A New Approach to Estimating Equilibrium Models for Metropolitan Housing Markets

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We provide a new estimator for a broad class of equilibrium models of metropolitan housing markets with housing differentiated by quality. Quality is a latent variable that captures all features of a dwelling and its environment. We estimate the model for Chicago and New York, obtaining hedonic housing price functions for each quality level for each metropolitan area, stocks of each quality, and compensating variations required for a household of a given income in Chicago to be equally well off in New York.

I. Introduction

The objective of this paper is to develop a new flexible estimation strategy for a broad class of equilibrium models with nonlinear pricing of housing

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quality. We apply our new estimator to study housing markets in Chicago and New York, using data from the American Housing Survey (AHS). We show that the estimated models provide a compelling explanation of observed patterns in the metropolitan housing markets of both cities. We obtain plausible estimates for demand and supply elasticities. We also estimate compensating variations required for a household of a given income in Chicago to be equally well off in New York. We find that these compensating variations range between 25% for low- and moderate-income households and 15% for high-income households. These measures provide valuable new input for measuring differences in agglomeration externalities. As a consequence, this article provides new and important insights into metropolitan housing markets.

We develop a hedonic framework that permits comprehensive coverage of all dwellings in a metropolitan area, whether owner occupied or rental. We employ a unidimensional quality index that captures all features of a dwelling and its environment. We control for observed and unobserved differences in preferences over housing, which proves to be essential for generating a realistic demand structure. We assume that housing supply depends on the asset price of the unit and not the current rental price, because the asset price of a dwelling is the return to a builder for constructing a dwelling. We introduce a flexible parameterization of investors' expectations over future interest and tax rates as well housing price appreciation. This approach allows us to express the current housing value as the product of the current rental price and a stochastic user cost of capital.[1] Hence, supply depends on credit-market conditions and expectations over future rental prices. The equilibrium in our model determines price as a nonlinear function of quality for each time period in each of the metropolitan areas.

Our approach to identification and estimation significantly differs from the previous empirical literature. It builds on four important insights. First, we depart from standard practice in estimating hedonic or locational equilibrium models by treating both housing quality and housing prices as latent. Our approach treats quality as consisting of both housing services and neighborhood amenities. This approach does not require decomposing quality into an observed and an unobserved component. This decomposition typically requires the availability of suitable instruments that are orthogonal to unobserved characteristics. It is often quite challenging to

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[1] Our approach to modeling expectation over future housing values extends the work on user costs of capital by Poterba (1992) and Poterba and Sinai (2008) to models with heterogenous housing types.

find good instruments. As we demonstrate below, there are important applications where such a decomposition is not essential. Our approach also does not require that we estimate prices per unit of housing services.[2]

Second, we show that we can compute the equilibrium for this class of hedonic models using a convenient discretization of the quality space. This idea is similar to the approach taken by Epple and Sieg (1999) and others. Note that computing equilibria in these discretized versions of the model is easier than solving for the system of partial differential equations that characterize the nonlinear pricing function that arises in models with continuous quality differentiation. This allows us to impose all equilibrium restrictions in estimation.

Third, we follow the insights in Landvoigt, Piazzesi, and Schneider (2015) and normalize housing quality using the distribution of rents in a baseline period. This normalization is feasible because rents are a monotonically increasing function of the single index of quality in equilibrium. Housing quality does not have an intrinsic cardinality. Identification of the model requires the observation of equilibria in multiple markets. We can observe (1) equilibria for the same metropolitan area over time, (2) equilibria in a cross section of metropolitan areas, or (3) both types of equilibria. In the use of data for multiple markets, our identification strategy is related to the early literature on identification of hedonic models, but, in contrast to that literature, we invoke all equilibrium implications of our model in estimating the model.

Finally, we provide a new nonparametric method for estimating rent-to-value functions that is consistent with the requirement that these functions depend on housing quality.[3] This allows us to incorporate owner-occupied housing into the empirical analysis without losing the internal consistency of the modeling approach. It also permits us to include imputed rents from owners' equity in the measurement of income. Finally, it allows us to estimate a model in which the supply of housing depends on housing values and not rents.

These four insights equip us to identify and estimate the structural parameters of a broad class of flexible parameterizations of our equilibrium model. The parameters include those of the utility function, the housing supply function, and the distribution of observed and unobserved household types. We develop a two-step, sequential estimation strategy. First, we estimate the rent-to-value function and impute rents for owner-occupied housing. Second, we estimate the remaining structural parameters of the

[2] See, e.g., Sieg et al. (2002, 2004) for a discussion of the problems that are encountered in such analysis.

[3] An alternative strategy to identify and estimate the rent-to-value function or user-cost locus is to use observations on units that were both rented and sold within a short period, as developed in Bracke (2015). Halket, Nesheim, and Oswald (2017) estimate user costs when tenure choice is endogenous.

model by matching the correlations between rent and income and the quantiles of the observed marginal distribution of rents conditional on observed household characteristics in each metropolitan housing market. We impose the market-clearing conditions for each quality and each time period that must hold in equilibrium.

To implement our approach we require data for a representative sample of housing units in multiple markets. To the best of our knowledge, the AHS is the only data source that meets these requirements. The AHS draws a new sample each time it surveys a market. Hence, the AHS does not provide repeat observations on either dwellings or occupants of dwellings. Our model permits us to accommodate these features of the AHS, enabling us to exploit the comprehensive multiperiod coverage of metropolitan housing markets afforded by the AHS.

We then provide a compelling application that illustrates the power of our new framework. We estimate a number of models, using data from Chicago and New York. We find that a model with five unobserved types provides a sufficiently flexible characterization of demand to capture all relevant features observed in the data while also providing quite plausible estimates of price and income elasticities for demand. There is much heterogeneity in housing demand. Our estimated supply elasticities are reasonable and consistent with the observed changes of the housing stock.

Estimation of our model does not require any assumptions regarding the extent to which households are mobile among metropolitan areas. However, with the further assumption that households are fully mobile across metropolitan areas, we can use our estimated model to calculate the compensating variation required for a household of a given income in a given metropolitan area to be equally well off in another metropolitan area. This calculation exploits the fact that housing quality incorporates not only structural housing characteristics but also all amenities and disamenities that affect the desirability of a dwelling. Thus, in addition to structural characteristics, quality incorporates the presence of a subway stop near a dwelling, an art museum in the metropolitan area in which the dwelling is located, balminess or harshness of climate, and so forth. We also compare compensating variation as a function of income across household types. We find, for example, that, for a household at the 50th income percentile in Chicago, a compensating variation of approximately 20% of income is required to make that household equally well off in New York. These measures are of interest in their own right and also provide valuable input for measuring agglomeration economies.

The rest of the paper is organized as follows. Section II provides a review of the literature and discusses how our paper differs from important previous papers. Section III presents our model. Section IV discusses identification and estimation. Section V discusses the data used in the analysis. Sections VI and VII provide the empirical results for the housing markets in Chicago and New York and provide a careful cross-metropolitan comparison of the two housing markets. Section VIII offers conclusions and discusses future research.

II. Literature Review

The pioneering work of Rosen (1974) transformed modeling of markets for differentiated products and inspired an extensive literature focused on applications and associated issues of identification and estimation. A great many fruitful applications have built on Rosen's framework, including extensive research applying Rosen's framework to the study of housing markets.[4]

Recent research on hedonic identification is particularly relevant to our work. Ekeland, Heckman, and Nesheim (2004) describe limitations of prior work that uses linear systems of equations to study identification. They demonstrate the payoff from fully exploiting all equilibrium implications of the hedonic framework. Investigating additive-utility models, they establish that nonparametric identification of the Rosen model is possible using data for a single market. Heckman, Matzkin, and Nesheim (2010) extend the analysis of nonparametric identification to nonadditive models utilizing a unidimensional quality scale with multidimensional household types. As advocated in these papers, we exploit the full set of hedonic equilibrium conditions in our model of metropolitan equilibrium. Also, as in Heckman, Matzkin, and Nesheim (2010), we use a unidimensional index of housing quality that encompasses all observed and unobserved housing characteristics.[5]

Bajari and Benkard (2005) develop identification results and counterpart estimation methods for hedonic models, focusing in particular on developing methods that incorporate product characteristics observed by the consumer but not the econometrician. They develop a semiparametric approach to estimating demand, exploiting the set of optimality conditions implied by consumer choice of product characteristics. While we employ a single index to achieve tractability for modeling supply and demand at the metropolitan level, we are mindful of the importance of unobserved characteristics demonstrated by the work of Bajari and Benkard (2005). Our latent-quality approach captures both observed and unobserved characteristics.

Our work is related to recent work by Bajari et al. (2012), who use repeat sales in a rational expectations framework to estimate the marginal

[4] An illuminating review of this literature is provided in Kuminoff, Smith, and Timmins (2013).

[5] See, also, Chernozhukov et al. (2017) for an extension to multiple dimensions of unobserved heterogeneity.

prices of changes in the housing bundle. As demonstrated in their application, this proves to be especially well suited to estimating the implicit prices of changes in environmental quality.

There have also been important recent advances in the study of the dynamics of housing markets.[6] We build on Landvoigt, Piazzesi, and Schneider (2015), who consider housing sales by owner-occupants while incorporating frictions in asset markets (e.g., collateral constraints, transaction costs, and idiosyncratic shocks to housing returns). An important feature of their framework, which we adopt, is treating housing as a differentiated product that varies along a single dimension. We follow them as well in defining housing quality using the value distribution in a baseline period. There are also significant differences between our work and theirs. As noted above, a key contribution of their work is study of frictions in the market for owner-occupied housing. We abstract from market frictions in order to undertake a unified treatment of both owner-occupied and rental dwellings in a metropolitan area. Our unified treatment of ownership and rental housing includes development of a new approach for estimating the mapping from value to implicit rentals for owner-occupied dwellings. In modeling households, we incorporate both observed and unobserved heterogeneity in order to obtain a rich demand model that can account for variation across observed household types in the correlation between income and housing expenditure. We model changes in housing supply, while Landvoigt, Piazzesi, and Schneider model matching in the market for the existing housing stock. A further difference is that we undertake estimation, while Landvoigt, Piazzesi, and Schneider use a method-of-moments approach to calibrate their model. The above differences in modeling reflect differing objectives of the their analysis relative to ours. Our framework permits us to pursue several objectives, including comparisons of distributions of housing quality across metropolitan areas, comparisons of prices as a function of quality across metropolitan areas, and calculation of the compensating variation that would make a household in one metropolitan area equally well off in another. The Landvoigt, Piazzesi, and Schneider framework enables them to provide a rich quantitative account of the factors, including market frictions, that drove the housing boom in San Diego while also permitting them to explain the pattern of limited price growth in the years following the boom.

