PROVIDENCE CHRISTIAN COLLEGE

SOUTHERN RIVER, WESTERN AUSTRALIA

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	6	
10	6	
11	8	
12	8	
13	8	
14	8	
15	9	
16	8	
17	8	
18	8	
19	7	
20	7	
21	7	
S2 Total	98	
S2 Wt ($\times 0.6633$)	65%	

See next page

Section Two: Calculator-assumed 65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

A survey of all apartments for sale in a particular suburb showed that 65% had a lift and 78% had a swimming pool. Of those with a lift, 80% had a swimming pool.

Determine the probability that a randomly selected apartment from those in the survey had

(a) a lift and a swimming pool. (2 marks)
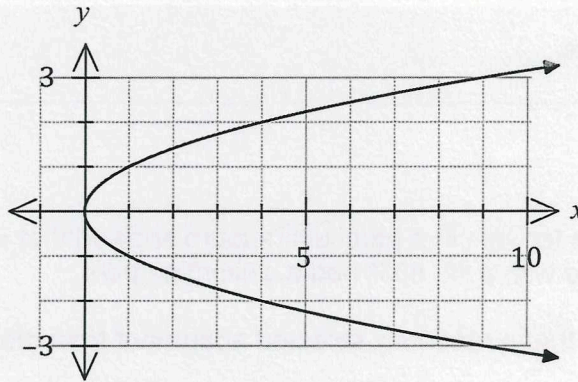
(b) a lift or a swimming pool. (2 marks)

(c) no lift or no swimming pool. (2 marks)

Question 10   **(6 marks)**

(a)   The parabolic graph of a relation is shown below.

(i)   State the equation of its axis of symmetry.   (1 mark)

(ii)   State the equation of the relationship between x and y.   (1 mark)

(b)   Points A and B have coordinates $(7, 8)$ and $(-3, 2)$ respectively. Determine the equation of the circle that has diameter AB.   (4 marks)
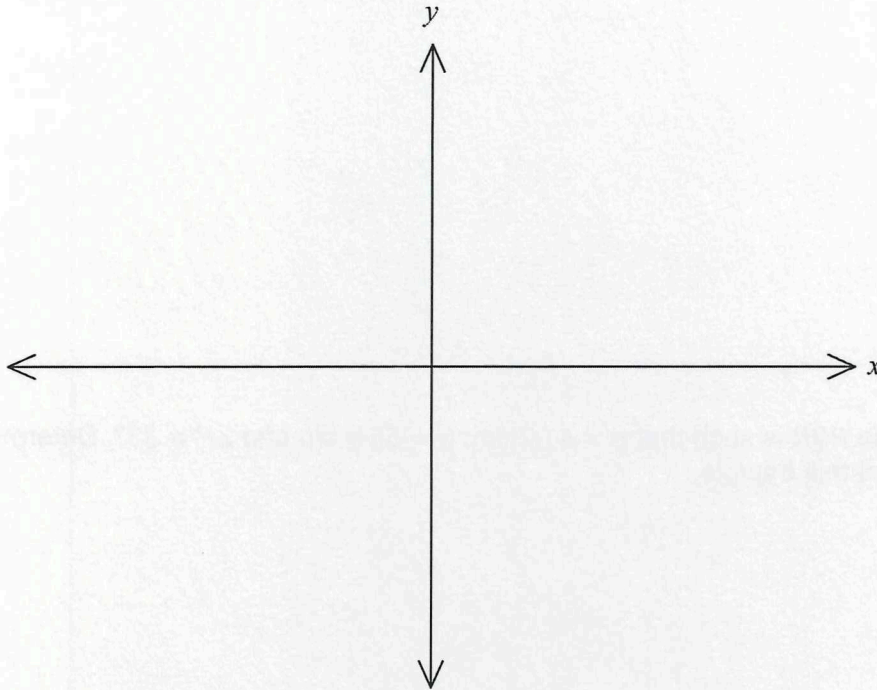
Question 11 (8 marks)

Let $f(x) = 2\sqrt{4 - x} - 2$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = f(x)$:

(i) $y = 2\sqrt{1 - x} - 2$. (2 marks)

(ii) $y = \sqrt{4 - x} - 1$. (2 marks)

Question 12 (8 marks)

(a) Triangle ABC is such that $b = 25$ cm, $c = 33$ cm and $\angle A = 142°$. Determine, with justification, the length of side a. (2 marks)

(b) Triangle PQR is such that $p = 41.2$ cm, $q = 55.3$ cm and $\angle P = 33°$. Determine all possible areas of this triangle. (6 marks)

Question 13 (8 marks)

A study of the achievements of 360 students enrolled in a university course yielded the following information:

