MALE SPEAKER:

PROFESSOR: So today I want to finish up thinking about circuits. And the major topic for today is just one -- thinking about abstractions that we can use for thinking about circuits. The attractions are things that capitalize on linearity. And they include things like [UNINTELLIGIBLE] and superposition. OK so what I want to do is finish up thinking about circuits. And just to get you thinking about circuits, let's think about where we are.

Last time we saw that one of the issues in thinking about circuit design, is the fact that every part, in principle, interacts with every other part. Which makes the design process harder than the design process was for things like linear systems, where we thought about boxes having inputs and outputs. In linear systems, when we thought about boxes having inputs and outputs, the output didn't necessarily have any affect on the input, unless there was an explicit path feedback.

In circuits, there's feedback always. There's no way to avoid it. And in fact, that coupling can make thinking about the circuits very difficult. And we introduce that idea by just thinking about, even if you wanted to close a switch, you know a circuit, that's logically the same as adding a new part. And when you do that, that's going to change the currents and voltages everywhere. So that's kind of a big thing-- that's kind of the thing that's different about circuits from what we've been thinking about before.

Because that complicates design, we'd like some way of dealing with it. And the way we introduced, and the way that you used in the lab, so far, has been to use a buffer. We can use an op-amp to make a buffer. And a buffer has this nice isolation property. In this particular circuit, it has the property that it copies this voltage to the output, regardless of what's at the output.

That means we could put a switch here, we could change the light bulb, we could do anything we wanted to, and we would still know, because of the properties of the op-amp, we would still know that this voltage is going to be 8 Volts, regardless of what we put there. So that's very nice, that gives us a modularity, that gives us a way to design things. It let's us design the left-- make a circuit that generates 8 Volts without knowing precisely what's going to be the ultimate circuit that that drives. So that's nice, it allows us to do modularity.

And in many instances, that's a very good solution. In fact in the solution-- in the problems that you are working on in lab, where you're trying to drive a motor-- that's an excellent solution. It's not always an excellent solution. In some sense, it's very expensive. An op-amp is a complicated part. If you were to look inside an op-amp, there's some two dozen transistors in most op-amps. So it's not an inexpensive part, especially when you think about this kind of a circuit that only has three parts to the left.

The op-amp is actually a more complex device than anything else up. So op-amps are wonderful, op-amps allow us to make buffers, buffers are wonderful, but they're not always the best solution for thinking about modularity. And, in fact, there's other ways. And so that's what we want to think about today is -- other ways for achieving modularity in circuit design.

And the key to thinking about this, is to think about, well, what's the worst thing that could happen? If I changed this part arbitrarily, just how bad can things get? So I'll let you answer that. Think about that circuit, and assume that this is a 90 Volt power supply, 3 Ohm, 6 Ohm, but this can change-- R0 can change. I've tabulated some values of R0 and putative, corresponding values for V0 and I0. Are my putative answers right?

So take a minute, talk to your neighbor, figure out how many of these-- let's see-- ten blue numbers are right or wrong.

It's very quiet, you are allowed to talk to people.

So how many of the numbers are wrong? So raise your hand, indicate by number fingers how many mistakes are in the table. More votes, more votes-- come on, come on. if you had talked more, you could blame it on your neighbor more easily, so talk quickly.

OK. So does everybody agree with their neighbor? OK, i don't see a single right answer. So take 30 more seconds and think about it again. I don't see any right answers. So assume your answer's wrong.

[CHUCKLES]

Color blind-- OK, so how many of the voltages and currents are incorrect?

OK, how about a re-vote? So how many of the blue numbers-- the currents and voltages, how many of V0 and I0 -- how many of those are incorrect? OK, not very many votes-- I'd say about 80% correct, now. That's definitely an improvement. What would have to be true? If I wanted to prove that some of these numbers were wrong, what could I do? Give me a condition that would have to be true if the numbers were correct.

AUDIENCE: V0 has to equal I0?

PROFESSOR: V0 has to equal I0. We know that this resistor better be-- better obey Ohm's Law. That's what we mean by that symbol. So it had better be the case that V0 is I0R0. So you you'd want this number to be the product of that and that. So 0 is the product of 0 and 30, that looks good. 30 is 2 times 15, 36 is 3 times 12, 48 is 6 times 8. 60 is-- well that's a little marginal. how about if I rearrange it a little bit, and say what if R is V/I?

