MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 3 - 2017

Discrete Random Variables and Binomial Distributions

Resource Free

Name: SOLUTIONS Teacher: ____________

Marks: 22

Time Allowed: 15 minutes

Instructions: You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1) [2,2,3 = 7 marks]

a) The table below shows the values taken by a function f(x).

x	-1	0	1	0.5
f(x)	0.2	0.6	0.1	0.1

John argues that f(x) cannot be a probability distribution function of a discrete random variable as x has a negative value. Comment on his answer.

x can be negative. It's f(x) that can't be negative ✓

JOHN is WRONG ✓

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b) The table below shows the values taken by a function f(x).

x	0	0.5	1	1.5	2
f(x)	0.2	0.5	a	b	0.1

i. Under what conditions can f(x) represent the probability distribution of a discrete random variable?

$a+b=0.2$ ✓

$a \geq 0,\ b \geq 0$ ✓

ii. If f(x) is the pdf of a discrete random variable X, find the values of a and b given that P(X=1) = P(X=1.5).

$a=b$ ✓

$a+b=0.2$

$\Rightarrow a=b=0.1$ ✓✓

2) [3,2,~~3~~,~~3~~ = 11 marks] 2,4

A certain tropical plant produces both white and pink orchid flowers. 20% of the flowers are white. The flower colour white forms a binomial distribution. One of these plants has 3 flowers.

[Note: You do not need to simplify your answers to parts b,c and d].

(a) What are the values of n, p and q?

$n=3$ ✓

$p=0.2$ ✓

$q=0.8$ ✓

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(b) What is the probability that all flowers are pink?

$(0 \cdot 8)^3$ ✓✓

(c) What is the probability that at least 1 flower is white?

$1 - P(\text{all pink})$ $\quad [\text{or } P(1) + P(2) + P(3)]$

$1 - (0 \cdot 8)^3$ ✓✓

(d) What is the probability that 2 flowers are pink given that at least 1 of them is pink?

$P(\text{at least 1 pink}) = 1 - P(\text{all white})$

$= \underline{1 - (0 \cdot 2)^3}$ ✓

$P(2 \text{ pink}) = \binom{3}{2}(0 \cdot 8)^2 (0 \cdot 2)^1$

$= \underline{3(0 \cdot 8)^2 (0 \cdot 2)^1}$ ✓

Thus $P(2 \text{ pink} \mid \geq 1 \text{ pink}) = \dfrac{3(0 \cdot 8)^2 (0 \cdot 2)^1}{1 - (0 \cdot 2)^3}$ ✓ ✓

$[0 \cdot 387097 \text{ approx}]$

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3) [4 marks]

It is know that 2% of all new electrical components produced on a mass production assembly line are defective. The components are packed in boxes of 6. Boxes are ‘passed’ only if they contain no defective items. What is the probability that a randomly chosen box is rejected?

$1 - P(0 \text{ defective})$ ✓

$n = 6, \ p = 0.02, \ q = 0.98$ ✓

$\underline{1 - (0.98)^6}$ ✓✓

i.e. very little chance!! $\approx \underline{0.114}$

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MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 3 - 2017
Discrete Random Variables and Binomial Distributions
Resource Rich

ALL SAINTS' COLLEGE

Name: ANSWERS **Teacher:** ____________

Marks: ~~40~~ 45

Time Allowed: **30 minutes**

Instructions: You are allowed a ClassPad and scientific calculator but NO notes.

You will be supplied with a formula sheet.

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1. [2,2,2,3 = 9 marks]

A probability distribution is defined by the following table:

x	0	1	2	3	4
P(X=x)	0.1	0.4	k 0.11	3k 0.33	0.06

a) Find the value of k

$0.56 + 4k = 1$ ✓

$\Rightarrow \quad 4k = 0.44$

$\therefore \quad k = 0.11$ ✓

b) Find the P(X>2)

$P(X=3) + P(X=4)$

$= 0.39$ ✓✓

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c) Find E[X]

$E[X] = (0 \times 0.1 + 1 \times 0.4 + 2 \times 0.11 + 3 \times 0.33 + 4 \times 0.06)$

$= \underline{1.85}$ ✓✓

d) Find the standard deviation of X

$E[(X^2)] = (0^2 \times 0.1 + 1^2 \times 0.4 + 2^2 \times 0.11 + 3^2 \times 0.33 + 4^2 \times 0.06)$

$= 4.77$ ✓

Hence $SD(X) = \sqrt{4.77 - (1.85)^2}$ ✓

$\approx \underline{1.161}$ ✓

2) [1,2,3 = 6 marks]

a) What is the probability of guessing the month in which a person is born?

