

## 9. Kompleksie skaitļi - trigonometriskais pieraksts

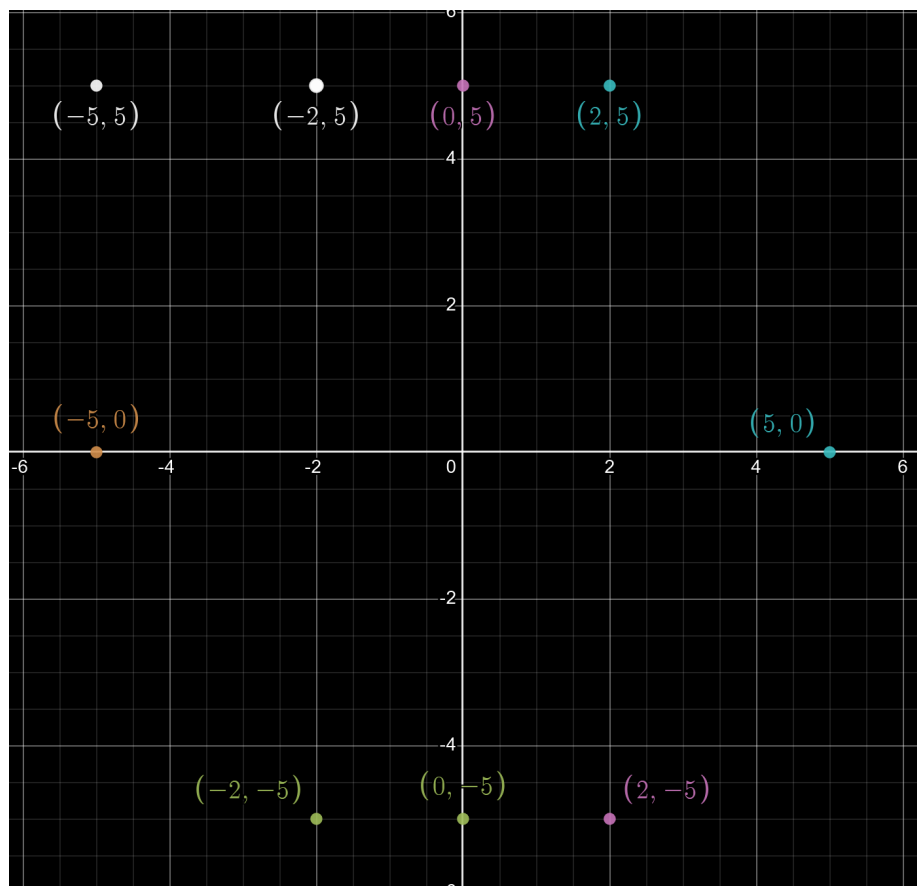
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### 1. uzdevums

Attēlojiet plaknē un atrodiet no attēla aptuvenus trigonometriskos pierakstus šādiem skaitļiem:  $5$ ,  $-5$ ,  $5i$ ,  $-5i$ ,  $-5+5i$ ,  $2+5i$ ,  $2-5i$ ,  $-2+5i$ ,  $-2-5i$ .

$$a + bi = r(\cos \varphi + i \sin \varphi)$$



$$\begin{aligned}
5 &= 5(\cos 0^\circ + i \sin 0^\circ) \\
-5 &= 5(\cos 180^\circ + i \sin 180^\circ) \\
5i &= 5(\cos 90^\circ + i \sin 90^\circ) \\
-5i &= 5(\cos 270^\circ + i \sin 270^\circ) \\
-5 + 5i &= 5\sqrt{2}(\cos 135^\circ + i \sin 135^\circ) \\
2 + 5i &\approx 5.4(\cos 70^\circ + i \sin 70^\circ) \\
2 - 5i &\approx 5.4(\cos 290^\circ + i \sin 290^\circ) \\
-2 + 5i &\approx 5.4(\cos 110^\circ + i \sin 110^\circ) \\
-2 - 5i &\approx 5.4(\cos 250^\circ + i \sin 250^\circ)
\end{aligned}$$

