Image Processing
- Intensity Transformations and Spatial Filtering -

Fundamentals of Spatial Filtering

Spatial filtering

- Principal tool to enhance image
- Name "filter" borrowed from frequency domain approach
  - Freq domain approach rejects/keeps only certain information
  - Smoothing (low-pass filtering), Edge enhancement (high-pass filtering)
- Filtering creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood, and whose value is the result of the filtering operation
- Mechanics:
  - Predefined neighborhood: 4 neighbor, 8 neighbor, squared region
  - Operation: mathematical operation

Fundamentals of Spatial Filtering

The mechanics of spatial filtering

- Linear spatial filter – linear operation
  - Consider a mask (filter) of odd size m × n (m = 2a + 1, n = 2b + 1 for a, b > 0 ∈ Z)

    *Mask = spatial filter, kernel, window, template

  - *In general*, linear filtering of an image *f* of size *M X N* with a filter mask of size *m X n* is given by the expression

called *convolution mask*

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

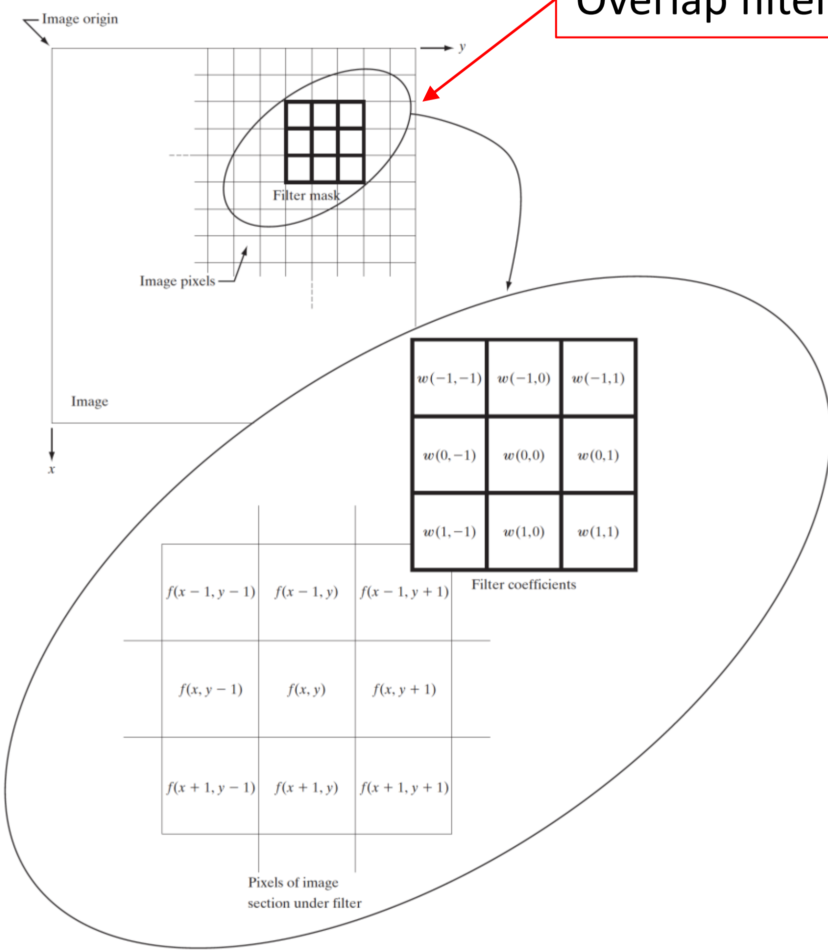
where $a = \frac{m-1}{2}$, b $= \frac{n-1}{2}$

$\Rightarrow$ *linear spatial filtering = convolving a mask with an image*

Fundamentals of Spatial Filtering

The mechanics of spatial filtering

Overlap filter on the image



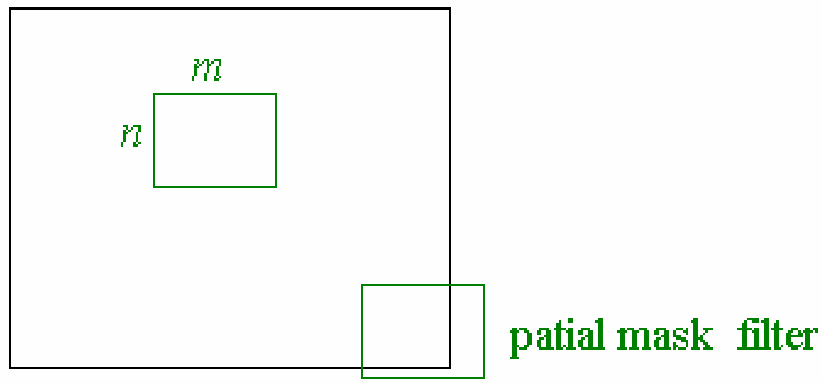
Filtered image g(x)

