Data-Driven Modeling of Landau Damping by Physics-Informed Neural Networks

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Kinetic approaches are generally accurate in dealing with microscale plasma physics problems but are computationally expensive for large-scale or multiscale systems. One of the long-standing problems in plasma physics is the integration of kinetic physics into fluid models, which is often achieved through sophisticated analytical closure terms. In this study, we successfully construct a multi-moment fluid model with an implicit fluid closure included in the neural network using machine learning. The multi-moment fluid model is trained with a small fraction of sparsely sampled data from kinetic simulations of Landau damping, using the physics-informed neural network (PINN) and the gradient-enhanced physics-informed neural network (gPINN). The multi-moment fluid model constructed using either PINN or gPINN reproduces the time evolution of the electric field energy, including its damping rate, and the plasma dynamics from the kinetic simulations. For the first time, we introduce a new variant of the gPINN architecture, namely, gPINNp to capture the Landau damping process. Instead of including the gradients of all the equation residuals, gPINNp only adds the gradient of the pressure equation residual as one additional constraint. Among the three approaches, the gPINNp-constructed multi-moment fluid model offers the most accurate results. This work sheds new light on the accurate and efficient modeling of large-scale systems, which can be extended to complex multiscale laboratory, space, and astrophysical plasma physics problems.

I. INTRODUCTION

Microscale kinetic physics is crucial for accurately modeling many laboratory, space, and astrophysical systems [1–6]. Unfortunately, for large-scale systems, the first-principle method, which is based on the direct numerical treatment of the kinetic equations, frequently incurs computational costs that are unaffordably expensive. To mitigate the computational cost of kinetic models, numerous attempts have been made to incorporate kinetic physics into the fluid framework that evolves a finite number of fluid moment equations constructed by taking velocity moments of the kinetic Vlasov equation [7–9]. In the area of plasma physics, one profound attempt is the Landau-fluid models pioneered by Hammett and Perkins [10], who derived analytical closure relations for the truncated plasma fluid equations by matching the exact linear response associated with Landau damping in a collisionless electrostatic plasma. A lengthy series of works have gone into devising variants of the fluid closures in different regimes that greatly determine the validity and accuracy of the resulting models [11]. Unfortunately, one major difficulty in constructing the fluid closures in collisionless plasmas is that it typically requires very nontrivial physical and mathematical analyses applied to the specific regime. With the prosperity of Artificial Intelligence in the past decade, naturally, the

question arises: Can Machine Learning assist in completing this challenging task by exploring the kinetic simulation data?

Indeed, conventional artificial neural networks (ANN) have been actively experimented for the discovery of fluid closures in collisionless plasmas. The earliest attempt was perhaps done by Ma et al. [12], who learned the Hammett-Perkins closure with the multilayer perceptron (MLP), the convolutional neural network (CNN), and the discrete Fourier transform (DFT) network. However, to the authors' knowledge, this and the subsequent attempts relied on training data from Landau fluid simulations with a known closure relation. Another promising progress is that of Laperre et al. [13] who used the MPL and a gradient boosting regressor to synthesize a local mapping from local information to local plasma pressure tensor and heat flux, using kinetic simulation data of a 2D magnetic reconnection problem as the input. In their work, nonlocal closures were not investigated, and the mapping differs from conventional closure concepts where the plasma pressure is used as an input.

One common issue in applying the traditional ANN to the discovery of physical relations is its strong reliance on large datasets and slow convergence since the complex underlying physical constraints are not properly imposed. As a remedy, machine learning techniques and methods such as symbolic regression [14, 15], sparse regression [16–18], and the physics-informed neural network (PINN) [19, 20] have been developed. In terms of theoretical plasma physics, some attempts have been recently made by distilling the data and selecting appropriate physical terms from a library of can-

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didate terms. For instance, Alves and Fiuza [21] explored sparsity-based model-discovery techniques in [17] to discover multi-fluid and magnetohydrodynamic equations from the kinetic simulation data. The mPDE-Net has also been used to discover multi-moment fluid equations together with an explicit heat flux closure from kinetic simulation data [22, 23]. However, such librarybased frameworks rely on pre-defined candidate terms that are not always known or well-understood. Among these venues, PINN is possibly one of the most influential examples. The physical partial differential equation (PDE) residuals are incorporated into the loss function of the neural network as regularization, transforming the process of solving PDEs into an optimization problem by constraining the space of permissible solutions. Since its introduction by Raissi et al. [19], PINN and its variants have been widely applied to fluid dynamics, plasma physics, electromagnetics, and many more areas [24–28]. One remarkable improvement of PINN was made by Yu et al. [29], who added additional gradient loss terms to construct the gradient-enhanced physics-informed neural network (gPINN) to improve the accuracy for largegradient shock-wave physics.