If I is 0, that would make the resistor infinity, so there's a way of thinking about that last line as correct. That's a little funny. Maybe the forward way of thinking about it is, well what if I made the resistor infinite? if I made the resistor infinite, what would be I? 0. And what would be V?

What would be the voltage difference if this were right, then? The voltage divider--

AUDIENCE: Oh, yeah.

PROFESSOR: So if there's no current here, then you can use the voltage relationship-- the voltage-divider relationship here. 6 over (3 plus 6) times 90 -- which is 60. So if you use straightforward reasoning saying, what if the resistor was infinitely large? What would happen? Then you would conclude that the bottom line is OK. If you choose-- if you show that R equals V over I, are you done? What else has to be true in order for the numbers to be true? It's not very hard.

AUDIENCE: The voltages drop out of the [UNINTELLIGIBLE]. The resistor has to be the same as the--

PROFESSOR: So the voltages-- that's an instance-- what you just said-- is an instance of KVL. Basically, the voltage around all of the loops better be 0. The sum of the voltages around all of the loops better be 0, and the sum of the currents in all of those closed surfaces better be 0, right? KVL better be satisfied everywhere, and KCL better be satisfied everywhere.

So, in particular, we can ask about this node. We could say, do the currents flowing into that node sum to 0? So if you take a line here-- let's take this line-- if V0 happened to be-- V0 happens to be the voltage here, right? So if V0 were 36, then that puts some voltage on this leg. 36 would what, 54? So then you'd have to say-- you'd have to compute, then, the current through here and the current through there, and see if those currents sum to 0.

And, in fact, if you do those calculations, you if you can convince yourself of those are all true. So the answer was 0. All of those answers were true. The point of doing the exercise was to just remind you about how you solve circuits, but also to let us look at patterns. The interesting thing in this problem is the pattern the results between the V's and the I's.

So if I were to make a plot, in fact, those V's and I's all fall on a straight line. Well that's pretty interesting. So if I plot just this V versus this I-- so V equals 0, 30, 36, 0, 30, 36, 48, 60 -- and the corresponding I's, 30,15,12,6,0 -- 30,15,12,6,0. Those points all fall on a straight line. That suggests that there's some pattern here. And if there's a pattern, then there might be a way to exploit it.

So that's what I'm trying to develop-- is a way to exploit the pattern that results when parts of circuits interact. The interesting thing that we-- so not only is it true that there's a simple pattern, but it turns out that the pattern is completely independent of the thing that I put on the right. The pattern is a property of the circuit to the left.

One way to convince yourself of that is to substitute-- take that resistor out, and put in a voltage source, and redo the problem. This time, instead of assuming that there's an Ohm's Law-type resistor here, assume there's a constant voltage, and that constant voltage is adjusted to 0,30,36,48,60. If you re-solve that problem, you get exactly these same currents. The answer continues to fall on exactly the same line.

So that's a very interesting pattern. The idea is that when the circuit on the left interacts with the circuit on the right, regardless of what the circuit on the right is, you get a simple relationship between the voltage and current that comes out of the circuit on the left. So that motivates the idea that we can think about the left-hand circuit as some kind of a generic part. We call that generic part a one-port.

Think about this circuit on the left-- the thing that's in the red box-- it's got at its terminals-- the terminals are the things that poke through the red box. First off it has two terminals, two terminals is just like all of our other parts. And it's like a resistor, it's like a voltage source, it's like a current source, it's a two terminal device. And just like a voltage source or a resistor or a current source, there's some voltage across those terminals, and there's some current that flows in those terminals.

So there's some current that goes in the plus. And that same current comes out the minus. That'll be true for this circuit, just the same as it's true for a resistor or voltage source or anything else, the only difference is, the thing that's inside the box-- the thing that's inside the one-port-- is more complicated than it was for a simple resistor or voltage source or current source. So what we can think about, is this whole red box just looks like a super part.

So the interesting thing that happens is, this red box behaves like a one-port, like a super part. And just like a resistor has a relationship between V and I, V equals IR, Or a voltage source has some kind of a constraint, V equals V0. Or a current source has some kind of a constraint, I equals I0. This funny part has a relationship between V and I that's given by that curve.