Finally, our work is also related to the class of locational sorting models estimated in Epple and Sieg (1999). There are four important differences between our current framework and theirs. First, the model of Epple and Sieg was designed to investigate household sorting in a system of local jurisdictions and the provision of local expenditures on public goods, via

[6] See also the discussion by Bayer et al. (2016), who estimate a dynamic model of household location choice and housing preferences.

majority rule within each community, including estimation of household preferences and the variation in housing prices across jurisdictions. The objectives of the current model are to estimate the equilibrium distribution of housing qualities at each point in time within each metropolitan area, to estimate the associated variation of price with quality, to compare the distributions of prices and quality over time both within and across metropolitan areas, and to calculate the compensating variation required for households of a given income and type to be equally well off in different metropolitan areas.

Second, Epple and Sieg (1999) explicitly differentiated between housing services and local neighborhood characteristics, with local neighborhood characteristics treated as observed. The Epple and Sieg approach is well suited to comparisons across jurisdictions within a metropolitan area. Here we bundle neighborhood and housing characteristics together in one variable, h, which we treat as latent. This approach is well suited to comparisons across metropolitan areas. It permits a comprehensive measure of the services provided by a dwelling and its environment while sidestepping the potentially intractable task of cataloguing, measuring, and comparing the factors that make a location in one metropolitan equivalent to a location in another metropolitan area.

Third, most previous hedonic or locational choice models treat the housing stock as fixed over time or, as in Epple and Sieg (1999), assume that aggregate housing supply in a community is a function of the current housing rental price prevailing in the community.[7] Our current approach seeks to capture the durability and heterogeneity of housing while also capturing dynamics of adjustment of housing stock. Finally, we use discrete types to capture preference heterogeneity in this model, while Epple and Sieg used continuous distributions to capture heterogeneity in preferences for neighborhood amenities and local public goods.

III. Housing Markets in Metropolitan Areas

We model housing as a differentiated product. Housing units differ by quality, which can be characterized by a one-dimensional ordinal measure denoted by h. We follow the hedonic literature allowing for nonlinear pricing in markets for housing services, $v_t(h)$. All households behave as renters; that is, households who are owner-occupants make decisions about housing consumption using an implicit rental rate that equals the amount the dwelling would command on the rental market.

To simplify exposition, we consider a metropolitan area at multiple points in time, but the model applies equally well to multiple metropolitan areas

[7] See, e.g., Rosen (1974) or Epple and Romer (1991), who assume that supply changes with current rental prices, or Nechyba (1997) and Bayer, Ferreira, and McMillan (2007), who use models with a fixed housing stock.

or multiple points in time for multiple metropolitan areas. There is a continuum of households with mass equal to one.[8] There are I types of households in the economy. Following Heckman and Singer (1984), we treat these types as unobserved by the econometrician in estimation. Households differ by an observed vector of characteristics x as well as by income y. The population share of households with characteristics x at time t is given by $s_t(x)$. We assume that x affects the probability of being each household type i. Hence, there exists a mapping $p_{it}(x)$ that maps observed characteristics x into types i; that is, the fraction of each type i with characteristics x in period t is given by $p_{it}(x)$.

We interpret income as a broadly defined measure that includes not just labor income but also income from asset holdings for wealthy households and transfer income for poor households.[9] Our approach thus implicitly accounts for differences in wealth. Let $F_{it}(y)$ be the metropolitan income distribution at time t of type i. Note that the income distributions of unobserved types are linked to the income distributions of observed types by the following identity:

$$F_{it}(y) = \frac{\sum_x s_t(x) p_{it}(x) F_t(y|x)}{\sum_x s_t(x) p_{it}(x)}, \tag{1}$$

where $F_t(y|x)$ is the (observed) distribution of income conditional on x.

Households have preferences defined over housing services h and a composite good z. Let $U_{it}(h, z)$ be the utility of a household of type i at time t. We invoke the exclusion restriction that, conditional on type i, preferences do not depend on x. Since housing quality is ordinal, housing quality is defined only up to a monotonic transformation. Given such a normalization, we can define a mapping $v_t(h)$ that denotes the period-t rental price of a housing unit that provides quality h. Here and subsequently, the rental price $v_t(h)$ denotes both the rental price of renter-occupied units of quality h and the implicit rental price of owner-occupied units of quality h. We assume that transaction costs are zero. Hence, whether owner-occupant or renter-occupant, a household can change its housing consumption on a period-to-period basis as rental rates change. It follows that, regardless of tenure choice, a household's optimal choice of housing maximizes its period utility at date t

[8] It is straightforward to allow for population growth in our model. In that case, population is given by N_t and $\{N_t\}_{t=1}^{\infty}$ is treated as an exogenous process. With a suitable rescaling of all key equations, the results go through as before.

[9] Broadly defined income measures are available in the AHS, which we use in the empirical application. We discuss how to account for owners' equity in detail below.

$$\max_{h_t, z_t} U_{it}(h_t, z_t),$$
$$\text{subject to} \quad y_t = v_t(h_t) + z_t, \tag{2}$$

where z_t denotes expenditures on a composite good.

The first-order condition for the optimal choice of housing consumption is

$$m_{it}(h_t, y_t - v_t) \equiv \frac{U_{it}^h(h_t, y_t - v_t)}{U_{it}^z(h_t, y_t - v_t)} = v_t'(h_t). \tag{3}$$

Solving this expression yields housing demand functions, $h_{it}(y_t, v_t(h))$. Integrating over the income distribution yields the aggregate housing demand of each type:

$$H_{it}^{\mathrm{d}}(h|v_t(h)) = \int_0^\infty 1\{h_{it}(y, v_t(h)) \leq h\}\, dF_{it}(y), \tag{4}$$

where $1\{\cdot\}$ denotes an indicator function. Total housing demand is given by

$$H_t^{\mathrm{d}}(h|v_t(h)) = \sum_x s_t(x) \sum_{i=1}^{I} p_{it}(x) H_{it}^{d}(h|v_t(h)). \tag{5}$$

Thus, $H_t^{\mathrm{d}}(h|v_t(h))$ is the fraction of households whose housing demand is less than or equal to h.

To characterize household sorting in equilibrium, we impose an additional restriction on household preferences.

Assumption 1. The utility function for each type of household i satisfies the following single-crossing condition:

$$\left.\frac{\partial m_{it}}{\partial y}\right|_{U_{it}(h, y - v(h)) = \bar{U}} > 0. \tag{6}$$

Assumption 1 states that, for each type, high-income households are willing to pay more for a higher-quality house than low-income households—a weak restriction on preferences. Single-crossing implies that, in equilibrium, the house rental expenditure at date t by income y of type i must satisfy

$$F_{it}(y) = G_{it}(v), \tag{7}$$

where $G_{it}(v)$ denotes the distribution of rents of type i. The single-crossing condition then implies the following result. If $F_{it}(y)$ is strictly monotonic, then there exists a monotonically increasing function $y_{it}(v)$, defined as

$$y_{it}(v) = F_{it}^{-1}(G_{it}(v)). \tag{8}$$

Note that $y_{it}(v)$ fully characterizes household i sorting in equilibrium. In equilibrium, rental expenditures are a deterministic function of income, conditional on each type. Hence, the model predicts that the correlation between income and rental expenditures is almost one, conditional on unobserved type. In the data, we observed that income and rental expenditures are positively, but far from perfectly, correlated. We use unobserved household types to explain this feature of the data. We show below that the model is sufficiently flexible to generate realistic correlation patterns, conditional on observed type.

To close the model, we need to specify the supply of housing units. Previous studies typically assume that housing supply depends on the current rental price $v_t(h)$. Our formulation reflects the fact that home builders produce and sell dwellings and hence are concerned about the market value of the dwelling and not the implicit rent. Moreover, our supply model allows for potential adjustment costs in the housing stock. In particular, we assume that the current supply of housing depends on the current and previous-period asset prices of housing, denoted by $V_t(h)$ and $V_{t-1}(h)$, respectively, and on the stock of housing in the previous period, $q_{t-1}(h)$.

ASSUMPTION 2. Let $q_t(h)$ denote the density of housing of quality h at date t. Taking the distribution of housing quality as given at time 0, the density of housing evolves over time according to the following law of motion:

$$q_t(h) = f(q_{t-1}(h), V_t(h), V_{t-1}(h)). \tag{9}$$

Including lagged values of quantity and price serves to capture potential adjustment costs. The distribution of housing supply in period t is

$$R_t(h|V_t(h), V_{t-1}(h), q_{t-1}(h)) = \int_0^h f(q_{t-1}(h), V_t(h), V_{t-1}(h))\, dx. \tag{10}$$

We thus obtain a recursive relationship governing the evolution of the supply of housing over time.

Finally, we endogenize the sequence of asset values $\{V_t(h)\}_{t=1}^{T}$ by explaining asset prices through the current endogenous prices of rental units and an exogenous sequence of the "user cost of capital." In particular, we assume that at each point of time, there is an asset market in which institutional investors can buy and sell houses at the beginning of each period. Let $V_t(h)$ denote the asset price of a house of quality h at time t. Let the one-period risk-adjusted interest rate be denoted by i_t. Investors are also responsible for paying property taxes to the city. The property tax rate is given by τ_t^{p}. Finally, owners have additional costs due to depreciation and maintenance that occur with rate δ_t.