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

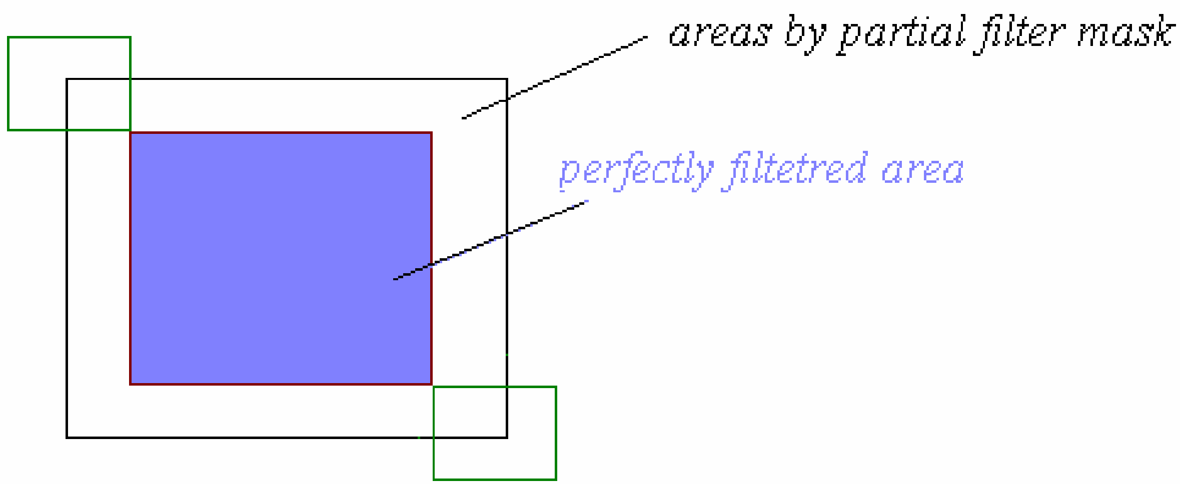
$f(x_0,y_0) \rightarrow g(x_0,y_0)$

Fundamentals of Spatial Filtering

The mechanics of spatial filtering



In general, images get small after filtering operation

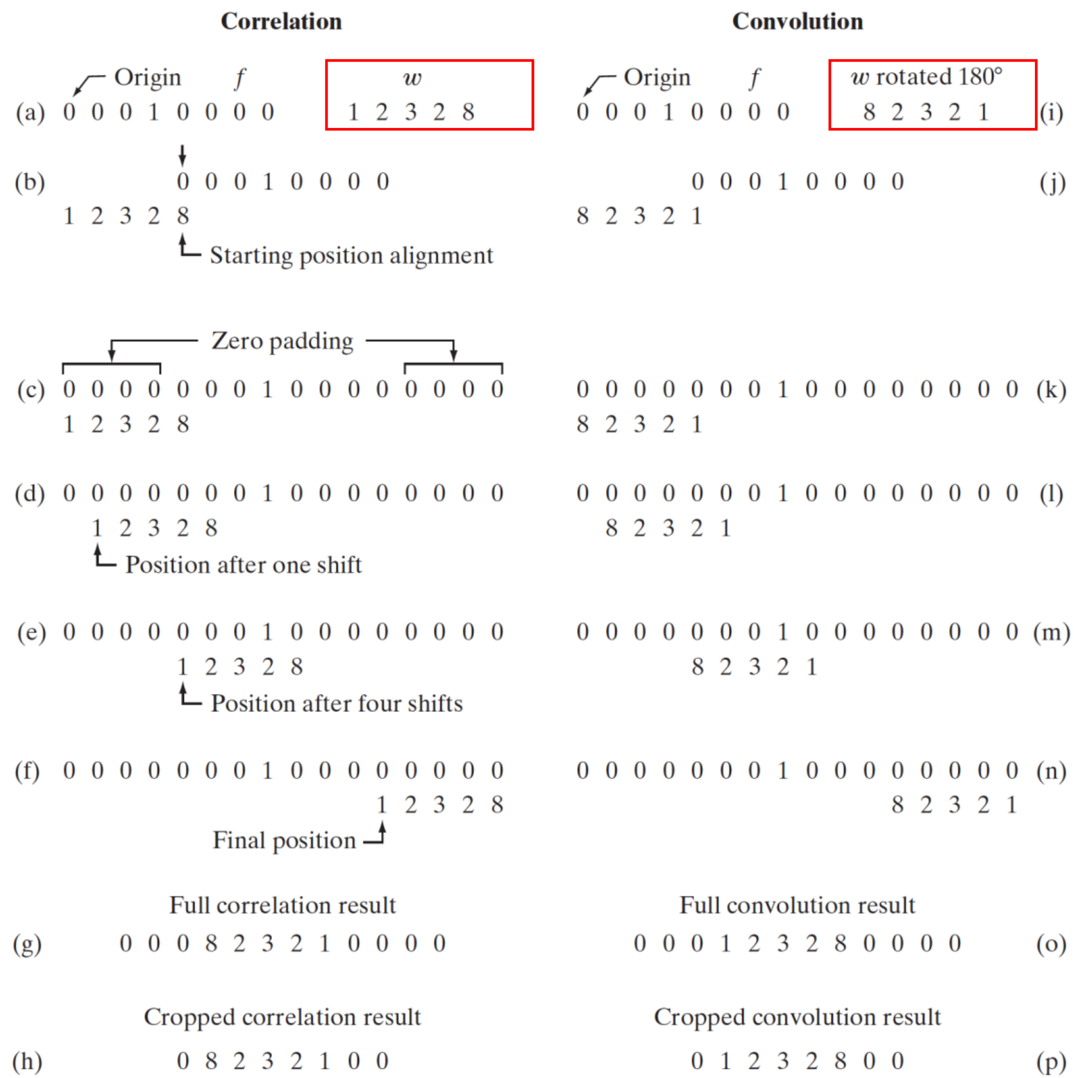


Fundamentals of Spatial Filtering

Correlation vs Convolution

Correlation is the process of moving a filter mask over the image and computing the sum of products at each location

The mechanics of convolution are the same, except that the filter is first rotated by 180°.



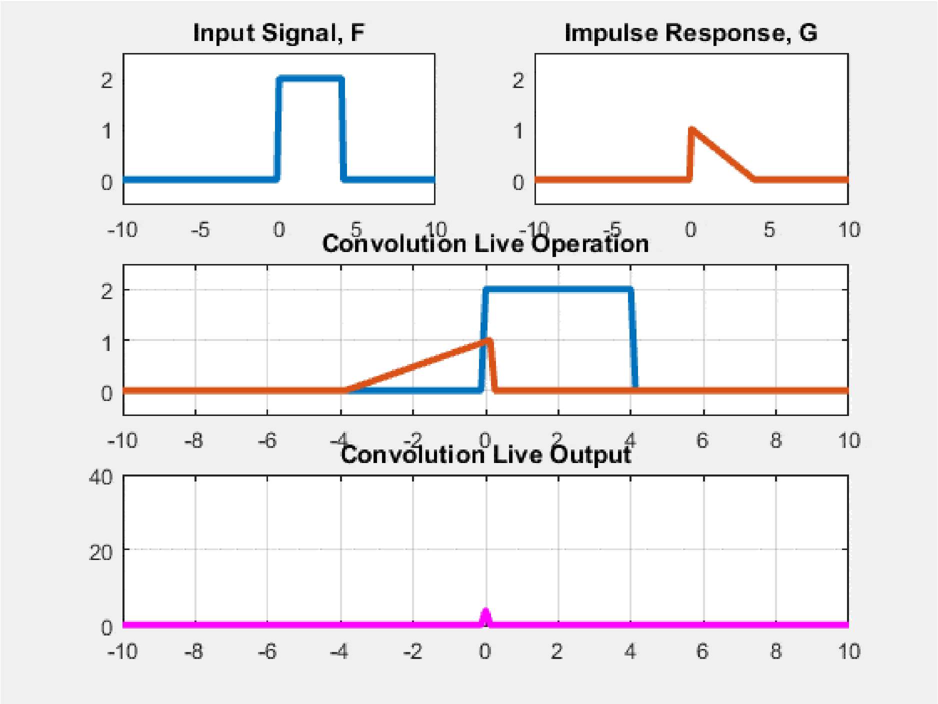
Fundamentals of Spatial Filtering

Correlation vs Convolution

Correlation is the process of moving a filter mask over the image and computing the sum of products at each location

The mechanics of convolution are the same, except that the filter is first rotated by 180°.