This work aims to explore the feasibility and effectiveness of capturing the hidden fluid closure using PINN without prescribing the form of the closure itself. A library of explicit candidate terms in the closure relations would not be necessary. The key point here is to use the kinetic simulation data that contains the complete closure information as the training datasets, and use fluid moment equations to constrain the training process. The trained neural network then embeds the closure information implicitly and can be used to close the multi-moment fluid equation system and incorporate desirable kinetic physics. As a first but critical step, we will use the example of Landau damping in a collisionless, electrostatic plasma, which is one of the most fundamental kinetic processes in a variety of plasmas. We will explore the performance of the original PINN and its variants, in particular, the gradient-enforced PINN (gPINN), in capturing the hidden fluid closure that can reproduce the Landau damping process. The results of this study could be extended to other more complex problems and be combined with more sophisticated, more general approaches.

II. METHODOLOGY

A. Physical Model

Consider a collisionless plasma in the absence of a magnetic field, the dynamics of the plasmas are governed by the Vlasov equation which describes the evolution of the particle distribution function in the phase space (\mathbf{r}, \mathbf{v}) ,

$$\frac{\partial f_s}{\partial t} + \mathbf{v_s} \cdot \nabla_{\mathbf{r}} f_s + \left(\frac{q_s}{m_s}\right) \mathbf{E} \cdot \nabla_{\mathbf{v}} f_s = 0 \tag{1}$$

where $f_s(\mathbf{r}, \mathbf{v_s}, t)$ is the velocity distribution function of particle species s in a plasma, q_s/m_s is the charge over the mass ratio of the particle species s, and the operators $\nabla_{\mathbf{r}} = (\partial_x, \partial_y, \partial_z)$ and $\nabla_{\mathbf{v}} = (\partial_{v_x}, \partial_{v_y}, \partial_{v_z})$ are the gradient operators in configuration space and velocity space, respectively. For simplicity, we consider a one-dimensional model in $x - v_x$ space. Additionally, $E_x(x,t)$ is the self-induced electric field, which satisfies the Poisson equation describing the electrostatic field:

$$E_x(x,t) = -\nabla\phi\tag{2}$$

$$\triangle \phi = -\frac{\rho}{\varepsilon_0} \tag{3}$$

Here, $\phi(x,t)$ is the electric potential, ε_0 is the vacuum permittivity, $\rho(x,t)$ denotes the charge density:

$$\rho = \sum_{s} q_{s} n_{s} \tag{4}$$

where q_s and n_s is the charge and number density of the particle species s, respectively.

In general, Vlasov models tend to become more memory-consuming and computationally demanding due to the high dimensionality of phase space, so we consider fluid models of plasma that involve only the evolution of macroscopic quantities. Consequently, we obtain some macroscopic fluid quantities by calculating the moments of $f_s(x, v_s, t)$ in the velocity space, and then extract the evolution of the moments from the Vlasov simulation data. In detail, macroscopic fluid quantities including the number density $n_s(x,t)$, the fluid velocity $u_s(x,t)$, the pressure $p_s(x,t)$, and the heat flux $q_s(x,t)$ can be derived from the first three moment equations:

$$n_s(x,t) = \int f_s(x,v_s,t)dv_s \tag{5}$$

$$u_s(x,t) = \frac{1}{n_s(x,t)} \int v_s f_s(x,v_s,t) dv_s \tag{6}$$

$$p_s(x,t) = m_s \int (v_s - u_s)^2 f_s(x, v_s, t) dv_s$$
 (7)

$$q_s(x,t) = m_s \int (v_s - u_s)^3 f_s(x, v_s, t) dv_s$$
 (8)

The set of multi-moment fluid PDEs for electron species, e, is expressed as follows (we drop the subscript, s, for the variables hereafter for brevity):

$$\frac{\partial n}{\partial t} + u \frac{\partial n}{\partial x} + n \frac{\partial u}{\partial x} = 0 \tag{9}$$

