

# Package ‘qfa’

January 12, 2023

**Type** Package

**Title** Quantile-Frequency Analysis (QFA) of Time Series

**Version** 1.2

**Date** 2023-01-06

**Maintainer** Ta-Hsin Li <thl@us.ibm.com>

**Description** Quantile-frequency analysis (QFA) of univariate or multivariate time series based on the method of trigonometric quantile regression. See Li, T.-H. (2012) “Quantile periodograms”, *Journal of the American Statistical Association*, 107, 765–776, <doi:10.1080/01621459.2012.682815>; Li, T.-H. (2014) *Time Series with Mixed Spectra*, CRC Press, <doi:10.1201/b15154>; Li, T.-H. (2022) “Quantile Fourier transform, quantile series, and nonparametric estimation of quantile spectra”, <doi:10.48550/arXiv.2211.05844>.

**Depends** R (>= 3.5)

**Imports** RhpcBLASctl,

doParallel,  
fields,  
foreach,  
mgcv,  
nlme,  
parallel,  
quantreg,  
splines,  
stats,  
graphics,  
colorRamps

**License** GPL (>=2)

**URL** <https://www.r-project.org>, <https://github.com/IBM/qfa>

**NeedsCompilation** yes

**Encoding** UTF-8

**RoxygenNote** 7.2.1

## R topics documented:

qdft . . . . .	2
qdft2qacf . . . . .	3

qdft2qper . . . . .	3
qdft2qser . . . . .	4
qfa.plot . . . . .	5
qkl.divergence . . . . .	6
qper2qcoh . . . . .	6
qsmooth.qdft . . . . .	7
qsmooth.qper . . . . .	8
qspec.lw . . . . .	9
sqdft . . . . .	10
sqr.fit . . . . .	10
tqr.fit . . . . .	11
tsqr.fit . . . . .	12

<b>Index</b>	<b>13</b>
--------------	-----------

---

qdft	<i>Quantile Discrete Fourier Transform (QDFT)</i>
------	---

---

## Description

This function computes quantile discrete Fourier transform (QDFT) for univariate or multivariate time series.

## Usage

```
qdft(y, tau, n.cores = 1, cl = NULL)
```

## Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

## Value

matrix or array of the quantile discrete Fourier transform of y

## Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y,tau)
# Make a cluster for repeated use
n.cores <- 2
cl <- parallel::makeCluster(n.cores)
parallel::clusterExport(cl, c("tqr.fit"))
doParallel::registerDoParallel(cl)
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y1,tau,n.cores=n.cores,cl=cl)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y2,tau,n.cores=n.cores,cl=cl)
parallel::stopCluster(cl)
```

qdft2qacf

*Quantile Autocovariance Function (QACF)***Description**

This function computes quantile autocovariance function (QACF) from QDFT.

**Usage**

```
qdft2qacf(y.qdft, return.qser = FALSE)
```

**Arguments**

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`  
`return.qser` if TRUE, return quantile series (QSER) along with QACF

**Value**

matrix or array of quantile autocovariance function if `return.qser = FALSE` (default), else a list with the following elements:

`qacf` matrix or array of quantile autocovariance function  
`qser` matrix or array of quantile series

**Examples**

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qacf <- qdft2qacf(y.qdft)
plot(c(0:9),qacf[c(1:10),1],type='h',xlab="LAG",ylab="QACF")
qser <- qdft2qacf(y.qdft,return.qser=TRUE)$qser
plot(qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
plot(c(0:9),qacf[1,2,c(1:10),1],type='h',xlab="LAG",ylab="QACF")
```

qdft2qper

*Quantile Periodogram and Cross-Periodogram (QPER)***Description**

This function computes quantile periodogram/cross-periodogram (QPER) from QDFT.