ASSUMPTION 3. Investors discount housing assets at a rate that reflects the perceived financial market risk of housing assets. The market for housing assets is competitive.

The expected profit, Π_t, of buying a house with quality h at the beginning of period t and selling it at the beginning of the next period is then given by

$$E_t[\Pi_t(h)] = E_t\left[-V_t(h) + v_t(h) + \frac{V_{t+1}(h)(1 - \tau^{\mathrm{p}}_{t+1} - \delta_{t+1})}{1 + i_t}\right], \quad (11)$$

where the first term reflects the initial investment, the second term the flow of profits from rental income at time t, and the last term the discounted expected value of selling the asset in the next period.[10]

In equilibrium, expected profits for investors must be equal to zero. Hence, housing values or asset prices must satisfy the following no-arbitrage condition:

$$0 = E_t\left[-V_t(h) + v_t(h) + \frac{V_{t+1}(h)(1 - \tau^{\mathrm{p}}_{t+1} - \delta_{t+1})}{1 + i_t}\right]. \quad (12)$$

Solving for $V_t(h)$, we obtain the following recursive representation of the asset value at time t:

$$V_t(h) = v_t(h) + E_t\left[\frac{(1 - \tau^{\mathrm{p}}_{t+1} - \delta_{t+1})}{(1 + i_t)} V_{t+1}(h)\right]. \quad (13)$$

By successive forward substitution of the preceding, we obtain

$$V_t(h) = v_t(h) + E_t \sum_{j=1}^{\infty} \beta_{t+j} v_{t+j}(h), \quad (14)$$

where the stochastic discount factor is given by

$$\beta_{t+j} = \prod_{k=1}^{j} \frac{1 - \tau^{\mathrm{p}}_{t+k} - \delta_{t+k}}{1 + i_{t+k-1}}. \quad (15)$$

This demonstrates that the asset value of a house of quality h is the expected discounted flow of future rental income. The discount factors β_{t+j} depend on interest rates, property tax rates, and depreciation rates. An alternative instructive way of writing this expression is as follows. Let $1 + \pi_{t+k}(h) = v_{t+k}(h)/v_{t+k-1}(h)$ denote the rate of housing inflation at date t. Define $\tilde{\beta}_{t+j}$ as follows:

$$\tilde{\beta}_{t+j}(h) = \prod_{k=1}^{j} \frac{(1 - \tau^{\mathrm{p}}_{t+k} - \delta_{t+k})(1 + \pi_{t+k}(h))}{1 + i_{t+k-1}}. \quad (16)$$

Then,

[10] For analytical convenience, we are assuming that property taxes and maintenance expenditures are due at the beginning of the next period.

$$V_t(h) = \frac{v_t(h)}{c_t(h)}, \tag{17}$$

where $c_t(h)$ is the user-cost ratio for housing capital:

$$c_t(h) = \frac{1}{1 + E_t \sum_{j=1}^{\infty} \tilde{\beta}_{t+j}(h)}. \tag{18}$$

Thus, $c_t(h)$ is a summary statistic that aggregates the impact of future interest rate, depreciation, and tax payment as well as expectations over future rental income growth into a single number. Note that this idea is similar to the approach of Landvoigt, Piazzesi, and Schneider (2015), who also parameterize expectations over future housing prices as functions of current prices.

Given $c_t(h)$, $V_t(h)$ is a function of $v_t(h)$. Hence, we write the supply function as

$$R_t = R_t(h | v_t(h) c_t(h), v_{t-1}(h) c_{t-1}(h), q_{t-1}(h)). \tag{19}$$

In equilibrium, the hedonic rental price function, $v_t(h)$, must clear the market for occupancy at each value of h; that is, for each level of housing quality h,

$$H_t^{\mathrm{d}}(h | v_t(h)) = R_t(h | v_t(h) c_t(h), v_{t-1}(h) c_{t-1}(h), q_{t-1}(h)). \tag{20}$$

Assumption 4. The sequence of user costs $\{c_t(h)\}_{t=1}^{T}$ is exogenously determined.[11]

Define $p_t(x) = (p_{1t}(x), \ldots, p_{It}(x))$ and $F_t(y) = (F_{1t}(y), \ldots, F_{It}(y))$. We can define an equilibrium of our model as follows.

Definition 1 (Equilibrium). Given initial supply conditions $(q_0(h), v_0(h), c_0(h))$, an exogenous process of households $\{s_t(x), p_t(x), F_t(y)\}_{t=1}^{T}$, and an exogenous process of user costs $\{c_t(h)\}_{t=1}^{T}$, an equilibrium of this hedonic model consists of a sequence of endogenous rental price functions $\{v_t(h)\}_{t=1}^{T}$ and endogenous housing and consumption choices for each household type $(h_{it}(y, v_t(h)), z_{it}(y, v_t(h)))$ such that at each point of time t the following five conditions hold: (1) $(h_{it}(y, v_t(h)), z_{it}(y, v_t(h)))$ are the arguments that solve the household's maximization problem in equation (2); (2) aggregate demand for each value of h is given by equation (5); (3) aggregate supply for each value of h satisfies the recursive formulation in

[11] As detailed in Gyourko and Sinai (2003), the user cost of owner-occupied housing is affected by the deductibility of property taxes and mortgage interest from federal taxes. They show that the federal subsidy arising from deductibility varies greatly across metropolitan areas. The subsidy also varies across individuals of differing incomes.

equation (10); (4) asset prices satisfy equation (17); and (5) the market for each quality h clears as in equation (20).

We offer four observations. First, the demand for each level of quality depends on the demand for all household types. The pricing function simultaneously clears the market for all housing quality types. Hence, the model simultaneously matches all household types to housing units on the basis of a common rental price function. Second, given the initial supply conditions, we can compute the housing price function that clears the market in period 1 and determines the initial conditions for period 2. We then compute the housing price function in period 2 that clears the markets in period 2, and so on. Hence, the equilibrium price functions can be computed using a forward-iteration algorithm. Third, given the exogenous process of user costs, the current rental price function determines the current home values and therefore housing supply. Finally, equilibria can be computed only numerically. To accomplish that task, it is useful to discretize housing quality. Let us define a grid for the housing quality variable, $h_1, \ldots, h_J$. As the grid gets finer and finer, the discretized sorting model will more closely approximate the continuum of choices available in our model. Similarly, let r_{jt} be the fraction of units in each quality bin h_{jt} available at time t:

$$r_{jt} = R_t(h_j \mid V_t, V_{t-1}, r_{t-1}) - R_t(h_{j-1} \mid V_t, V_{t-1}, r_{t-1}). \tag{21}$$

Finally, we define the pricing function for the discrete model as follows:

$$v_{jt} = v\left(h_{t,j}\right). \tag{22}$$

Define the vector of prices $v_t = (v_{1t}, .., v_{Jt})$. By discretizing the quality space, we follow Epple and Sieg (1999) in treating households as making choices among discrete alternatives. Note that optimal household behavior in the discretized version of the model implies that there exist type-specific cutoff points $\hat{y}_{ijt}$ such that the demand by type i for quality h_j in period t as a function of rental prices is given by

$$H_{ijt}\left(v_{1t}, \ldots, v_{Jt}\right) = F_i\left(\hat{y}_{ijt}\right) - F_i\left(\hat{y}_{i,j-1,t}\right), \tag{23}$$

where $\hat{y}_{ijt}$ is defined as the level of income such that household type i is indifferent between consuming quality h_j and quality h_{j+1} at their corresponding prices in period t; that is, it is the level of income for type i that satisfies the following condition:

$$U_i\left(h_j, \hat{y}_{ijt} - v_{jt}\right) = U_i\left(h_{j+1}, \hat{y}_{ijt} - v_{j+1t}\right). \tag{24}$$

Equilibrium requires that demand equals supply for each housing type at each point of time. Hence, housing prices are the solution to the following system of nonlinear equations:

$$r_{jt}(v_t, c_t, v_{t-1}, c_{t-1}, r_{t-1}) = \sum_x s_t(x) \sum_i p_{it}(x) H_{ijt}(v_t) \quad \forall\, j, t. \tag{25}$$

IV. Identification and Estimation

A. A Parameterization of the Model

To identify and estimate the model, it is necessary to introduce flexible parameterizations of the key functions of interest.

ASSUMPTION 5. Let the utility provided by housing quality h for household type i at each period t be

$$U = u_i(h) + \frac{1}{\alpha_i} \ln(y_t - v_t(h)), \tag{26}$$

with $u(h) = \ln(1 - \phi_i(h + \eta_i)^{\gamma_i})$, where $\alpha_i > 0$, $\gamma_i < 0$, $\phi_i > 0$, and $\eta_i > 0$.

This utility function requires that the following two assumptions be satisfied: $1 - \phi_i(h + \eta_i)^{\gamma_i} > 0$ and $y_t - v_t > 0$. We impose these structural constraints in estimation. Note that we also assume that the parameters of the utility function are time invariant.

This utility function gives rise to flexible forms for the price and income elasticity equations. It also allows us to obtain a closed-form solution for the income cutoffs in equation (23):

$$\hat{y}_{ijt} = \frac{v_{jt} - e^{(M_{i,j+1} - M_j)\alpha_i} v_{j+1,t}}{1 - e^{(M_{i,j+1} - M_j)\alpha_i}}, \tag{27}$$

where $M_{i,j} = \ln(1 - \phi_i(h_j + \eta_i)^{\gamma_i})$.

Note that our approach to estimation is flexible and does not require this specific specification of utility. It can be applied to many other utility functions as well. We find that the specification we have fits our data well and also provides quite reasonable estimates of income and price elasticities.

With respect to the unobserved household types, we assume that x is discrete and hence that $p_{it}(x)$ can be treated as a discrete distribution. Hence, we do not need any functional form assumptions.[12]

Finally, we need to parameterize the law of motion for the housing supply.