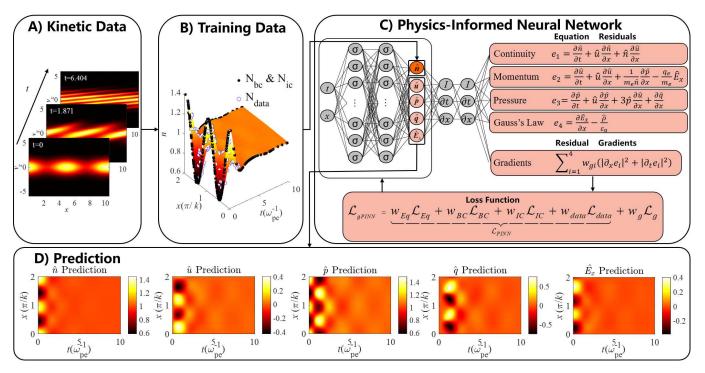


FIG. 1: Physics-informed neural network (PINN) architecture for the multi-moment fluid model with an implicit fluid closure learned from the kinetic simulation data. The whole procedure includes A) kinetic simulation data generation, B) sparse sampling of training data, C) PINN construction with the constraints of different moment equation residuals and their gradients, and D) parameter prediction.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{m_e n} \frac{\partial p}{\partial x} = \frac{q_e}{m_e} E_x \tag{10}$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + 3p \frac{\partial u}{\partial x} + \frac{\partial q}{\partial x} = 0 \tag{11}$$

$$\frac{\partial E_x}{\partial x} = \frac{\rho}{\varepsilon_0} \tag{12}$$

These are the electron continuity, momentum, pressure, and Gauss's law equations, respectively. Clearly, the update of the lower-order moment equations (e.g., the pressure equation) depends on the evolution of the next higher-order moment (e.g., the heat flux q); therefore, a comprehensive multi-moment fluid model must include a closure relation to close the system of equations. Because of the absence of the evolution of the fourth-order moment equation (or the heat flux equation) in the preceding equations, a closure relation for the heat flux q is required for the multi-moment fluid model.

B. PINN and Gradient-enforced PINN Architectures

The schematic diagram of the whole architecture depicted in Fig. 1 includes four parts, A) kinetic simula-

tion data generation, B) sparse sampling of the training data, C) physics-informed neural network construction, and D) parameter prediction. Beginning with the generation of kinetic simulation data by numerically solving the equations of the Vlasov-Poisson system, as depicted in Fig. 1A, we take snapshots of the velocity distribution $f(x, v_x)$ in phase space at several time steps to characterize these data. Secondly, for boundary and initial conditions, all physical variables (n, u, p, q, E_x) are sampled, but only density n is also sparsely sampled from the simulation data at the first few time steps as hypothetical known observations to train the neural network as shown in Fig. 1B (or Fig. 2). Finally, the neural network with multi-moment fluid equation residual constraints is built to recover and forecast the number density $\hat{n}(x,t)$, the fluid velocity $\hat{u}(x,t)$, the pressure $\hat{p}(x,t)$, the heat flux $\hat{q}(x,t)$, and the electric field $\hat{E}_x(x,t)$ across the entire spatial and temporal range (Figs. 1C-D). A schematic illustration of the proposed PINN and gPINN, composed of a fully connected feedforward neural network (FNN) with multiple hidden layers and a residual network with the fluid moment equation and their gradient constraints, is depicted in Fig. 1C.

For the one-dimensional fluid model described by Eqs. (9)-(12) on a spatial domain $\Omega \subset \mathbb{R}$, we first define the multi-moment fluid system deduced from the kinetic simulation data as the set $\mathcal{F}(x,t) = \{n(x,t), u(x,t), p(x,t), q(x,t), E_x(x,t)\}$ and then con-

struct a neural network with the trainable parameters θ to approximate the solution. The neural network as a parametric function approximator can be represented by a nonlinear function:

$$\hat{n}, \hat{u}, \hat{p}, \hat{q}, \hat{E}_x = \hat{\mathcal{F}}_i(x, t; \boldsymbol{\theta}), \quad x \in \Omega, t \in [0, T]$$
 (13)

where $\theta = \{W, b\}$ is the weight matrix and the bias vector. We take the derivatives of $\hat{\mathcal{F}}$ with respect to x and t by applying automatic differentiation. PINN encodes professional physical priors into the loss function. These physical priors, which are expressed as a set of PDEs with appropriate initial and boundary conditions, are highly condensed knowledge of physical mechanisms that can inform the neural network. Then we utilize the constraints implied by the PDEs, the initial conditions, the boundary conditions, and some extra measurements of the density n as labeled data to train the neural network. The whole loss function is defined as follows:

$$\mathcal{L}_{PINN} = w_{Eq} \mathcal{L}_{Eq} + w_{BC} \mathcal{L}_{BC} + w_{IC} \mathcal{L}_{IC} + w_{data} \mathcal{L}_{data}$$

$$(14)$$

where w_{Eq} , w_{BC} , w_{IC} and w_{data} are the weights of each loss function respectively. In this study, we choose the weights $w_{Eq} = w_{BC} = w_{IC} = w_{data} = 1$. In particular, we seek to minimize the residuals of the fluid moment equations, which are given as follows:

