영상처리
- Intensity Transformations and Spatial Filtering -

Histogram Processing

Histogram

- Histogram of digital image: occurrence of pixel intensity (밝기값의 발생 빈도)

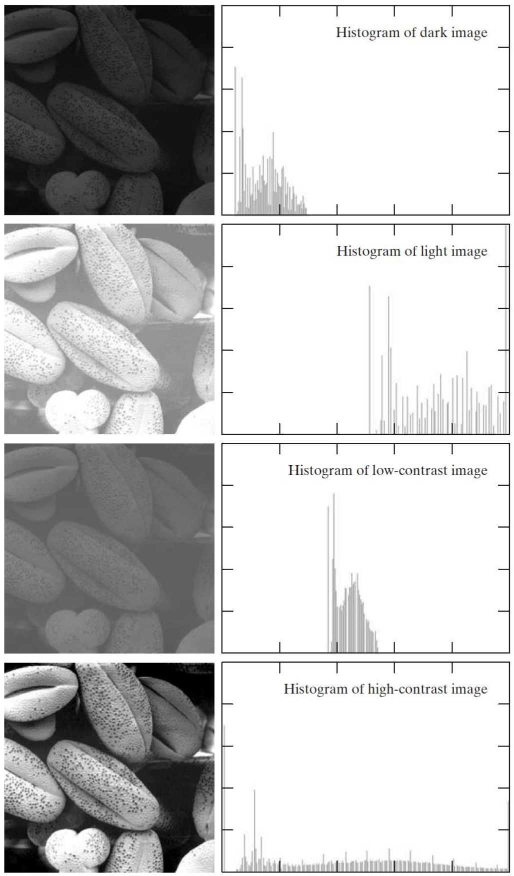


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

normalized histogram

= ***probability of occurrence of r_k***

$$h(r_k) = n_k$$

$$p(r_k) = n_k / n \quad \text{for } k = 0,1,\ldots,L-1$$

$$\text{where } r_k \text{ the } k^{th} \text{ gray level}$$

$$n_k \text{ the \# of pixels having } r_k$$

$$n \text{ the total \# of pixels}$$

narrow and

centered toward the middle of the gray scale

(*dull and washed-out gray look*)

broad and *nearly uniformly distributed*

(*abundant gray-level detail and high dynamic range*)

Histogram Processing

Histogram

- Histogram
  - 여러 공간 도메인 프로세싱의 기본
  - Histogram manipulation
    → can be used for image enhancement
  - 여러 유용한 통계값을 제공해줌
  - 압축, segmentation가능
  - Histogram is simple to calculate (계산이 쉬움)
    → real-time image processing

