ADA511: Data science and data-driven engineering

Steffen Mæland PierGianLuca Porta Mana

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Preface

**WARNING: THIS IS A WORKING DRAFT. TEXT WILL CHANGE A LOT. MANY PASSAGES ARE JUST TEMPORARY, INCOHERENT, AND DISJOINTED.

To be written.

1 Introduction

To be written: motivation and structure of this course.

2 Data: use and communication

2.1 Sentences – or, what is "data"?

What is "data"?

"Data" (from Latin "given") is used more or less in the same sense as "information", and in these notes we'll use the two words as synonyms.

"Data" is often presented as numbers; but it's obviously more than that. I give you this number: "8". Is it "data"? what is it about? what should you do with it? We can hardly call this number a piece of information, since we have no clue what we could do with it. Instead, if I tell you: "The number of official planets in the solar system is 8", then we can say that I've given you data. So "data" is not just numbers. A number is not "data" unless there's some verbal, non-numeric context associated with it – even if this context is only implicitly understood.

Data can also be completely non-numeric. A clinician saying "The patient has fully recovered from the disease" (we imagine to know who's the patient and what was the disease) is giving us a piece of information that we could further use, for instance, to make prognoses about other, similar patients. The clinician's statement surely is "data". It is essentially non-numeric data, even if in some situations we can represent it as "1", say, while "0" would represent "not recovered".

From these two examples, and with some further thought, we realize that "data" – and in general any piece of information or hypothesis – can universally be represented and communicated by sentences, also called statements or $propositions^1$. In some

¹ These terms are not equivalent in Logic, but sometimes we'll use them as synonyms.

cases we can summarize or represent such sentences as numbers. But the numbers alone, by themselves, are not data.

So our conclusion is that *information* or *data* is represented by *sentences*.

Recognizing that data and information are ultimately sentences has important practical consequences:

Clarity and goal-orientation. As a data engineer you'll have to acquire information and convey information. Acquiring information is not simply making some measurement or counting something: you must understand what you are measuring and why. If you gather data from third parties, you have to ask what exactly the data mean and how they were acquired. In designing and engineering a solution, you'll have to understand what information or outcomes the end user exactly wants. It will often happen that you ask "wait, what do you mean by that?"; this question is not just an unofficial parenthesis in the official data-transfer workflow between you and someone else: it is an integral part of that workflow, it means that the data has not been completely transferred yet.

Artificial Intelligence Sentences are the central components of knowledge representation and inference in artificial-intelligence agents.

2.2 Well-posed and ill-posed sentences

We face problems when the sentences that should convey information and data are not clear. Suppose that an electric-car model consumes 150 Wh/km and has a range of 200 km; a second car model consumes 250 Wh/km and has a range of 600 km. Someone says "I think the second model is better; what do you think?". It isn't clear how we should answer; what does "better" mean? If it refers to consumption, then the first car model is "better". If it refers to range, then the second model is "better". If it refers to a combination of these two characteristics, or to something else, then we simply can't answer. Here we have a

Reading material

§ 7.1 in Artificial Intelligence

problem with querying and giving data, because the sentence underlying such query is not clear.

We say that such sentences are **not well-posed**, or that they are **ill-posed**.

This may seem an obvious discussion to you. Yet you'd be surprised by how often unclear sentences appear in scientific papers about data engineering! Not seldom we find discussions and disagreements that actually come from unclear underlying sentences, that two parties interpret in different ways.

As a data engineer, you'll often have the upper hand if you are on the lookout for ill-posed sentences. Whenever you face an important question, or you're given an important piece of information, or you must provide an important piece of information, always take a little time to examine whether the question or information is actually well-posed.

• [TODO] Exercise: give actual paper to analyse

Reading list

3 Inference

3.1 What is inference?

The first core problem in all data-driven engineering applications – and in daily life too – is that we would like to know something we don't. In other words, we'd like to acquire information. We may wish to acquire information out of simple curiosity, or we may need it for some specific engineering reason or goal, as we'll discuss later. Examples:

- 1) We'd like to know whether it'll rain today, so we can decide whether to get an umbrella or rain clothes.
- 2) A clinician would like to know which disease affects a patient, so as to decide for the optimal treatment.
- 3) The X-player of this game of Xs & Os: needs to know where put the next **X** in order to win.
- 4) The computer of a self-driving car needs to know whether a particular patch of colours in the visual field is a person, so as to slow down the car and stop.
- 5) A rocket engineer needs to know, within two significant digits, how much is the velocity $\sqrt{2\,G\,M/r}$, where $G=6.67\cdot 10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}}$, and $M=5.97\cdot 10^{24}\,\mathrm{kg}$ and $r=6.37\cdot 10^6\,\mathrm{m}$ are the mass and radius of the Earth, in order to launch a rocket to the Moon.
- 6) We'd like to know whether the coin will come down heads, so we can win a bet.
- 7) An aircraft's autopilot system needs to predict how much the aircraft's roll will change by increasing the right wing's angle of attack by 0.1 rad.