ASSUMPTION 6. The law of motion for housing supply is given by

$$r_{jt} = A_t r_{jt-1} \left(\frac{V_{jt}}{V_{jt-1}} \right)^{\zeta} \quad \forall\, j, t, \tag{28}$$

where A_t is a scalar that guarantees that the fractions of housing types sum to one.

[12] It is straightforward to extend our approach treating x as a continuous random variable.

This function has some attractive properties. It is parsimonious; it introduces only one additional parameter, ζ. If the value of housing type j rises, then the quantity rises as a constant elasticity function of the proportion by which the value increases. If the value of housing type j falls, the quantity declines, reflecting depreciation and reduced incentive to invest in maintaining the housing stock. The magnitude of the response depends on the elasticity ζ.

B. *Rent-To-Value Functions*

In the data, we observe rents for rental units and values for owner-occupied units. We never observe rents and values for the same unit. As a consequence, we have to impose some assumptions to identify the rent-to-value functions.

Assumption 7. All households are indifferent between renting and owning.

Consider the simple case with just one discrete household type ($I = 1$). The model implies that households with income y consume the same housing quality, independently of whether they live in a rental unit, for which we observe $v_t(y)$, or live in an owner-occupied unit, for which we observe $V_t(y)$. For each housing quality, h, we can find the income, y, of the household renting a house of that quality, $y(h)$. This and the observed rental expenditure, $v_t(y)$, of a household of income y, we obtain $v_t(y(h))$. Indifference between owning and renting then implies that the corresponding housing value must satisfy $V_t(y) \equiv V_t(y(h))$. With these functions in hand, we have user-cost function $c_t(h) = v_t(y(h))/V_t(y(h))$. Alternatively, we think about this matching algorithm as identifying a function $V_t(v)$ that can be used to evaluate the housing supply equation. Recall that supply depends on values, not rents. But with this function in hand, we can express supply as a function of rents.

With multiple types, the expected rent for a household of type (x, y) is given by

$$v_t(x, y) = \sum_i p_{it}(x) v_t(h_{it}(y)), \tag{29}$$

and the expected housing value is

$$V_t(x, y) = \sum_i p_{it}(x) V_t(h_{it}(y)). \tag{30}$$

We can estimate the conditional expectations above using a nonparametric estimator. For each type x, fix quality at h and find the value of y such that $v_t(h) = v_t(y, x)$. Then define $V_t(h) = V_t(y, x)$. This matching algorithm defines a mapping, $v_t^x(V)$, for each observed household type x.

Equilibrium requires that these mappings be the same for all x. Hence, we define the rent-to-value function, $v_t(V)$, as the following projection:

$$\min_{v_t(V)} \sum_x s_t(x) \int \| v_t(V) - v_t^x(V) \| \ dV. \tag{31}$$

For the discretized version of the model, this equation can be written as

$$\min_{v_{1t},\ldots,v_{Jt}} \sum_x s_t(x) \sum_j (v_{jt} - v_{jt}^x)^2. \tag{32}$$

Summarizing the discussion above, we have the following result:

PROPOSITION 1. The rent-to-value function $v_t(V)$ and its inverse function $V_t(v)$ are nonparametrically identified.

Having identified the rent-to-value functions, we can convert values for owner-occupied houses into rents and hence construct the complete rental distribution for each market and each time period.

Finally, note that we can extend our approach to account for income that is due to owners' equity. Imputed rental income is typically not included in broadly defined income measures. Suppose that we have some measure of owners' housing equity. To generate a more comprehensive income measure, we can use an iterative procedure. We start with the income measure used above. We then estimate the rent-to-value function, using our baseline income measure that ignores imputed rental income for owners. Given the initial estimate of the rent-to-value function, we can impute the rental income for owners, using the observed housing equity in the data. We then use the comprehensive income measure to reestimate the rent-to-value function. We iterate this procedure until it converges. Hence, this procedure extends our approach to estimating rent-to-value functions to provide more comprehensive income measures that account for owners' equity.

C. Latent Housing Quality and Prices

Since housing quality is ordinal and latent, there is no intrinsic unit of measurement for housing quality. The implications of the latent-quality measure for identification are formalized by the following proposition.

PROPOSITION 2. For any model with equilibrium rental price function $v_1(h)$, there exists a monotonic transformation of h, denoted by h^*, such that the resulting equilibrium pricing function is linear in h^*; that is, $v_1(h^*) = h^*$.

We can use an arbitrary monotonic transformation of h and redefine the utility function and supply function accordingly. The proposition then implies that if we observe data in only one housing market and one time period, we cannot identify the utility function separately from the pricing function. A corollary of this proposition is that we can normalize housing quality by setting $h = v_1(h)$. Recall that Landvoigt, Piazzesi, and Schneider (2015) use the same approach for normalizing housing quality in their analysis.

Given our estimated user-cost function $c_1(h)$, we can compute the initial housing supply in period 1, denoted by $q_1(h)$, by imposing the market-clearing condition. This normalization then provides us with initial supply conditions $v_1(h)$, $c_1(h)$, and $q_1(h)$ that are consistent with the observed equilibrium outcomes in period 1. We then compute equilibria for periods $t = 2, 3, \ldots$, using the forward-iteration algorithm discussed above. In other words, the normalization of housing quality is just a useful way to generate initial conditions for our model that are consistent with observed outcomes.

Note that the equilibrium rental price functions are functions of the structural parameters of the model. For some simple models, we can actually compute a closed-form solution for the nonlinear pricing functions. In that case, we can provide an analytic proof of identification of the structural parameters of both the utility function and the supply function.[13] For more realistic models we have to rely on numerical algorithms to compute equilibrium housing prices. As a consequence, we do not have a constructive proof of identification for these models.

Nevertheless, key elements of the logic captured by the proof for the simpler models carry over to more general model specifications. In particular, the predicted demand for each period depends on the latent equilibrium prices, which in turn depend on the supply elasticity. There is then a natural exclusion restriction that helps with identification. Given our normalization of housing quality and prices in the first period, the sorting in the first period depends only on the preference parameters. Moreover, supply is observed in the first period in one metropolitan area. In contrast, sorting in all subsequent periods and all other metropolitan areas also depends on the parameters of the housing supply function. This exclusion restriction that follows from our normalization of quality in one market in one time period allows us to disentangle demand from supply parameters, which is the most fundamental problem encountered in the analysis.

Of course, another main source of identification of the supply function comes from exogenous changes in the user costs of capital that are captured by the rent-to-value functions that we observe. Changes in the user cost of capital drive a wedge between rents and values. Demand in our model depends on rents, while supply depends on values and, as a consequence, on the user cost of capital. A decrease in the user cost of capital

[13] Appendix A (apps. A–C are available online) contains a constructive proof of identification for a simplified version of our model that allows for a closed-form solution of the equilibrium pricing function.

then leads to an increase in values, holding rents constant. A decrease in the user cost, therefore, affects supply, but not demand. This additional exclusion restriction also helps us distinguish between demand- and supply-side parameters.

The main difference between the model with unobserved types and the one without them is the following. In the model with only observed types, housing demand is a deterministic function of income for each observed type. As a consequence, the model is not well suited to capturing the correlations of incomes and rents observed in the data. Adding unobserved types to the model specification generates more realistic correlation patterns between income and rent. In that sense, unobserved heterogeneity in housing plays the same role as the unobserved heterogeneity for public goods in Epple and Sieg (1999), which is needed to generate realistic household sorting patterns by income among neighborhoods and communities.

D. A Method-of-Moments Estimator

The intuition behind our estimator is then the following. We can estimate the rent-to-value functions and impute rents for owner-occupied housing for each time period that we have data for. We then estimate the remaining structural parameters of the model by matching the correlations between rent and income and the quantiles of the observed marginal distribution of rents conditional on observed household characteristics in each metropolitan housing market. We impose the market-clearing conditions for each quality and each time period that must hold in equilibrium. More formally, we estimate the structural parameters of the model, using the following estimation algorithm:

1. Estimate for each t the rent-to-value function, $v_t(V)$, and its inverse, using a nonparametric matching estimator.
2. Use the rent-to-value function to impute rents for owner-occupied units, obtain the market distribution for rents, and compute the joint distributions for income and rent conditional on x, denoted by $F_t^N(y, v|x)$.
3. Discretize the rent distribution in period 1 into J intervals indexed by j, and normalize housing quality to obtain h_j and r_{j1} for all j.
4. Choose a vector of structural parameters, denoted by θ, that includes the parameters of the type distribution. Compute the type-specific income distributions $F_{it}(y)$, using equation (1) and the observed distributions $F_t(y|x)$.
5. Solve for the implied equilibrium prices in all periods $t > 1$.
   a. Guess values of v_{jt}.

b. Calculate the implied income cutoffs $\hat{y}_{ijt}$ such that

$$U_i(h_j, \hat{y}_{ijt} - v_{t,j}) = U_i(h_{j+1}, \hat{y}_{i,j} - v_{t,j+1}) \quad \forall\ j, it. \tag{33}$$

Note that these cutoff points depend on $F_{it}(y|\theta)$.

c. Calculate the implied demands:

$$H^{\mathrm{d}}_{ijt}(v_t) = F_{it}(\hat{y}_{ijt}) - F_{it}(\hat{y}_{i,j-1,t}) \quad \forall\ j, it. \tag{34}$$

d. Calculate the supplies:

$$r_{jt}(v_t) = A_t r_{jt-1}\left(\frac{V_{jt}(v_{jt})}{V_{jt-1}(v_{jt-1})}\right)^{\zeta} \quad \forall\ j, t > 1. \tag{35}$$

e. Check whether equilibrium holds

$$\sum_x s_t(x)\sum_{i=1}^{I} p_{it}(x) H^{\mathrm{d}}_{ijt}(v_t) = r_{jt}(v_t) \quad \forall\ j, t > 1. \tag{36}$$

f. Repeat until the solution of equilibrium prices has been found for each time period.