$$e_{1} = \frac{\partial \hat{n}}{\partial t} + \hat{u}\frac{\partial \hat{n}}{\partial x} + \hat{n}\frac{\partial \hat{u}}{\partial x}$$

$$e_{2} = \frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \hat{u}}{\partial x} + \frac{1}{m_{e}\hat{n}}\frac{\partial \hat{p}}{\partial x} - \frac{q_{e}}{m_{e}}\hat{E}_{x}$$

$$e_{3} = \frac{\partial \hat{p}}{\partial t} + \hat{u}\frac{\partial \hat{p}}{\partial x} + 3\hat{p}\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{q}}{\partial x}$$

$$e_{4} = \frac{\partial \hat{E}_{x}}{\partial x} - \frac{\hat{p}}{\varepsilon_{0}}$$

$$(15)$$

$$\mathcal{L}_{Eq} = \frac{1}{N_{eq}} \sum_{i=1}^{N_{eq}} \sum_{j=1}^{4} |e_i(x_j, t_j)|^2, x_j \in \Omega, t_j \in [0, T] \quad (16)$$

Here e_1 denotes the continuity equation residual, e_2 the momentum equation residual, e_3 the pressure equation residual, and e_4 the Gauss's law equation residual. N_{eq} is the number of the trained data for \mathcal{L}_{Eq} . In fact, we want to conduct a forward problem using PINN, where the fluid closure is implicitly included in the neural network, assuming that both the initial and boundary conditions are known and sparsely sampled,

$$\mathcal{L}_{BC} = \frac{1}{N_{bc}} \sum_{j=1}^{N_{bc}} \sum_{i=1}^{5} \left| \hat{\mathcal{F}}_i \left(x_j, t_j \right) - \mathcal{F}_i \left(x_j, t_j \right) \right|^2,$$

$$x_j \in \partial \Omega, t_j \in [0, T]$$

$$(17)$$

$$\mathcal{L}_{IC} = \frac{1}{N_{ic}} \sum_{j=1}^{N_{ic}} \sum_{i=1}^{5} \left| \hat{\mathcal{F}}_i \left(x_j, t_j \right) - \mathcal{F}_i \left(x_j, t_j \right) \right|^2,$$

$$x_j \in \Omega, t_j = 0$$
(18)

Although the model inputs should ensure there is enough information for the neural network to accurately capture the governing equations of the system, the amount of input information should be minimized. Therefore, we only sample the kinetic simulation data in the first few time steps as labels (see Fig. 2) to allow the network to capture the fluid closure that incorporates the kinetic effects. In this study, only the density n is sampled.

$$\mathcal{L}_{data} = \frac{1}{N_{data}} \sum_{j=1}^{N_{data}} |\hat{n}(x_j, t_j) - n(x_j, t_j)|^2,$$

$$x_j \in \Omega, t_j \in [0, t'], t' \le \frac{T}{5}$$
(19)

Meanwhile, other studies have demonstrated that gPINN improves the accuracy of PINN, especially when applied to PDEs with steep gradients [29]. Thus, we introduce gPINN to capture the structures with large gradients. The main idea of gPINN embeds the gradient information into the loss function by enforcing the derivatives of the moment equation residuals to be the minimum. Assuming the gradient of the equation residual ∇e exists, the loss function of gPINN is:

$$\mathcal{L}_{gPINN} = w_{Eq} \mathcal{L}_{Eq} + w_{BC} \mathcal{L}_{BC} + w_{IC} \mathcal{L}_{IC} + w_{data} \mathcal{L}_{data} + \mathbf{w}_q \mathcal{L}_q$$
 (20)

For the 1X1V case, the additional loss term is,

$$\mathbf{w}_{g}\mathcal{L}_{g} = \frac{1}{N_{g}} \sum_{j=1}^{N_{g}} \sum_{i=1}^{4} w_{gi} \left(\left| \partial_{x} e_{i} \left(x_{j}, t_{j} \right) \right|^{2} + \left| \partial_{t} e_{i} \left(x_{j}, t_{j} \right) \right|^{2} \right),$$

$$x_{j} \in \Omega, t_{j} \in [0, T]$$
(21)

The weight $\mathbf{w}_g = \{w_{g_1}, w_{g_2}, w_{g_3}, w_{g_4}\}$ is an extra hyper-parameter in the gPINN architecture for optimization.

In the traditional gPINN architecture [29], it incorporates the gradient terms of all the equation residuals and adds them to the loss function. In this study, we make a new attempt to only include the gradient of a specific equation residual as the additional constraint, which also reduces the computational cost compared with the traditional gPINN. Here, we define a new variant of gPINN that only includes the gradient of the pressure equation residual, namely, gPINNp. The idea of gPINNp is motivated by the fact that the heat flux q and the pressure p are closely related in the residual e_3 of Eq. 15.