- 8) An archaeologist would like to know whether the fossil bone just dug out belonged to a Tyrannosaurus rex.
- 9) An automated system in an assembly line needs to predict whether an electric component of a widget will fail within the next two years.

Note how each of these inferences boils down to wanting to know whether some sentences are true or false. In example 1., we want to know whether the sentence "It rains today" is true or not. In example 2., the clinician wants to know which of the sentences "The patient has pneumonia", "The patient has asthma", "The patient has bronchitis", and so on, are true (several can be true at the same time). In example 5., the rocket engineer wants to know which among the sentences "The velocity is 1.0 m/s", "The velocity is 1.1 m/s", "The velocity is 1.2 m/s", and so on, is true.

The example inferences above present very different levels of difficulty.

Example inferences 3. and 5. are special because they can actually be made exactly. In example 3., we trivially see that putting the next \mathbf{X} in the mid-right slot makes the X-player win. In example 5., a couple of mathematical operations give the approximate value $11 \, \mathrm{km/s}$ for the velocity. When we can obtain data we want from the data we have by using "only" logic and mathematical operations, we call the inference a deduction. We say "only" in quotation marks because the logical analysis and operations leading to the answer can still be computationally very expensive. But every deduction can be basically made by repeatedly applying the rules of logic.

The other example inferences cannot be made exactly, in the sense that we cannot get



Try to identify which sentences characterize the other example inferences

4 Truth inference and probability inference

- 4.1 Truth, falsity, and their consistency
- 4.2 Inferences without uncertainty: the truth calculus
- 4.3 Making room for uncertainty:Plausibility, credibility, degree of belief, probability
- 4.4 Inferences with uncertainty: the probability calculus
- 4.4.1 The Three Fundamental Laws of inference
 - $\bullet \ \ \textit{Exercise: Monty-Hall problem \& variations}$
 - Exercise: clinical test & diagnosis
- 4.4.2 Bayes's theorem
- 4.5 Common points of certain and uncertain inference

No premises? No conclusions!

5 Data and information

- 5.1 Kinds of data
- **5.1.1** Binary
- 5.1.2 Nominal
- 5.1.3 Ordinal
- 5.1.4 Continuous
 - unbounded
 - \bullet bounded
 - \bullet censored

5.1.5 Complex data

2D, 3D, images, graphs, etc.

5.1.6 "Soft" data

- orders of magnitude
- physical bounds

5.2 Data transformations

- \bullet log
- probit
- \bullet logit

6 Allocation of uncertainty among possible data values: probability distributions

6.1 The difference between Statistics and Probability Theory

Statistics is the study of collective properties of collections of data. It does not imply that there is any uncertainty.

Probability theory is the quantification and propagation of uncertainty. It does not imply that we have collections of data.

6.2 What's "distributed"?

Difference between distribution of probability and distribution of (a collection of) data.

6.3 Distributions of probability

6.3.1 Representations

- Density function
- Histogram
- Scatter plot

Behaviour of representations under transformations of data.

6.4 Summaries of distributions of probability

6.4.1 Location

Median, mean

6.4.2 Dispersion or range

Quantiles & quartiles, interquartile range, median absolute deviation, standard deviation, half-range

6.4.3 Resolution

Differential entropy

6.4.4 Behaviour of summaries under transformations of data and errors in data

6.5 Outliers and out-of-population data

(Warnings against tail-cutting and similar nonsensepractices)

6.6 Marginal and conditional distributions of probability

6.7 Collecting and sampling data

6.7.1 "Representative" samples

Size of minimal representative sample = $(2^\text{nerropy})/\text{precision}$

• Exercise: data with 14 binary variates, 10000 samples

6.7.2 Unavoidable sampling biases

In high dimensions, all datasets are outliers.

Data splits and cross-validation cannot correct sampling biases

6.8 Quirks and warnings about high-dimensional data

7 Making decisions

- 7.1 Decisions, possible situations, and consequences
- 7.2 Gains and losses: utilities
- 7.2.1 Factors that enter utility quantification

Utilities can rarely be assigned a priori.

7.3 Making decisions under uncertainty: maximization of expected utility

8 The most general inference problem