- 50% of all students achieved a distinction
- 60% of those who did not achieve a distinction studied part-time
- 45% of those who studied full-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Full-time	Part-time	Totals
Distinction			
No distinction			
Totals			360

(b) Determine the probability that a randomly chosen student from the study

(i) achieved a distinction and studied full-time. (1 mark)

(ii) achieved a distinction or studied full-time. (1 mark)

(c) Explain whether this study provides any evidence that achieving a distinction is independent of study mode. (2 marks)

Question 14 (8 marks)

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
White	280	370	200
Black	250	310	190

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine

(i) the most likely size and colour of this T-shirt and the probability that this T-shirt is selected. (2 marks)

(ii) the probability that the T-shirt selected is not a medium. (2 marks)

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are

(i) both large. (2 marks)
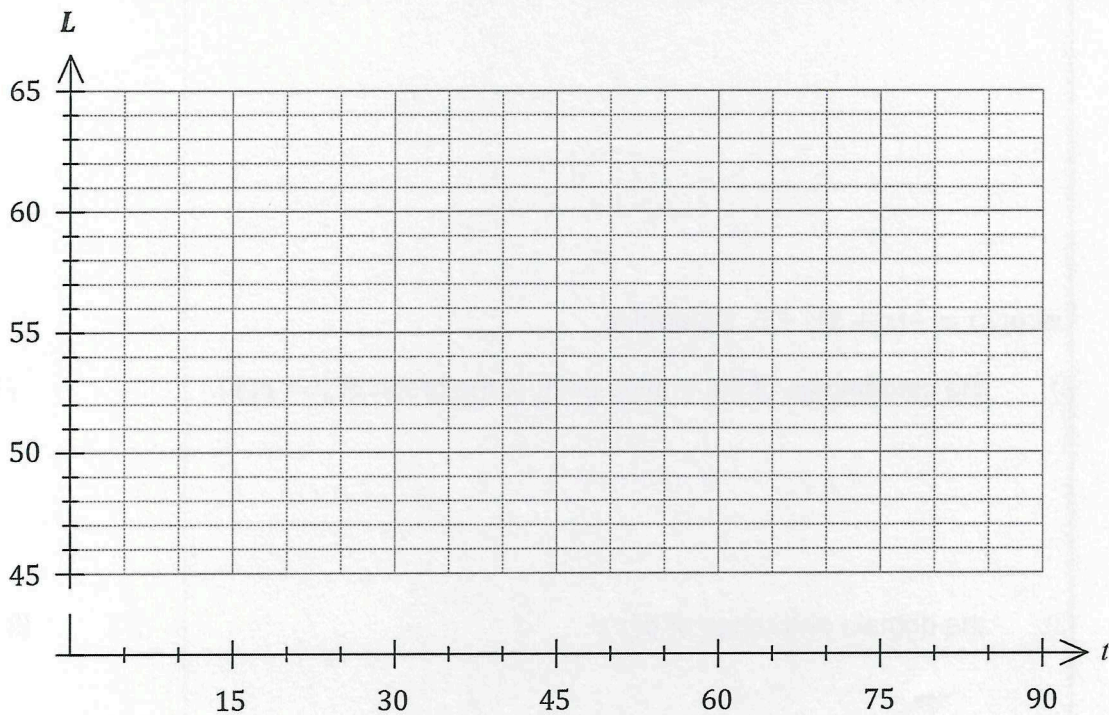
(ii) of different colours. (2 marks)

Question 15 **(9 marks)**

The loudness L of sound, in decibels, emitted by a machine t minutes after it is switched on can be modelled by

$$L = 55 - 8\cos\left(\frac{\pi t}{15}\right)$$

(a) Determine the initial loudness emitted by the machine. (1 mark)

(b) Draw the graph of L against t on the axes below for the first 90 minutes. (3 marks)

(c) State the maximum loudness emitted by the machine and the time this maximum was first reached. (2 marks)

(d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 60 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable. (3 marks)

Question 16   **(8 marks)**

(a)   Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = 4$ and an axis intercept at $(0, 6)$.

      (i)   State the value of the constant c.   (1 mark)

      (ii)   Determine the value of the constant b.   (2 marks)

(b)   Let $g(x) = -(x + 3)^2 + 5$. Determine

      (i)   the coordinates of the turning point of the graph of $y = g(x)$.   (1 mark)

      (ii)   the domain and range of $g(x)$.   (2 marks)

      (iii)   the coordinates of the turning point of the graph of $y = g(x + 2) - 3$.   (2 marks)

Question 17 **(8 marks)**

The events S and T are such that $P(S) = 0.4$ and $P(T) = 0.3$.