III. SIMULATION

A. Synthetic Model Setup

1. Kinetic simulation data generation

This section describes the kinetic Vlasov-Poisson simulations used to generate the training data. The physical problem under investigation is the Landau damping in a collisionless, electrostatic plasma. The initial setup consists of an immobile, neutralizing ion background and two perturbation modes applied to the electron density,

$$n_e(x, t = 0) = n_0(1 + A_1 \cos(k_1 x) + A_2 \cos(k_2 x + \varphi))$$
(22)

$$n_i(x, t = 0) = n_0 (23)$$

where n_0 is the initial density of each species, k_1 and k_2 are the wavenumbers of the two modes, A_1 and A_2 are their amplitudes, and φ is a random phase.

We use the open-source continuum Vlasov code Gkeyll [30] for this study. The simulation employs a periodic configuration domain, $0 < x < \frac{2\pi}{k_1}$, discretized to 128 cells, and a plasma velocity space $-6v_{th_s} < v_x < 6v_{th_s}$ with 128 cells. A fixed time step size $\Delta t = 0.001\omega_{pe}^{-1}$ is used and the simulation takes 10000 steps before it stops at $t = 10\omega_{pe}^{-1}$. The numerical scheme being used is a discontinuous Galerkin method with second-order Serendipity polynomial bases [31]. The specific simulation parameters are summarized in Table I.

To construct the training datasets, the electron density n, velocity u, pressure p, and heat flow q are extracted from the phase-space data following Eqs. (5)-(8).

TABLE I: Summary of the initial setup parameters

k_1	k_2	A_1	A_2	φ
0.6	1.2	0.05	0.4	0.38716

Here we want to point out that the damping rates of these two modes are different; the mode with a short wavelength predominates but decays fast, and the one with a long wavelength of low energy decays slowly.

2. Deep neural network setup

Both PINN and gPINN involve neural network architecture selection since it has a significant impact on the prediction precision. The parameters of the neural network utilized are shown in Table II. All of them are searched by trial and error while taking into account the solution precision, convergence, and computational efficiency.

The artificial neural network consists of the input layer, the hidden layer, and the output layer. The hidden layer

TABLE II: Parameter setting of PINN (and gPINN)

Hidden layers, and No. of neurons	Optimizer	Learning rate	Activation function	Batch size
5 and 50	Adam	0.01	Swish	10000

that contains five layers and fifty neurons provides the best accurate solutions. In addition, we choose this network structure in order to reduce additional computational requirements or over-fitting. We choose a nonlinear activation function Swish [32] to retain information about the gradient of the data with respect to the input variables (x, t). In each iteration of the Adam optimizer [33], the mini-batch of data and the residual points used to penalize the equation are processed and have a size of 10000. To avoid an unstable training process caused by a rapid change in the learning rate, we use a constant learning rate of 0.01. In addition, we utilize weight normalization to accelerate the training of PINN (and gPINN) [34].

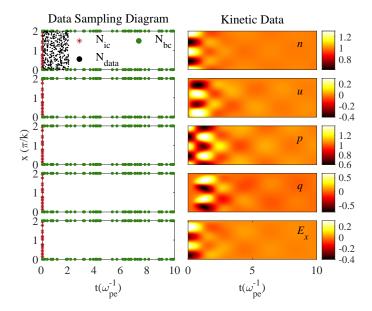


FIG. 2: The schematic diagram of training data sampling from the simulation domain. Each row of the right panel shows the kinetic simulation data of density n, velocity u, pressure p, heat flux q, and electric field E_x , respectively.

The data sampling diagram of the neural network is displayed in Fig. 2. The training data are composed of the randomly sampled $N_{ic}=200$ initial conditions and $N_{bc}=300$ boundary points from the five quantities, i.e., $n,\,u,\,p,\,q,\,E_x$. For the electron density n, we also sample a small set of points from 0 to $2\omega_{pe}^{-1}$ as extra measurements, totaling $N_{data}=23863$ sampling points, which correspond to the sampling rate approximately $\sim 3.125\%$ within $t'=2\omega_{pe}^{-1}$.