Image Convolution mask



Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w	1	2	3					

g

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w	1	2	3					
x=2								
g		0						

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w		1	2	3				
x=3								
g		0	3					

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w			1	2	3			
x=4								
g		0	3	2				

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w				1	2	3		
x=5								
g		0	3	2	1			

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w					1	2	3	
x=6								
g		0	3	2	1	0		

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w						1	2	3
x=7								
g		0	3	2	1	0	0	

Fundamentals of Spatial Filtering

Correlation

f	0	0	0	1	0	0	0	0
w	1	2	3					
g		0	3	2	1	0	0	

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}	3	2	1					

g

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}	3	2	1					
x=2								
g		0						

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}		3	2	1				
$x=3$								
g		0	1					

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}			3	2	1			
x=4								
g		0	1	2				

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}				3	2	1		
x=5								
g		0	1	2	3			

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}					3	2	1	
x=6								
g		0	1	2	3	0		

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}						3	2	1
$x=7$								
g		0	1	2	3	0	0	

Fundamentals of Spatial Filtering

Convolution

f	0	0	0	1	0	0	0	0
w^{flip}	3	2	1					
g		0	1	2	3	0	0	

Fundamentals of Spatial Filtering

Correlation vs Convolution

f	0	0	0	1	0	0	0	0
w	1	2	3					

g_{corr}		0	3	2	1	0	0
g_{conv}		0	1	2	3	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

0	0	0	0	0
0	9			0
0				0
0				0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

0	0	0	0	0
0	9	8		0
0				0
0				0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

0	0	0	0	0
0	9	8	7	0
0				0
0				0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

0	0	0	0	0
0	9	8	7	0
0	6			0
0				0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

f(x,y)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

w(x,y)

1	2	3
4	5	6
7	8	9

g(x,y)

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Convolution

$f(x,y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w_{flip}(x,y)$

9	8	7
6	5	4
3	2	1

$g(x,y)$

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

Correlation

$$w(x, y) \star f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

Convolution

$$w(x, y) \bigstar f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

Fundamentals of Spatial Filtering

Correlation vs Convolution (2D)

- In practice, we frequently use a symmetric filter
- Such as,

1/16

1	2	1
2	4	2
1	2	1

1/273

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

1/1003

0	0	1	2	1	0	0
0	3	13	22	13	3	0
1	13	59	97	59	13	1
2	22	97	159	97	22	2
1	13	59	97	59	13	1
0	3	13	22	13	3	0
0	0	1	2	1	0	0

- So, convolution and correlation operation are not different in many cases

Fundamentals of Spatial Filtering

Vector representation of linear filtering

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3x3 filter mask

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 \\ &= \sum_{k=1}^{9} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$

Where, z is corresponding pixel intensity

Fundamentals of Spatial Filtering

Generating spatial filter mask

- coefficients are selected based on what the filter is supposed to do
- Ex> replace the pixels in an image by the average of a 3x3 neighbor

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

- Ex> spatial mask based on continuous function

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Smoothing spatial filters

Smoothing Linear Filters

- Reduces sharp transitions but has the undesirable side effect that they blur edges
- Used for blurring and for noise reduction
  - prior to object extraction and bridging of small gaps in lines or curves

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

normalizing

weigh the center pt the highest

Smoothing spatial filters

Smoothing Linear Filters

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

weigh the center pt the highest

normalizing

box filter

whose all coeff areequal

weighted averaging filter

it reduces blurring in the smoothing process

$$R = \frac{1}{9} \sum_{i=1}^{9} z_i$$

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)}$$

the sum of the mask coeff

Smoothing spatial filters

Smoothing Linear Filters

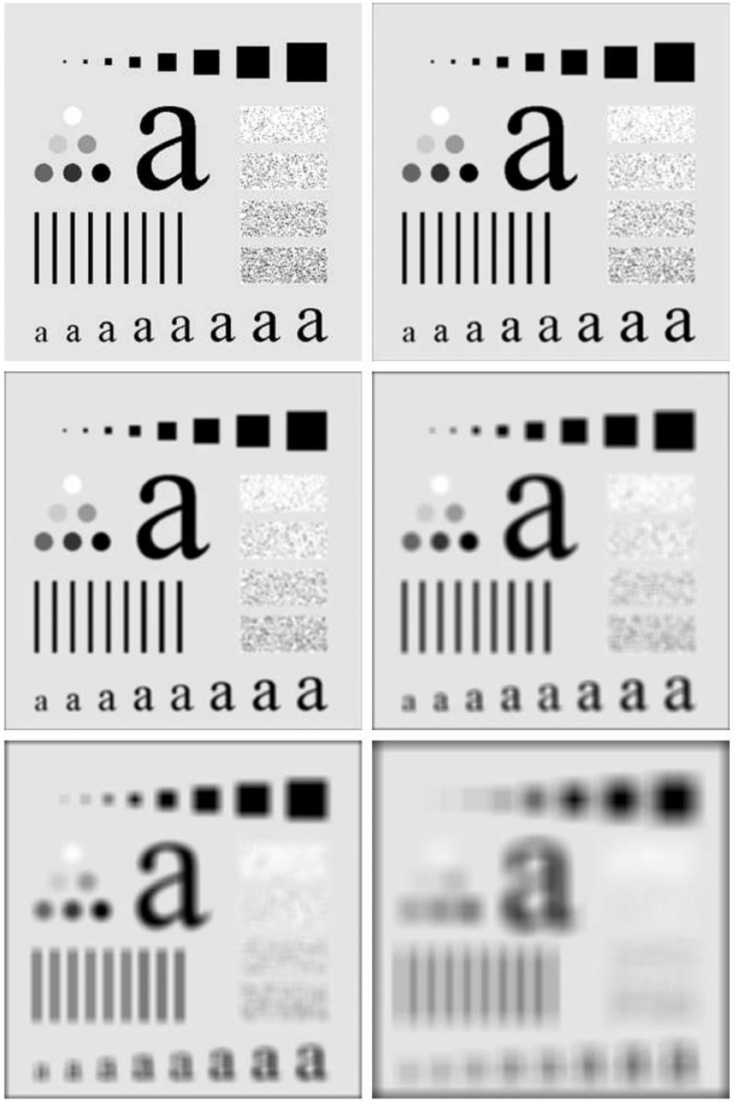
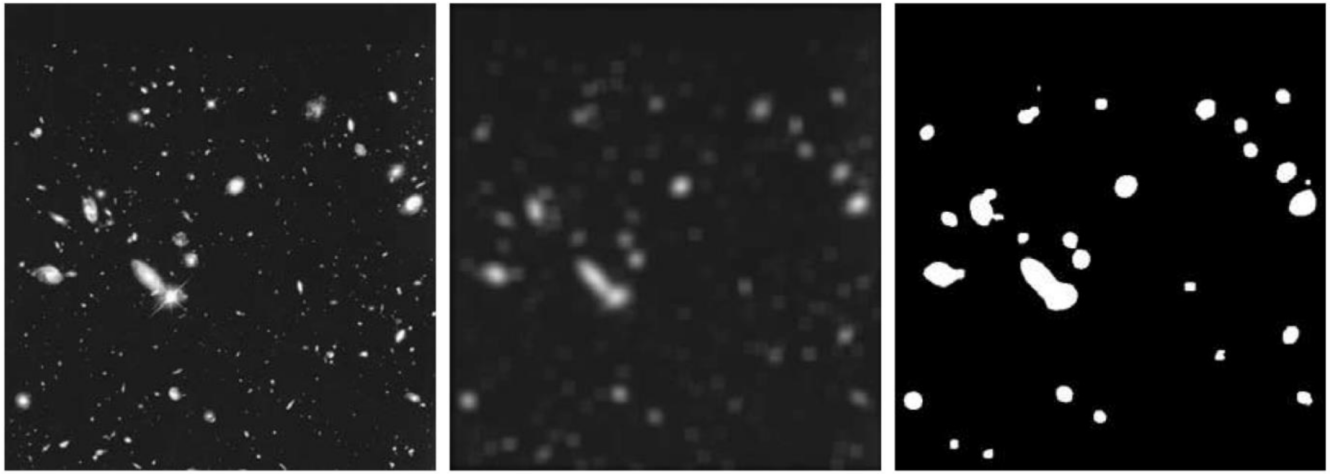


FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Smoothing spatial filters

Smoothing Linear Filters

- Ex) blurring coupled with thresholding
  - Blurring:
    The intensity of small objects blends with the background and larger objects become “bloblike”. And thus easy to detect.
  - Choose:
    the size of mask ≈ size of the objects that will be blended with the background



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing spatial filters

Order-statistic (Nonlinear) Filters

- Order-statistic filter are nonlinear spatial filter whose response is based on ordering (ranking)

- Ex) Median filter for smoothing impulse noise (Salt & pepper noise)
  - *popular because* they provide excellent noise reduction capabilities with less blurring than linear spatial filter

  - Suppose $A = \{a_1, a_2, \ldots, a_K\}$ are the pixel values in a neighborhood of a given pixel with $a_1 \leq a_2 \leq \ldots \leq a_K$. Then

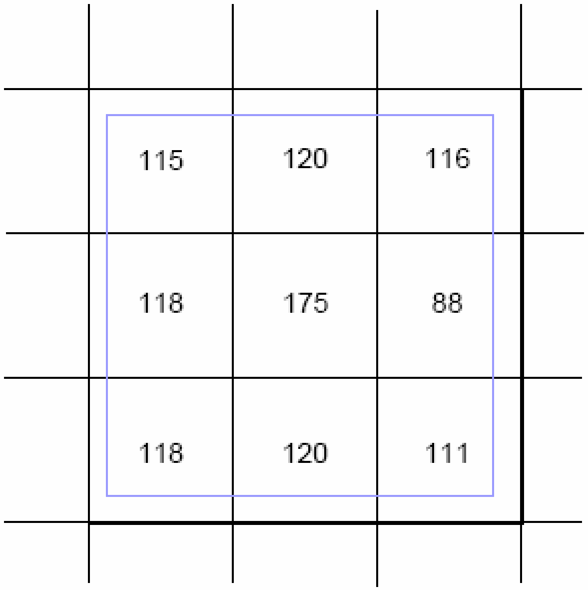
$$median(A) = \begin{cases} a_{K/2} & \text{for K even} \\ a_{(K+1)/2} & \text{for K odd} \end{cases}$$

  - Note: Median of a set of values is the "center value," after sorting.

Order-statistic (Nonlinear) Filters

- Median filtering example

$$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{median}[g(s,t)]$$

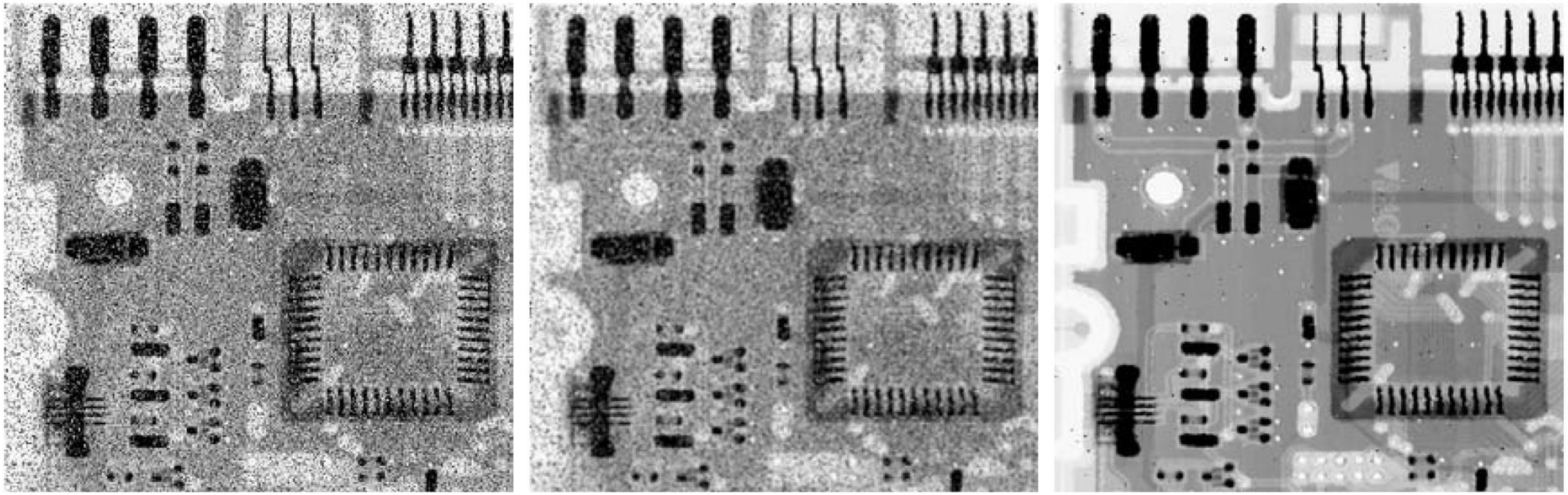


- Example:
  - Sort: 88, 111, 115, 116, 118, 118, 120, 120, 175.
  - Median = 118.

