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The Concept of a 'Unit of Degradation of Energy' in Fluid Flow

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Abstract

By analogy with the concept of the Number of Transfer Units (NTU) introduced by Colburn, the definition of the Number of Units of Energy (NEU) degraded in an apparatus in which flows a fluid at steady state is proposed. The two modes of mechanical energy degradation are: by viscous friction and by inertia (starting and stopping the fluid's motion). The 'Number of Energy degradation Units' is explicitly defined as the addition of two terms corresponding respectively to each of these modes and a third term which takes into account deviation from simple additivity of the two first terms.

The additivity is shown to be well observed for flow through a porous medium (Ergun's equation), this approach being reasonably correct for flow around a convex obstacle (free sedimentation). The additivity is no more correct for flow in empty pipes or in tanks with stirrers.

The Number of Energy degradation Units is evaluated for several geometrical configurations, for example in the wake of an obstacle, in each cavern of a porous medium, etc. *In each case the various usual dimensionless numbers (friction factor, drag coefficient, power number,* etc.*) are expressed in terms of a single parameter (the NEU).*

1. DEFINITION OF THE 'UNIT OF DEGRADATION OF ENERGY'

Consider an object of mass M at rest in vacuum. We wish to transport this object a given distance L, in a given time t. How can we move it in order to degrade the least possible amount of energy?

There is an infinite number of possible trajectories (see Fig. 1(a)); the velocity increases and passes through a maximum at time t_m. The object undergoes, therefore, acceleration during time 0 to t_m, at which point the velocity has its maximum value V_m, followed by deceleration between t_m and t.

Whatever the form of the acceleration–deceleration, between times $t = 0$ and $t = t_m$ the body receives from the surroundings a quantity of kinetic energy $mV_m^2/2$ which it gives back during the time between t_m and t.

The energy exchanged between the object and its environment is therefore $2 \times 1/2 \times mV_m^2$. This energy is a minimum for $V_m = u = L/t$, *i.e.* for the process called 'double-impulsion' in which the surroundings provide an initial impulse to the body, an energy of $Mu^2/2$, in an infinitely short time. The object continues on its course with constant velocity (assuming no friction). At the final instant, the object dissipates its kinetic energy into the surroundings via a second impulse in the reverse direction (Fig. 1(b)). This double transfer of energy is an irreversible process; this means, therefore, that a certain amount of potential energy is degraded; this energy was contained in the surroundings and served to transport the object.

Imagine, for example, the environmental potential energy produced by the compression of an elastic material (a spring). The release of such a spring gives the object its initial impulsion. Also, the final braking of the object could be due to the compression of a second spring. The potential energy will thus be given back to the environment, but only part of it. In fact the two operations of compression and release of the springs cause a localised heating, due to internal friction within the metal (Fig. 1(c)).

Let E_d be the quantity of mechanical energy degraded as heat. We shall evaluate E_d by comparing it with the energy Mu^2, exchanged with the environment, by writing

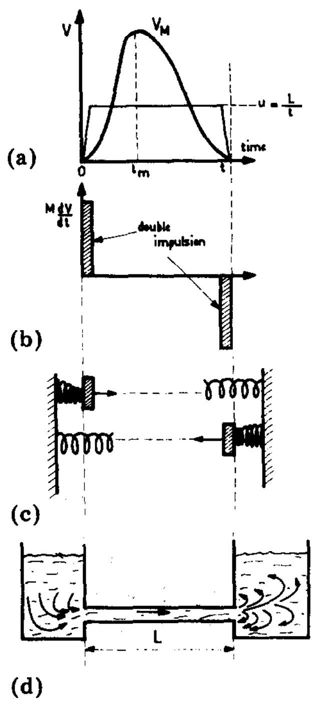

Fig. 1. Definition of the 'unit of degradation of energy'. The 'Number of Energy degradation Units', NEU, is $\sim 10^{-4}$ for elastic springs and ~ 1 for a fluid flowing from one reservoir to another through a tube.

$$E_d = \text{NEU} \times Mu^2 \tag{1}$$

NEU is the Number of Energy Units degraded during this operation, and Mu^2 is the 'Unit' of Energy degraded in the transport. Thus

$$\text{one unit of energy degraded} \equiv Mu^2$$

$$= M(L/t)^2 \tag{2}$$

This definition of NEU is proposed in analogy with the notion of the 'Number of Transfer Units' (of mass or heat), NTU, introduced by Colburn in 1933 [1, 2]. It will be shown later that these notions are very similar — they are even numerically equal in the particular case of the Reynolds analogy.

In the case of very elastic springs, NEU can have a very low value, often of the order of 10^{-4}. However, if the environment is a fluid, NEU is of the order of unity. In a fluid the mechanical potential energy per unit volume is the driving pressure $\hat{p} = p + \rho g Z$, and the conversion of potential energy into kinetic energy is very inefficient (see below). The energy degraded is of the same order as the energy transferred.

2. APPLICATION TO A FLOWING FLUID

We shall now apply the above concept to each unit of volume of a fluid which flows from one reservoir to another via a long cylindrical tube of length L (Fig. 1(d)).

