

## 2. Gausa metode. Determinanti.

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### **Lekcijas konspekts**

Tika stāstīta par determinantiem. Kā tos aprēķināt. To īpašības.

## 1. Uzdevums

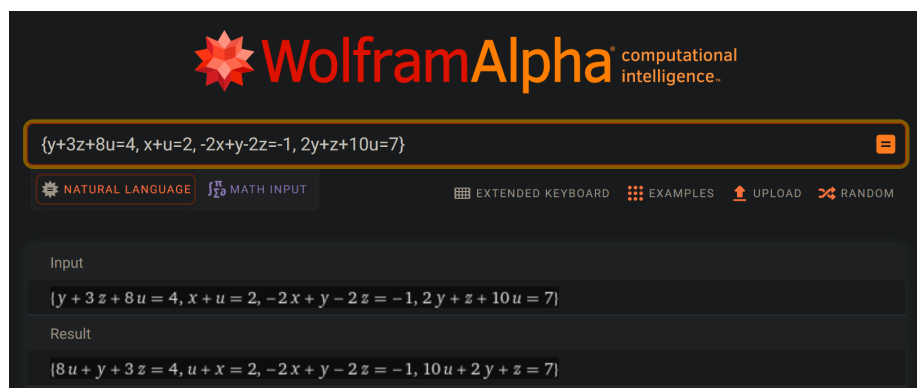
Parādot katru risinājuma soli, atrisiniet ar Gausa metodi šādu vienādojumu sistēmu:

$\{y + 3z + 8u = 4, x + u = 2, -2x + y - 2z = -1, 2y + z + 10u = 7\}$ . Rezultātu pārbaudiet ar WolframAlpha.

$$\begin{cases} y + 3z + 8u = 4 \\ x + u = 2 \\ -2x + y - 2z = -1 \\ 2y + z + 10u = 7 \end{cases}$$

$$\begin{pmatrix} 0 & 1 & 3 & 8 & | & 4 \\ 1 & 0 & 0 & 1 & | & 2 \\ -2 & 1 & -2 & 0 & | & -1 \\ 0 & 2 & 1 & 10 & | & 7 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 3 & 8 & | & 4 \\ -2 & 1 & -2 & 0 & | & -1 \\ 0 & 2 & 1 & 10 & | & 7 \end{pmatrix} \xrightarrow{r_3 += 2r_1} \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 3 & 8 & | & 4 \\ 0 & 1 & -2 & 2 & | & 1 \\ 0 & 2 & 1 & 10 & | & 7 \end{pmatrix} \xrightarrow{\substack{r_3 -= 3r_2 \\ r_4 -= (-2r_2)}} \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 3 & 8 & | & 4 \\ 0 & 0 & -5 & -6 & | & -1 \\ 0 & 0 & -5 & -6 & | & -1 \end{pmatrix} \xrightarrow{r_3 /= (-5)} \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 3 & 8 & | & 4 \\ 0 & 0 & 1 & 1.2 & | & 0.2 \\ 0 & 0 & -5 & -6 & | & -1 \end{pmatrix} \xrightarrow{\substack{r_2 -= 3r_3 \\ r_4 += 5r_3}} \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 \\ 0 & 1 & 0 & 4.4 & | & 3.4 \\ 0 & 0 & 1 & 1.2 & | & 0.2 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\begin{cases} x + u = 2 \\ y + 4.4u = 3.4 \\ z + 1.2u = 0.2 \end{cases}$$



The screenshot shows the WolframAlpha interface. At the top is the logo "WolframAlpha" with the tagline "computational intelligence". Below the logo is a search bar containing the input:  $\{y+3z+8u=4, x+u=2, -2x+y-2z=-1, 2y+z+10u=7\}$ . Below the search bar are several buttons: "NATURAL LANGUAGE", "MATH INPUT", "EXTENDED KEYBOARD", "EXAMPLES", "UPLOAD", and "RANDOM". Below these buttons is the "Input" section, which shows the same input as the search bar. Below the "Input" section is the "Result" section, which shows the output:  $\{8u+y+3z=4, u+x=2, -2x+y-2z=-1, 10u+2y+z=7\}$ .

## 2. Uzdevums

Aprēķiniet determinantus, parādot visus aprēķina soļus:

$[\{8, 5\}, \{12, 9\}]; [\{-2, 5\}, \{-3, -4\}];$

$[\{a, 2a\}, \{-2a, -3a\}]; [\{a + b, b\}, \{-b, a - b\}].$

$$\det \begin{pmatrix} 8 & 5 \\ 12 & 9 \end{pmatrix} = (8 \cdot 9) - (5 \cdot 12) = 72 - 60 = 12$$

$$\det \begin{pmatrix} -2 & 5 \\ -3 & -4 \end{pmatrix} = (-2 \cdot (-4)) - (5 \cdot (-3)) = 8 - (-15) = 23$$

$$\det \begin{pmatrix} a & 2a \\ -2a & -3a \end{pmatrix} = (a \cdot (-3a)) - (2a \cdot (-2a)) = -3a^2 - (-4a^2) = a^2$$

$$\det \begin{pmatrix} a+b & b \\ -b & a-b \end{pmatrix} = ((a+b) \cdot (a-b)) - (b \cdot (-b)) = (a^2 - b^2) - b^2 = a^2$$

### 3. Uzdevums

Atrisiniet vienādojumu sistēmu,  $\{13x - 16y = 4, -6x + 7y = 2\}$ , izmantojot Kramera formulas. Parādiet visus aprēķina soļus.

$$\begin{cases} 13x - 16y = 4 \\ -6x + 7y = 2 \end{cases}$$

$$d = \begin{vmatrix} 13 & -16 \\ -6 & 7 \end{vmatrix} = 91 - 96 = -5$$

$$d_1 = \begin{vmatrix} 4 & -16 \\ 2 & 7 \end{vmatrix} = 28 - (-32) = 60$$

$$d_2 = \begin{vmatrix} 13 & 4 \\ -6 & 2 \end{vmatrix} = 26 - (-24) = 50$$

$$x = \frac{d_1}{d} = \frac{60}{-5} = -12$$

$$y = \frac{d_2}{d} = \frac{50}{-5} = -10$$