Smoothing spatial filters

Order-statistic (Nonlinear) Filters

- Median filtering example



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Smoothing spatial filters

DLIP_practice5.ImageFiltering1.ipynb

Sharpening spatial filter

Foundation

Blurring ⟷ Sharpening

Spatial averaging
(Integration)

Spatial differentiation
(Derivatives)

- 1st order derivative of 1-d f(x):

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Can be used to extract edge component

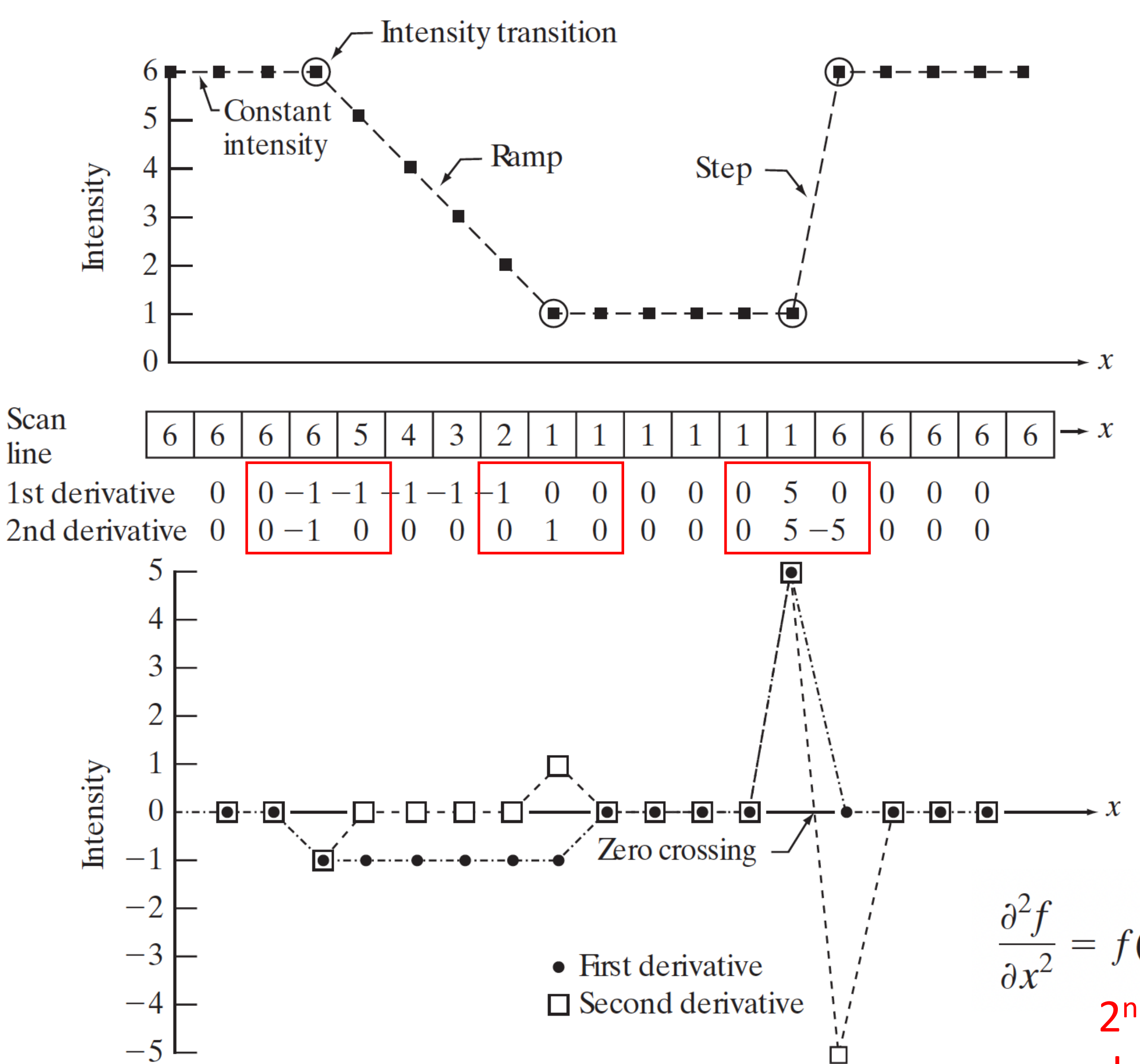
- 2nd order derivative of 1-d f(x)

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Sharpening spatial filter

Foundation

Edge(fine detail) → Where the intensity profile changes rapidly



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

2nd derivative enhance fine detail much better

Using the 2nd derivative for image sharpening (The Laplacian)

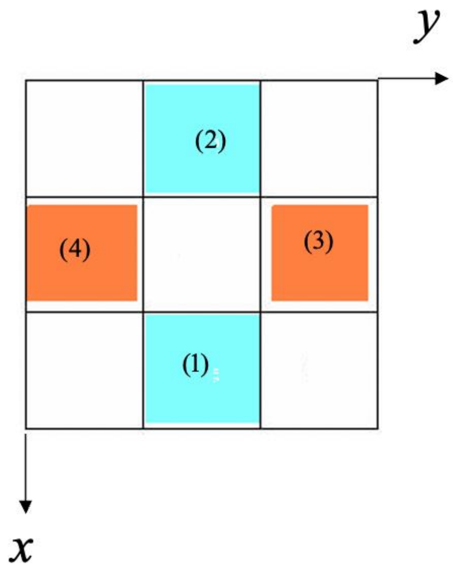
- Laplacian ← isotropic derivative operator
  - isotropic filter :
    its response is independent of the direction (i.e., rotation invariant)
  - Laplacian for an image *f(x,y)*

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

where,

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$



  - *Thus.*

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y) \tag{3.6-6}$$

Sharpening spatial filter

Using the 2nd derivative for image sharpening **(The Laplacian)**

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y) \quad (3.6\text{-}6)$$

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37
(a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Sharpening spatial filter

Using the 2nd derivative for image sharpening **(The Laplacian)**

gives an isotropic filter for increments of 90^0

gives an isotropic filter for increments of 45^0

Laplacian masks

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Sharpening→

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \textit{if the center coeff is negative} \\ f(x, y) + \nabla^2 f(x, y) & \textit{if the center coeff is positive} \end{cases}$$