In some sense, it's not very different. So we think that-- so what we want to do now is figure out the rules that govern the currents and voltages that flow through one-ports. And in particular, how special was this straight line thing? I mean if they were always a straight line, that would be really easy, right? So the question-- so the next question I'd like to ask is, just how often are we expecting to see straight lines there?

I already said, the primitive elements that we think about-- Ohm's Law, resistors, voltage sources, current sources-- they have straight line constraints between the voltages and currents that they can generate. So think about what I'm doing. I'm trying to think about a rule that's going to let me describe the currents and voltages into that red box from the previous slide, much like I would describe the voltages and currents in an Ohm's Law a resistor.

I can tell you the voltage-current relationship, V equals IR for an Ohm's resistor, independent of what it's connected to. That's the reasoning that I'm using here. I'm going to try to figure out, independent of what it is connected to, what will be the voltage-current relationship for the red box? So the question is, when are we expecting straight lines, and when are we not expecting straight lines?

So here's a simple circuit. Here's a super part made out of one linear resistor, one Ohm's Law resistor, and one voltage source. What's the current-voltage relationship for that part? What if I put a red box around the whole thing, and asked you to draw the I-V curve, the current voltage curve? Which would it look like, A,B,C,D -- which should have been (1), (2), (3), (4) -- so you can raise fingers, or none of the above?

So take 30 seconds. Figure out what would be the I-V curve for this part.

So if you map A to D into (1) to (4) -- which of those plots describes the current-voltage relationship for that circuit? Map A to D to (1) to (4). OK, you're quiet, and the success rate is smaller than usual, about 50% correct. Take 30 seconds, reconsider your answer, try to get this up to 85% or so.

[CHATTER]

So which is the best plot? Which plot best characterizes the circuit on the top? OK, that's much better. That's about 75%. Certainly not 100%, but better. OK so how do you think about this? One way to think about it is special cases. Can you think about any special cases that are particularly easy to check? 0 -- OK, what equals 0?

AUDIENCE: V or I --

PROFESSOR: V or I -- yes. So what if you were to make -- if you were to make I equal to 0? First off, if you made I equal to 0, what's special in the plots? If I equals 0, then you round up x-axis. Sorry, if you make I 0, what will V be? If you made I be 0, how big will be V? 5 Volts, right? If you make I be 0, then there's no voltage across the resistor. So the total voltage here will be the same as the voltage across the voltage source, V equals 5.

So the intersection on the x-axis should be at the point V equals 5. Makes sense? Now if you said V equals 0, how big would I be? If you set V equal to 0 -- how do you set V equal to 0?

AUDIENCE: Negative 2.5.

PROFESSOR: Negative 2.5, so what is negative 2.5?

AUDIENCE: The value of I.

PROFESSOR: So do you need-- how do we set V equal to 0? What's the circuit way of saying, set V equal to 0? You set V equals 0 in a circuit by--

AUDIENCE: Grounding.

PROFESSOR: --grounding, by setting it-- by putting in a wire. So you run a wire from here to here. Voltage across a wire is always 0, right? So you set something to 0 by putting a wire across it. We call that a short circuit. OK, there's a short path for the voltage travel. So you put a short circuit here, and then I becomes V over R, and the only trick is that it's up, right? Current likes to flow down the electrochemical gradient. So it likes to go down the electrical gradient. So the current is going to go up, but the reference direction for this I is down, so I is minus 2.5.

So the intersection on this axis is minus 2.5. So we know that it has to go through a negative on the bottom, and it has to go through a positive V on the x-axis, and the only curve that does that is (A). And if we wanted to be a little bit more fancy, we could figure out the general rule. We could just write an expression for Vr. Well by KVL, Vr is always the difference between V and 5 Volts.

And we could write an expression for the current through the resistor. That's just V over R. Vr is V minus 5, R is 2. So I get some expression which is, lo and behold, linear in V. So just like that more complicated circuit that I'd looked at, I ended up with a straight line relationship in the current voltage-- the relationship between the current and voltage falls on a straight line.

So how special is that? Well it actually happens pretty robustly. Think about what would happen if I had two parts, the generically-- so I'm thinking about these being generic boxes, inside could be anything. There could be a current source, a voltage source, a linear resistor, or some other one-port. So if I had a generic box here, and a generic box here, and I connected them in parallel, under the condition that each of the generic boxes had a straight line current-voltage relationship, it's easy to argue that the resulting relationship between current and voltage for the parallel combination is also a straight line.