Knowing that the driving pressure $\hat{p}$ is the mechanical potential energy per unit volume, we shall write the quantity of energy degraded (per unit volume) as

$$\hat{p}_1 - \hat{p}_2 = \text{NEU} \times \rho u_m^2 \tag{3}$$

where u_m is the average flow velocity in the pipeline; $u_m = L/t$, ρu_m^2 is the Unit of Energy of Transport per unit volume, and NEU is the Number of Energy Units degraded in the whole pipe.

We see that NEU acts as a dimensionless number which is usually called Euler's number [3] : $\text{NEU} \equiv \text{Eu}$.

The quantity NEU has three components: one for each extremity and one for the pipe itself. This latter component will be dealt with in Section 4.

It is known that the quantity of energy degraded at the entrance to, or exit from, a cylindrical tube depends on the shape of the entrance/exit. For example, for a cylindrical tube which opens abruptly into a large reservoir, we have

for the exit: $\text{NEU}_x = 0.50$

and

for the entrance: $\text{NEU}_e = 0.25$ to 0.50

depending on whether or not the tube penetrates into the reservoir. Thus we have a total

$$\text{NEU}_e + \text{NEU}_x = 0.75 \text{ to } 1 \tag{4}$$

It is important to note that, in a perfectly *macro*-mixed tank (see Fig. 2), the *'ordered'* kinetic energy of the incident jet is entirely converted into random kinetic energy of eddies, but this energy is not necessarily converted into random kinetic energy of molecules (*i.e.* heat) — there is not necessarily *micro*-mixing (on a molecular scale) (*cf.* ref. 4). This point will be re-examined in the following article [5].

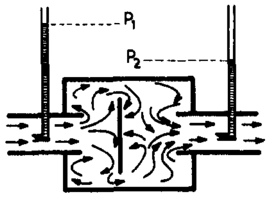

Fig. 2. One 'Unit of Energy' is degraded in a fluid flowing through a perfectly macro-mixed tank.

3. THE TWO MODES OF DEGRADATION OF ENERGY: BY INERTIA AND BY VISCOUS FRICTION

Consider now the transport of the object at constant velocity u (on average), over a distance L, and assuming the existence of friction between the body and its environment. It is known that two modes of degradation of mechanical energy result, that is, two irreversible phenomena, which lead to two terms in the expression for NEU. We shall only treat the case of a Newtonian fluid here.

The two terms are:

— on the one hand, an inertia term. If the body is stopped a certain number of times during its trajectory, each time *one* Unit of Energy must be supplied to stop and start it. Let A be the number of *stop-and-starts* per unit length of trajectory. The energy exchange is then $LA\ Mu^2$. This model can be used to represent the turbulent flow of a fluid: each object is an element of fluid which is alternately stopped and started in the eddies;

— on the other hand, a viscous friction term. We postulate that the friction force of the object on the surrounding fluid is proportional to the velocity u and to a dimension 'd' which is characteristic of the object (Stokes' law for a Newtonian fluid). Thus F_f = (const.)μud.

The work effected by this force on the trajectory L is therefore $b\mu uLd$. b is a coefficient which depends only on the shape of the object ($b = 3$ for a sphere; for any shape we write $b = 3\psi$, where ψ is a sphericity coefficient).

If we postulate that these terms can simply be added together, the energy consumed is then

$$E_d = LAMu^2 + b\mu uLd \tag{5}$$

We now introduce the Reynolds number relative to the object:

$$\mathrm{Re} \equiv \rho ud/\mu \tag{6}$$

where ρ is the density of the fluid.

The energy degraded is now

$$E_d = LMu^2(A+B/\mathrm{Re})$$

where

$$B \equiv (18\psi/\pi d)(\rho/\rho_o) \tag{7}$$

B is thus a characteristic constant of the system; ρ/ρ_o is the ratio of the densities of the fluid and of the body. From this expression we can deduce the Number of Energy Units degraded per unit length:

$$\mathrm{NEU}/L = A + B/\mathrm{Re} \tag{8}$$

As can be seen in the log–log plot (Fig. 3), three domains can be distinguished, separated by two critical values of the Reynolds number:

— for $\mathrm{Re} > \mathrm{Re}_2$, the first term in (8) is at least ten times greater than the second. This means that more than 90% of the energy is degraded in the inertia mode.

— for $\mathrm{Re} < \mathrm{Re}_1$, the second term is at least ten times greater than the first. This means that more than 90% of the energy is degraded through viscous friction.

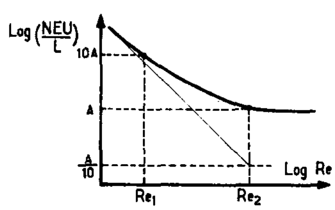

Fig. 3. The Number of Energy degradation Units per unit tube length as a function of the Reynolds number.

We propose to show that, in nearly all industrial equipment, the energy degraded can be expressed in this way. However, the simple addition of the two terms describing the inertia and the viscous friction will only be verified approximately and only in certain cases.