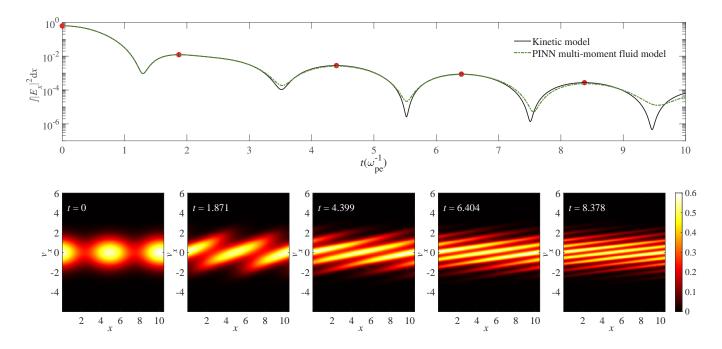


FIG. 3: Top: The evolution of the electric field energy predicted by the multi-moment fluid model constructed using PINN (green dashed line) and the kinetic model (black solid line), with wave peaks at time t = 0, $1.871\omega_{pe}^{-1}$, $4.399\omega_{pe}^{-1}$, $6.404\omega_{pe}^{-1}$, $8.378\omega_{pe}^{-1}$ (red dots). Bottom: The velocity distribution $f(x, v_x)$ in phase space at these peak points (see red dots on the top panel).

B. Data-Driven Modeling Results

Based on the kinetic simulation data, we construct the multi-moment fluid model using PINN and gPINN, respectively. When training is converged, the neural network simultaneously predicts the values of \hat{n} , \hat{u} , \hat{p} , \hat{q} , and $\hat{E_x}$ for the whole time period up to $10\omega_{pe}^{-1}$. We use the diagnostic variable $|E_x|^2$ which is the integral of the electric field square over the entire configuration space $\int |E_x|^2 dx$ to evaluate the accuracy of PINN (and gPINN) on capturing the Landau damping process.

Fig. 3 shows the evolution of the electric field energy predicted by the PINN-constructed multi-moment fluid model over time, and the temporal evolution of velocity phase space distribution $f(x, v_x)$ at several fixed time steps from the kinetic simulation data. Based on the sparsely sampled electron density n in time period $0 < t < 2\omega_{pe}^{-1}$, the PINN-constructed multi-moment fluid model recovers and reconstructs the electric field energy evolution during the time period $t = [0, 10\omega_{pe}^{-1}]$. The predicted electric field energy oscillates and decays as time progresses, with wave peaks at time $t = 0, 1.871\omega_{pe}^{-1}, 4.399\omega_{pe}^{-1}, 6.404\omega_{pe}^{-1}, 8.378\omega_{pe}^{-1}$ (labeled as red dots), which agrees with the kinetic simulation data. The evolution of the complicated velocity distribution $f(x, v_x)$ in phase space from the kinetic simulation data at these time steps (labeled as red dots) is depicted in the bottom panels of Fig. 3. The good agreement be-

tween the kinetic simulation data and PINN-generated data indicates that the PINN-constructed multi-moment fluid model is capable of accurately representing the complicated evolution of the plasma dynamics and capturing the Landau damping process even without directly evolving the distribution function in the phase space.

For quantitative assessment, we define the absolute error (AE) as the evaluation metrics, which is expressed as:

$$AE(\hat{y}, y) = |\hat{y}(x, t) - y(x, t)| \tag{24}$$

where $AE(\hat{y}, y)$ is defined as the difference between the outputs $\hat{y}(x, t)$ of the neural networks and the kinetic simulation data y(x, t).

The introduction of equation residuals into the loss function (see Eq.(16)) is the most important component of our scheme, while the equations provide the necessary physical priors and serve as a roadmap for network optimization. It is noteworthy that our method does not involve any explicit fluid closure equations, but the fluid closure relation is implicitly included in the neural network.

In Fig. 4a, we compare the performance of the PINN-constructed multi-moment fluid model using different combinations of moment equation residual as constraints (see Fig. 1C or Eq.(15)). The goal is to determine the minimum requirement to accurately capture the Landau damping process. In more detail, the electric field equa-

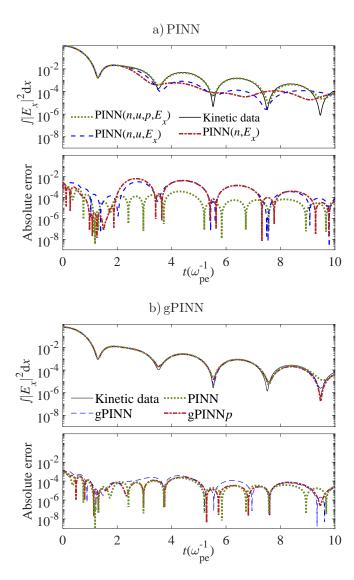


FIG. 4: Temporal evolution of electric field energy and absolute error using (a) PINN and (b) gPINN with different moment equation residuals as constraints. The weight $w_{gi} = 0.01$ (i = 1, 2, 3, 4) and $w_{g3} = 0.01$ are adopted for gPINN and gPINNp, respectively.

