

2. Gausa metode. Determinanti.

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Lekcijas konspekts

Tika stāstīts par determinanta īpašībām. Parādīja kā to aprēķināt. Kramera formulas.

1. Uzdevums

Aprēķiniet determinantu $\text{Det}[\{6, 5, 7\}, \{4, 3, 1\}, \{3, 2, 3\}]$, izmantojot lekcijā doto 3×3 determinanta definīciju (ne savādāk). Parādiet katru aprēķina soli. Rezultātu pārbaudiet ar WolframAlpha.

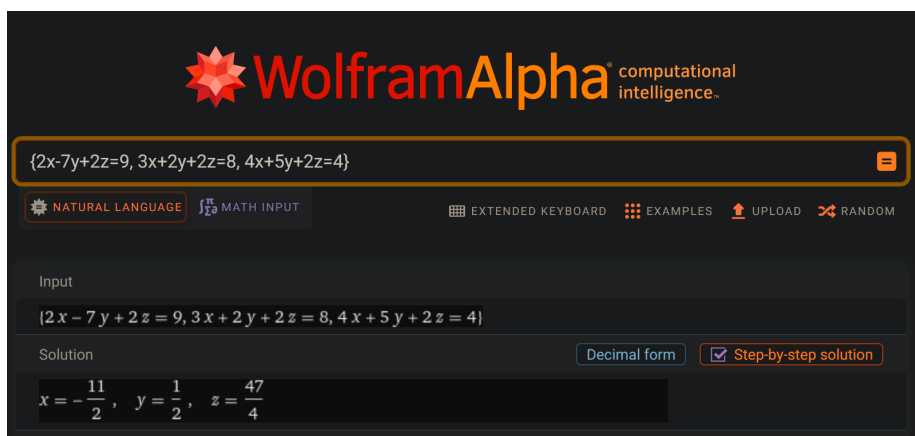
$$\begin{aligned}\det \begin{pmatrix} 6 & 5 & 7 \\ 4 & 3 & 1 \\ 3 & 2 & 3 \end{pmatrix} &= \begin{vmatrix} 6 & 5 & 7 \\ 4 & 3 & 1 \\ 3 & 2 & 3 \end{vmatrix} = 6 \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} - 5 \begin{vmatrix} 4 & 1 \\ 3 & 3 \end{vmatrix} + 7 \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = \\ &= 6 \det \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix} - 5 \det \begin{pmatrix} 4 & 1 \\ 3 & 3 \end{pmatrix} + 7 \det \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix} = \\ &= 6((3 \cdot 3) - (1 \cdot 2)) - 5((4 \cdot 3) - (1 \cdot 3)) + 7((4 \cdot 2) - (3 \cdot 3)) = \\ &= 6 \cdot 7 - 5 \cdot 9 + 7 \cdot (-1) = 42 - 45 - 7 = (-10)\end{aligned}$$

The screenshot shows the WolframAlpha web interface. At the top is the WolframAlpha logo with the tagline "computational intelligence". Below the logo is a search bar containing the input "Det[{6,5,7},{4,3,1},{3,2,3}]," with a small menu icon on the right. Below the search bar are two tabs: "NATURAL LANGUAGE" (selected) and "MATH INPUT". To the right of these tabs are links for "EXTENDED KEYBOARD", "EXAMPLES", "UPLOAD", and "RANDOM". Below the tabs is the "Input interpretation" section, which displays the matrix $\begin{vmatrix} 6 & 5 & 7 \\ 4 & 3 & 1 \\ 3 & 2 & 3 \end{vmatrix}$. Below the matrix is the text "|m| is the determinant". Below the input interpretation is the "Result" section, which shows the value "-10". To the right of the result is a checkbox labeled "Step-by-step solution" which is checked.

2. Uzdevums

Izmantojiet Kramera formulas, lai atrisinātu vienādojumu sistēmu $\{2x-7y+2z=9, 3x+2y+2z=8, 4x+5y+2z=4\}$. Vajadzīgos determinantus uzrakstiet, bet aprēķiniet ar WolframAlpha. Risinājuma pareizību pārbaudiet ar WolframAlpha.