4. FLOW IN LONG CYLINDRICAL PIPES

By comparing this energy balance with the conventional balance of the work done by fluid forces acting on the walls of the pipe, the following expression is obtained:

$$\mathrm{NEU} = (S/\Omega)(f/2) \quad (9)$$

where f is the friction factor, S the surface area of the walls and Ω the average cross-sectional area of the pipe.

It is important to note that this is a general expression and applies to any pipe which is macroscopically cylindrical, even if the walls are very rough, or if the pipe is made up of a sequence of cavities and narrow constrictions. This remark will be useful later when we deal with flow through porous media.

In Fig. 4 are reproduced several curves of Moody's classic diagram [6] for pipes with circular section ($S/\Omega = 4L/d$). It shows the friction factor $f/2$ as a function of the Reynolds number, for various values of the relative rugosity e/d. The three solid curves are experimental results. The three dashed curves represent expression (8) with the same asymptotes. That is, they represent the assumption of additivity of the two energy degradation modes:

$$f/2 = (d/4)(A + B/\mathrm{Re})$$

We see that for relatively large values of the rugosity ($e/d \geqslant 5 \times 10^{-3}$) the experimental curve lies entirely below the curve corresponding to the addition: the deviation from additivity is always negative; inversely, for the lowest values of rugosity, the deviation is negative in the laminar flow region and positive in the turbulent flow region. For example, for $e/d = 10^{-4}$, the relative deviation attains a maximum value of the order of +30% for $\mathrm{Re} \simeq 30\,000$.

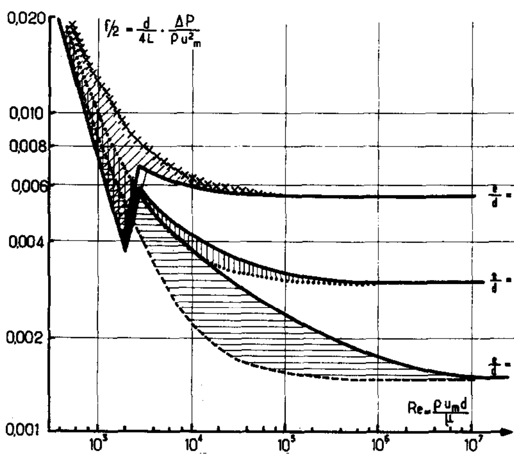

Fig. 4. Deviation from the single addition of the modes of energy degradation in long cylindrical tubes.

The existence of non-additivity signifies that the appearance of turbulence and viscous friction are not independent phenomena. As the flow speed is increased (and therefore Re) the system swings from the laminar flow regime into the turbulent flow regime, and this happens over a very small range of Re. There is an abrupt changeover from one mode of energy degradation to the other, and not coexistence of the two with progressive replacement of one by the other, contrary to what happens in a porous medium (see later).

In analogy with the notion of 'Height of a Transfer Unit' introduced by Chilton and Colburn [2] for the phenomena of transfer of mass and heat to the walls (see the following article, ref. 5), it is interesting to introduce here the notion of 'Height of Energy degradation Unit' (or better the 'Length' of Energy degradation Unit) by putting

$$L = \mathrm{NEU} \times \mathrm{HEU} \quad (10)$$

HEU is then, by definition, the length of tube in which one unit of energy is degraded.

In the general case,

$$\mathrm{HEU} = \frac{\Omega Z}{Sf/2}$$

and for a tube of diameter d,

$$\mathrm{HEU} = d/2f \quad (11)$$

HEU is thus expressed as a multiple of the diameter of the tube.

We know that the friction factor changes relatively little: it is always in the range 10^{-2} - 10^{-3}, while the Reynolds number varies from 1600 to 2×10^8 for all values of the rugosity of the walls. Similarly, the Height of Energy degradation Units varies little: it is always between 25 and 250 diameters within this same interval, as can be seen in Fig. 5. A uniform average value of the order of 50 to 100 diameters fits well the practical situation in industry.

Note: A classical calculation of the total pressure drop in a long pipeline including fittings (elbows, tees, valves, constrictions,

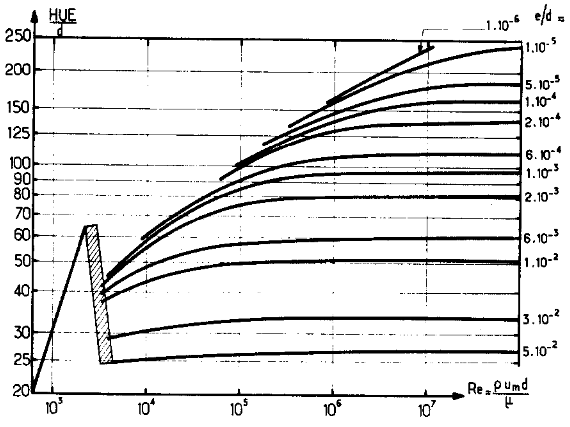

Fig. 5. The 'Height of Energy degradation Unit' for long pipes as a function of the Reynolds numbers for different values of the relative rugosity of the walls.

etc.) is often made by adding to the real length of pipe the equivalent lengths of each of the singularities. This procedure is equivalent to adding on Number of Energy degradation Units.