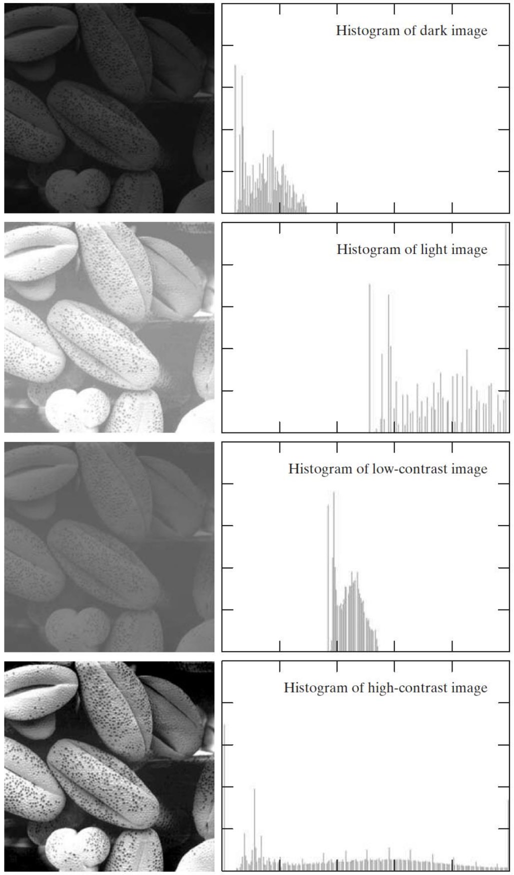


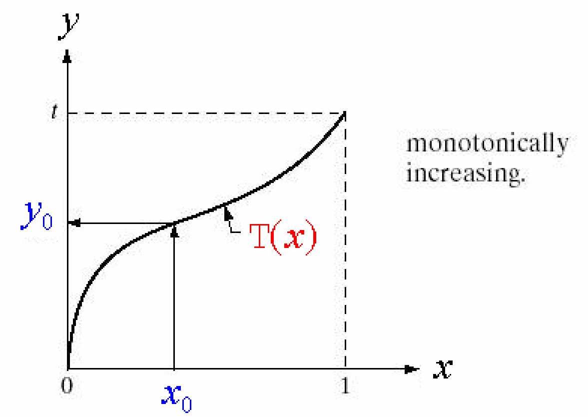
FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Processing

Review: Monotonic transformation of a continuous Random variable

- x, y: random variable

$$Y = T(X)$$



- T: monotonically increasing
  - If $T(x_1) < T(x_2)$ for any $x_1 < x_2$
- T: monotonically decreasing
  - If $T(x_1) > T(x_2)$ for any $x_1 < x_2$

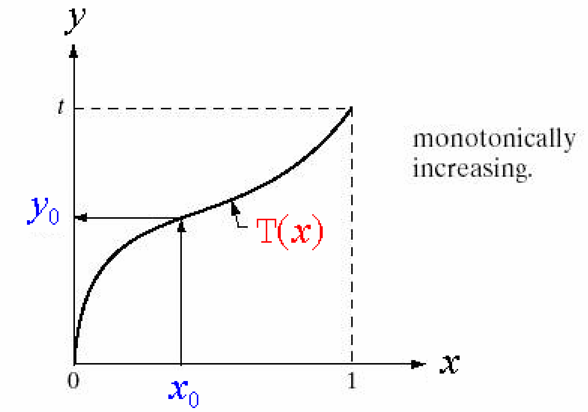
- $y_0 = T(x_0)$ *or* $x_0 = T^{-1}(y_0)$

Histogram Processing

Review: Monotonic transformation of a continuous Random variable

- When $f_X(x)$ and $F_X(x)$ are known, we can find $f_Y(y)$ and $F_Y(y)$
  - For monotonically increasing T
  - $F_Y(y_0) = P\{Y \le y_0\} = P\{X \le x_0\} = F_X(x_0)$

$$\therefore \int_{-\infty}^{y_0} f_Y(y)dy = \int_{-\infty}^{x_0} f_X(y)dy$$



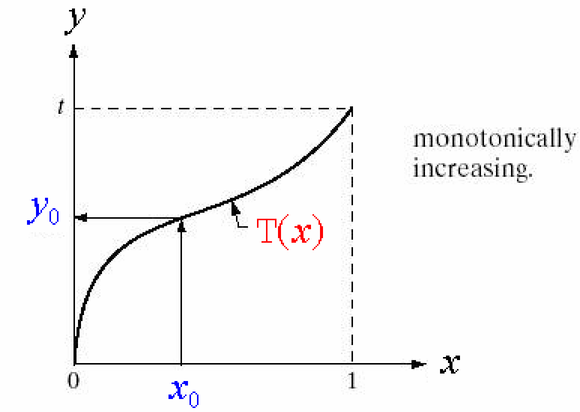
  - Differentiating both sides w.r.t y_0
  - $f_Y(y_0) = f_X(x_0) \cdot \frac{dx_0}{dy_0}$
  - $\frac{dx_0}{dy_0} > 0$, because monotonically increasing
  - So, $f_Y(y) = f_X(x) \cdot \frac{dx}{dy}$

