9. Kompleksie skaitļi - trigonometriskais pieraksts

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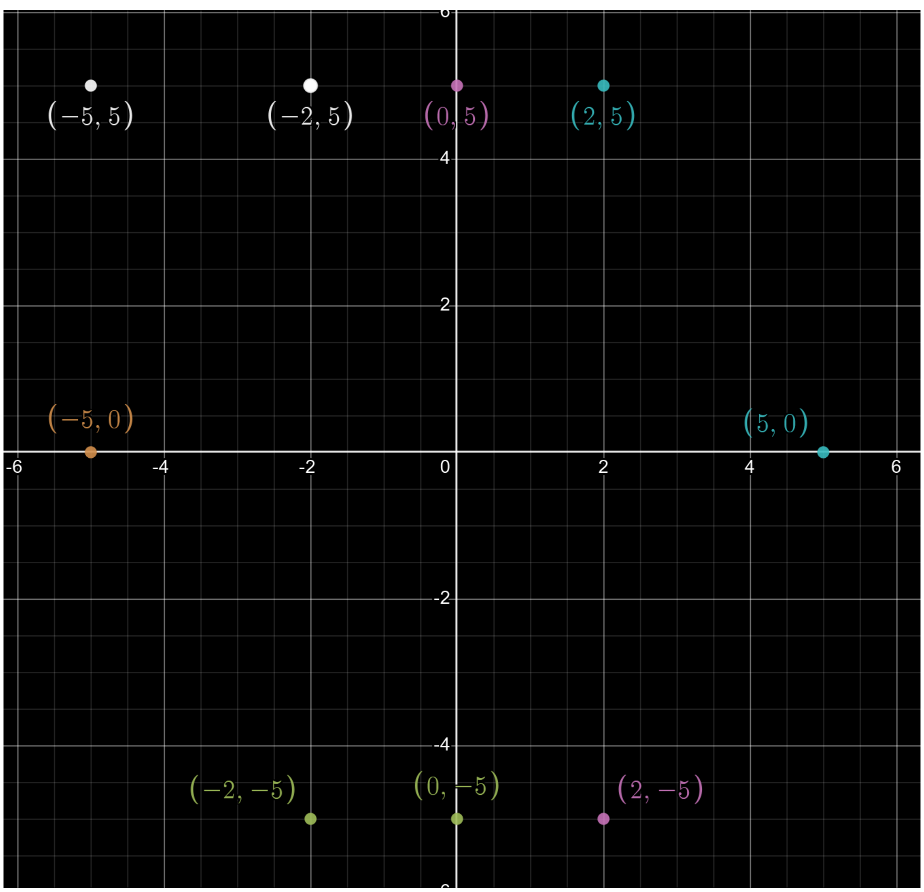
1. uzdevums

Attēlojiet plaknē un atrodiet no attēla aptuvenus trigonometriskos pierakstus šādiem skaitļiem: 5, -5, 5i, -5i, -5+5i, 2+5i, 2-5i, -2+5i, -2-5i.

$$a + bi = r(\cos\varphi + i\sin\varphi)$$



$$5 = 5(\cos 0^\circ + i \sin 0^\circ)$$
$$-5 = 5(\cos 180^\circ + i \sin 180^\circ)$$
$$5i = 5(\cos 90^\circ + i \sin 90^\circ)$$
$$-5i = 5(\cos 270^\circ + i \sin 270^\circ)$$
$$-5 + 5i = 5\sqrt{2}(\cos 135^\circ + i \sin 135^\circ)$$
$$2 + 5i \approx 5.4(\cos 70^\circ + i \sin 70^\circ)$$
$$2 - 5i \approx 5.4(\cos 290^\circ + i \sin 290^\circ)$$
$$-2 + 5i \approx 5.4(\cos 110^\circ + i \sin 110^\circ)$$
$$-2 - 5i \approx 5.4(\cos 250^\circ + i \sin 250^\circ)$$

2. uzdevums

Aprēķiniet izteiksmes (7+4i)/(2+3i) līdz simtdaļām precīzu vērtību algebriskajā pierakstā divos veidos: a) uzreiz, dalot skaitītāja un saucēja algebriskos pierakstus, b) vispirms iegūstot trigonometriskos pierakstus.

$$\frac{7+4i}{2+3i} = \frac{(7+4i)(2-3i)}{(2+3i)(2-3i)} = \frac{14-21i+8i+12}{4-6i+6i+9} = \frac{26-13i}{13} = 2.00 - i$$

$$r_1 = \sqrt{49+16} = \sqrt{65} = \sqrt{13} \cdot \sqrt{5}$$
$$\varphi_1 = \arctan(4/7)$$
$$r_2 = \sqrt{4+9} = \sqrt{13}$$
$$\varphi_2 = \arctan(3/2)$$

$$\frac{\sqrt{13} \cdot \sqrt{5}(\cos \varphi_1 + i \sin \varphi_1)}{\sqrt{13}(\cos \varphi_2 + i \sin \varphi_2)} = \sqrt{5}(\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2))$$

$$\sqrt{5}\cos(\arctan(4/7) - \arctan(3/2)) = 2$$
$$\sqrt{5}\sin(\arctan(4/7) - \arctan(3/2))i = -i$$