5. FLOW THROUGH A POROUS MEDIUM

Consider a fluid flowing through a porous medium at a volume flowrate $\dot{V}$. As a model, the porous medium can be represented by a collection of N_h parallel cylindrical tubes, of 'hydraulic' diameter d_h, such that the cross-sectional free area for flow is

$$\Omega = N_h \pi d_h^2/4 \tag{12}$$

In order to preserve the general character of previous definitions we shall define u_m as the average flowrate in the body of the porous medium:

$$u_m \equiv \dot{V}/\Omega$$

We now define the 'tubular' Reynolds number Re_t and the corresponding friction factor:

$$Re_t \equiv \rho u_m d_h/\mu$$

$$\frac{f}{2} \equiv \frac{\Omega}{S} \, \text{NEU} = \frac{d_h}{4L} \, \frac{P_1 - P_2}{\rho u_m^2} \tag{13}$$

The experimental results on the pressure loss in porous media are described well by the classic formula of Ergun [7]. Using the present notation, this is written

$$f/2 = 0.29 + 16.7/Re_t \tag{14}$$

We note here the additivity of the two modes of energy degradation.

From this we can deduce the expression for the Height of Energy degradation Unit from (10):

$$\text{HEU} = \frac{d_h}{1.16 + 66.7 Re_t^{-1}} \tag{15}$$

It would be interesting to compare this value of HEU with that which corresponds to flow in an 'equivalent' tube which is really cylindrical with the same diameter d_h and various rugosities. This is shown in Fig. 6 — in the range of Re_t considered ($\simeq 1$ to $\simeq 5000$) flow in a true cylindrical tube is laminar and the Height of Energy degradation Units, deduced from Poiseuille's law, is $\text{HEU}/d_h = Re_t/32$, whatever the rugosity of the walls.

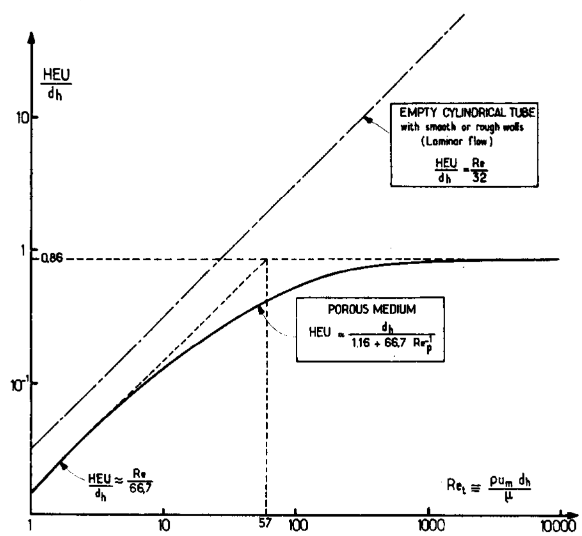

Fig. 6. Height of Energy degradation Unit/hydraulic diameter as a function of the Reynolds number for an empty tube and for a porous medium.

We see that in the porous medium HEU increases almost linearly for low values of Re, being always a little smaller than the value for an equivalent true cylindrical tube. Afterwards it tends asymptotically to a limiting value which is the order of magnitude of d_h:

$$\text{HEU}/d_h \to 0.86 \quad \text{for} \quad Re_t \geqslant 1000$$

This means that at high flowrates the fluid is stopped and re-started each time it travels a distance of the order of the hydraulic diameter d_h.

In order to visualise the physical interpretation of this phenomenon, consider the particular case where the porous medium is made up of a pile of spherical particles of diameter d_p. It is simple to express the hydraulic diameter as a function of d_p:

$$d_h = \frac{2}{3} \frac{\epsilon}{1-\epsilon} d_p \quad (16)$$

where ϵ is the porosity (void fraction).

From this we can deduce the expression for the Number of Energy degradation Units:

$$\mathrm{NEU} = \frac{L}{d_p} \frac{1-\epsilon}{\epsilon} \left(1.75 + \frac{100}{\mathrm{Re_t}}\right) \quad (17)$$

When the fluid moves at high speed ($\mathrm{Re_t} \gg 100$) over a length L equal to one diameter of a particle, in a porous medium of porosity $\epsilon = 0.5$, 1.75 units of energy will be degraded. This is an evaluation of the number of times that the fluid is stopped in the eddies in the caverns and then re-accelerated in the constrictions which join the caverns (see Fig. 7).

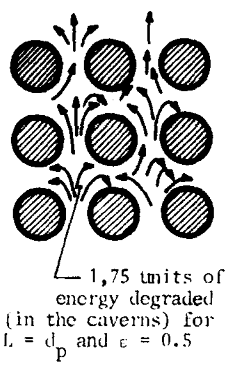

Fig. 7. Fluid flowing at a high velocity through a stack of spheres.