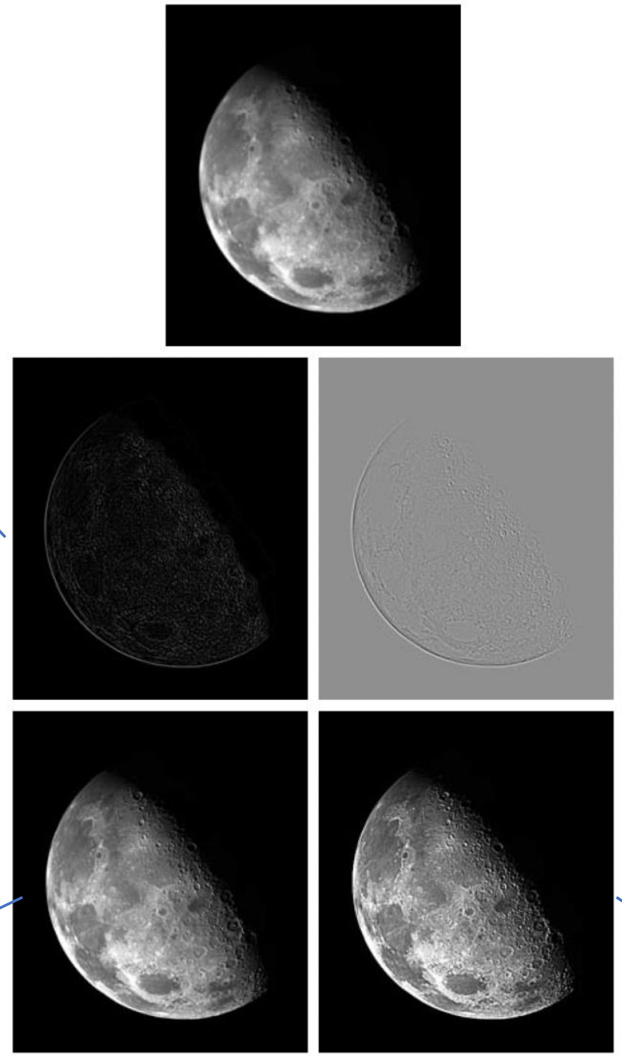
Sharpening spatial filter

Using the 2nd derivative for image sharpening (The Laplacian)

- *Ex)*

$\nabla^2 f(x,y)$

0	1	0
1	−4	1
0	1	0



$\nabla^2 f(x,y)$

1	1	1
1	−8	1
1	1	1

$f(x,y) - \nabla^2 f(x,y)$

$f(x,y) - \nabla^2 f(x,y)$

a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

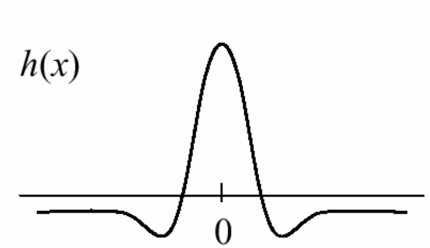
Sharpening spatial filter

Using the 2nd derivative for image sharpening **(The Laplacian)**

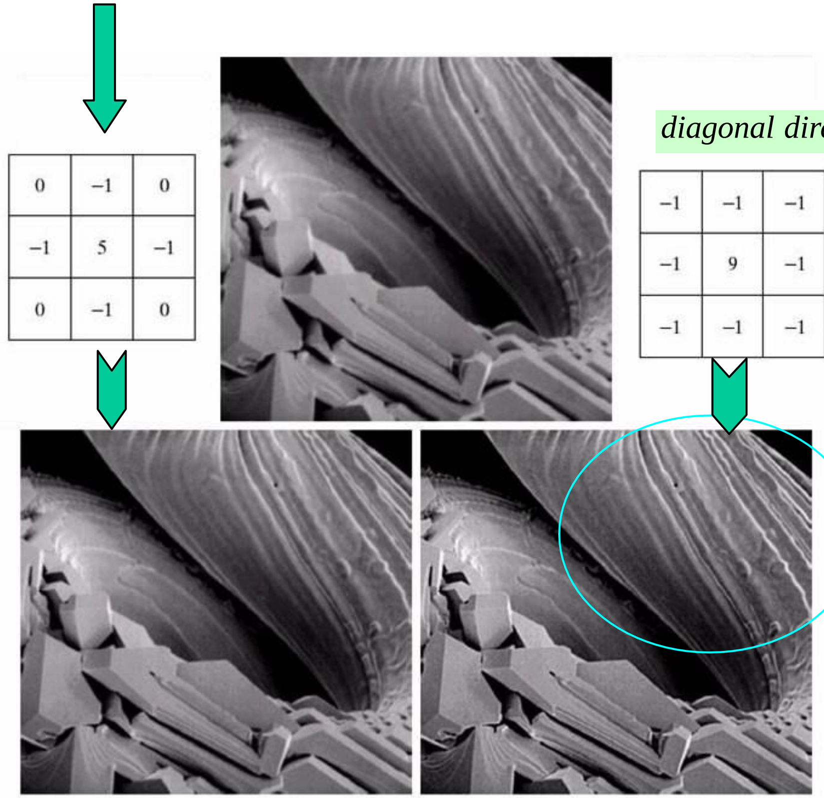
- *Ex2)*

$$g(x,y) = f(x,y) - \nabla^2 f$$

$$\therefore \quad = 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$



diagonal direction



diagonal components are sharper than those in (d)

a b c
d e

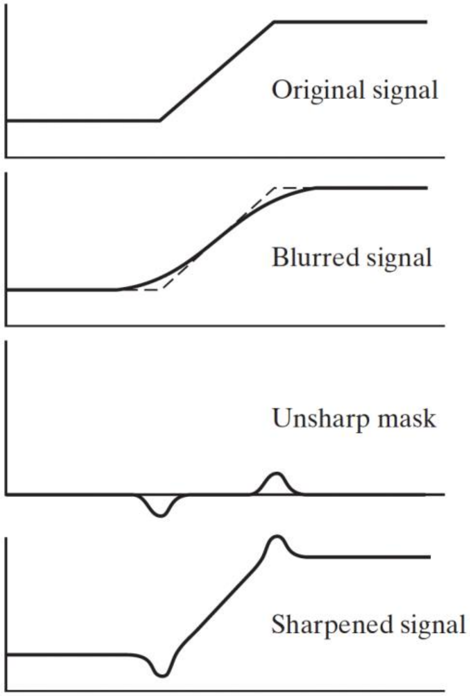
FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Sharpening spatial filter

Unsharp masking and Highboost Filtering

- Image sharpening by subtracting smoothed version of an image from original image

  1. Blur the original image
  2. Subtract the blurred image from the original → result = edge component
  3. Add the 2 to the original



$f(x,y) + k \cdot g(x,y)$

If k=1, Unsharp masking

If k > 1, Highboost filtering

Sharpening spatial filter

Unsharp masking and Highboost Filtering

- Ex)

size 5×5 with $\sigma = 3$

a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

Sharpening spatial filter

Using 1st order derivative for image sharpening (The Gradient)

- Gradient

$$\nabla f \equiv \text{grad}(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

(Non-isotropic linear operator)

- Magnitude

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2} \approx \boxed{|g_x| + |g_y|}$$

simple to implement, *isotropic for multiples of* 90^0

(Isotropic, not linear operator)