**Usage**

```
qdft2qper(y.qdft)
```

**Arguments**

`y.qdft` matrix or array of QDFT from `qdft()`

**Value**

matrix or array of quantile periodogram/cross-periodogram

**Examples**

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qper <- qdft2qper(y.qdft)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(qper[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qper <- qdft2qper(y.qdft)
qfa.plot(ff[sel.f],tau,Re(qper[1,1,sel.f,]))
qfa.plot(ff[sel.f],tau,Re(qper[1,2,sel.f,]))
```

---

qdft2qser	<i>Quantile Series (QSER)</i>
-----------	-------------------------------

---

**Description**

This function computes quantile series (QSER) from QDFT.

**Usage**

```
qdft2qser(y.qdft)
```

**Arguments**

`y.qdft` matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`

**Value**

matrix or array of quantile series

**Examples**

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qser <- qdft2qser(y.qdft)
plot(qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
```

```

y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qser <- qdft2qser(y.qdft)
plot(qser[1,,1],type='l',xlab="TIME",ylab="QSER")

```

qfa.plot

*Quantile-Frequency Plot*

## Description

This function creates an image plot of quantile spectrum.

## Usage

```

qfa.plot(
  freq,
  tau,
  qper,
  rg.qper = range(qper),
  rg.tau = range(tau),
  rg.freq = c(0, 0.5),
  color = colorRamps::matlab.like2(1024),
  ylab = "QUANTILE LEVEL",
  xlab = "FREQUENCY",
  tlab = NULL,
  set.par = TRUE,
  legend.plot = TRUE
)

```

## Arguments

freq	sequence of frequencies in (0,0.5) at which quantile spectrum is evaluated
tau	sequence of quantile levels in (0,1) at which quantile spectrum is evaluated
qper	real-valued matrix of quantile spectrum evaluated on the freq x tau grid
rg.qper	zlim for qper (default = range(qper))
rg.tau	ylim for tau (default = range(tau))
rg.freq	xlim for freq (default = c(0, 0.5))
color	colors (default = colorRamps::matlab.like2(1024))
ylab	label of y-axis (default = "QUANTILE LEVEL")
xlab	label of x-axis (default = "FREQUENCY")
tlab	title of plot (default = NULL)
set.par	if TRUE, par() is set internally (single image)
legend.plot	if TRUE, legend plot is added

## Value

no return value

---

qkl.divergence	<i>Kullback-Leibler Divergence of Quantile Spectral Estimate</i>
----------------	--

---

**Description**

This function computes Kullback-Leibler divergence (KLD) of quantile spectral estimate.

**Usage**

```
qkl.divergence(qper, qspec, sel.f = NULL, sel.tau = NULL)
```

**Arguments**

qper	matrix or array of quantile spectral estimate from, e.g., <code>qspec.lw()</code>
qspec	matrix of array of true quantile spectrum/cross-spectrum (same dimension as qper)
sel.f	index of selected frequencies for computation (default = NULL: all frequencies)
sel.tau	index of selected quantile levels for computation (default = NULL: all quantile levels)

**Value**

real number of Kullback-Leibler divergence

---

qper2qcoh	<i>Quantile Coherence Spectrum (QCOH)</i>
-----------	---

---

**Description**

This function computes quantile coherence spectrum (QCOH) from quantile spectrum and cross-spectrum.

**Usage**

```
qper2qcoh(qspec, k = 1, kk = 2)
```

**Arguments**

qspec	array of quantile spectrum and cross-spectrum
k	index of first series (default = 1)
kk	index of second series (default = 2)

**Value**

matrix of quantile coherence evaluated at Fourier frequencies in (0,0.5)

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qcoh <- qper2qcoh(qper.lw,k=1,kk=2)
qfa.plot(ff[sel.f],tau,Re(qcoh))

```

qsmooth.qdft

*Quantile Smoothing of Quantile Discrete Fourier Transform***Description**

This function computes quantile-smoothed version of quantile discrete Fourier transform (QDFT).

**Usage**

```

qsmooth.qdft(
  y.qdft,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)

```

**Arguments**

y.qdft	matrix or array of QDFT from qdft()
method	smoothing method: "gamm" for mgcv::gamm(), "sp" for stats::smooth.spline()
spar	smoothing parameter in smooth.spline() (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix or array of quantile-smoothed QDFT

**Examples**

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)

```

```

y.qdft <- qsmooth.qdft(y.qdft,method="sp",spar=0.9)
qacf <- qdft2qacf(y.qdft)
qper.qslw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.qslw[1,1,sel.f,]))

```

---

qsmooth.qper

*Quantile Smoothing of Quantile Periodogram or Spectral Estimate*


---

## Description

This function computes quantile-smoothed version of quantile periodogram/cross-periodogram (QPER) or other quantile spectral estimate.