tion residual is always retained, and the number of equation residuals is always one less than the number of the network predictions. As an example, PINN(n, u, E_x) in Fig. 4a indicates the use of continuity and momentum equation residuals, as well as the electric field equation residual, while the model outputs are $\hat{n}, \hat{u}, \hat{p}$, and \hat{E}_x . In order to compare and show more clearly the differences between the results, we give the absolute errors between the predictions using different numbers of equation residuals and the kinetic simulation data. The overall absolute error of PINN(n, u, p, E_x) is less than 10^{-3} , which is smaller than the other two cases using fewer numbers of equation residuals, i.e., PINN(n, E_x) and PINN(n, u, E_x).

The results obtained by the PINN-constructed multi-

moment fluid model in Fig. 4a without using the pressure equation residual as a constraint have seriously deviated from the true value (or the kinetic simulation data). Consequently, we draw the conclusion that it is necessary to use at least the first three moment equations as constraints and such a fluid system contains five variables (n, u, p, q, E_x) to accurately capture the Landau damping process. When this condition is not satisfied, i.e., the number of constraints is less than the minimum requirement, PINN-constructed multi-moment fluid model is not able to capture the specific kinetic effects due to the lack of sufficient input information. Meanwhile, Fig. 4a also demonstrates that the results begin to deteriorate at later stages, particularly in the wave troughs of the electric field energy curve, where large deviations are observed. Therefore, we adopt gPINN by adding the gradients of the moment equation residuals, which has been demonstrated to be more effective than PINN [29] when addressing similar issues.

The performance of the PINN-constructed and gPINNconstructed multi-moment fluid models to capture the Landau damping process is compared in Fig. 4b. Here, for the first time, we introduce a new variant of gPINN, namely gPINNp which only incorporates the gradient of the pressure equation residual as an additional constraint, while the traditional gPINN includes the gradients of all the equation residuals (see Fig.1C or Eq.(21)). The idea of gPINNp is motivated by the heat flux q and the pressure p are closely related in the residual e_3 of Eq. 15. Meanwhile, gPINNp is computationally cheaper than the traditional gPINN with more constraints. In both gPINN and gPINNp, the gradient weight w_{gi} is a hyperparameter, filtered by the optimized tests with $w_{qi} =$ 0.01 (i = 1, 2, 3, 4 for gPINN and i = 3 for gPINNp). In Fig. 4b, the absolute error of electric field energy between predicted and true value reaches 2.22×10^{-5} using PINN, 2.00×10^{-6} using gPINN, and 4.68×10^{-7} using gPINNp at the last wave trough approximately at time $t = 9.5\omega_{pe}^{-1}$. The multi-moment fluid model constructed using standard gPINN fits the kinetic simulation data better than that using PINN, while gPINNp-constructed multi-moment fluid model provides the most refined results, especially at later stages $t > 8\omega_{pe}^{-1}$ when the electric field energy decays to relatively low values. The best performance of gPINNp-constructed multi-moment fluid model indicates that the evolution of the heat flux q heavily relies on the pressure p and its gradients, consistent with the theoretical expectation [11].

In Fig. 5, we recorded the aggregate losses \mathcal{L}_{PINN} and \mathcal{L}_{gPINN} of PINN and gPINNp during the whole training process, as well as each component of the loss function. To filter out the oscillations in the time series for the loss values, we employ a centered moving average by sliding a window of length 100 iterations. \mathcal{L}_{PINN} and \mathcal{L}_{gPINN} showed a general downward trend, with a gentle trend after 40,000 iterations and they converge to roughly 10^{-5} after a total of 216000 iterations. For the definition of the loss function for PINN and gPINNp, see Eq. (14)

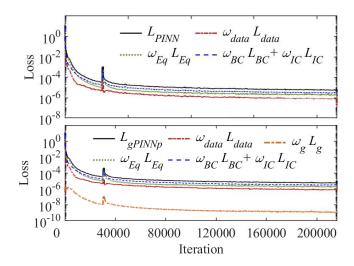


FIG. 5: History of the aggregate losses and loss of various components of PINN (top) and gPINNp with $\omega_{g_3} = 0.01$ (bottom). A moving average filter with a moving window of length 100 time iterations is used to smooth losses.

and Eq. (20) respectively. As for the training procedure, all convergent results are obtained after 2×10^5 steps of iterative optimization. Among the total loss \mathcal{L}_{PINN} and \mathcal{L}_{gPINN} , the data loss, \mathcal{L}_{data} , is the lowest, and then the equation loss, \mathcal{L}_{Eq} , while the loss at boundary and initial conditions, $\mathcal{L}_{BC} + \mathcal{L}_{IC}$, is the maximum, which actually controls the total loss through selecting the best weights for each component.