All you need to do is realize that if you hook two things in parallel, the parallel voltage is the same as V1 and V2. So all the voltages are the same. And the currents add. So Ip, the current end of the parallel combination is the sum of the two I's. So if you think about the relationship between V1 and I1, and the relationship between V2 and I2, you could derive the relationship between Ip and Vp by just adding.

V1 equals v2 equals Vp, that's what we saw here. And the current at the bottom is the sum of the other two currents. So, in particular, if this was a straight line, and that was a straight line, the sum of two straight lines is a straight line, right? If you add two straight lines, you get a new straight line. So what this shows is that if you started with parts that were themselves straight lines, parallel combinations would generate a new part with another straight line.

Same sort of thing happens if the two parts we're in series. In series we would have Iseries -- it's the same as I1 equals I2-- so that's the equivalent of the y-axes, where previously we had equivalents of the x-axes. And now if both of these boxes had linear I-V curves, I would now add horizontally rather than adding vertically. But I have the same result. If the two, individual one-ports had linear I-V curves, and if I add horizontally, I'll get a new linear I-V curve.

And in fact, if you start-- if you put any combination of parts that have linear I-V curves together to form a new circuit-- a new one-port, the I-V curve for that new one-port will be a linear function. And the way to see that, is to think about linear equations. Remember when we solve a circuit, we have to have one law for every element, Like Ohm's Law-- it's V equals IR, or voltage source is V equals V0 or something like that-- we need one law for every element.

And then we have KVL and KCL. Well all of the-- if we start with the assumption that each component has a straight line relationship between voltage and current, which is true for linear resistors, voltage sources, and current sources. Those equations that describe those parts are what we call linear equations. Linear equation is an equation where the function of the unknowns is a quote -- linear function.

It has the form-- some constant times an unknown, plus some other constant times some other unknown, plus, in principle, any number of those. But it has to be a linear function, so you're not allowed to have things like V-squared in there. But if you think about things like voltage sources and current sources and Ohm's Law, they don't have squares in there, they're linear equations.

And the idea then, when we're solving for the I-V relationship for this new one-port thing, all we're doing is we're solving a system of linear equation. Sort of in the abstract, the idea is that we write down all of the component equations inside the box, we write down all of the relevant KVL and KCL, and then we solve. So they're all going to have-- each one of those equations is going to be linear, so it's going to be something like, say, a0x0 if x is my unknown. 'x' could be a current or a voltage.

And then I might have sum of x1, then I might have sum of x2, and I might have a whole bunch of things, and they all add together to be some constant, like that. So that might represent part one-- that might represent one of the components in the box. Then I would have another component, which would be some other linear equation.

Then I'd have a KCL equation, well KCL is easy, because that only has currents in it, and the multipliers are all 1 or minus 1. Right, so that's clearly linear. And I have KVL equations. Those are linear. And the point is, that when you solve linear equations, you get a new linear equation. So think about solving this by the method of substitution. I could figure out what is x0 here.

If I use this equation to figure out x0. x0 would be a linear function of all the other x's. So then if I plug that linear function into here, I replace this linear equation with a new linear equation with one fewer unknown. if I keep doing that, I just keep replacing linear equations with other linear equations. When I'm all done, I'm left with a new linear equation. That's why, if you start with parts that linear I-V curves, you'll end up with a straight line I-V curve.

So that idea that the I-V function is a straight line is quite robust. It will happen anytime you go to circuit out of ideal parts-- whereby ideal parts, I mean Ohm's Law resistors, voltage sources, and current sources. So that has a very interesting circuit interpretation. If I know that an arbitrary circuit can be represented by a straight line-- if the I-V curve can be represented by a straight line-- well that generates an equivalent set.

There's obviously more than one circuit that could generate the same straight line. Here's a circuit that can generate that straight line. In fact, it will always be true that I can generate a circuit with one voltage source and one Ohm's Law resistor that will mimic the behavior of any arbitrary combination of resistors, voltage sources, and current sources. All I need to do is think about, you take the complicated thing, figure out its straight line plot, now you read off some critical numbers from this plot.

You say, OK, well what if the current were 0? Well if the current were 0, I'd be on this axis. If the current were 0 in the circuit, V would be V0. So that means you look over here, you figure out the x-intercept, and the x-intercept is the value of the voltage source. Similarly, if you figure out the rate of growth of I -- so more generally, if you solve for I, I will be the difference between V and V0 divided by R, that's just Ohm's law for the resistor.