## Usage

```

qsmooth.qper(
  qper,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)

```

## Arguments

qper	matrix or array of quantile periodogram/cross-periodogram or spectral estimate
method	smoothing method: "gamm" for <code>mgcv::gamm()</code> , "sp" for <code>stats::smooth.spline()</code>
spar	smoothing parameter in <code>smooth.spline()</code> (default = "GCV")
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

## Value

matrix or array of quantile-smoothed quantile spectral estimate

## Examples

```

y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.lw[1,1,sel.f,]))
qper.lwqs <- qsmooth.qper(qper.lw,method="sp",spar=0.9)
qfa.plot(ff[sel.f],tau,Re(qper.lwqs[1,1,sel.f,]))

```



---

qspec.lw	<i>Lag-Window Estimator of Quantile Spectrum and Cross-Spectrum (QSPEC)</i>
----------	---

---

## Description

This function computes lag-window (LW) estimate of quantile spectrum/cross-spectrum (QSPEC) from QACF.

## Usage

```
qspec.lw(qacf, M = NULL)
```

## Arguments

qacf	matrix or array of QACF from <code>qdft2qacf()</code>
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)

## Value

A list with the following elements:

spec	matrix or array of LW estimate
lw	lag-window sequence

## Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(qper.lw[sel.f,]))

# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
qacf <- qdft2qacf(y.qdft)
qper.lw <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.lw[1,2,sel.f,]))
```

sqdft

*Spline Quantile Discrete Fourier Transform (SQDFT)***Description**

This function computes spline quantile discrete Fourier transform (SQDFT) for univariate or multivariate time series.

**Usage**

```
sqdft(y, tau, c0 = 0.02, d = 4, weighted = FALSE, n.cores = 1, cl = NULL)
```

**Arguments**

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

**Value**

matrix or array of the spline quantile discrete Fourier transform of y

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sqdft <- sqdft(y,tau,c0=0.02,d=4)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qacf <- qdft2qacf(y.sqdft)
qper.sqr1w <- qspec.lw(qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(qper.sqr1w[sel.f]))
```

sqr.fit

*Spline Quantile Regression (SQR)***Description**

This function computes the spline quantile regression (SQR) solution given response vector and design matrix. It uses the code rqfnb.f in the "quantreg" package with the permission of Dr. R. Koenker.

**Usage**

```
sqr.fit(y, X, tau, c0, d = 1, weighted = FALSE, mthreads = FALSE)
```

**Arguments**

y	response vector
X	design matrix (nrow(X) = length(y))
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
mthreads	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)

**Value**

A list with the following elements:

coefficients	matrix of regression coefficients
nit	number of iterations

---

tqr.fit

*Trigonometric Quantile Regression (TQR)*


---

**Description**

This function computes trigonometric quantile regression (TQR) for univariate time series at a single frequency.

**Usage**

```
tqr.fit(y, f0, tau, prepared = TRUE)
```

**Arguments**

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

**Value**

object of rq() (coefficients in \$coef)

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
plot(tau,fit$coef[1,],type='o',pch=0.75,xlab='QUANTILE LEVEL',ylab='TQR COEF')
```

tsqr.fit

*Trigonometric Spline Quantile Regression (TSQR)***Description**

This function computes trigonometric spline quantile regression (TSQR) for univariate time series at a single frequency.

**Usage**

```
tsqr.fit(y, f0, tau, c0, d = 1, weighted = FALSE, prepared = TRUE)
```

**Arguments**

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

**Value**

object of `sqr.fit()` (coefficients in `$coef`)

**Examples**

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
fit.sqr <- tsqr.fit(y,f0=0.1,tau=tau,c0=0.02,d=4)
plot(tau,fit$coef[1,],type='p',xlab='QUANTILE LEVEL',ylab='TQR COEF')
lines(tau,fit.sqr$coef[1,],type='l')
```

# Index

qdft, [2](#)  
qdft2qacf, [3](#)  
qdft2qper, [3](#)  
qdft2qser, [4](#)  
qfa.plot, [5](#)  
qkl.divergence, [6](#)  
qper2qcoh, [6](#)  
qsmooth.qdft, [7](#)  
qsmooth.qper, [8](#)  
qspec.lw, [9](#)  
  
sqdft, [10](#)  
sqr.fit, [10](#)  
  
tqr.fit, [11](#)  
tsqr.fit, [12](#)