6. FLOW AROUND AN OBSTACLE

Consider an obstacle, having cylindrical symmetry, in a flowing fluid at steady state. Let us think about a stream tube with the main cross-sectional area of the obstacle (Fig. 8).

The mechanical energy balance per unit volume, between two points, one far upstream and the other far downstream from the obstacle, leads to eqn. (3) once again:

$$P_1 - P_2 = \mathrm{NEU} \times \rho u^2$$

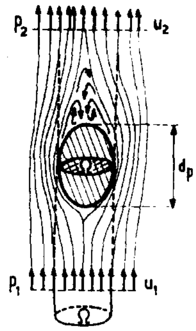

Fig. 8. The NEU is half the drag coefficient for a fluid flowing around an obstacle.

Comparison of this with the conventional definition of the drag coefficient C_D leads to

$$\mathrm{NEU} = C_D/2 \quad (18)$$

A great deal of theoretical and experimental work concerns the value of C_D as a function of the particle Reynolds number $\mathrm{Re_p} \equiv \rho u d_p/\mu$ (where d_p is the apparent diameter of the obstacle) and various form factors which characterise the deviation of the obstacle from sphericity.

In the special case of a spherical obstacle (= one particle), the experimental results, for $\mathrm{Re_p} \leqslant 300\,000$, can be represented by the following function:

$$\mathrm{NEU} = 0.21 + 12\mathrm{Re_p^{-1}} + \Delta(\mathrm{NEU}) \quad (19)$$

The first term corresponds to energy degraded due to inertia, in the turbulent wake of the obstacle. This term dominates for $\mathrm{Re_p} \geqslant 1000$ (Newton regime).

The second term describes the viscous friction (Stokes regime).

The third term is a correction term which accounts for the 'non-additivity' of the first two terms. It is of importance only in the intermediate region, as shown in Fig. 9 and Table 1. It has a maximum value of 39% of the total value of NEU for $\mathrm{Re_p} = 50$.

In agreement with Kaskas [8], cited by Molerus [9], we can express this correction term in the following form:

$$\Delta(\mathrm{NEU}) = 2\mathrm{Re}^{-0.50} \quad (20)$$

The corresponding curve is also shown in Fig. 9.

It is interesting to note that, even at very high speed (Newton regime), only 0.21 unit of energy is degraded in the turbulent wake

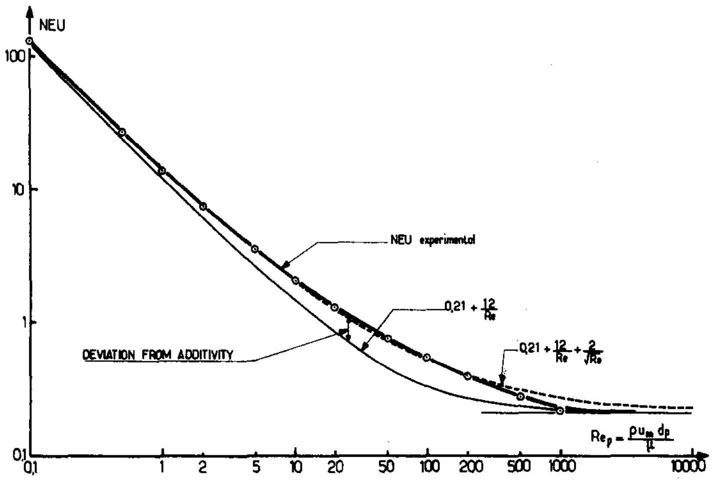

Fig. 9. Number of Energy degradation Units in flow around a sphere.

TABLE 1

Re	$\Delta(NUE)$	$\frac{\Delta(NUE)}{NUE}$ (%)
0.1	−0.22	− 1.8
0.5	+ 0.50	+ 2
1	+ 1.03	+ 8.6
10	+ 0.63	+30
50	+ 0.29	+39
100	+ 0.19	+36
1000	−0.004	− 0.8

produced by a spherical obstacle. We know that the drag coefficient takes different numerical values for non-spherical obstacles. For example, for a thin disc of diameter d perpendicular to the flow, we measure $C_D = 2$ which gives NEU = 1. *One* unit of energy is thus degraded in its wake. On the other hand, for a streamlined object, the inertia term can be less than 0.05. In the Appendix is proposed an interpretation of the value NEU = 0.21 for a sphere, based on the double-impulsion model given earlier.

7. COMPARISON BETWEEN ISOLATED AND PILED SPHERES

Let us compare the energy degraded around a single sphere in free space and that around the same body in the middle of a pile of spheres whose void fraction is ϵ. The number of spheres per unit volume is

$$n = 6(1-\epsilon)/\pi d_p^3$$

Suppose that this assembly of N particles acts as an *ideal mixture* (in a thermodynamical sense). This would mean that the total energy degraded would be N *times* that due to a single particle. By applying (19) and (20) we can thus calculate the Number of Units of Energy degraded per unit length L:

$$(\mathrm{NEU})_{\mathrm{ideal}} = \frac{L}{d_p}\,\frac{3}{2}\,(1-\epsilon)(0.21 + 12\mathrm{Re}_p^{-1} + 2\mathrm{Re}_p^{-0.50}) \tag{21}$$

The dashed curves in Fig. 10 give $(\mathrm{NEU})_{\mathrm{ideal}}$ for five values of ϵ (0.70, 0.60, 0.50, 0.40 and 0.33) for a porous bed with a given thickness, *i.e.* $L = 100\, d_p$.