And that then lets you figure out the law for the slope. The slope over here is going to turn out to be one over this resistor-- I should have had-- this should be R0, these two should match. So the slope of this line is one over that resistor. So in general, regardless of how complicated the box is, figure out the straight line and I-V curves, read off the x-axis, read off the slope, and that lets you construct a simple circuit that has the same Vi curve. We call that circuit the Thevenin Equivalent.

What that means is, you can think about a complicated circuit, regardless of how many parts, by a circuit it just has two. That's an abstraction that lets us think more simply about complicated circuits even if there's no buffers. This is true always. OK, it's not true-- it's not it's not the case that if I change the thing that I put here-- if I change the thing I put here, this relationship is still true. It's the equivalent of Ohm's Law for complicated circuits.

Ohm's Law is what I get if what's in the box is a single resistor. If what's in the box is complicated, more generally this is what I get, the Thevenin Equivalent. Of course there's lots of circuits that have this I-V curve. So a different one is a current source with a resistor. That's called a Norton Equivalent. You do the same sort of thing. What would happen if here, if I set V to be 0? Well how do I make V be 0? I make V be 0 -- Did I write that on the slide? no I didn't write it on the slide. I make V be 0 by putting a short circuit here. I connect a wire.

If I put a wire here, then how big is the current I? Well if I put a wire here, it's easier for this current to go through the wire than it is to go through that resistor. So all of this current goes through the wire. All of this current is I0. So if I put a short circuit across V -- if I make V be 0, how big is i? Minus I0 right? All the current I0 goes through, it just goes to backwards.

So that's how I get this point. So if I wanted to replace some complicated circuit with a Norton Equivalent, I would take the complicated system, to figure out the I-V curve, read off the intercept on this axis, change the sign-- that's the most confusing part, by the way. This is the error that you all make. There's a minus sign in the current relationship. We like to draw-- it just makes us feel good to have the arrow go up.

And if you make the arrow go up, it's in the wrong direction to I. So when we did the Thevenin equivalent, there was no sign from it, this was V0. So the voltage on the voltage source is equal to the x-intercept. But when we do the Norton, the current in the current source is minus the y-intercept. Same idea, though. OK? So the idea is that Thevenin and Norton equivalent circuits are equivalent, in the sense that they generate the same voltage. that the more complicated circuit did.

So that means for thinking about the circuit we can ignore the complicated stuff, and just know two numbers, V0 and R0, or I0 and R0. So one more step, what this all means is, that if you can represent the current voltage relationship for an arbitrary circuit in terms of a straight line, that means that when we're trying to characterize an arbitrary circuit, it doesn't matter how complicated it is. It could have 100 parts in it.

Regardless of how complicated it is, I only need to measure two things in order to fully characterize it. If a circuit is made out of linear resistors, voltage sources, and current sources, three special linear parts-- if a circuit is composed entirely out of linear parts, doesn't matter how many parts are in it, there could 100, there could be

1,000-- I only need to measure two things to get a complete description. And that's because two points determine a straight line.

I know, by having proved it by using linear algebra, I know by using linear algebra that the solution is a straight line. I know from geometry that two points determine a straight line. I only need to find out two points. So by convention, the easiest two points is usually the simplest cases. Set the voltage to 0 and set the current to be 0. So that motivates the idea that if I have an arbitrary circuit, if I want to figure out this reduced complexity abstraction-- say I want to make a Thevenin Equivalent-- what I would do is first to ask the question, how big is the open circuit voltage?

Open circuit means there's no connection between the terminals. That means the current is 0. If the current is 0, I'm on the x-axis. So over here I'm on the x-axis, so I'm thinking about the red point. Open circuit over here means there's no connection here, which means the current is 0, and all I need to ask is-- regardless of how complicated it is the circuit-- how big is the voltage that I would measure here?

So in this circuit, if I have a-- yeah, I confused myself for a moment. if this current is 0, then this voltage drop is 0. This voltage source prescribes the voltage between these nodes to be 1 Volt, so V0 is 1 Volt. So I just fell one point. I set I to be 0, I found the open circuit voltage. Then I need to find one other point, because I only to find two. So I'll find the short circuit current.