Histogram Processing

Review: Monotonic transformation of a continuous Random variable

- When $f_X(x)$ and $F_X(x)$ are known, we can find $f_Y(y)$ and $F_Y(y)$
  - For monotonically decreasing T
  - $F_Y(y_0) = P\{Y \leq y_0\} = P\{X \geq x_0\} = 1 - F_X(x_0)$

$$\therefore \int_{-\infty}^{y_0} f_Y(y)dy = 1 - \int_{-\infty}^{x_0} f_X(y)dy$$



  - Differentiating both sides w.r.t y_0 gives:
  - $f_Y(y_0) = -f_X(x_0) \cdot \frac{dx_0}{dy_0}$
  - $\frac{dx_0}{dy_0} < 0$, because monotonically decreasing
  - So, in general $f_Y(y) = f_X(x) \cdot \left|\frac{dx}{dy}\right|$

Histogram Processing

Histogram equalization (히스토그램 이퀄라이제이션, 평활화)

- Automatically determines a transformation function that seeks to produce an output image that has a **uniform histogram (automatic enhancement)**

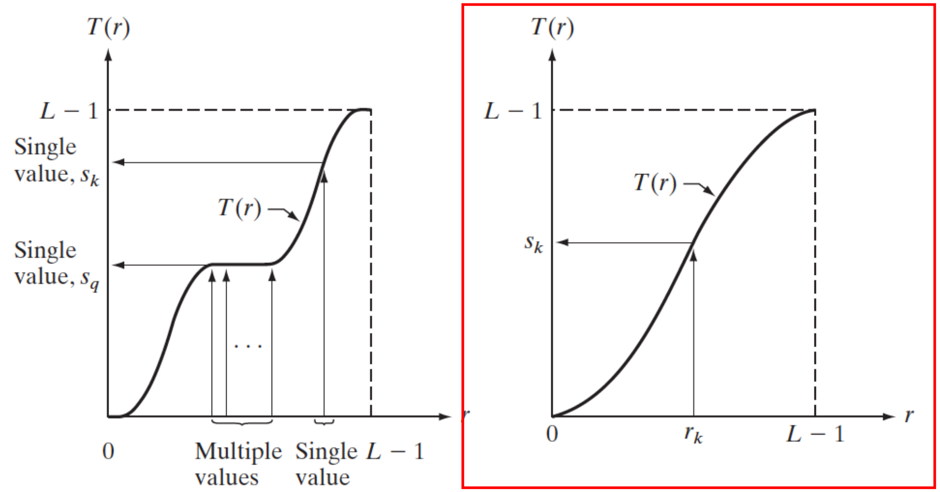


FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

†Recall that a function $T(r)$ is *monotonically increasing* if $T(r_2) \geq T(r_1)$ for $r_2 > r_1$. $T(r)$ is a *strictly monotonically increasing* function if $T(r_2) > T(r_1)$ for $r_2 > r_1$. Similar definitions apply to monotonically decreasing functions.

r: gray levels to be enhanced

$S=T(r)$, $0 \leq r \leq L\text{-}1$

Assumption of T:

1) Strictly monotonically increasing
2) Single value
   → Its inverse exists

$r = T^{-1}(S)$ and

$$P_S(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

Histogram Processing

Histogram equalization

If transformation T(r)=CDF of r,

Note.
r = intensity values in histogram
s = 0~1 (CDF value)

$$s = T(r) = \int_0^r P_r(w)dw$$

$$\rightarrow \frac{ds}{dr} = P_r(r)$$

With equation,

$$P_S(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

$$\rightarrow P_s(s) = P_r(r)\left|\frac{1}{P_r(r)}\right| = 1$$

$P_s(s)$ is uniform PDF!!