## 2. uzdevums

Aprēķiniet izteiksmes  $(7+4i)/(2+3i)$  līdz simtdaļām precīzu vērtību algebriskajā pierakstā divos veidos: a) uzreiz, dalot skaitītāja un saucēja algebriskos pierakstus, b) vispirms iegūstot trigonometriskos pierakstus.

$$\frac{7+4i}{2+3i} = \frac{(7+4i)(2-3i)}{(2+3i)(2-3i)} = \frac{14-21i+8i+12}{4-6i+6i+9} = \frac{26-13i}{13} = 2.00 - i$$

$$r_1 = \sqrt{49+16} = \sqrt{65} = \sqrt{13} \cdot \sqrt{5}$$

$$\varphi_1 = \arctan(4/7)$$

$$r_2 = \sqrt{4+9} = \sqrt{13}$$

$$\varphi_2 = \arctan(3/2)$$

$$\frac{\sqrt{13} \cdot \sqrt{5}(\cos \varphi_1 + i \sin \varphi_1)}{\sqrt{13}(\cos \varphi_2 + i \sin \varphi_2)} = \sqrt{5}(\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2))$$

$$\sqrt{5} \cos(\arctan(4/7) - \arctan(3/2)) = 2$$

$$\sqrt{5} \sin(\arctan(4/7) - \arctan(3/2))i = -i$$

### 3. uzdevums

$z = -4 + 7i$ . Aprēķiniet  $z^3$  (kubā) un  $z^{-2}$  (pakāpē mīnus 2). Vispirms kāpināmo skaitli pārveidojiet trigonometriskajā pierakstā, tad izpildiet kāpināšanas, rezultātus attēlojiet plaknē un no attēla atrodiat aptuvenus algebriskos pierakstus.

$$a + bi = r(\cos \varphi + i \sin \varphi)$$

$$r_z = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$$

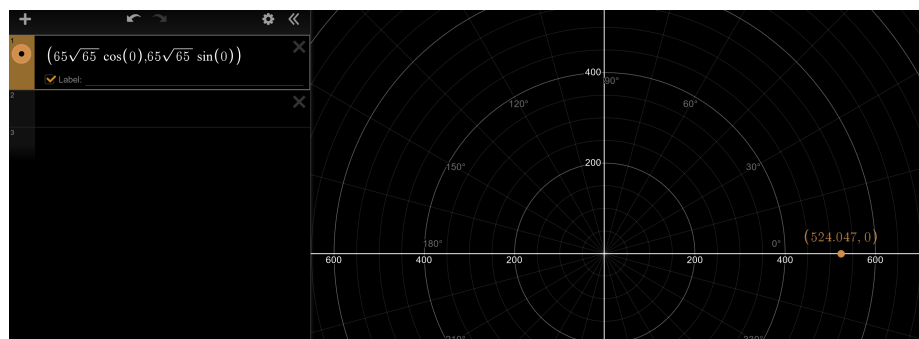
$$\varphi_z = 180^\circ - \arctan\left(-\frac{7}{4}\right) \approx 120^\circ$$

$$z \approx \sqrt{65}(\cos 120^\circ + i \sin 120^\circ)$$

Pēc Muavra likuma  $(r(\cos \varphi + i \sin \varphi))^n = r^n(\cos n\varphi + i \sin n\varphi)$ .

$$(\sqrt{65}(\cos 120^\circ + i \sin 120^\circ))^3 = \sqrt{65}^3 (\cos 3 \cdot 120^\circ + i \sin 3 \cdot 120^\circ) = 65\sqrt{65}(\cos 0^\circ + i \sin 0^\circ)$$

$$65\sqrt{65}(\cos 0^\circ + i \sin 0^\circ) = 65\sqrt{65}$$



$$(\sqrt{65}(\cos 120^\circ + i \sin 120^\circ))^{-2} = \frac{1}{\sqrt{65}^2}(\cos -2 \cdot 120^\circ + i \sin -2 \cdot 120^\circ) = \frac{1}{65}(\cos 120^\circ + i \sin 120^\circ)$$

$$\frac{1}{65}(\cos 120^\circ + i \sin 120^\circ) = \frac{1}{65}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -\frac{1}{130} - \frac{\sqrt{3}}{130}i$$