Imagine that I put a wire between the input and the output, and I'll compute how much current that circuit generates in that wire. Again the only confusing part is that the reference directions are backwards to the way you might have expected them to be. If I put a short circuit here, then the voltage between these nodes is still 1 Volt. That's what the voltage source always says.

So the current that flows through this wire, the short circuit current, is just this V over that R, except it's in the negative direction. And the way you can think about that is that the slope of this curve has to be positive. I need-- because of the way Ohm's Law works-- increasing the voltage better tends to increase the current, because that's what Ohm's Law resistors do. So I need to have this slope be positive if it's going to be Ohm's Law.

So I characterize just those two points, and then the resistance is simply the ratio of the two. So the resistance is related to the slope. It's 1 over the slope, you don't need to worry about that. The resistance is always V over I, so you just take the open circuit voltage and divide by the short circuit current, minus sign, and you get that the resistor must be 2 Ohms. Is that all clear?

The idea is that we're trying to build an abstraction that lets us simplify the way we think about circuits, without introducing buffers. So that means then that these two circuits are equivalent to that circuit, in the sense that they all share the same I-V curve. If you substituted one server for the other, you couldn't tell from outside the red box which was on the inside of the red box.

OK so I'll do an example now. Think about, what if I want to find the equivalent-- the Thevenin equivalent for this circuit. I just do the things I just told you to do. First thing I think about is, what's the open circuit voltage? So If I think about open circuit, there's no connection here. That means this current is 0. That means that the voltage that develops is the voltage divider. So I get 7 and a 1/2 volts-- 3 over (1 plus 3) times 10, right.

So that gives me one point-- that tells me the voltage source for the Thevenin equivalent. Then for the second point, I want to think about the short circuit current. So I consider putting a wire here, and then I compute the amount of current that flows in that wire. And in this circuit, this wire shorts out that resistor, so all the current goes through this wire, and none of the current goes through that resistor.

So that means that the total current that flows is 10 Volts divided by 1 Ohm, which is 10 Amps. And then I know that the equivalent resistance is the ratio of the open circuit voltage to the short circuit current, except I have to worry about the minus sign. And so I end up with the equivalent resistance being the 7.5 Volts, which is the open circuit voltage, divided by 10 Amps, so I get 7.5 Ohms.

And so the answer then, is that here is the circuit I started with, here's the Thevenin equivalent circuit, they're identical in the sense that they have the same I-V curve. And you can just sort of see why that has to be true. If you think about, here's the two circuits, and if you think about the simple cases, if you set I to be 0, the voltages better be the same. Well over here it's 7.5 by the voltage divider, over here it's 7.5 by the fact that there's no current going through that resistor.

And the short circuit current better be the same. So over here, if you short this out, you're going to get 10 Amps, over here if you short it out, you're going to get 10 Amps. So you can always go back and forth. The point is, that you can substitute the simpler circuit for the more complex circuit, because they have the same I-V curve. OK, just to make sure that you're following me, here's a question that has to do with taking the same circuit, but considering the Thevenin equivalent-- Thevenin or Norton equivalent-- at three different ports.

Here I'm thinking about what would happen if I looked in terminal A, what if I looked in terminal B, or looked in terminal C? Figure out the Thevenin and Norton parameters, and see if there's an error in the table.

So how many errors are in the table? About 50% correct again. So which entry don't you like.

AUDIENCE: 2D?

PROFESSOR: That's exactly right, so 2D is wrong. How do I figure out 1A? How do I figure out V0 for the a circuit? So the definition of V0 is the open circuit voltage. So I need to figure out, for the a circuit, how big would be the voltage across A if there was no current flowing in the leg of A. Everybody clear on that? So the thing that I would do is, I would think about, there's no connection here. What's the voltage here? So how would I calculate that? Yeah.

AUDIENCE: You would use the current-divider relationship to figure out that current flowing through that is going to be 4 Amps.

PROFESSOR: Precisely, so first I need to take-- here are the current sources, so I have two resistor lengths, so that's a perfect set up for the current divider. So the amount of current that goes in this leg compared to that leg is the ratio of this resistance to the sum of resistances, right? And if you work that out, you're going to get 4 Amps coming through here. So then after you know the current through this leg, it's an easy matter to take the current and turn it into a voltage, so the voltage at the a port is 4 Amps times 5 Ohms is 20 Volts. Is that clear?