$$\begin{cases} 2x - 7y + 2z = 9 \\ 3x + 2y + 2z = 8 \\ 4x + 5y + 2z = 4 \end{cases}$$
$$x = \frac{\det \begin{pmatrix} 9 & -7 & 2 \\ 8 & 2 & 2 \\ 4 & 5 & 2 \end{pmatrix}}{\det \begin{pmatrix} 2 & -7 & 2 \\ 3 & 2 & 2 \\ 4 & 5 & 2 \end{pmatrix}} = \frac{66}{-12} = -\frac{11}{2}$$
$$y = \frac{\det \begin{pmatrix} 2 & 9 & 2 \\ 3 & 8 & 2 \\ 4 & 4 & 2 \end{pmatrix}}{\det \begin{pmatrix} 2 & -7 & 2 \\ 3 & 2 & 2 \\ 4 & 5 & 2 \end{pmatrix}} = \frac{-6}{-12} = 0.5$$
$$z = \frac{\det \begin{pmatrix} 2 & -7 & 9 \\ 3 & 2 & 8 \\ 4 & 5 & 4 \end{pmatrix}}{\det \begin{pmatrix} 2 & -7 & 2 \\ 3 & 2 & 2 \\ 4 & 5 & 2 \end{pmatrix}} = \frac{-141}{-12} = \frac{47}{4}$$



The screenshot shows the WolframAlpha website interface. At the top is the WolframAlpha logo with the tagline "computational intelligence". Below the logo is a search bar containing the input "{2x-7y+2z=9, 3x+2y+2z=8, 4x+5y+2z=4}". Below the search bar are several tabs: "NATURAL LANGUAGE", "MATH INPUT", "EXTENDED KEYBOARD", "EXAMPLES", "UPLOAD", and "RANDOM". The "MATH INPUT" tab is selected. Below the tabs is the "Input" section, which shows the input "{2 x - 7 y + 2 z = 9, 3 x + 2 y + 2 z = 8, 4 x + 5 y + 2 z = 4}". Below the input section is the "Solution" section, which shows the solution "x = -11/2, y = 1/2, z = 47/4". There are two buttons in the "Solution" section: "Decimal form" and "Step-by-step solution". The "Step-by-step solution" button is selected.

3. Uzdevums

Izgudrojiet un uzrakstiet 3x3 determinantu piemērus, kas parādītu, kā Jūs saprotat determinantu īpašības 6, 7, 8. Nepieciešamos determinantus aprēķiniet ar WolframAlpha.

6. Īpašība

Ja matricā kāda rinda ir iegūta no citas rindas, reizinot ar kādu skaitli (t.i. divas rindas ir proporcionālas), tad matricas determinanta vērtība ir 0.

$$\begin{vmatrix} 1 & (2 \cdot 1) & 11 \\ 2 & (2 \cdot 2) & 13 \\ 3 & (2 \cdot 3) & 17 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 13 \\ 3 & 6 & 17 \end{vmatrix} = 0$$

7. Īpašība

Ja matricas A i-jā rindā katrs elements ir divu skaitļu summa $a_{ij} = b_j + c_j$, tad $\det(A) = \det(B) + \det(C)$, kur matricas B un C ir iegūtas no A, aizstājot i-jā rindā katru a_{ij} attiecīgi ar b_j vai c_j .

$$\begin{vmatrix} 1 & 6 & 11 \\ 2 & 4 & 13 \\ 3 & 2 & 17 \end{vmatrix} = 16$$

$$\begin{vmatrix} 1 & 6 & (4+7) \\ 2 & 4 & (10+3) \\ 3 & 2 & (7+10) \end{vmatrix} = \begin{vmatrix} 1 & 6 & 4 \\ 2 & 4 & 10 \\ 3 & 2 & 7 \end{vmatrix} + \begin{vmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \\ 3 & 2 & 10 \end{vmatrix} = 72 + (-88) = -16$$

8. Īpašība

Ja determinantā kādai rindai pieskaita vai atņem citu rindu, pareizinātu ar kādu skaitli, tad determinanta vērtība nemainās

$$\begin{vmatrix} 1 & 6 & 4 \\ 2 & 4 & 10 \\ 3 & 2 & 7 \end{vmatrix} = 72$$

$$\begin{vmatrix} 1 & 6 & 4 \\ 2 & 4 & 10 \\ 3 & 2 & 7 \end{vmatrix} = \begin{vmatrix} 1 & (6+1) & 4 \\ 2 & (4+2) & 10 \\ 3 & (2+3) & 7 \end{vmatrix} = \begin{vmatrix} 1 & 7 & 4 \\ 2 & 6 & 10 \\ 3 & 5 & 7 \end{vmatrix} = 72$$