2. matematiskas loģikas majadarbs

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7. Dedukcijas teorēma. Formulu izvešana izteikumu loģikā

Uzdevums 2.2.4 (a)

1.4.2 d

$$[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \leftrightarrow (A \wedge B \to C)$$
$$L_1 : B \to (C \to B)$$
$$L_2 : (B \to (C \to B)) \to ((B \to D) \to (B \to B))$$
$$L_3 : B \wedge C \to B$$
$$L_4 : B \wedge C \to C$$
$$L_5 : B \to (C \to B \wedge C)$$

$$[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \to (A \wedge B \to C)$$

1. $A \to (B \to C)$ (Pieņemta hipotēze)

2. $A \wedge B$ (Pieņemta hipotēze)

3. $A \wedge B \to A$ (L$_3$)

4. $A \wedge B \to B$ (L$_4$)

5. A (MP 2, 3)

6. B (MP 2, 4)

7. $B \to C$ (MP 1, 5)

8. C (MP 6, 7)

9. $[L_1 - L_5, \mathrm{MP}] : A \to (B \to C), A \wedge B \vdash C$ (Pierādīts)

10. $[L_1 - L_5, \mathrm{MP}] : A \to (B \to C) \vdash A \wedge B \to C$ (DT$_1$)

11. $[L_1 - L_5, \mathrm{MP}] : (A \to (B \to C)) \to (A \wedge B \to C)$ (DT$_1$)

$$[L_1 - L_5, \mathrm{MP}] : (A \wedge B \to C) \to (A \to (B \to C))$$

1. $A \wedge B \to C$ (Pieņemta hipotēze)

2. A (Pieņemta hipotēze)

3. B (Pieņemta hipotēze)

4. $A \to (B \to A \wedge B)$ (L$_5$)

5. $B \to A \wedge B$ (MP 2, 4)

6. $A \wedge B$ (MP 3, 5)

7. C (MP 1, 6)

8. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C, A, B \vdash C$ (Pierādīts)

9. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C, A \vdash B \to C$ (DT$_1$)

10. $[L_1 - L_5, \mathrm{MP}] : A \wedge B \to C \vdash A \to (B \to C)$ (DT$_1$)

11. $[L_1 - L_5, \mathrm{MP}] : (A \wedge B \to C) \to (A \to (B \to C))$ (DT$_1$)

Uzdevums 2.2.4 (b)

$$[L_1 - L_5, \text{MP}] : (A \to B) \to (A \wedge C \to B \wedge C)$$
$$L_1 : B \to (C \to B)$$
$$L_2 : (B \to (C \to B)) \to ((B \to D) \to (B \to B))$$
$$L_3 : B \wedge C \to B$$
$$L_4 : B \wedge C \to C$$
$$L_5 : B \to (C \to B \wedge C)$$

1. $A \to B$ (Pieņemta hipotēze)

2. $A \wedge C$ (Pieņemta hipotēze)

3. $A \wedge C \to A$ (L_3)

4. $A \wedge C \to C$ (L_4)

5. A (MP 2, 3)

6. B (MP 1, 5)

7. C (MP 1, 4)

8. $B \to (C \to B \wedge C)$ (L_5)

9. $C \to B \wedge C$ (MP 6, 8)

10. $B \wedge C$ (MP 7, 9)

11. $[L_1 - L_5, \text{MP}] : A \to B, A \wedge C \vdash B \wedge C$ (Pierādīts)

12. $[L_1 - L_5, \text{MP}] : A \to B, A \wedge C \vdash B \wedge C \to B \wedge C$ (DT_1)

13. $[L_1 - L_5, \text{MP}] : A \to B \vdash A \wedge C \to B \wedge C$ (DT_1)

14. $[L_1 - L_5, \text{MP}] : (A \to B) \to (A \wedge C \to B \wedge C)$ (DT_1)