

4. Determinanti. Matricu reizināšana

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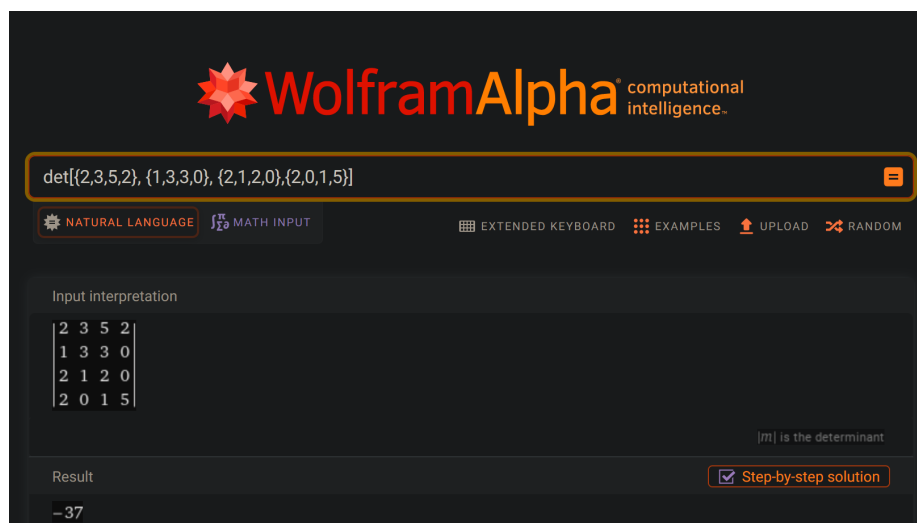
Lekcijas konspekts

Matricu īpašības. Kad drīkts reizināt. Efektīva determinanta aprēķināšana.

1. Uzdevums

Izmantojot Gausa metodi (ne savādāk), aprēķiniet determinantu $\text{Det} [\{2,3,5,2\}, \{1,3,3,0\}, \{2,1,2,0\}, \{2,0,1,5\}]$. Mēģiniet pārkārtot rindas un/vai kolonnas, lai ietaupītu darbības. Rezultātu pārbaudiet ar WolframAlpha.

$$\begin{aligned} \det \begin{pmatrix} 2 & 3 & 5 & 2 \\ 1 & 3 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 0 & 1 & 5 \end{pmatrix} &= \begin{vmatrix} 2 & 3 & 5 & 2 \\ 1 & 3 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 0 & 1 & 5 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 3 & 0 \\ 2 & 3 & 5 & 2 \\ 2 & 1 & 2 & 0 \\ 2 & 0 & 1 & 5 \end{vmatrix} = \\ &= - \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & -3 & -1 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -6 & 5 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & -3 & -1 & 2 \\ 0 & -6 & 5 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 2 & -13 \\ 0 & 0 & 1 & -25 \end{vmatrix} = \\ &= - \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & -25 \\ 0 & 0 & 2 & -13 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & -25 \\ 0 & 0 & 0 & 37 \end{vmatrix} = -(1 \cdot 1 \cdot 1 \cdot 37) = -37 \end{aligned}$$



The screenshot shows the WolframAlpha interface. At the top is the WolframAlpha logo with the tagline "computational intelligence". Below the logo is a search bar containing the input: `det[{2,3,5,2}, {1,3,3,0}, {2,1,2,0}, {2,0,1,5}]`. Below the search bar are several tabs: "NATURAL LANGUAGE" (selected), "MATH INPUT", "EXTENDED KEYBOARD", "EXAMPLES", "UPLOAD", and "RANDOM". Below the tabs is a section titled "Input interpretation" which displays the input as a 4x4 matrix: $\begin{vmatrix} 2 & 3 & 5 & 2 \\ 1 & 3 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 0 & 1 & 5 \end{vmatrix}$. Below the matrix is a note: "|m| is the determinant". At the bottom is a "Result" section showing the answer: `-37`. To the right of the result is a checkbox labeled "Step-by-step solution" which is checked.

2. Uzdevums

- a) Aprēķiniet 5-dimensiju vektoru $\{2,-3,5,6,-1\}$, $\{-1,2,2,-3,1\}$ skalāro reizinājumu.
b) Sareiziniet matricu $[\{1,-2,3\},\{4,-5,6\},\{0,8,7\}]$ ar vertikālo vektoru $\{-1,3,2\}$.

$$\begin{pmatrix} 2 \\ -3 \\ 5 \\ 6 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot (-1) \\ -3 \cdot 2 \\ 5 \cdot 2 \\ 6 \cdot (-3) \\ -1 \cdot 1 \end{pmatrix} = (-2 + -6 + 10 + -18 + -1) = (-17)$$

$$\begin{pmatrix} 1 & -2 & 3 \\ 4 & -5 & 6 \\ 0 & 8 & 7 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \cdot (-1) + (-2) \cdot 3 + 3 \cdot 2 \\ 4 \cdot (-1) + (-5) \cdot 3 + 6 \cdot 2 \\ 0 \cdot (-1) + 8 \cdot 3 + 7 \cdot 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 9 \\ 36 \end{pmatrix}$$

3. Uzdevums

Dotas matricas $A = [\{-1, 2\}, \{3, -5\}]$, $B = [\{1, 2\}, \{3, 4\}]$, $C = [\{1, -2\}, \{-3, 3\}]$, . Aprēķiniet reizinājumus AB , BA , $(AB)C$, $A(BC)$. Pārliecinieties, ka $AB \neq BA$ un $A(BC) = (AB)C$. Ko no tā var secināt?

$$\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 \cdot 1 + 2 \cdot 3 & (-1) \cdot 2 + 2 \cdot 4 \\ 3 \cdot 1 + (-5) \cdot 3 & 3 \cdot 2 + (-5) \cdot 4 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ -12 & -14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} = \begin{pmatrix} 1 \cdot (-1) + 2 \cdot 3 & 1 \cdot 2 + 2 \cdot (-5) \\ 3 \cdot (-1) + 4 \cdot 3 & 3 \cdot 2 + 4 \cdot (-5) \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 9 & -14 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 6 \\ -12 & -14 \end{pmatrix} \neq \begin{pmatrix} 5 & -8 \\ 9 & -14 \end{pmatrix}$$

$$\begin{aligned} & \left(\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ -12 & -14 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix} = \\ & = \begin{pmatrix} 5 \cdot 1 + 6 \cdot (-3) & 5 \cdot (-2) + 6 \cdot 3 \\ (-12) \cdot 1 + (-14) \cdot (-3) & (-12) \cdot (-2) + (-14) \cdot 3 \end{pmatrix} = \begin{pmatrix} -13 & 8 \\ 30 & -18 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} & \begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} \cdot \left(\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix} \right) = \\ & = \begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix} \cdot \begin{pmatrix} -5 & 4 \\ -9 & 6 \end{pmatrix} = \begin{pmatrix} -13 & 8 \\ 30 & -18 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} -13 & 8 \\ 30 & -18 \end{pmatrix} = \begin{pmatrix} -13 & 8 \\ 30 & -18 \end{pmatrix}$$

Matricu reizināšana nav komutatīva, bet ir distributīva.