6. For each unobserved type i, compute the predicted joint $F^{\theta}_{it}(y, v)$ as well as

$$F^{\theta}_t(y, v|x) = \sum_i p_{it}(x) F^{\theta}_{it}(y, v). \tag{37}$$

7. Form orthogonality conditions that are based on the difference between the observed joint distributions of income and rents, denoted by $F^N_t(y, v|x)$, and their predicted counterparts, $F^{\theta}_t(y, v|x)$. Specifically, we use quantiles of the marginal distributions and correlations between y and v conditional on x. Evaluate the objective function $Q^N_t(\theta)$.
8. Update θ until $Q^N_t(\theta)$ is minimized.

We use a standard bootstrap procedure to estimate the standard errors. We use clustering algorithms to reduce the dimensionality of x. By combining clustering strategies with unobserved heterogeneity along the lines suggested by Heckman and Singer (1984), we obtain a relatively parsimonious model that is sufficiently flexible to capture all relevant dimensions of sorting observed in the data.

V. Data

We use data from the AHS, the most comprehensive national housing survey in the United States. It is conducted in the field from May 30 through September 8. There is a national version and a metropolitan version, and, in selected years, also an extended representative metropolitan component for some metropolitan areas in the national version. There are surveys conducted every year, but the metropolitan areas covered in the metropolitan version change in each year. There is no fixed interval over which a given metropolitan area is resurveyed. The unit of observation in the survey is the housing unit, together with the household. The same housing unit is followed through time, but the sample of households may change.

Fortunately, the AHS conducted surveys in both Chicago and New York for both 1999 and 2003. We exploit data from Chicago for our first application to illustrate our new method. We then use both surveys and jointly estimate the model for those two metropolitan areas. One of the most advantageous features of our model is its capacity to separate quality from price by identifying the prices for different levels of the quality distribution for each market at each point in time.

Households also differ in income. We interpret income as a broadly defined measure that includes not just labor income. Income in the AHS is based on the respondent's reply to a number of detailed questions about different income categories for the 12 months before the interview. Income is the sum of the amounts reported for wage and salary income, net self-employment income, Social Security, public assistance or welfare payments, and all other money income, which includes income from assets. The figures represent the amount of income received before deductions for personal income and payroll taxes. Accuracy of income data has been studied for this survey. Although individual over- and underreporting has been found, the accuracy is similar to information included in other surveys, such as the Current Population Survey.

AHS definitions of each of these metropolitan areas are unchanged across these two periods.[14] We use the Chicago metropolitan area in 1999 as the base for our normalization. We use data from the extended metropolitan surveys conducted in the Chicago and New York metropolitan areas in 1999 and 2003.

For implementation of our model, we reduce the dimensionality of potential household types, using k-means clustering. This is a standard method in data mining initially used by MacQueen (1967) as a method to group observations into clusters based on some similarity measurement criterion. In a two-dimensional space, the method is intuitive. First, K points are selected randomly, and then each observation is assigned to its nearest point. Next,

[14] The Chicago metropolitan area is defined by the US Census Bureau in 1999 and 2003 to consist of the following counties: Cook, Du Page, Kane, Lake, McHenry, and Will. The New York metropolitan area consists of Bronx, Kings, Nassau, New York, Putnam, Orange, Queens, Richmond, Rockland, Westchester, and Suffolk counties.

TABLE 1
k-Means Clustering Centroids

Cluster	No. of Children	Age	$s_t(x)$
	A. Chicago		
1	.287	29.13	.260
2	1.390	45.00	.433
3	.299	71.00	.307
	B. Chicago and New York		
1	.345	21.50	.256
2	1.660	46.50	.475
3	.314	69.49	.268

Note.—The table provides some descriptive statistics for our clusters for the two samples used in the empirical analysis.

centroids are computed for the K groups, and observations are reassigned to the nearest centroid, which will cause some observations to move from their original clusters to a new cluster. The algorithm iterates until no more reallocation of observations into clusters happens.

It is well known that k-means clustering results are sensitive to the choice of initial points; that is, the local minimum determined by the iterative procedure is not necessarily a global minimum. The usual practice to address this problem is to perform the algorithm multiple times with different randomly selected starting points and select the solution with the smallest squared error (Bernhardt and Robinson 2007). We follow this procedure and use 10,000 repetitions of the algorithm. This iterative partitioning minimizes the sum, over all clusters, of the within-cluster sums of point-to-cluster-centroid distances.

The results presented here are for squared Euclidean distances and a multidimensional data matrix with variables that influence housing consumption.[15] This method allows us to reduce dimensionality on the basis of observable characteristics that are relevant to the preferences for housing without imposing ad hoc thresholds, using an algorithm that is efficient in large data sets and easy to implement. When we implement the cluster analysis for our two samples, we obtain three clusters, which is the optimal number of clusters calculated with our sample data employing the silhouette criterion.[16]

Panel A of table 1 shows estimated cluster shares and centroids for Chicago, pooling the data for 1999 and 2003. An intuitive interpretation of

[15] Bishop (2016) provides more details of this method. We used the Matlab package kmeans.

[16] The silhouette of a data point is a measure of the average distance to other points in its cluster less the average distance to points in the alternative closest cluster, normalized by the maximum of both distances (Kaufman and Rousseeuw 2009).

the three groups is the following: type 1 consists of young households with few or no children, type 2 of middle-aged households with more than one child, and type 3 of older households with no children residing in the household. To estimate the model jointly, we also conducted a joint cluster analysis, using data for both cities. The results are shown in panel B of table 1. The results for the two samples are relatively similar.

VI. Empirical Results

Here we report the empirical results when we estimate the model using data for Chicago in 1999 and 2003. We turn to the joint model of Chicago and New York in the next section. To incorporate owner-occupied housing into our analysis, we estimate the user-cost function and convert home values into imputed rents. We consider two specifications that differ by the income measure. The first model is based on the commonly used income measure reported by the AHS. The second model uses a comprehensive income measure that also accounts for owners' equity, adding imputed rental income to the first measure.[17] The results are illustrated in figure 1: *A* shows the results for 1999, and *B* focuses on 2003. We plot the two estimated user-cost functions and the difference between the two functions.

We find that the average user cost is approximately 0.06 in 1999 in Chicago in the baseline model. Between 1999 and 2003 the user costs moderately decreased for most units, indicating that home ownership became more expensive. Comparing the results across income measures, we find that income increases for the majority of owners as we account for owners' equity. Since imputed rents added to income are mostly positive, owners are matched with higher-income renters, which increases the estimated user costs. Hence, the estimated rent-to-value functions that are based on the second income measure are above the ones based on the first measure, which ignores owners' equity. Moreover, the user-cost functions are more convex once we account for owner's income, highlighting the fact that user costs are not necessarily constant across units.

We then estimate the remaining parameters of the equilibrium model. We estimated different versions with up to five unobserved types of households. As we explain in detail below, our preferred model specification has five unobserved types. Table 2 shows the estimated probabilities of each type and the estimated standard errors. The baseline model uses the conventional income measure. The second model accounts for owners' equity.

Table 3 shows the implied probabilities of each observed-unobserved type pair in the population as well as the marginal probability for each unobserved type. Our results imply that observed type 1 is primarily matched

[17] See app. B for more details and differences between the two income distributions.

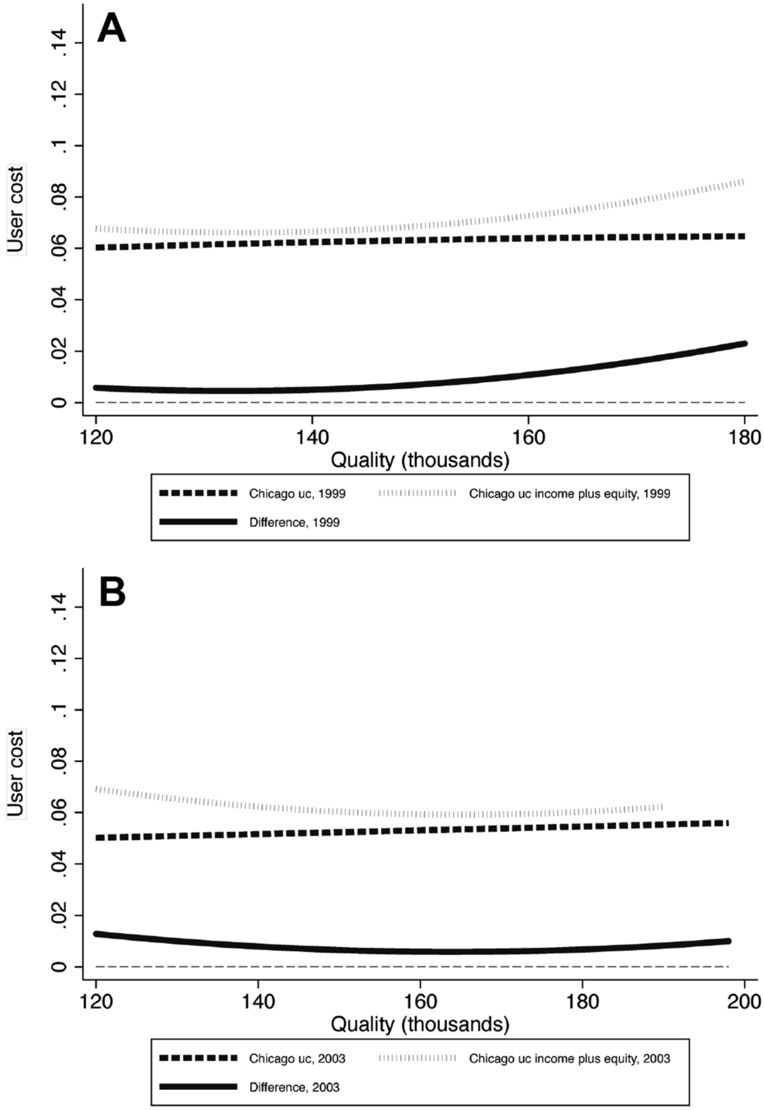

FIG. 1.—Estimated user costs (UC) for Chicago in 1999 (*A*) and 2003 (*B*).

into unobserved types 1 and 4, while observed type 2 is primarily matched into types 2 and 5. Observed type 3 is matched primarily into unobserved types 2 and 3. These results are stable across specifications.

