2. Gausa metode. Determinanti.

Gunārs Ābeltiņš

Lekcijas konspekts

Tika stāstīta par determinantiem. Kā tos aprēķināt. To īpašības.

1. Uzdevums

Parādot katru risinājuma soli, atrisiniet ar Gausa metodi šādu vienādojumu sistēmu:
$\{y + 3z + 8u = 4, x + u = 2, -2x + y - 2z = -1, 2y + z + 10u = 7\}$. Rezultātu pārbaudiet ar WolframAlpha.

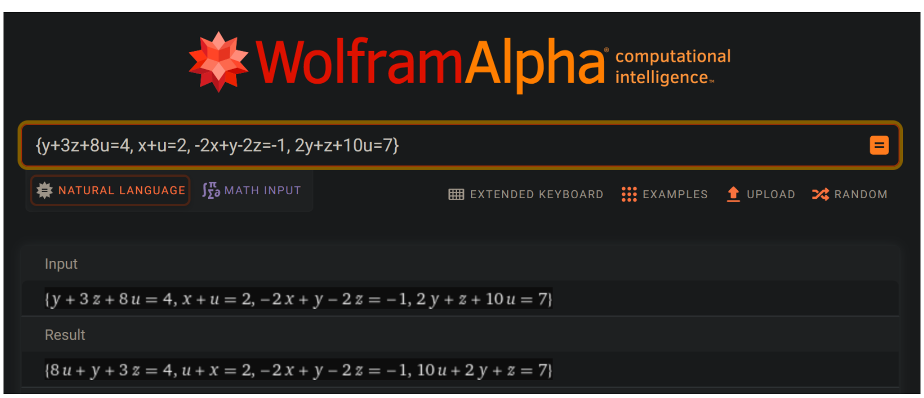
$$\begin{cases} y + 3z + 8u = 4 \\ x + u = 2 \\ -2x + y - 2z = (-1) \\ 2y + z + 10u = 7 \end{cases}$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 3 & 8 & 4 \\ 1 & 0 & 0 & 1 & 2 \\ -2 & 1 & -2 & 0 & -1 \\ 0 & 2 & 1 & 10 & 7 \end{array}\right) \xrightarrow{r_1 \leftrightarrow r_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 8 & 4 \\ -2 & 1 & -2 & 0 & -1 \\ 0 & 2 & 1 & 10 & 7 \end{array}\right) \xrightarrow{r_3 += 2r_1}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 8 & 4 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 2 & 1 & 10 & 7 \end{array}\right) \xrightarrow[r_4 -= (-2r_2)]{r_3 -= 3r_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 8 & 4 \\ 0 & 0 & -5 & -6 & -1 \\ 0 & 0 & -5 & -6 & -1 \end{array}\right) \xrightarrow{r_3 /= (-5)}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 8 & 4 \\ 0 & 0 & 1 & 1.2 & 0.2 \\ 0 & 0 & -5 & -6 & -1 \end{array}\right) \xrightarrow[r_4 += 5r_2]{r_2 -= 3r_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 4.4 & 3.4 \\ 0 & 0 & 1 & 1.2 & 0.2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x + u = 2 \\ y + 4.4u = 3.4 \\ z + 1.2u = 0.2 \end{cases}$$



2. Uzdevums

Aprēķiniet determinantus, parādot visus aprēķina soļus:
$[\{8, 5\}, \{12, 9\}]$; $[\{-2, 5\}, \{-3, -4\}]$;
$[\{a, 2a\}, \{-2a, -3a\}]$; $[\{a + b, b\}, \{-b, a - b\}]$.

$$\det \begin{pmatrix} 8 & 5 \\ 12 & 9 \end{pmatrix} = (8 \cdot 9) - (5 \cdot 12) = 72 - 60 = 12$$

$$\det \begin{pmatrix} -2 & 5 \\ -3 & -4 \end{pmatrix} = (-2 \cdot (-4)) - (5 \cdot (-3)) = 8 - (-15) = 23$$

$$\det \begin{pmatrix} a & 2a \\ -2a & -3a \end{pmatrix} = (a \cdot (-3a)) - (2a \cdot (-2a)) = -3a^2 - (-4a^2) = a^2$$

$$\det \begin{pmatrix} a + b & b \\ -b & a - b \end{pmatrix} = ((a + b) \cdot (a - b)) - (b \cdot (-b)) = (a^2 - b^2) - b^2 = a^2$$

3. Uzdevums

Atrisiniet vienādojumu sistēmu, $\{13x - 16y = 4, -6x + 7y = 2\}$, izmantojot Kramera formulas. Parādiet visus aprēķina soļus.

$$\begin{cases} 13x - 16y = 4 \\ -6x + 7y = 2 \end{cases}$$

$$d = \begin{vmatrix} 13 & -16 \\ -6 & 7 \end{vmatrix} = 91 - 96 = -5$$

$$d_1 = \begin{vmatrix} 4 & -16 \\ 2 & 7 \end{vmatrix} = 28 - (-32) = 60$$

$$d_2 = \begin{vmatrix} 13 & 4 \\ -6 & 2 \end{vmatrix} = 26 - (-24) = 50$$

$$x = \frac{d_1}{d} = \frac{60}{-5} = -12$$

$$y = \frac{d_2}{d} = \frac{50}{-5} = -10$$