The solid lines in Fig. 10 show the experimental values of NEU for the same values of ϵ. These are represented by Ergun's equation, after having expressed NEU in terms of the particle Reynolds number Re_p. The latter is deduced from the tubular Reynolds number Re_t (eqn. (13)) in the following manner:

$$\mathrm{Re}_p = \frac{3}{2}\,\frac{1-\epsilon}{\epsilon}\,\mathrm{Re}_t \tag{22}$$

It then follows that

$$(\mathrm{NEU})_{\mathrm{real}} = \frac{L}{d_p}\left[1.75\,\frac{\epsilon}{1-\epsilon} + 150\left(\frac{\epsilon}{1-\epsilon}\right)^2 \mathrm{Re}_p^{-1}\right] \tag{23}$$

This is a well-known modified form of Ergun's equation [7].

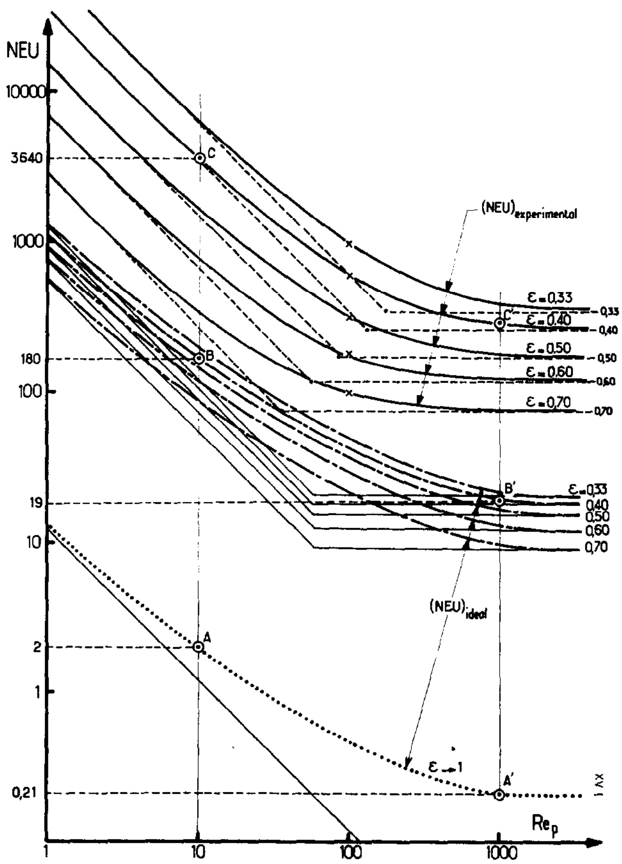

Fig. 10. The extra degradation energy due to the interaction between particles (non-ideality term for a mixture of particles).

The difference $(NEU)_{real} - (NEU)_{ideal}$ which is shown in Fig. 10 represents the *non-ideality* term. It is the extra amount of energy degraded because the particles interact; each particle is in the wake of one or more particles upstream from itself. Thus Fig. 10 illustrates several interesting properties of the non-ideality term. For example, consider the three points A, B and C corresponding to the same value of Re_p. For a thin slice of porosity $\epsilon = 0.40$ and thickness equal to the diameter of a particle ($\Delta L = d_p$), cut within the body of the porous bed:

for $Re_p = 10$

NEU = 1.80 for an ideal mixture
NEU = 36.40 for a real mixture

for $Re_p = 1000$

NEU = 0.19 for an ideal mixture
NEU = 2.96 for a real mixture

We see that the non-ideality term is considerably greater than the ideality term (15 - 20 times greater). The degradation of energy is mainly due to the interaction between the particles and the wake in which they are situated.

A second conclusion is that the two curves, ideal and real, for the same value of ϵ, seem to be deductable from one another by a simple affinity of ordinates — in other words the interaction between particles increases the quantity of energy degraded approximately in the same ratio, irrespective of the value of Re_p, in the range 0.1 - 1000. This means that the influences of the interaction between particles on the viscous term and on the inertia term are proportional. This result was unexpected *a priori*.

8. TANK WITH ROTATING STIRRER

Consider a cylindrical tank in which there is a rotating stirrer, *i.e.* a wheel of diameter d, equipped with blades, having height h and width b. The tangential velocity of rotation of the blades is

$$V = \pi \dot{N} d$$

where $\dot{N}$ is the number of revolutions per second (see Fig. 11).