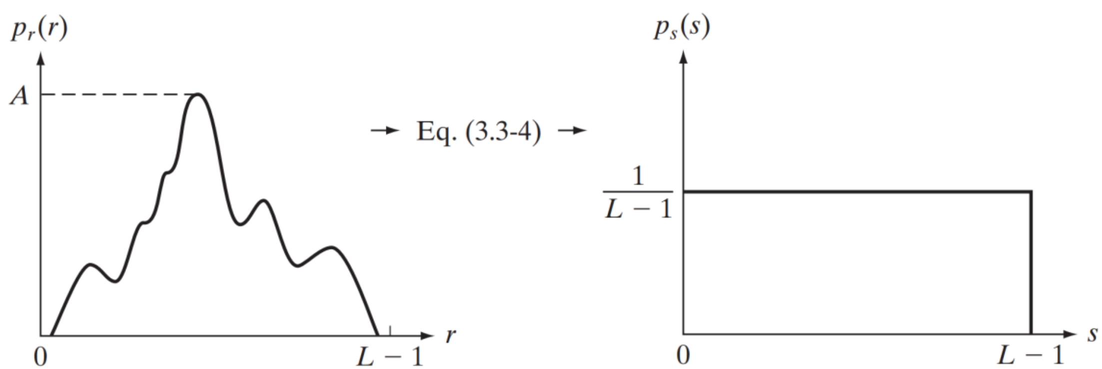
Histogram Processing

Histogram equalization

If transformation T(r)=CDF of r,

Note.
r = intensity values in histogram
s = 0~L-1 (CDF value)

$$s = T(r) = (L-1)\int_0^r P_r(w)dw$$



a b

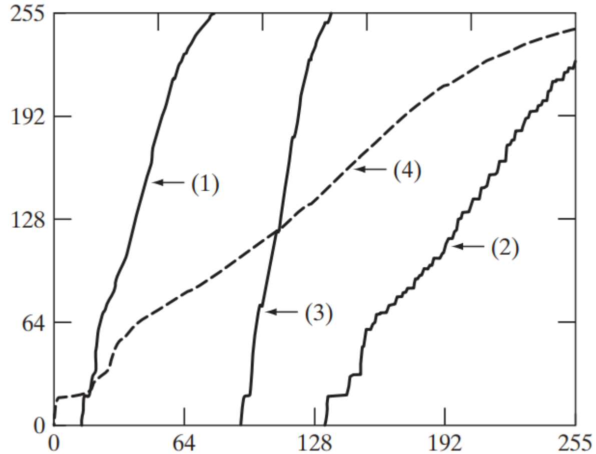
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Processing

Histogram equalization

Y-axis: s (new intensity)

FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).



X-axis: r (input intensity)