Determine $P(S \cup T)$ in each of the following cases:

(a) S and T are mutually exclusive. (1 mark)

(b) $P(\bar{S} \cup T) = 0.85$. (2 marks)

(c) S and T are independent. (2 marks)

(d) $P(T|\bar{S}) = 0.25$. (3 marks)

Question 18   (8 marks)

A chess club has 13 members, of which 6 are beginners, 4 are intermediate and the rest are advanced. The club has to select a group of 3 members at random to assist with a regional tournament.

(a)   Determine the number of different groups that can be selected.   (2 marks)

(b)   Determine the number of different groups that can be selected which contain at least 2 intermediate members.   (2 marks)

(c)   Determine the probability that the group contains

(i)   no intermediate members.   (2 marks)

(ii)   exactly one advanced member.   (2 marks)
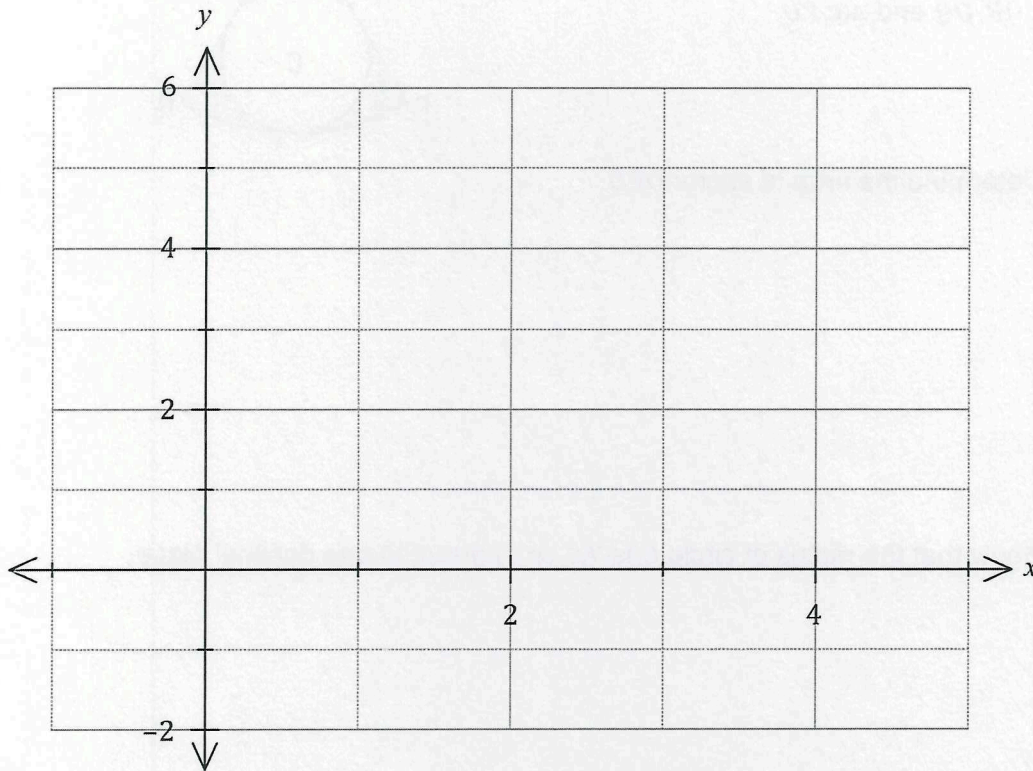
Question 19 (7 marks)

The equation of a parabola is $y = \frac{1}{2}(x^2 - 4x + 6)$.

(a) Sketch the parabola on the axes below. (3 marks)

All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x - p)^2 + q$, the focal point is at $\left(p, q + \frac{1}{4a}\right)$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where a, p and q are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)
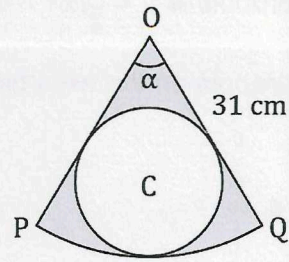
Question 20   **(7 marks)**

The diagram shows sector OPQ of a circle centre O of radius 31 cm and $\alpha = 40°$.

Circle C is inside the sector and just touches OP, OQ and arc PQ.

(a)   Determine the area of sector OPQ.   (2 marks)

(b)   Show that the radius of circle C is 7.9 cm, correct to one decimal place.   (3 marks)

(c)   Determine the area of the shaded region, inside sector OPQ but outside circle C.

           (2 marks)

Question 21   **(7 marks)**

The equation $f(x) = k$ has two solutions, where $f(x) = ax^3 + bx^2 - 12x + 8$ and a, b and k are constants.

The graph of $y = f(x)$ cuts the x-axis at $x = 2$, $x = -2$, and at one other point.

Determine the value(s) of the constant k, rounded to 2 decimal places. Explain your reasoning.

End of questions

Supplementary page

Question number: _____