Consider now a unit volume of fluid transported by a blade, at this velocity V. In analogy with (3), we define the Unit of Energy of Transport of this volume element:

$$\rho V^2 = \rho \pi^2 \dot{N}^2 d^2 \tag{24}$$

The mechanical power supplied to the shaft of rotation $\dot{E}_m$ is transmitted to the fluid, which is swept along by the wheel and then degraded in the tank. Let us make the hypothesis that this power is transmitted to the ring of fluid transported by the blades whose volume is

$$v \simeq \pi d b h$$

(the width b of the blades is considered small compared with the diameter d).

Lastly, the energy supplied to each volume element of this fluid, for each revolution, is

$$e_m = \dot{E}_m / v\dot{N}$$

The Number of Energy degradation Units in the tank is then

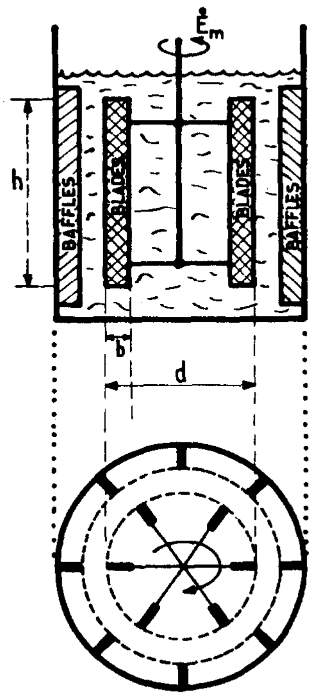

Fig. 11. A tank with a rotating stirrer.

$$\text{NEU} \equiv e_{\text{m}}/\rho V^2 = \dot{E}_{\text{m}}/\rho \pi^3 \dot{N}^3 d^3 bh \tag{25}$$

It is known that the 'power number' Po is a dimensionless number usually defined as follows:

$$\text{Po} \equiv \dot{E}_{\text{m}}/\rho \dot{N}^3 d^5$$

Thus

$$\text{Po} = \text{NEU} \times \psi_{\text{p}}$$

with

$$\psi_{\text{p}} = \pi^3 bh/d^2 \tag{26}$$

ψ_{p} is the geometric form factor for a blade. It depends only on the dimensions of the blade.

The power number is thus simply proportional to NEU. It is important to note the physical significance of the quantity NEU; it is the ratio of the energy degraded by each unit volume of fluid for each complete revolution to the kinetic energy of rotation of the same unit volume of fluid.

$$\text{NEU} = \frac{\text{energy degraded per unit volume and per revolution}}{\text{kinetic energy of rotation of unit volume}} \tag{27}$$

In applying the general relation (8) we shall write (provisionally) NEU as the sum of two terms:

$$\text{NEU} = A + B/\text{Re}_{\text{r}} \tag{28}$$

where Re_{r} is the rotation Reynolds number defined by

$$\text{Re}_{\text{r}} \equiv \rho \dot{N} d^2/\mu \tag{29}$$

$$\text{Po} = A' + B'/\text{Re}_{\text{r}}$$

with

$$A' = A\psi_{\text{f}} \quad \text{and} \quad B' = B\psi_{\text{f}} \tag{30}$$

Figure 12, taken from refs. 10 and 11, shows the experimental values of Po as a function of Re, for wheels having blades of various sizes and shapes. These are the solid lines. For the two extremes we have also traced, as dashed lines, the curves based on the same asymptotes, but with the law of additivity given by (28). We note that for the lower curve there is a small positive deviation from simple additivity and for the upper curve the deviation is negative and much greater, of the order of 50% at the maximum.

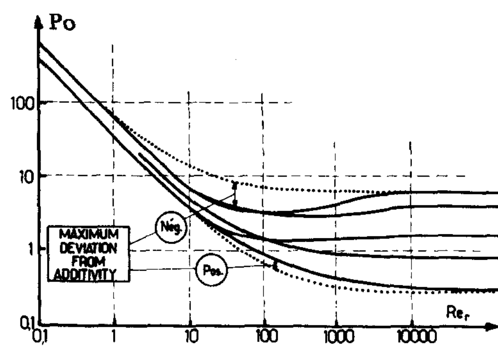

Fig. 12. Power number as a function of the Reynolds number for a rotating stirrer.

9. A PROPOSED INTERPRETATION OF THE COEFFICIENT A

Recall the significance of the coefficient A in (28): A is the number of stops and starts experienced by each fluid element per revolution of the wheel.

It is reasonable to assume that this number is proportional to the number of blades p on the wheel. In fact, the baffles on the walls of the tank keep a quasi-stationary ring of fluid in contact with the walls.

Each element of this fluid, immobilised by a baffle, experiences a shock each time that a blade passes by. A fraction of this fluid is then dragged into the ring of rotating fluid. Eventually, the element returns to the immobile ring, undergoing a second shock, of the same energy but in the reverse sense, which stops it.