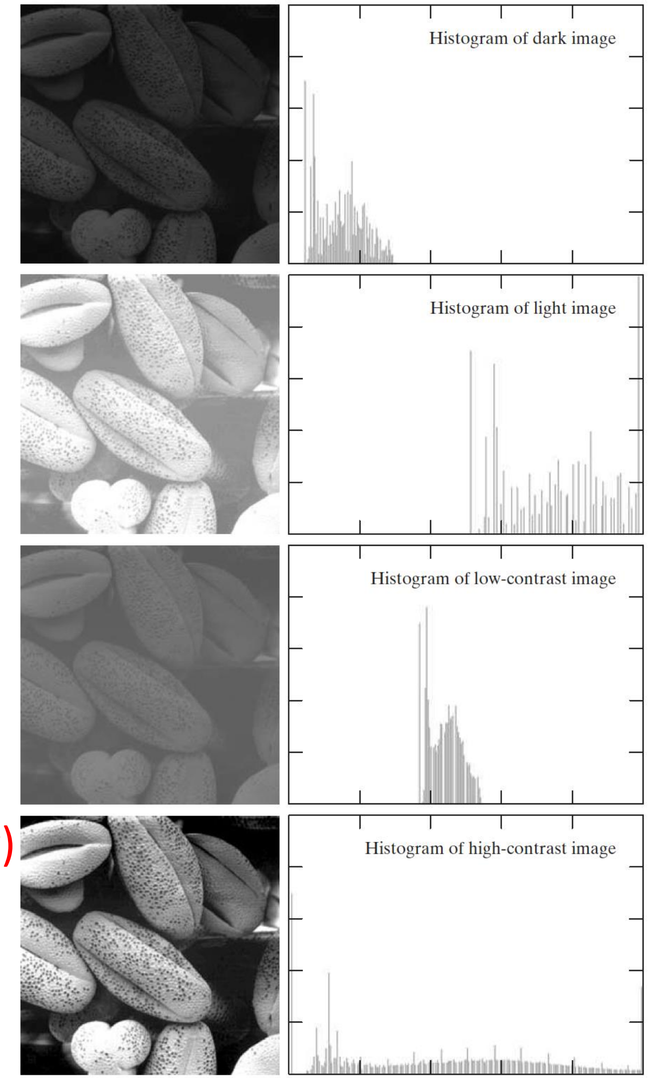


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Processing

Histogram equalization

Before / After

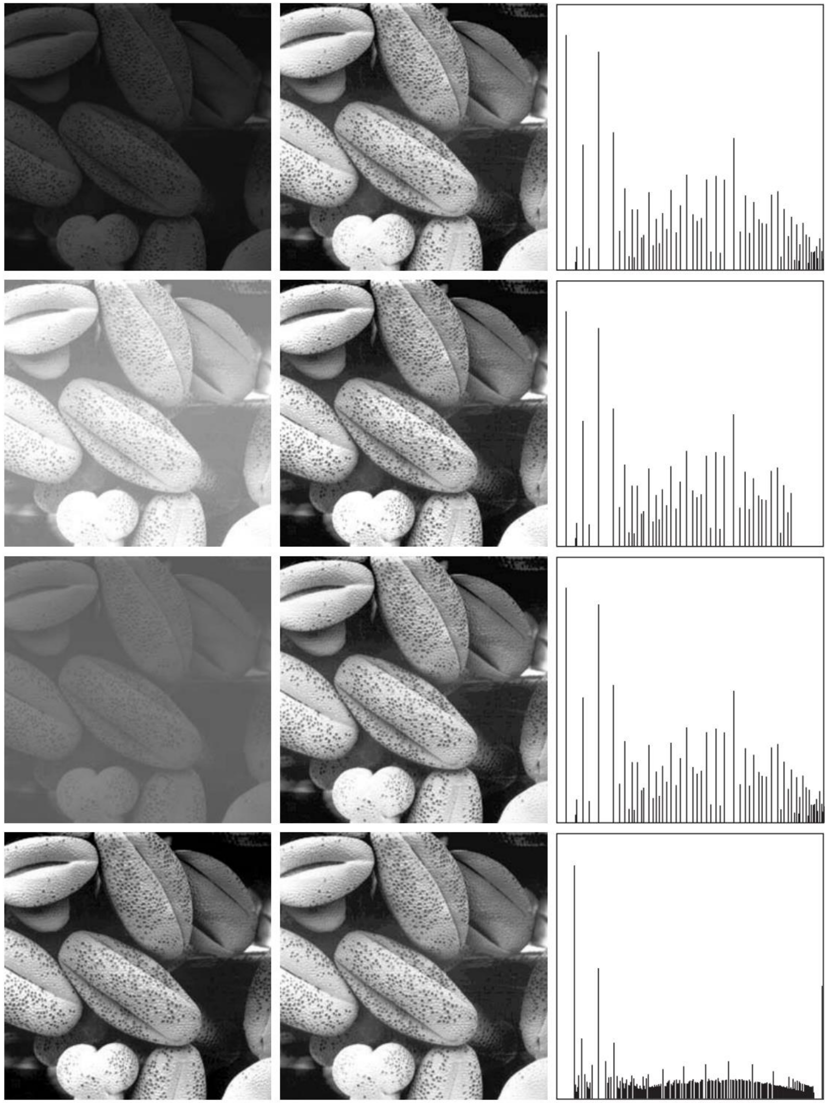


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Histogram Processing

Histogram equalization

Example) 64x64 3-bit image (Dynamic range 0~7)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$$\begin{aligned} s_2 &= T(r_2) = 4.55 \\ s_3 &= T(r_3) = 5.67 \\ s_4 &= T(r_4) = 6.23 \\ s_5 &= T(r_5) = 6.65 \\ s_6 &= T(r_6) = 6.86 \\ s_7 &= T(r_7) = 7.00 \end{aligned}$$