And similarly, but with a different answer, the voltage at the B terminal is the same 4 Amps, but now times 10 Ohms. So that's how we got 40. And at C, it's the same 4 Amps, but now it's times the sum, 4 times 15 is 60. Everybody's happy about that? So the point is that when you generate an equivalent circuit, it depends upon which set of terminals you're using. You can't just take a circuit and say, give me the Thevenin Equivalent.

You have to say, give me the Thevenin equivalent looking somewhere. So I could look in the A port, the B port, or the C port, and I get different V0's. How would I compute the I0? Short circuit current-- so what I would do is I would short this out. When I do that, all the current flows in this leg and one of the current flows that leg. So that means I have, equivalently, 10 Ohms in parallel with 10 Ohms. Then by the current divider, how much current goes down one leg?

Half of it. I get a different answer over here, because now I short out that node, which shorts out that resistor, so now I get a different ratio of resistors. It's not the same as the first, so I know that this answer can't be 5. And, in fact, if you work it out, the answer is 20 over 3. And finally, if you short here, then you know that that short circuit shorts out both this series combination and that one.

So all of the 10 Amps goes through that, so you get that 10 Amps. Then how do you get R0, which is the ratio of V0 over I0. So if you take V0 over I0, you get 4. Here if you do right answer, you get 6, and here if you do that you get 6. The point is, that the Thevenin equivalent you get is different. The Norton equivalent that you get is different, depending on which ports you're looking at.

OK there's two reasons for thinking about this. One-- so why am I thinking about all of these equivalent circuits? So I wanted to have an abstraction that was useful for thinking about how parts interact. I wanted a way of thinking about what would happen if I changed the load on the circuit without having to recalculate all the voltages and currents throughout the circuit. And so this Thevenin and Norton idea is a way of doing that.

That's important from a practical sense, because when you buy a part-- when you buy an electronic part, they tell you how it works by telling you the Thevenin equivalent or the Norton equivalent, or whatever is the easy way to think about it. So there's a practical reason-- when you buy an op-amp, they tell you how good the op-amp is by telling you how big is the equivalent resistance at the output.

So it has a practical value, because it lets you-- it's the way you specify an electronic part. You can't -- when the manufacturer makes a part, they can't know what you're going to do with it. So they tell you how it works by telling you something about the equivalent circuit. There's also a different reason for thinking about this, and that is because it's conceptually simplifying to think about Thevenin and Norton equivalents.

So here's an example that's very much like the first problem that I worked out. What would be the effect of closing this switch on the current I? We solved the problem very much like this last time, and one way you can solve it is you figure out I in two cases, when the switch is open and when the switch is closed. And you figure out whether went up or down, and you know the answer.

The point is, that if you think about this in terms of equivalent circuits, it's completely trivial. If I think about what would happen-- I'm interested in what happens at I, what I do is split the circuit into two pieces, the stuff to the left of I, and the stuff to the right to of I. I make a Thevenin equivalent for both of them, and then I jam together the two Thevenin equivalents. What I get, in detail, is showed here.

So if I look left, I see a 20 Volt source with a 4, 4, and a 2. That has a Thevenin equivalent that is showed here, which is independent of the state of the switch. So the same Thevenin occurs on the two sides, switch open, or switch closed. If i make a Thevenin over here, there's no sources. So it's just going to be a resistance. So when the switch is open, I have to 2 Ohms, when the switch is closed, I have 1 Ohm.

I don't even need to figure out what are these part values to see that, if I make this resistance smaller, which happens if I close the switch, the current goes up. I mean it's true that I showed here that the Thevenin voltage is 10 and the Thevenin resistor is 4. I don't even care. Regardless of what the Thevenin voltage was, regardless of what the Thevenin current is, it's going to be the same when the switch is open and closed. And that's a powerful statement.

I know it's the same, so when I close this which, all I've really done is I've made that resistor smaller. The net series resistance is down, the current is up. So I have two reasons for thinking about Thevenin and Norton equivalence. One is practical value, the other is conceptual simplicity. It lets you simplify way you think about a circuit, and it's a way of gaining intuition without ever even solving the equations.

Most circuit designers don't solve certain equations. They know what it's going to do just by looking at it, and this is the kind of reasoning that they use. OK, there's one more topic-- the idea of Thevenins and Nortons really derived from linear algebra. The basic parts of most interest are linear. And when you put together a system out of linear parts, you get a linear system.

