2. matematiskas loģikas majadarbs

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7. Dedukcijas teorēma. Formulu izvešana izteikumu loģikā

Uzdevums 2.2.4 (a)

1.4.2 d

$$[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \leftrightarrow (A \wedge B \to C)$$
$L_1 : B \to (C \to B)$
$L_2 : (B \to (C \to B)) \to ((B \to D) \to (B \to B))$
$L_3 : B \wedge C \to B$
$L_4 : B \wedge C \to C$
$L_5 : B \to (C \to B \wedge C)$

$$[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \to (A \wedge B \to C)$$

1. $A \to (B \to C)$ (Pieņemta hipotēze)
2. $A \wedge B$ (Pieņemta hipotēze)
3. $A \wedge B \to A$ (L_3)
4. $A \wedge B \to B$ (L_4)
5. A (MP 2, 3)
6. B (MP 2, 4)
7. $B \to C$ (MP 1, 5)
8. C (MP 6, 7)
9. $[L_1 - L_5, \mathrm{MP}] : A \to (B \to C), A \wedge B \vdash C$ (Pierādīts)
10. $[L_1 - L_5, \mathrm{MP}] : A \to (B \to C) \vdash A \wedge B \to C$ (DT_1)
11. $[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \to (A \wedge B \to C)$ (DT_1)

$$[L_1 - L_5, \mathrm{MP}] : (A \wedge B \to C) \to (A \to (B \to C))$$

1. $A \wedge B \to C$ (Pieņemta hipotēze)
2. A (Pieņemta hipotēze)
3. B (Pieņemta hipotēze)
4. $A \to (B \to A \wedge B)$ (L_5)
5. $B \to A \wedge B$ (MP 2, 4)
6. $A \wedge B$ (MP 3, 5)
7. C (MP 1, 6)
8. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C, A, B \vdash C$ (Pierādīts)
9. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C, A \vdash B \to C$ (DT_1)
10. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C \vdash A \to (B \to C)$ (DT_1)
11. $[L_1 - L_5, \mathrm{MP}] : (A \wedge B \to C) \to (A \to (B \to C))$ (DT_1)

Uzdevums 2.2.4 (b)

$[L_1 - L_5, \mathrm{MP}] : (A \to B) \to (A \wedge C \to B \wedge C)$
$L_1 : B \to (C \to B)$
$L_2 : (B \to (C \to B)) \to ((B \to D) \to (B \to B))$
$L_3 : B \wedge C \to B$
$L_4 : B \wedge C \to C$
$L_5 : B \to (C \to B \wedge C)$

1. $A \to B$ (Pieņemta hipotēze)
2. $A \wedge C$ (Pieņemta hipotēze)
3. $A \wedge C \to A$ (L_3)
4. $A \wedge C \to C$ (L_4)
5. A (MP 2, 3)
6. B (MP 1, 5)
7. C (MP 1, 4)
8. $B \to (C \to B \wedge C)$ (L_5)
9. $C \to B \wedge C$ (MP 6, 8)
10. $B \wedge C$ (MP 7, 9)
11. $[L_1 - L_5, \mathrm{MP}] : A \to B, A \wedge C \vdash B \wedge C$ (Pierādīts)
12. $[L_1 - L_5, \mathrm{MP}] : A \to B, A \wedge C \vdash B \wedge C \to B \wedge C$ (DT_1)
13. $[L_1 - L_5, \mathrm{MP}] : A \to B \vdash A \wedge C \to B \wedge C$ (DT_1)
14. $[L_1 - L_5, \mathrm{MP}] : (A \to B) \to (A \wedge C \to B \wedge C)$ (DT_1)