$\frac{1}{12}$ ✓

b) What is the percentage probability of correctly guessing the month of birth of 3 students form a group of 10 students?

$\binom{10}{3}\left(\frac{1}{12}\right)^3\left(\frac{11}{12}\right)^7 \approx \underline{0.03777}$ ✓

$\therefore \underline{3.78\%}$ ✓

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c) If the probability of correctly guessing the birthday of 5 students from a group of n students is approximately 0.1595 what is the value of n?

$$\binom{n}{5}\left(\frac{1}{12}\right)^5\left(\frac{11}{12}\right)^{n-5} \approx 0.1595 \checkmark$$

From CALC guess and check

$\underline{n = 47}$ ✓✓

3) [2,3,3 = 8 marks]

A certain binomial experiment has 15 trials. The probability of success in any trial is 0.315. The random variable X is the number of successes. Calculate the probability of

a) X=11

$p \approx 0.0009106$ ✓✓

b) X is at most 9

$P(x \leq 9) \approx 0.9946$ ✓✓✓

c) X is between 5 and 8 inclusive.

$P(x \leq 8) - P(x \leq 4)$

$= 0.9787892 - 0.4644469$

$= \underline{0.5143423}$ ✓✓✓

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4) [4,2,6 = 12 marks]

Three marbles are drawn one at a time from a bag containing 5 blue marbles and 7 green marbles. The marble is replaced after each draw. Find:

a) The probability distribution for the random variable X, the number of blue marbles drawn,

x	0	1	2	3
$P(X=x)$	0.1985	0.4253	0.3038	0.0723

3G 2G 1G 0G

✓ ✓ ✓ ✓

b) The probability that all three marbles were the same colour,

$$\left(\frac{5}{12}\right)^3 + \left(\frac{7}{12}\right)^3$$

$$= \frac{13}{48} \quad ✓✓$$

(−1 for ANY decimal EXCEPT $0.2708\bar{3}$ if '=' sign used)

c) The probability that at most 1 green marble was drawn given that at least one marble was blue.

$$P(\text{at least 1 BLUE}) = 1 - P(0 \text{ BLUE})$$
$$\approx 1 - 0.1985$$
$$\approx 0.8015 \quad ✓✓$$

$$P(\text{at most 1 green} \mid \geq 1 \text{ blue})$$

$$\approx \frac{0.0723 + 0.3038}{0.8015} \quad ✓✓$$

$$\approx 0.4692 \quad ✓✓ \quad \left[\text{EXACT: } \frac{434}{1145}\right]$$

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5) [4 marks]

A binomial distribution has a mean of 4.8 and a standard deviation of approximately 1.833. Find the number of trials, n, and the probability of success, p.

$np = 4.8$

$\sqrt{npq} = 1.833$

$npq \approx 3.359889$ ✓

$\therefore \quad q = \dfrac{3.359889}{4.8}$ ✓

$\Rightarrow \quad q \approx 0.7$

$\therefore \quad p = 0.3$

If $\underline{p = 0.3}$ ✓

$\underline{n = 16}$ ✓

6) [6 marks]

The probability that Jordy tosses a coin into a container from 2 metres is 0.2.

a) If she tosses 9 coins, what is the probability of her getting at least two coins into the container?

$P \approx 0.5637924$ (CLASSPAD) ✓

b) How many coins would she need to toss so that the probability of getting at least 1 coin into the container is greater than 0.65?

$n = ?$

$P(x \geq 1) > 0.65$

$1 - P(x=0) > 0.65$

$P(x=0) < 0.35$ ✓

$\binom{n}{0}(0.2)^0(0.8)^n < 0.35$ ✓

$n \ln 0.8 < \ln 0.35$

$\Rightarrow \quad n > \dfrac{\ln 0.35}{\ln 0.8}$

$n > 4.7$ ✓

$\therefore \quad \underline{n = 5 \text{ coins}}$ ✓✓

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