Sharpening spatial filter

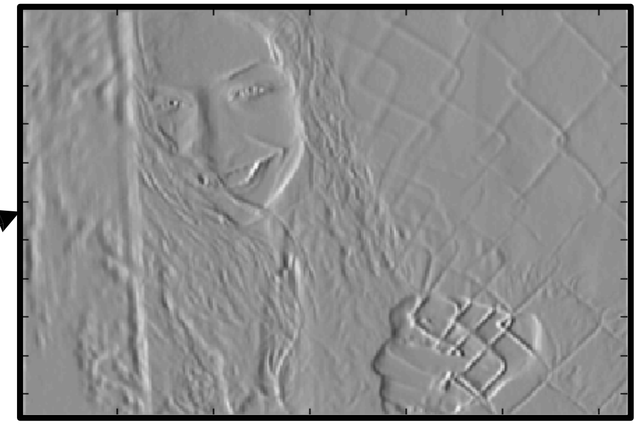
Using 1st order derivative for image sharpening (The Gradient)

- Ex)

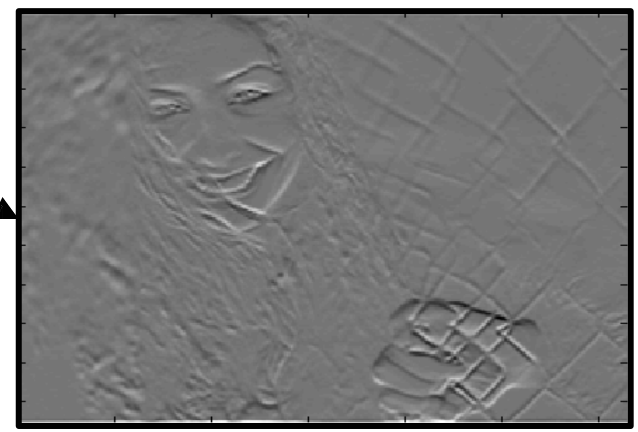
$$\frac{\partial f}{\partial x}$$

Vertical edge enhanced

Vertical edge



$$\frac{\partial f}{\partial y}$$

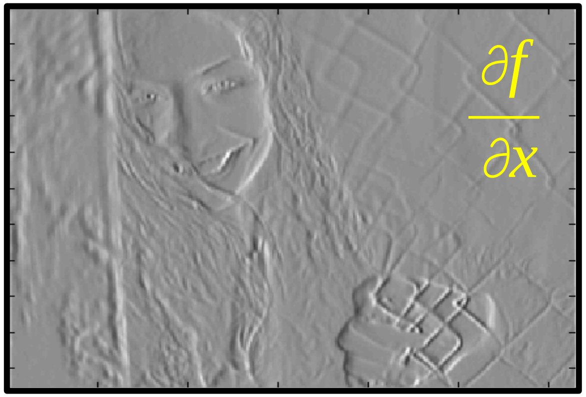


Horizontal edge

Sharpening spatial filter

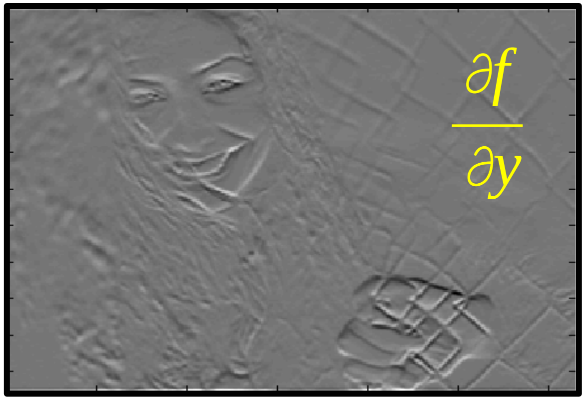
Using 1st order derivative for image sharpening (The Gradient)

- Ex)



$$: \sqrt{\frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2}$$

final edge



Sharpening spatial filter

Sobel filter (1st derivative)

$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$\begin{aligned} &\nabla f(x,y) \\ &= |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \\ &+ |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \end{aligned}$$

$$h_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad h_2 = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Vertical edge Horizontal edge

the coeff ' s sum is to zero

$\Rightarrow$ *gives a response of* 0 *in an area of const gray level*

(as expected of *a derivative operator*)

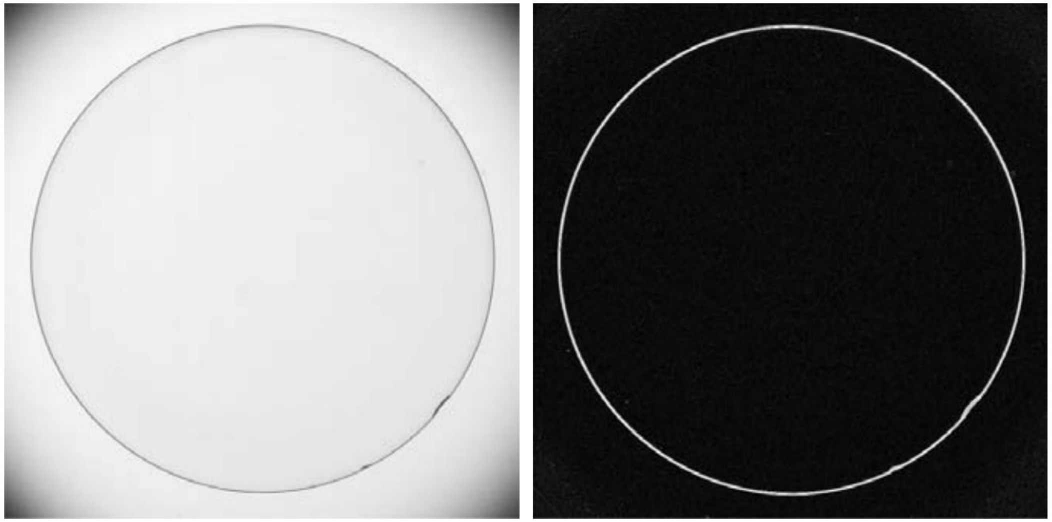
Sharpening spatial filter

Sobel filter (1st derivative)

- Ex) use of gradient for edge enhancement

a b

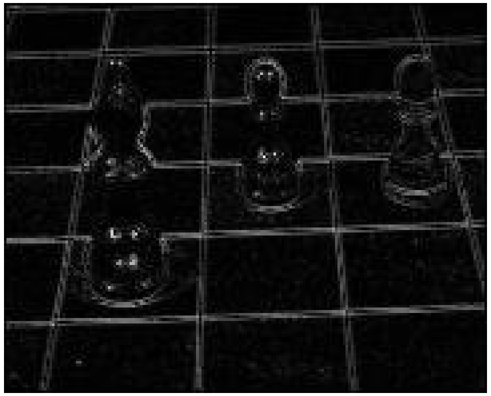
FIGURE 3.42
(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Pete Sites, Perceptics Corporation.)