There's one more consequence of linearity that is terribly useful, and that's the idea of super position. If you have a system of equations that is linear, and if you have multiple sources, things are very simple. You can see that over here. If I had multiple sources, the system of equations that I get wouldn't look quite like this. These terms, the constant terms, they're the ones that come from the drives, the voltage sources, the current sources.

So if I had two drives, I would get something else here. I would get plus a at N+1 for example-- source one, source two. And I would get source one, source two. So the idea is that, if I have a system that has multiple sources, I know from the structure of the linear equations, there is just more constant terms. OK, well what's that mean? That means you can just use linear algebra to see that the answer to this problem is the sum of the answers to that problem plus the answers to that problem.

That's just linear algebra. What that means in terms of circuits is, I can figure out the response to a circuit by turning on the sources one at a time. That's called superposition. And generally speaking, it's a lot easier than solving the circuit out the long way. So here I've got two sources So say I wanted to compute I, in response to V0 and I0.

What I would do, is I would turn off the I0 source, and calculate the voltage that results just from the voltage source. That's the same as setting this to 0 and finding the response to this. So if I want to set I to 0, the way you set I to be 0 is open circuit. If you open circuit something, there's no current going to flow through it. So I replace the current source with an open circuit, and compute the i that would be result when the current source isn't there. OK, well that's easy.

If their current source we're not there, this would be open circuit. The current, i, would just be the total voltage going through R1 and R2. So the answer, I1, the first component of the current I, would be V0, the result of the voltage source divided by the sum of R1 and R2. Now that's not the whole answer. That would be the whole answer if I0 weren't there.

So now I have to worry about the other case. What if only I0 were there? Well now I have to set V to be 0. Well setting V to 0 is not open circuiting it. You can't just reach in, grab the voltage source, and throw it away, because that'll make the current 0. If I want the voltage to be 0, I have to short circuit it. So what I do then, is I leave the current source alone, and I replace the voltage source with a short circuit, guaranteeing that V is 0. Then I ask, how big is I2, the component of I that results from the current source?

Well if I've short circuited this, then I just get a current divider. This current has to do with how readily the current divides between R1 and R2. The amount that goes through the R1 side is in proportion to R2 make R2 bigger, more of it goes through R1. Standard current divider, except for the slippery minus sign. So if I only had the current source, the current I2 would have been current divider operating I0 with the slippery minus sign.

So that means, by superposition, that the result of having both sources on is just the sum of those answers. That's a very big simplification. If you've got multiple sources in the circuit, you can think about them all at once. And if you really good at writing linear equations and solving them with pencils, you'll get the right answer. But there's an enormous simplification if you just turn off all but one, and do them one at a time.

So here's a problem. Here's a very simple circuit, compute V by using super position. V is the voltage across the resistor. How big would V be if you used the idea of super position?

So what's the answer? 50% correct, roughly speaking. OK I want to use superposition. So how big would the voltage v be, if all I had was the voltage source? One. How big would the voltage be if i only had the current source? Ah, got half of you. That explains the 50% correct. How big would the voltage be, if I only had the current source? It's tempting to say (1). I say tempting, because that's the wrong answer.

[LAUGHTER]

What is wrong about the answer (1)? What does the voltage do to the current source? The voltage due to the current source is the voltage that the current source would have generated if the voltage source weren't there. If the voltage source weren't there, then-- half of you got it right, so half of you can shout the answer. So if the voltage source weren't there, the voltage source would be 0.

If the voltage source were 0, then V would be 0. So if the voltage due to the voltage source is 1, the voltage due to the current source is 0. The sum of the two is 1. The answer is 1. OK? Make sense? So, very closely related problem. What's the current I? Solve that by superposition.

So how big is the current I according to superposition? Wonderful, so how big is the current i generated by the voltage source? 1. How big is the I generated by the current source? Negative 1, the sum is 0. All right, so those two problems were trivial by superposition. They're not too hard by non-superposition. But the point is, that they're trivial by super position.

So what we saw today-- the goal for today was to generate some abstractions that let you think about the way parts interact with each other, because that's a central issue when you're thinking about circuit design And we built the idea of Thevenins and Nortons and super positions. That was the basic idea. And the sub-theme-- or maybe I should say the major theme is really that.

It's the importance of linear algebra. So the take home message is probably, take 18.06. So this was all about the application of 18.06 to the solving of circuits. See you later.