The parameter estimates and estimated standard errors for the preference of the different types are reported in table 4 for both income measures. The bootstrapped standard errors account for the sequential nature of our estimator. We find that all parameter estimates have the correct

TABLE 2
Unobserved Type Probabilities $p_{it}(x)$

	Baseline Model			Model with Equity		
	x_1	x_2	x_3	x_1	x_2	x_3
i_1	.41	.08	.04	.45	.09	.02
	(.01)	(.05)	(.02)	(.02)	(.09)	(.02)
i_2	.07	.50	.17	.05	.49	.20
	(.02)	(.07)	(.02)	(.01)	(.04)	(.01)
i_3	.03	.03	.65	.04	.03	.60
	(.00)	(.05)	(.01)	(.02)	(.06)	(.01)
i_4	.40	.09	.10	.37	.04	.10
	(.01)	(.04)	(.04)	(.01)	(.06)	(.06)
i_5	.09	.30	.04	.09	.35	.08
	(.03)	(.05)	(.02)	(.03)	(.01)	(.02)

Note.—The table reports estimates and estimated standard errors (in parentheses) for the probabilities of the unobserved types.

algebraic signs and have reasonable magnitudes. We find that the unobserved types have different preference parameters and hence different housing demands. Since each observed type matches into more than one unobserved type, our model can generate realistic correlations between income and housing demand, as explained in detail below.

Price and income elasticities implied by our preference parameters estimates are reported in table 5. We find that income elasticities vary across type, with types 1 and 5 having the lowest income elasticities and type 3 the highest. With the exception of type 3, income elasticities do not vary much across income within type. For type 3, the income elasticity declines from 0.8 to 0.5 as income rises. We also find variation across type in price elasticities and modest variation across income within type, except for type 3.[18] Types 1 and 5 exhibit the lowest price sensitivity. Type 3 exhibits the greatest sensitivity to price and has declining price sensitivity as income rises. Once we account for owners' equity, income and price elasticities are slightly lower than in the baseline model.

Recall that the changes over time in the supply of housing of a given quality depend on changes in values and on the estimated supply elasticity. Our point estimate for the annual supply elasticity from the model with five household types is 0.072, with a standard error of 0.018.[19] The implied supply growth rates of quality from our parameter and value estimates are between 4.05% and 6.70% for the 4-year period, which correspond to average annualized changes of approximately 1.1% and 1.6%, respectively. The estimated annual growth in total number of units was 5.48% per year over the 4-year period, with the largest number of additional units being

[18] Note that the price elasticities are calculated on the basis of the quality normalization adopted in estimating the model.

[19] We obtain a similar estimate when we use the more comprehensive measure of income and when we estimate the joint model for Chicago and New York.

TABLE 3
Implied Probability or Fraction of Type (i, k) at Time $t\pi_{ikt}$

	Baseline Model				Model with Equity			
	x_1	x_2	x_3	Aggregate	x_1	x_2	x_3	Aggregate
i_1	.107	.035	.012	.154	.117	.037	.006	.160
i_2	.018	.217	.052	.287	.014	.212	.061	.288
i_3	.008	.013	.200	.220	.009	.015	.184	.208
i_4	.104	.039	.031	.174	.096	.017	.030	.143
i_5	.023	.130	.012	.166	.024	.152	.025	.201

Note.—The table reports estimates of the implied household types.

created in the qualities located around the middle of the distribution. Our supply elasticity estimates are broadly consistent with the estimates of supply elasticities summarized in Glaeser (2004).

Next, we study the goodness of fit of our model. Here we focus on our baseline model. The results are similar for the model with equity. Figures 2 and 3 show the fit of the model to the income and rent distributions. The fitted lines are from structural estimation that imposes all equilibrium conditions. Figure 2 shows the fit to income distributions in Chicago in 1999 and 2003, respectively, while figure 3 shows the rental distributions in 1999 and 2003. These graphs illustrate that the model fit of the income and rent data is quite good in both time periods.

Table 6 provides a detailed analysis of the observed and predicted correlations between income and rent. We report both unconditional correlations for each year and correlations conditional on observed type for each year. Standard errors are reported in parentheses.

Recall that the correlation between income and rent conditional on unobserved type is high. It typically exceeds 0.92. However, each observed type is mapped in more than one unobserved type, as shown in table 3. As a consequence, the model can generate much lower correlations between

TABLE 4
Preference Parameter Estimates

	Baseline Model				Model with Equity			
	α	ϕ	η	γ	α	ϕ	η	γ
i_1	1.22	9.29	.42	−1.77	1.212	10.92	.29	−1.51
	(.32)	(3.32)	(.10)	(.34)	(.10)	(2.22)	(.03)	(.22)
i_2	1.61	4.32	1.19	−1.22	1.12	5.55	1.45	−1.11
	(.23)	(3.10)	(.19)	(.25)	(.18)	(2.23)	(.10)	(.22)
i_3	2.12	1.15	5.71	−1.618	2.25	1.73	4.04	−1.92
	(.30)	(1.61)	(.22)	(.39)	(.17)	(1.99)	(.11)	(.30)
i_4	1.11	9.11	.99	−1.51	1.18	8.09	1.45	−1.32
	(.29)	(5.21)	(.18)	(.33)	(.13)	(4.01)	(.09)	(.23)
i_5	1.33	7.77	1.98	−1.99	1.41	8.10	2.09	−1.95
	(.42)	(5.51)	(.33)	(.23)	(.23)	(5.02)	(.18)	(.20)

Note.—The table reports estimates and estimated standard errors (in parentheses) for the structural parameters of the model.

TABLE 5
ESTIMATED ELASTICITIES

	Baseline			With Equity		
Type i	\$18,000	\$50,000	\$120,000	\$18,000	\$50,000	\$120,000
	A. Income Elasticity					
1	.39	.39	.39	.44	.43	.44
2	.54	.52	.52	.59	.56	.56
3	.73	.60	.55	.56	.48	.45
4	.47	.45	.45	.54	.51	.51
5	.44	.40	.39	.45	.41	.40
	B. Price Elasticity					
1	−.38	−.37	−.37	−.43	−.41	−.41
2	−.54	−.50	−.48	−.58	−.53	−.52
3	−.74	−.59	−.52	−.57	−.47	−.43
4	−.46	−.43	−.42	−.53	−.49	−.47
5	−.44	−.39	−.37	−.45	−.40	−.38

income and rents conditional on observed types. Overall, we find that our model explains the unconditional correlation between income and rental expenditures in both periods reasonably well. We also explain the average correlation conditional on observed type. To obtain reasonable correlation structures we need at least four types. Going from four to five types improves the fit significantly. Allowing for more than five types does not significantly improve the overall fit. Our model does not perform as well period by period. There is some variation in correlation conditional on type between 1999 and 2003. In our model, we assume that preferences and the fractions of each unobserved type are time invariant. As a consequence, our model faces more difficulties explaining variation in correlations across time.[20]

The graphs in figure 4 illustrate the resulting equilibrium in the housing markets. We plot supply and demand for each quality level in the two time periods in Chicago. Figure 4*A* is for 1999, and figure 4*B* is for 2003. As these graphs illustrate, our approach results in close correspondence of supply and demand over the quality range in Chicago in both time periods.

VII. Cross-Metropolitan Comparison

In this section, we highlight the usefulness of our approach by comparing the rental markets in New York with those in Chicago. To start the analysis, we estimate our baseline model, using data from both metropolitan

[20] Once we account for owners' equity, correlations among owners are, by construction, higher than those reported in table 6, since the imputation mechanism induces higher correlations between income and rents.

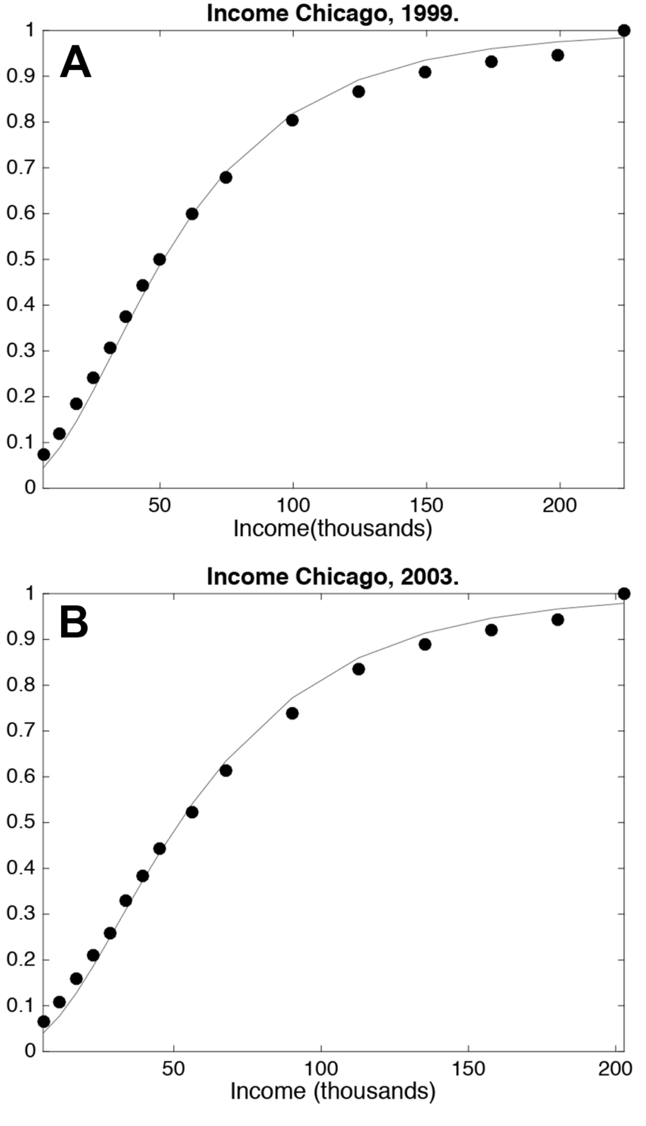

FIG. 2.—Estimated and predicted income distribution for Chicago in 1999 (*A*) and 2003 (*B*).

areas.[21] Overall, we find that our model fits the data in both periods and both metropolitan areas well. If anything, joint estimation improves the accuracy of the estimation procedure and yields smaller estimated standard

[21] Tables that report the parameter estimates and estimated standard errors for the joint model are available in app. C.

