

12. Polinomu LKD un saknes.

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1. Uzdevums

1. Parādot visus aprēķina soļus, atrodiet lielāko kopīgo dalītāju šādam polinomam pārim: $2x^5 - 5x^4 + 6x^3 + 3x^2 - 2x + 8$, $2x^5 - 7x^4 + 12x^3 - 8x^2 + 7x - 4$. Rezultātu pārbaudiet ar WolframAlpha.

$$P(x) = 2x^5 - 5x^4 + 6x^3 + 3x^2 - 2x + 8$$

$$Q(x) = 2x^5 - 7x^4 + 12x^3 - 8x^2 + 7x - 4$$

$$LKD(P(x), Q(x))$$

$$P_1 = P - Q =$$

$$\begin{aligned} &= (2x^5 - 5x^4 + 6x^3 + 3x^2 - 2x + 8) - (2x^5 - 7x^4 + 12x^3 - 8x^2 + 7x - 4) = \\ &= 2x^4 - 6x^3 + 11x^2 - 9x + 12 \end{aligned}$$

$$LKD(2x^4 - 6x^3 + 11x^2 - 9x + 12, 2x^5 - 7x^4 + 12x^3 - 8x^2 + 7x - 4)$$

$$Q_1 = Q - xP_1 =$$

$$\begin{aligned} &= (2x^5 - 7x^4 + 12x^3 - 8x^2 + 7x - 4) - x(2x^4 - 6x^3 + 11x^2 - 9x + 12) = \\ &= -x^4 + x^3 + x^2 - 5x - 4 \end{aligned}$$

$$LKD(2x^4 - 6x^3 + 11x^2 - 9x + 12, -x^4 + x^3 + x^2 - 5x - 4)$$

$$P_2 = P_1 + 2Q_1 =$$

$$\begin{aligned} &= (2x^4 - 6x^3 + 11x^2 - 9x + 12) + 2(-x^4 + x^3 + x^2 - 5x - 4) = \\ &= -4x^3 + 13x^2 - 19x + 4 \end{aligned}$$

$$LKD(-4x^3 + 13x^2 - 19x + 4, -x^4 + x^3 + x^2 - 5x - 4)$$

$$Q_2 = 4Q_1 - xP_2 =$$

$$\begin{aligned} &= 4(-x^4 + x^3 + x^2 - 5x - 4) - x(-4x^3 + 13x^2 - 19x + 4) = \\ &= -9x^3 + 23x^2 - 24x - 16 \end{aligned}$$

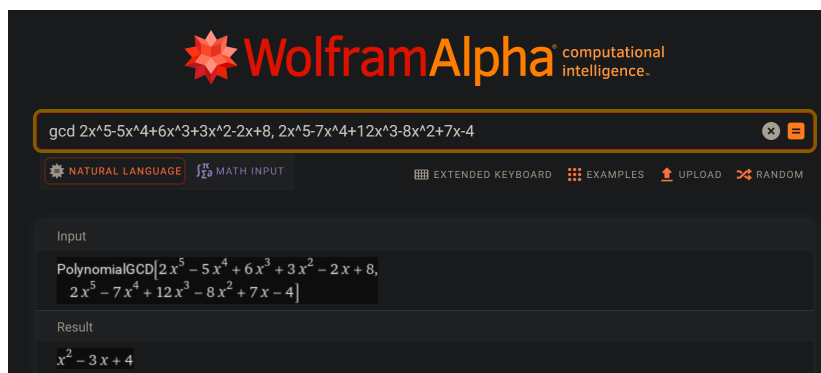
$$\begin{aligned}
& LKD(-4x^3 + 13x^2 - 19x + 4, -9x^3 + 23x^2 - 24x - 16) \\
& P_3 = 9P_2 - 4Q_2 = \\
& = 9(-4x^3 + 13x^2 - 19x + 4) - 4(-9x^3 + 23x^2 - 24x - 16) = \\
& = 25x^2 - 75x + 100
\end{aligned}$$

$$\begin{aligned}
& LKD(25x^2 - 75x + 100, -9x^3 + 23x^2 - 24x - 16) \\
& Q_3 = Q_2 + \frac{9}{25}xP_3 = \\
& = (-9x^3 + 23x^2 - 24x - 16) + \frac{9}{25}x(25x^2 - 75x + 100) = \\
& = -4x^2 + 12x - 16
\end{aligned}$$

$$\begin{aligned}
& LKD(25x^2 - 75x + 100, -4x^2 + 12x - 16) \\
& P_4 = \frac{1}{25}P_3 + \frac{1}{4}Q_3 = \\
& = \frac{1}{25}(25x^2 - 75x + 100) + \frac{1}{4}(-4x^2 + 12x - 16) = \\
& = 0
\end{aligned}$$

$$\frac{25x^2 - 75x + 100}{25} = x^2 - 3x + 4$$

$$LKD(P(x), Q(x)) = x^2 - 3x + 4$$



2. Uzdevums

a) Uzrakstiet kanoniskajā pierakstā divus 3.pakāpes polinomus: P , kura saknes ir 1, 2 un 5; Q , kura saknes ir 2, 3 un 5. Atrodiet $LKD(P, Q)$.

$$P(x) = (x - 1)(x - 2)(x - 5)$$

$$Q(x) = (x - 2)(x - 3)(x - 5)$$

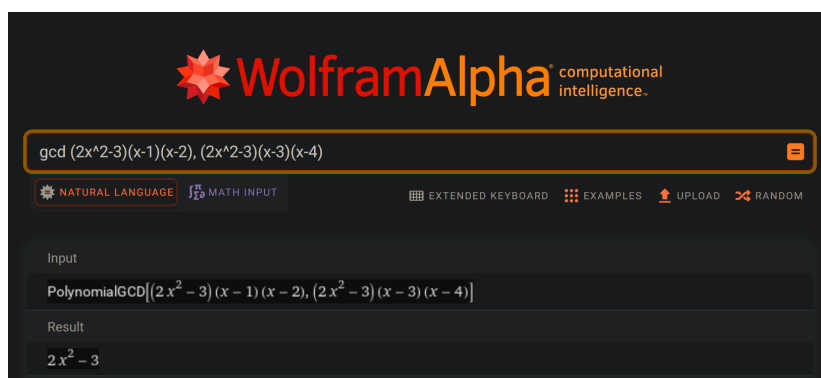
$$LKD(P(x), Q(x)) = (x - 2)(x - 5)LKD(x - 1, x - 3) = (x - 2)(x - 5)$$

b) (LABOTS 11.05 19:40) Uzrakstiet divus 4.pakāpes polinomus, kuriem visas saknes ir reālas un kuru LKD ir $2x^2 - 3$. Rezultātus pārbaudiet ar WolframAlpha.

$$P(x) = (2x^2 - 3)(x - 1)(x - 2)$$

$$Q(x) = (2x^2 - 3)(x - 3)(x - 4)$$

$$LKD(P(x), Q(x)) = (2x^2 - 3)$$



3. Uzdevums

Izmantojot WolframAlpha vai citu līdzekli, sadaliet lineāros reizinātājos slaveno polinomu $x^5 - x + 1$. Saknes ņemiet ar precizitāti 0,01.

$$\begin{aligned} x^5 - x + 1 &= \\ &= (x - 1.17)(x - (0.76 - 0.35i))(x - (-0.18 + 1.08i)) \\ &\quad (x - (-0.18 - 1.08i))(x - (0.76 + 0.35i)) \end{aligned}$$