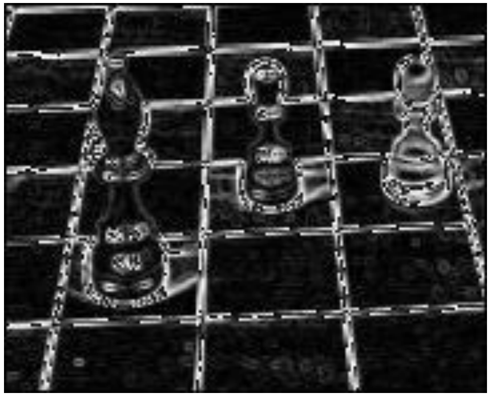
Sharpening spatial filter

Laplacian vs Sobel

Laplacian



Sobel

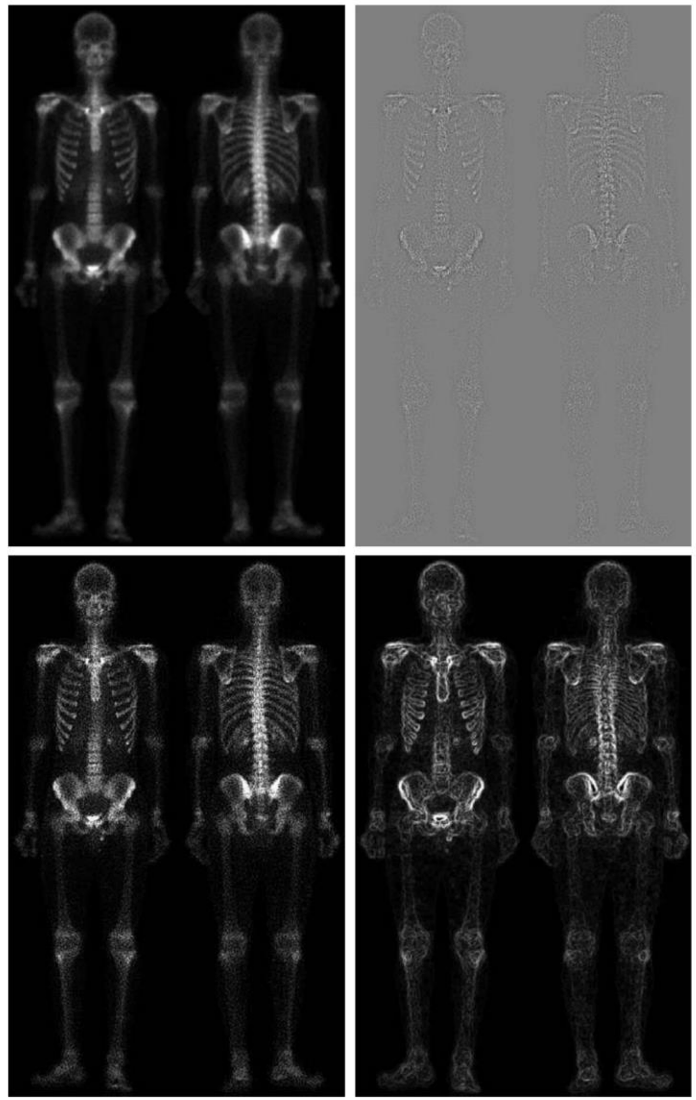


Sharpening spatial filter

Combining spatial enhancement methods

a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).



−1	−1	−1
−1	8	−1
−1	−1	−1

narrow dynamic range
& high noise

the noises are also enhanced !!

scaled only for display

$$g(x, y) = f(x, y) + \nabla^2 f$$

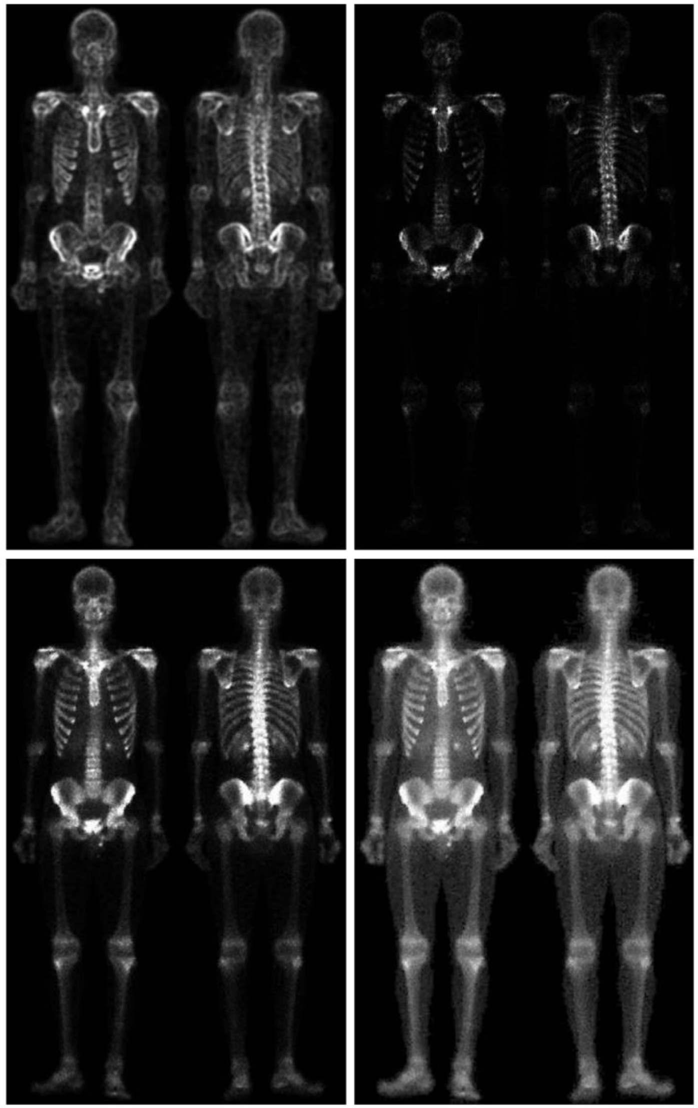
⇒ *noisy sharpened image*

includes the noises

→ median filter for removing theses noises
is unacceptable in medical I.P
since it is a non − linear process

Sharpening spatial filter

Combining spatial enhancement methods



e f
g h

FIGURE 3.43 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

dark !

$\gamma = 0.5, c = 1$

Sharpening spatial filter

DLIP_practice6.ImageFiltering2.ipynb