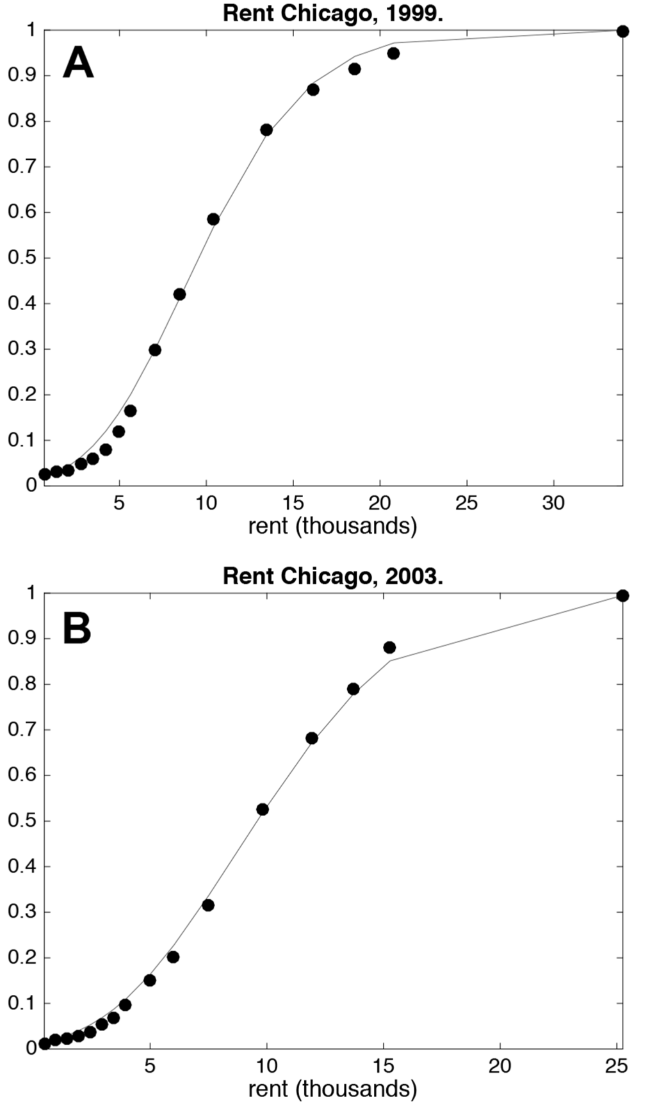

FIG. 3.—Estimated and predicted rent distribution for Chicago in 1999 (A) and 2003 (B).

errors. Moreover, we obtain a good fit of the correlations between income and rent by types. As a shorthand, we sometimes refer to the two metropolitan areas as CHI and NYC, respectively.

Figure 5 shows the hedonic price functions in 2003 for CHI and NYC. The steeper curve shows the hedonic price functions for NYC, while the

TABLE 6
Correlations between Rent and Income

	1999				2003				
	1	2	3	All	1	2	3	All	Aggregate
Data: all	.274	.404	.539	.398	.505	.318	.324	.418	.398
Data: owners	.181	.377	.562	.380	.480	.265	.313	.399	.379
Data: renters	.307	.425	.336	.369	.487	.364	.343	.405	.380
Model	.273	.429	.411	.38	.307	.410	.392	.375	.383
	(.01)	(.08)	(.00)	(.04)	(.02)	(.07)	(.01)	(.04)	(.04)

Note.—The table reports the estimated and predicted correlations between income and rents. Standard errors are reported in parentheses.

shallower curve shows the estimates for CHI. Points a, b, c, and d show housing quality and annualized rent paid by households at the 20th, 40th, 60th, and 80th percentiles of the income distribution, respectively, in New York at their optimally chosen housing consumption levels for those households. The corresponding values A–D show qualities and prices that households with those incomes would optimally choose if they were located in Chicago. At each income level, households pay more in New York than in Chicago and consume lower-quality housing in New York than Chicago. It is important to keep in mind that our housing quality measure is comprehensive and includes all locational amenities in addition to the housing structure itself.

As we have just seen, at every income level, a household in NYC consumes lower quality than the corresponding household in CHI. The effect of this difference in consumption levels is to shift the distribution of quality in NYC to the left relative to that in CHI. This effect is augmented to some extent by differences in the income distributions in the two metropolitan areas. The cumulative distribution function for income for the Chicago metro area is slightly shifted to the right relative to that for the New York metro area. Median income was slightly higher in Chicago than in New York in 1999 and 2003. In addition, New York also had more inequality than Chicago. The relatively higher concentration of low-income households in New York accentuates the leftward shift in the quality distribution in New York relative to that in Chicago. The estimated numbers of housing units by quality are shown in figure 6 for each city. CHI has relatively more high-quality housing. Given its larger population, however, NYC has a larger number of housing units overall than CHI.

We can compute compensating variations (CVs) required for a household of a given income in New York to be equally well off in Chicago. Housing at each quality level is more expensive in the New York metro area than in Chicago. To be equally well off in the two metro areas, a given household must then earn more in New York metro area than in the Chicago

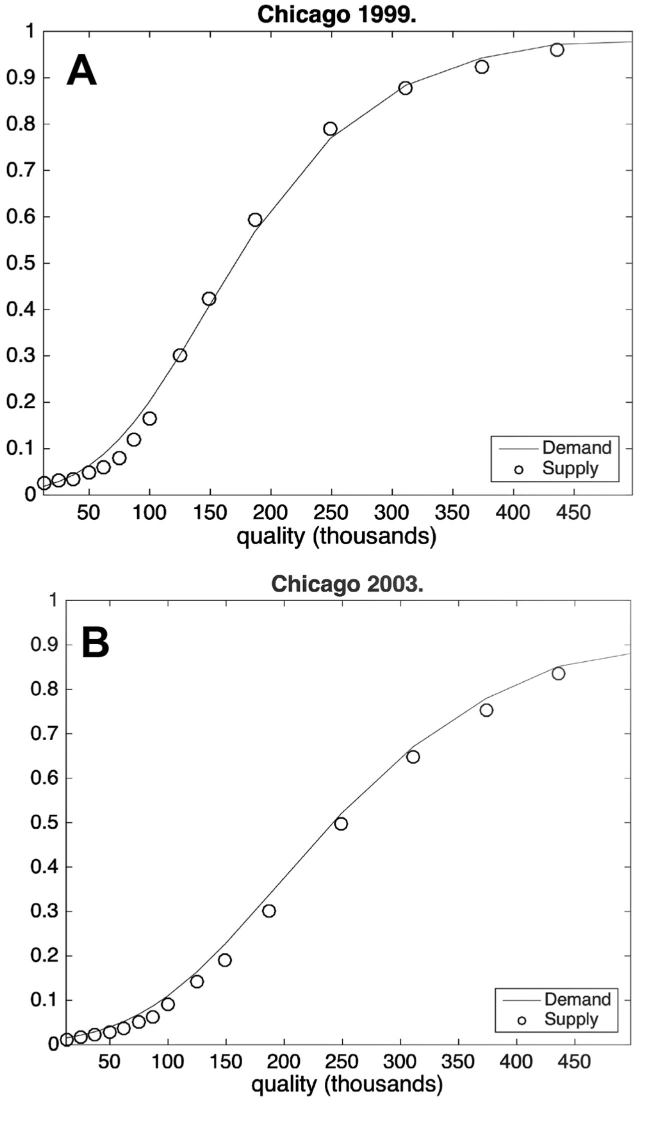

FIG. 4.—Supply and demand for Chicago in 1999 (*A*) and 2003 (*B*).

metro area. Hence, the compensation required for a household living in Chicago metro area to be as well off as its counterpart in New York metro area is a measure of the reduction in earnings required in the Chicago metro area; that is, the compensating variation is negative.

In figure 7*A*, we plot the CV in percent as a function of income. We find that CV as a share of income declines with income, reflecting the declining

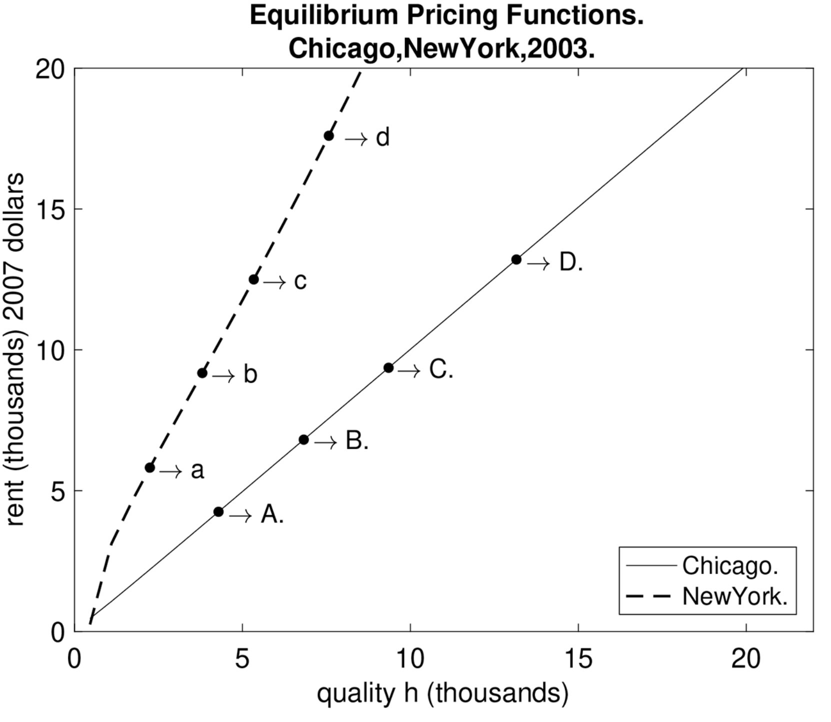

FIG. 5.—Equilibrium price function for Chicago and New York in 2003.

share of income spent on housing as income rises.[22] In figure 7*B*, we plot the CV aggregated across types. For a household earning \$24,000 in CHI (the 20th income percentile in Chicago), a CV of approximately 25% of the household's income (\$4,000) would be required. For a household at the 80th percentile, a CV of approximately 14% of income (\$14,000) would be required.