Histogram Processing

Histogram equalization

Example) 64x64 3-bit image (Dynamic range 0~7)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
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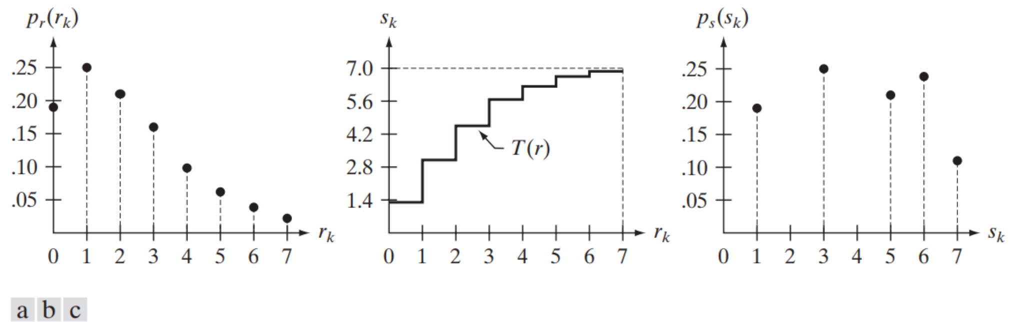


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$

$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$

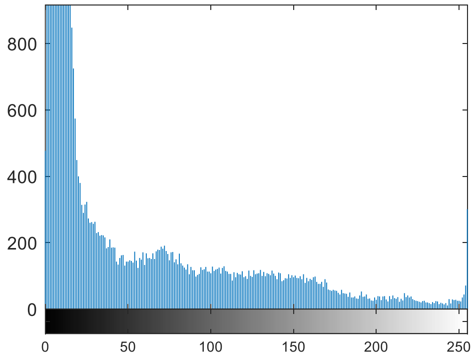
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$

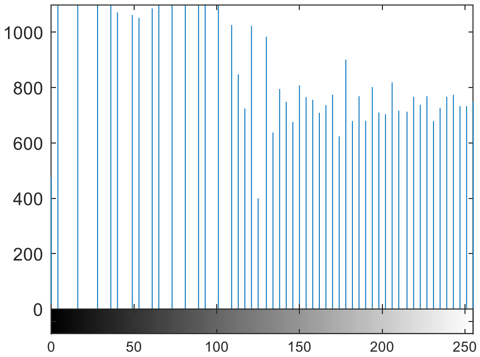
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

Histogram Processing

Histogram equalization

Example)

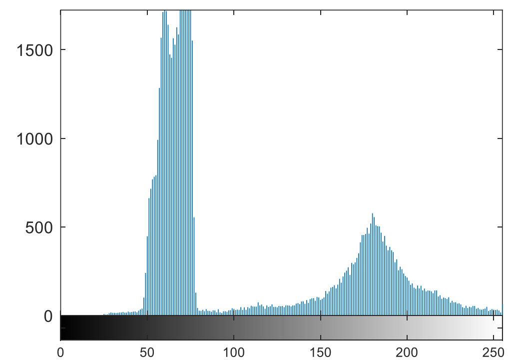


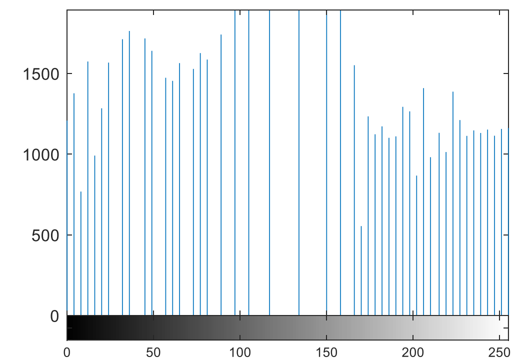


Histogram Processing

Histogram equalization

Example)





- Histogram equalization 의 결과는 항상 바람직한 결과물을 주지는 않음, 특히 원본 histogram이 매우 좁을 때.

