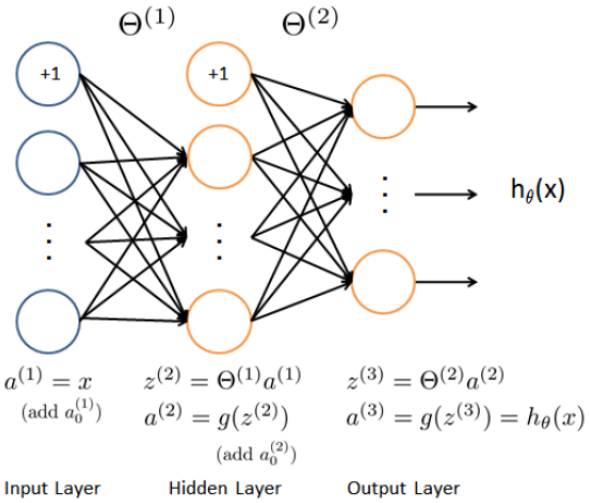

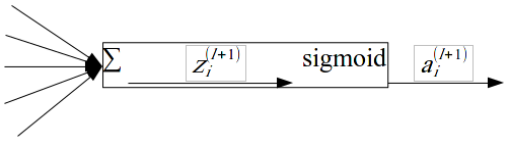

l layer (j)　　l+1 layer (i)　　l+2 layer (k)

$\delta_i^{(l+1)}$ ：第 l+1 层第 i 个单元的计算误差（delta）　$\delta_i^{(l+1)} = \frac{\partial J}{\partial z_i^{(l+1)}}$

s_l ：第 l 层的单元数（不包括偏置单元）。

假设当前为第 l+1 层，单元下标为 i。其中 l 层单元下标为 j，l+1 层单元下标为 k。

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}[y_k^{(i)}\log((h_\Theta(x^{(i)}))_k) + (1-y_k^{(i)})\log(1-(h_\Theta(x^{(i)}))_k)] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l-1}}(\Theta_{j,i}^{(l)})^2$$

$$\frac{\partial J}{\partial \Theta_{ji}^{(l)}} = \frac{\partial J}{\partial z_i^{(l+1)}} * \frac{\partial z_i^{(l+1)}}{\partial \Theta_{ji}} = \delta_i^{(l+1)} * \frac{\partial z_i^{(l+1)}}{\partial \Theta_{ji}^{(l)}} \qquad \frac{\partial J}{\partial \Theta_{ji}^{(l)}} = \frac{\partial J}{\partial a_i^{(l+1)}} * \frac{\partial a_i^{(l+1)}}{\partial z_i^{(l+1)}} * \frac{\partial z_i^{(l+1)}}{\partial \Theta_{ji}^{(l)}}$$

① $\frac{\partial J}{\partial a_i^{(l+1)}}$

若 l+1 层为输出层：

$\frac{\partial J}{\partial a_i^{(l+1)}} = a_i - d_i$ ，　d_i 为第 i 个样本标签

若 l+1 层为隐藏层：

$$\frac{\partial J}{\partial a_i^{(l+1)}} = \sum_{k=1}^{s_{l+2}} \frac{\partial J}{\partial z_k^{(l+2)}} * \frac{\partial z_k^{(l+2)}}{\partial a_i^{(l+1)}} = \sum_{k=1}^{s_{l+2}} \delta_k^{(l+2)} * \frac{\partial z_k^{(l+2)}}{\partial a_i^{(l+1)}}$$

$$\frac{\partial J}{\partial a_i^{(l+1)}} = \sum_{k=1}^{s_{l+2}} \delta_k^{(l+2)} * \frac{\partial\left(\Theta_{0k}^{(l+1)} * a_0^{(l+1)} + \Theta_{1k}^{(l+1)} * a_1^{(l+1)} + \ldots + \Theta_{s_{l+1}k}^{(l+1)} * a_{s_{l+1}}^{(l+1)}\right)}{\partial a_i^{(l+1)}}$$

$$\frac{\partial J}{\partial a_i^{(l+1)}} = \sum_{k=1}^{s_{l+2}} \delta_k^{(l+2)} * \Theta_{ik}^{(l+1)}$$

② $\frac{\partial a_i^{(l+1)}}{\partial z_i^{(l+1)}} = \frac{\partial g\left(z_i^{(l+1)}\right)}{\partial z_i^{(l+1)}} = g\left(z_i^{(l+1)}\right) * \left(1 - g\left(z_i^{(l+1)}\right)\right) = a_i^{(l+1)} * \left(1 - a_i^{(l+1)}\right)$

③ $\frac{\partial z_i^{(l+1)}}{\partial \Theta_{ji}^{(l)}} = \frac{\partial\left(\sum_{j=0}^{s_l} \Theta_{ji}^{(l)} * a_j^{(l)}\right)}{\partial \Theta_{ji}^{(l)}} = \frac{\partial\left(\Theta_{0i}^{(l)} * a_0^{(l)} + \Theta_{1i}^{(l)} * a_1^{(l)} + \ldots + \Theta_{s_l i}^{(l)} * a_{s_l}^{(l)}\right)}{\partial \Theta_{ji}^{(l)}} = a_j^{(l)}$

总结：

$\frac{\partial J}{\partial \Theta_{ji}^{(l)}}$ = ① * ② * ③

①：传播到隐藏层的误差（errors propagated to the hidden layer）

②：隐藏层梯度（hidden layer gradients）

① * ②：隐藏层计算误差（hidden layer delta）

若 l+1 层为输出层：

$\delta^{(l+1)} = \frac{\partial J}{\partial z^{(l+1)}}$ = ① * ② = $\left(a_i - d_i\right) * a^{(l+1)} * \left(1 - a^{(l+1)}\right)$

若 l+1 层为隐藏层：

$\delta^{(l+1)} = \frac{\partial J}{\partial z^{(l+1)}}$ = ① * ② = $\left(\Theta^{(l+1)}\right)^T \circ \delta^{(l+2)} * a^{(l+1)} * \left(1 - a^{(l+1)}\right)$

$\frac{\partial J}{\partial \Theta^{(l)}} = \delta^{(l+1)} \circ \left(a^{(l)}\right)^T$

$\Delta\Theta = \eta * \frac{\partial J}{\partial \Theta^{(l)}}$ ， η 是学习率（learning_rate）

参考索引

https://my.oschina.net/findbill/blog/529001

http://blog.csdn.net/qrlhl/article/details/50885527