These CV measures provide valuable insights into agglomeration economies. Productivity, and hence earnings, in NYC, arising from greater agglomeration economies in NYC, would need to be higher by these amounts to compensate a household for the differences in housing price functions between the two metropolitan areas.[23]

[22] Rent stabilization in New York City may result in an estimated hedonic price function that to some degree understates the price conditional on quality in New York City. However, our model is estimated using data for the metropolitan area and not just New York City. Ignoring rent stabilization in New York City may result in underestimation of the CV required for a household in Chicago to be as well off in the New York metropolitan area.

[23] Rosenthal and Strange (2003) provide an in-depth analysis of the spatial and organizational features of agglomeration economies and a discussion of alternative approaches to measuring agglomeration economies.

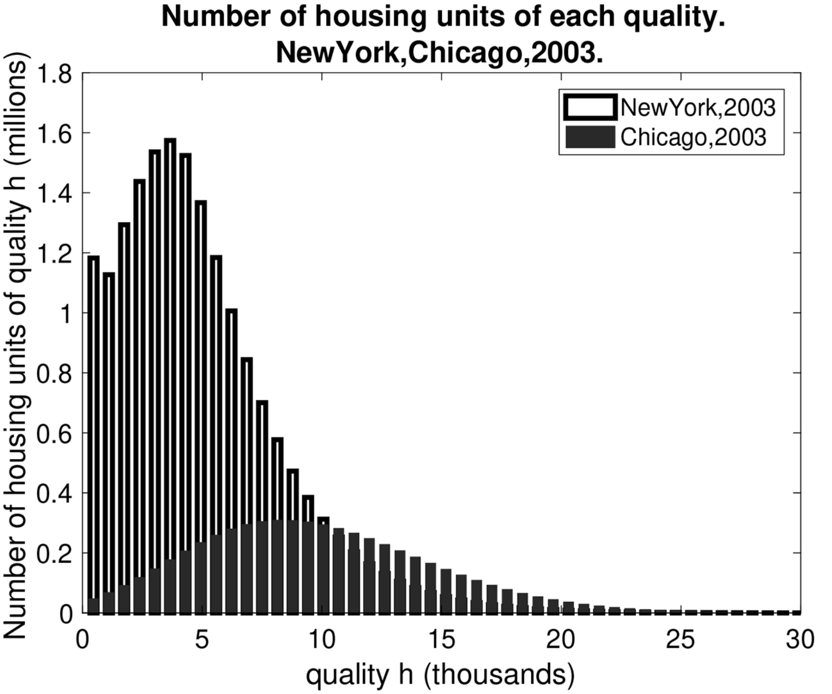

FIG. 6.—Housing stock in Chicago and New York in 2003.

We can test these predictions of our model using data obtained from the harmonized samples in the Integrated Public Use Microdata Series (IPUMS) in 2000. The IPUMS provides a 5% sample for the 2000 census and includes information on both current location and location 5 years before. We estimated household earnings regressions for two samples, controlling for household size, educational achievement, and age. Our main focus is on comparing movers and stayers. First, we considered a sample of households that either stayed in CHI or moved to NYC. Comparing movers to stayers, we find that movers had a significant positive increase in their incomes of around 25%. Second, we considered a sample of NYC households that either stayed in NYC or moved to CHI. As predicted by our model, the coefficient for movers is negative. However, the estimated coefficient is not statistically different from zero. Overall, we conclude that these findings are supportive of our modeling approach.

Finally, we can also compute the CV for price changes over time within metropolitan areas. For example, our estimated hedonic price functions show that there were price increases in New York between 1999 and 2003 for low- and medium-quality housing. As a consequence, intertemporal CVs in New York are positive for most low- and moderate-income households.

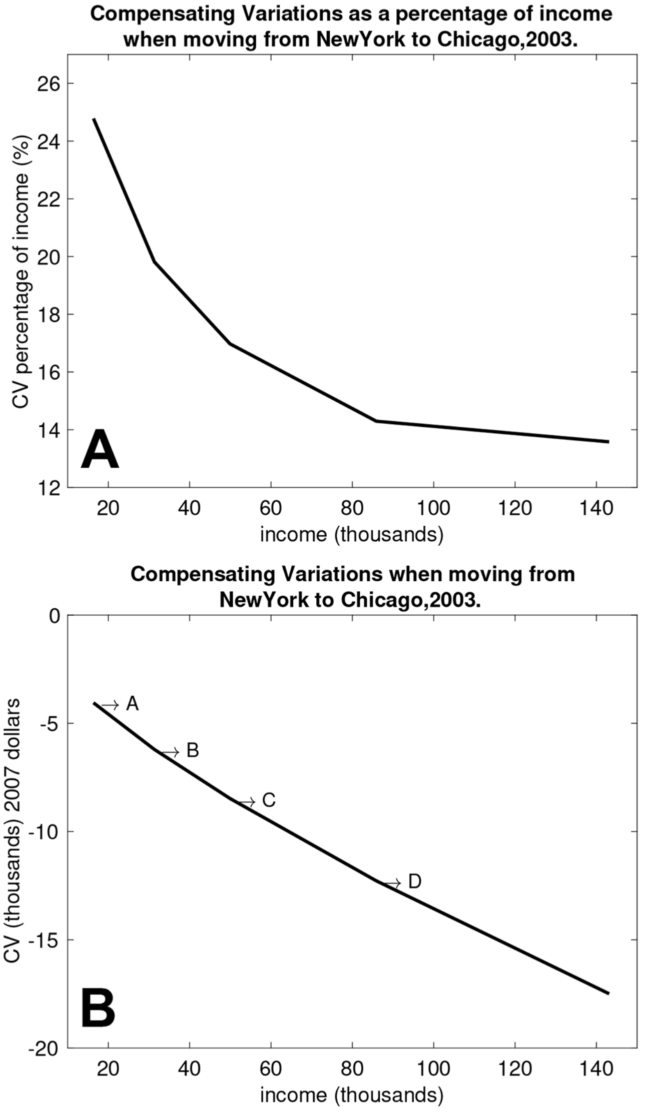

FIG. 7.—Compensation required for a household living in the Chicago metro area to be as well off as its counterpart in New York metro area as percentage of income (*A*) and absolute levels (*B*). CV = compensating variation.

For example, we find that the price increase experienced by a household at the 20th income percentile was equivalent to a 2.5% reduction in household income and the price increase experienced by a household at the 60th percentile was equivalent to a 1.5% reduction in household income.

VIII. Conclusions

We have developed a new approach for estimating hedonic equilibrium models in metropolitan housing markets. Our method has a number of advantages. First, it does not require any a priori assumptions about the characteristics that determine housing quality. Second, it is easily implementable with metropolitan-level data on the distribution of housing values and rents as well as the distribution of household income. Third, it provides a straightforward summary of the changes in prices across the housing quality distribution, complementing single-index measures such as the Case-Shiller index. Fourth, it is comprehensive in incorporating all location-specific amenities in addition to services provided by the dwelling. Fifth, it provides a new, comprehensive approach to measuring CVs, which in turn provides insights into agglomeration economies. Sixth, it gives new insights into the mechanism that generates housing price changes.

Estimating our model for Chicago, we find that a model with five unobserved types captures the observed heterogeneity in rental market conditions in 1999 and 2003. Not surprisingly, some differences in housing demand are driven by income. However, there is also much heterogeneity in price and income elasticities for housing among unobserved household types. The joint analysis for New York and Chicago provides a contrast of hedonic price functions and housing quality distributions across the two metropolitan areas. We also calculate CVs that leave households indifferent between the two areas. These estimates can be interpreted as measures of agglomeration externalities.

There are a variety of other potential applications of our approach. Our framework permits investigation of how changes in the real interest rate affects prices, rentals, and quantities across the quality spectrum in a metropolitan area—via its impact on the user cost of capital. By incorporating multiple household types, our framework also permits analysis of how changes in demographic composition and the income distribution affect housing prices and rents across the quality spectrum in a metropolitan area and the associated impact on supply across the quality distribution. Similarly, the model can be used to study how housing price changes from growth in size or income distribution of one demographic group impact welfare of other demographic groups. Data are available that permit applying the model to make comparisons across other metropolitan housing markets, such as London and Amsterdam. More challenging generalizations are also of interest. For example, it may be feasible to extend the model to incorporate tenure choice. This would permit investigation of how demographic composition, income distributions, and population size, via impacts on equilibrium prices and rents, affect tenure composition across the housing quality spectrum in a metropolitan area.

We have primarily focused on comparisons across metropolitan areas, since these data are more readily available from the AHS. We also think that there is some scope for extending our framework to make it suitable to study sorting within metropolitan areas. We also think that we can apply our methods to study policy interventions that affect equilibria across time and space, in the spirit of more traditional difference-in-differences hedonic papers. These topics provide ample scope for future research.

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