We shall therefore write $A = \alpha p$, where α is a coefficient which expresses the probability of this double jump of a fluid element into and out of the rotating stream. Finally, we have

$$A' = A\psi_p = \alpha\pi^3 pbh/d^2 \tag{31}$$

This expression for A' is reasonably analogous to that established by Le Lan and Angelino [12]. It would be worth collating with experiment, in a later study. Note, however, that from now on we have [13] $p = 6$, $b = 0.2\,d$ and $h = 0.2\,d$ for a standard wheel of this type. Substituting these values into (31) gives

$$A' = 5\alpha$$

In order that this value will be in agreement with experimental values of A' (between 4 and 6), we must take $\alpha = 1$. In other words, one unit of energy is degraded *each time* that a fluid element is knocked by a blade, which confirms very nicely the model proposed *a priori*.

10. GENERAL CONCLUSION

For four types of fluid flow, we have defined the *Unit of Mechanical Energy of unit volume* of fluid, that is ρu^2, twice its kinetic energy. We have then evaluated the energy degraded by this fluid, during its movement over a distance L as a multiple of this Unit of Energy; this is the Number of Energy degradation Units: NEU.

This concept has the advantage of providing a link between four dimensionless numbers, each of which is in current and classical use in a distinct domain of fluid mechanics.

(1) For flow in a long, empty, cylindrical tube, it is the friction factor:

$$f/2 = (d/4L)\mathrm{NEU}$$

(2) For flow through a porous medium, it is once again a friction factor:

$$f/2 = (d_h/4L)\mathrm{NEU}$$

(3) For flow around a spherical object, it is the drag coefficient:

$$C_D = 2\mathrm{NEU}$$

(4) For rotating flow in a tank with stirrer, it is the power number:

$$\mathrm{Po} = (\pi^3 bh/d^2)\mathrm{NEU}$$

We see that the dimensionless number used in each of these cases can be expressed simply as a function of the Number of Energy degradation Units. The proportionality constant is a geometric form factor, that is, a factor which depends only on the relative dimensions of the walls on which the energy is degraded.

This unitary presentation of the phenomena has allowed us to compare the densities of power degradation, at least in the first three cases, and notably to specify quantitatively the influence of the porous texture of a pile of particles on the energy degraded in a porous medium.

The concept of a 'Unit of Energy degraded' is apparently applicable to other problems: in another article we shall use it to evaluate the performance of heat and mass exchangers [5].

In a previous article [14] we used it, without clearly explaining it however, to evaluate the energetic performance of mixing operations by diffusion, by turbulent convection and by viscous elongation.

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APPENDIX

Number of Energy degradation Units around an obstacle in the Newton regime

It is known that in the Newton regime, for $500 \leqslant \mathrm{Re} \leqslant 300\,000$, the drag coefficient is a quasi-constant, of the order of 0.40 - 0.44 for a sphere. We deduce from this that the Number of Energy degradation Units is of the order of 0.20 - 0.22. We shall show here that it is possible to find this value 'theoretically' by a simple application of the double-impulsion model, presented in Section 1.

When the object moves a distance L equal to its own length, it forces a volume v of fluid to move aside. In a time $t = L/u$, each volume element of fluid must move laterally by a distance $\bar{y}$, whose average value is easily calculated:

$$\bar{y} = (2\sqrt{2} - 1)d/6 \simeq 0.31d$$

Then, as the object moves forward again a distance L, the volume v of fluid moves back to the axis of the trajectory (see Fig. A1).

The minimum energy to be given to the fluid in order to effect this double displacement is given by the procedure of double shock described in the first section: the fluid is assumed to move sideways with a constant speed $\bar{y}/t$.

When the volume v of fluid goes from position 1 to position 3, it exchanges with the 'surroundings', that is the rest of the fluid, the mechanical energy

$$E_i = 2v\rho(\bar{y}/t)^2$$

Imagine that a fraction α of this energy is degraded to heat. We obtain, for each unit of volume of fluid,

$$E_d = 0.19\alpha(d^2/L^2)\rho u^2$$

For a spherical object $d = L$, thus

$$\mathrm{NEU} = E_d/\rho u^2 = 0.19\alpha$$

Comparison with the experimental value NEU = 0.20 - 0.22 then gives $\alpha \simeq 1.1$.

Note. We have just shown that over a large range of velocities the energy degraded is almost equal to the minimum energy needed for the lateral movement, forwards and backwards of the volume of fluid displaced by the object. But it is important to note that this degraded energy is not immediately converted into heat at the end of the displacement $2L$, *i.e.* in the wake of the object; this energy is in the form of kinetic energy of packets of fluid, with more or less coherent velocities. The volume v of fluid must travel the order of 5 - 10 times L before the scale of turbulence reaches molecular dimensions. At this point the energy is regarded as having been converted to heat.

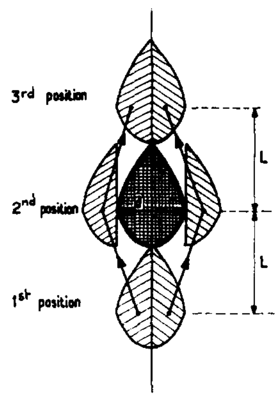

Fig. A1. Interpretation of the 'Number of Energy degradation Units' by the lateral movement of fluid around an obstacle.