b) Show that the radius of circle C is 7.9 cm, correct to one decimal place. (3 marks)

$$OM = 31 \text{ cm}$$

$$\tan 20 = \frac{MB}{31}$$

$$MB = 11.28$$

$$\tan\left(\frac{70}{2}\right) = \frac{r}{11.28}$$

$$\therefore \ r = 7.9 \text{ cm}$$

✓ Creates isosceles triangle

✓ Calculates MB

✓ Calculates radius

$$\underline{\underline{OR}} \quad \text{solve}\left(\sin(20) = \frac{r}{31-r}, r\right)$$

$$\therefore r = 7.9 \text{ cm}$$

(c) Determine the area of the shaded region, inside sector OPQ but outside circle C. (3 marks)

$$\text{Area} = \pi (7.9)^2$$

$$= 196.067$$

$$335.45 - 196.067 \approx 139.39 \text{ cm}^2$$

$$\text{or } 139.4 \text{ cm}^2$$
$$\text{or } 139 \text{ cm}^2$$

✓ Calculates area of circle

✓ Calculates shaded area

✓ Units used

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____