Histogram Processing

Practice

DLIP_practice3.HistogramProcessing.ipynb

Histogram Processing

Histogram matching (specification)

$$r \xrightarrow{\;\;\boldsymbol{T}\;\;} s \xrightarrow{\;\;\boldsymbol{G^{-1}}\;\;} z$$

Histogram-equalized image   Histogram-matched image

- Histogram equalization 은 무조건 uniform distribution에 밝기값의 분포를 맞추어 버림

- 이것이 항상 바람직한 것은 아님. 대신 이미지 histogram의 모양을 우리가 원하는 histogram모양으로 맞추는 것이 필요할 수 있음.

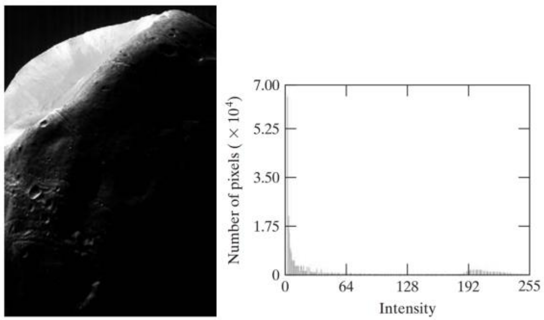
- This technique is called **histogram matching (specification)**.

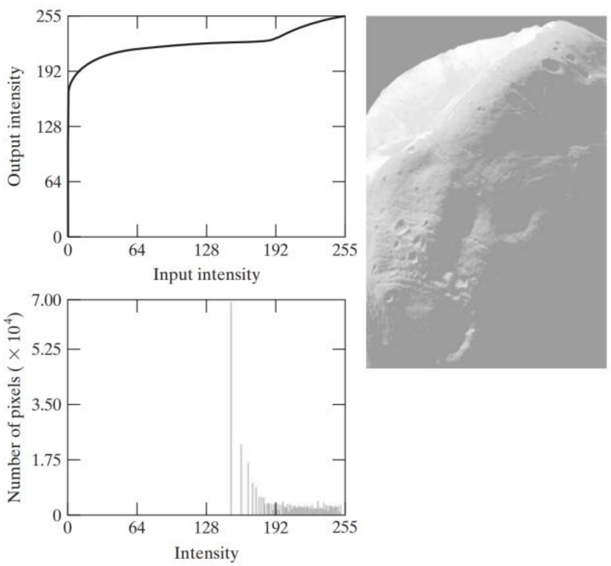
Histogram Processing

Histogram matching (specification)

a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)