In Fig. 6, we present the temporal-spatial evolution of the physical quantities predicted by the multimoment fluid models constructed using PINN, gPINN, and gPINNp, and the true value from the kinetic simulation. The corresponding row in each column displays the relevant values for the density n, velocity u, pressure p, heat flux q, and electric field E_x , respectively. As shown in Fig. 6, the PINN, gPINN, and gPINNp architectures have the ability to accurately reconstruct and predict those physical quantities with an implicit fluid closure included in the neural network. It is noteworthy that the accurate prediction of these quantities only relies on sparse sampling of a small fraction of the kinetic simulation data (see Fig.2). Most importantly, the kinetic phenomenon of Landau damping is successfully captured while the field reconstruction is also implemented simultaneously.

In order to compare the accuracy of the prediction, Fig. 7 depicts the absolute errors of the predicted quantities from the multi-moment fluid models constructed using PINN, gPINN, and gPINNp. All physical quan-

tities predicted by the three models are in good agreement with kinetic simulation data. Overall, the physical quantities predicted by the gPINNp-constructed multimoment fluid model are more accurate than the other two, especially at later stages. Therefore, the proposed gPINNp architecture, which uses only the gradient of pressure equation residual as one additional constraint, is the best option to study kinetic physics such as Landau damping based on all the results shown above.

IV. CONCLUSION AND DISCUSSION

In conclusion, we construct multi-moment fluid models using PINN and gPINN, where the fluid closure is learned from the kinetic simulation data and is implicitly included in the neural networks. The neural networks use the physical constraints of the multi-moment fluid equation residuals and their gradients. In order to accurately capture the Landau damping process, PINN and gPINN need to include the first three moment equations (i.e., equations of n, u, and p) as constraints. Meanwhile, the PINN and gPINN architectures are capable of accurately predicting all these physical quantities concurrently.

In addition, for the first time, we propose and explore a new variant of gPINN, namely, gPINNp. Different from the traditional gPINN that uses the gradients of all the moment equation residuals as additional constraints, gPINNp architecture only adopts the gradient of the pressure equation as the additional constraint. Compared with the results from the cases using PINN and gPINN, the gPINNp-constructed multi-moment fluid model provides the most accurate predictions, especially at later stages. The best performance of gPINNp indicates that the evolution of the heat flux q heavily relies on the pressure p and its gradients, consistent with the theoretical expectation.

In the future, we intend to expand the extrapolation capabilities of the neural networks in order to apply PINNs to higher-dimensional and more intricate multiscale problems.

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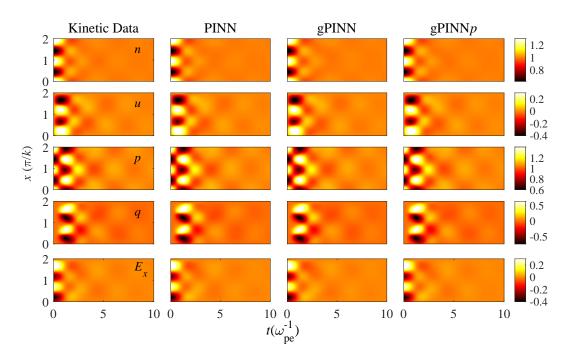


FIG. 6: Comparison of the predicted physical quantities (from the multi-moment fluid models constructed using PINN, gPINN, and gPINNp) with respect to the kinetic simulation data. Each panel from top to bottom shows the density n, velocity u, pressure p, heat flux q, and electric field E_x , respectively.

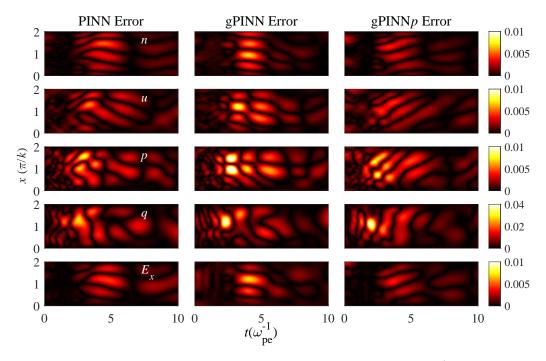


FIG. 7: Comparison of the absolute errors between the predicted physical quantities (from the multi-moment fluid models constructed using PINN, gPINN, and gPINNp) and the kinetic simulation data. Each row shows the absolute error of the density n, velocity u, pressure p, heat flux q, and electric field E_x , respectively.

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