3. uzdevums

$z = -4+7i$. Aprēķiniet z^3 (kubā) un z^{-2} (pakāpē mīnus 2). Vispirms kāpināmo skaitli pārveidojiet trigonometriskajā pierakstā, tad izpildiet kāpināšanas, rezultātus attēlojiet plaknē un no attēla atrodiet aptuvenus algebriskos pierakstus.

$$a + bi = r(\cos\varphi + i\sin\varphi)$$

$$r_z = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$$

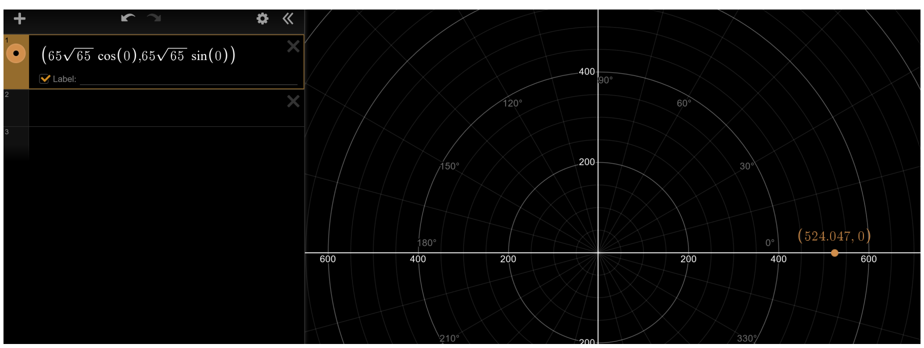
$$\varphi_z = 180^\circ - \arctan(\frac{7}{4}) \approx 120^\circ$$

$$z \approx \sqrt{65}(\cos 120^\circ + i\sin 120^\circ)$$

Pēc Muavra likuma $(r(\cos\varphi + i\sin\varphi))^n = r^n(\cos n\varphi + i\sin n\varphi)$.

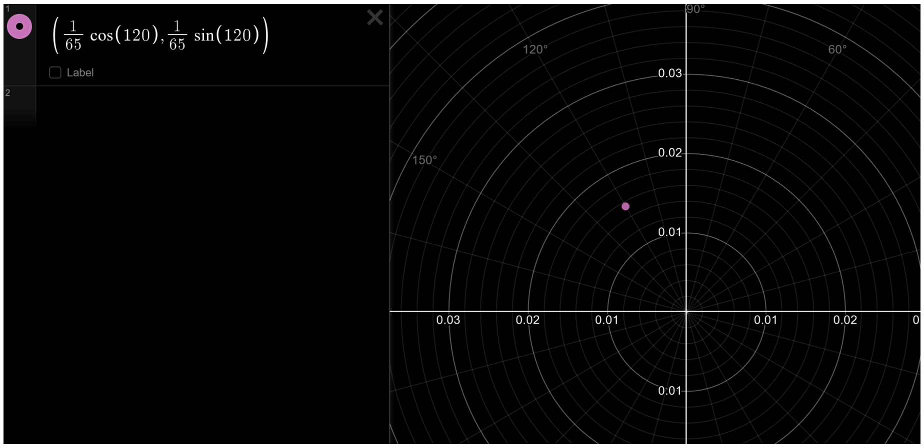
$$(\sqrt{65}(\cos 120^\circ{+}i\sin 120^\circ))^3 = \sqrt{65}^3(\cos 3{\cdot}120^\circ{+}i\sin 3{\cdot}120^\circ) = 65\sqrt{65}(\cos 0^\circ{+}i\sin 0^\circ)$$

$$65\sqrt{65}(\cos 0^\circ + i\sin 0^\circ) = 65\sqrt{65}$$



$$(\sqrt{65}(\cos 120^\circ{+}i\sin 120^\circ))^{-}2 = \frac{1}{\sqrt{65}^2}(\cos -2{\cdot}120^\circ{+}i\sin -2{\cdot}120^\circ) = \frac{1}{65}(\cos 120^\circ{+}i\sin 120^\circ)$$

$$\frac{1}{65}(\cos 120^\circ + i\sin 120^\circ) = \frac{1}{65}(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) = -\frac{1}{130} - \frac{\sqrt{3}}{130}i$$


$\left(\frac{1}{65}\cos(120), \frac{1}{65}\sin(120)\right)$
Label
90°
120°
60°
150°
0.03
0.02
0.01
0.03
0.02
0.01
0.01
0.02
0.01