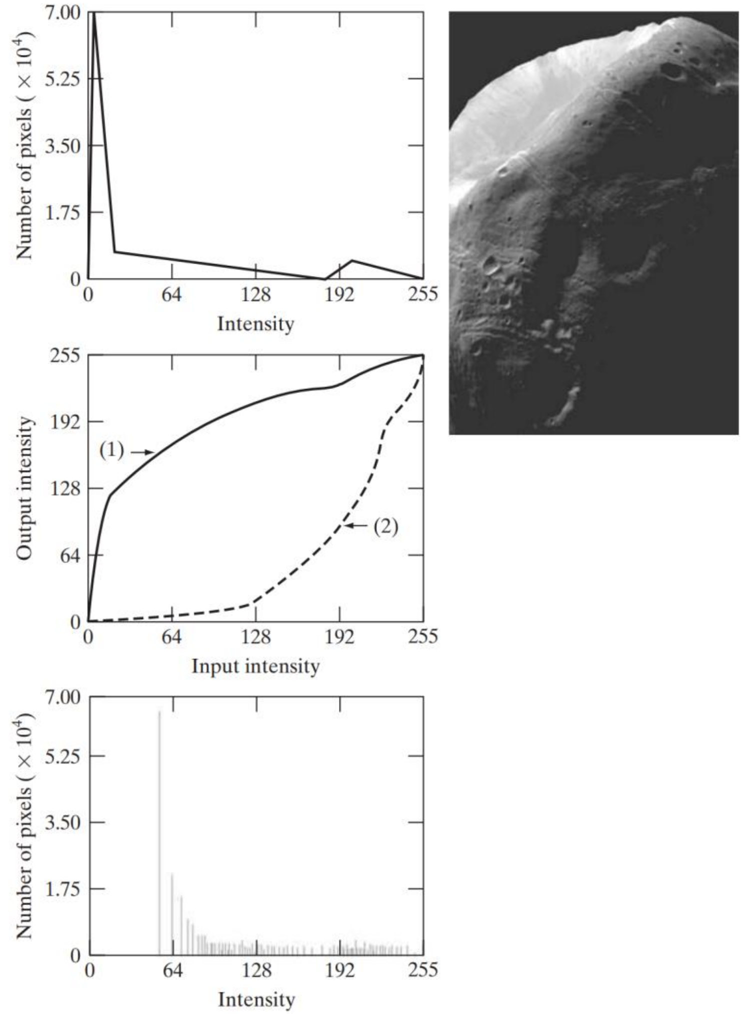


a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).



Histogram Processing

Histogram matching (specification)

$$r \xrightarrow{T} s \xrightarrow{G^{-1}} z$$

- Consider continuous gray level r and z
  - PDF of r: $P_r(r)$, PDF of z: $P_z(z)$

$$s = T(r) = \int_0^r P_r(w)dw$$

→ Histogram equalization

$$G(z) = \int_0^z P_z(w)dw = s$$

→ Pre-defined histogram to equalized histogram

$$\therefore G(z) = T(r)$$

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

Histogram Processing

Histogram matching (specification)

$$r \xrightarrow{T} s \xrightarrow{G^{-1}} z$$

- Histogram matching algorithm

  1. Find $T(r)$
  2. Find $G(z)$
  3. Find $G^{-1}(z)$
  4. Obtain the output image by applying $z = G^{-1}[T(r)]$ to all the pixels in the input image

Histogram matching (specification)

- Example

a b
c d

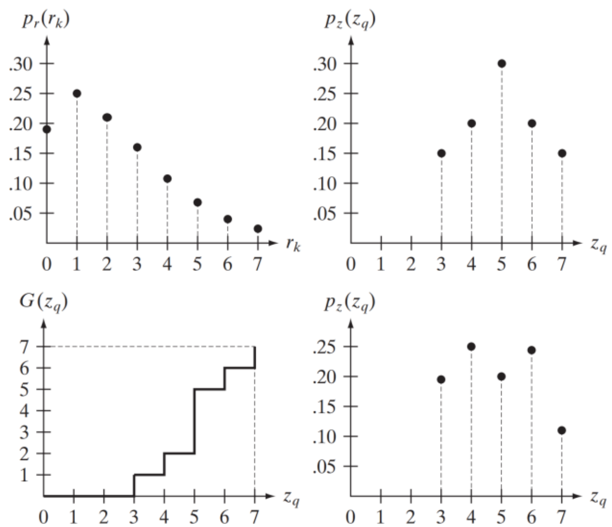


FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$
$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$
$$\boxed{G(z_3) = 1.05} \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$

0.15x7

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

Histogram Processing

Histogram matching (specification)

- Example

T

$$s_0 = 1.33 \rightarrow 1 \quad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \quad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \quad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \quad s_7 = 7.00 \rightarrow 7$$

G

$$G(z_0) = 0.00 \rightarrow 0 \quad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_1) = 0.00 \rightarrow 0 \quad G(z_5) = 4.55 \rightarrow 5$$
$$G(z_2) = 0.00 \rightarrow 0 \quad G(z_6) = 5.95 \rightarrow 6$$
$$G(z_3) = 1.05 \rightarrow 1 \quad G(z_7) = 7.00 \rightarrow 7$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
2	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Histogram matching

r: 0→ z: 3
r: 1→ z: 4
r: 2→ z: 5
r: 3→ z: 6
r: 4→ z: 6
r: 5→ z: 7
r: 6→ z: 7
r: 7→ z: 7

Histogram Processing

Practice

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