Package ‘qfa’

August 21, 2023

Type Package

Title Quantile-Frequency Analysis (QFA) of Time Series

Version 2.1

Date 2023-08-21

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Description
Quantile-frequency analysis (QFA) of univariate or multivariate time series based on trigonometric quantile regression. See Li, T.-H. (2012) ``Quantile periodograms'', Journal of the American Statistical Association, 107, 765–776, <doi:10.1080/01621459.2012.682815>; Li, T.-H. (2014) Time Series with Mixed Spectra, CRC Press, <doi:10.1201/b15154>; Li, T.-H. (2022) ``Quantile Fourier transform, quantile series, and nonparametric estimation of quantile spectra'', <doi:10.48550/arXiv.2211.05844>.

Depends R (>= 3.5)

Imports RhpcBLASctl,
doParallel,
fields,
foreach,
mgcv,
nlme,
parallel,
quantreg,
splines,
stats,
graphics,
colorRamps,
MASS

License GPL (>=2)

URL https://www.r-project.org, https://github.com/IBM/qfa

NeedsCompilation yes

Encoding UTF-8

RoxygenNote 7.2.1

R topics documented:

`ar2qspec` *Quantile Spectrum from AR Model of Quantile Series*

Description

This function computes quantile spectrum/cross-spectrum (QSPEC) from an AR model of quantile series (QSER).

Usage

```
ar2qspec(fit, freq = NULL)
```

Arguments

`fit`	object of AR model from `qser2sar()` or `qser2ar()`
`freq`	sequence of frequencies in [0,1) (default = `NULL`: all Fourier frequencies)

Value

a list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
freq	sequence of frequencies

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qacf *Quantile Autocovariance Function (QACF)*

Description

This function computes quantile autocovariance function (QACF) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qacf(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

y	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
n.cores	number of cores for parallel computing of QDFT if `y.qdft` = `NULL` (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

matrix or array of quantile autocovariance function

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qacf <- qacf(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qacf <- qacf(y.qdft=y.qdft)
```

qdft — *Quantile Discrete Fourier Transform (QDFT)*

Description

This function computes quantile discrete Fourier transform (QDFT) for univariate or multivariate time series.

Usage

```
qdft(y, tau, n.cores = 1, cl = NULL)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`n.cores`	number of cores for parallel computing (default = 1)
`cl`	pre-existing cluster for repeated parallel computing (default = `NULL`)

Value

matrix or array of quantile discrete Fourier transform of `y`

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y,tau)
# Make a cluster for repeated use
n.cores <- 2
cl <- parallel::makeCluster(n.cores)
parallel::clusterExport(cl, c("tqr.fit"))
doParallel::registerDoParallel(cl)
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y1,tau,n.cores=n.cores,cl=cl)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y.qdft <- qdft(y2,tau,n.cores=n.cores,cl=cl)
parallel::stopCluster(cl)
```

qdft2qacf — *Quantile Autocovariance Function (QACF)*

Description

This function computes quantile autocovariance function (QACF) from QDFT.

Usage

```
qdft2qacf(y.qdft, return.qser = FALSE)
```

Arguments

`y.qdft`	matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`
`return.qser`	if `TRUE`, return quantile series (QSER) along with QACF

Value

matrix or array of quantile autocovariance function if `return.sqer = FALSE` (default), else a list with the following elements:

`qacf`	matirx or array of quantile autocovariance function
`qser`	matrix or array of quantile series

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[c(1:10),1],type='h',xlab="LAG",ylab="QACF")
y.qser <- qdft2qacf(y.qdft,return.qser=TRUE)$qser
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
plot(c(0:9),y.qacf[1,2,c(1:10),1],type='h',xlab="LAG",ylab="QACF")
```

`qdft2qper` *Quantile Periodogram and Cross-Periodogram (QPER)*

Description

This function computes quantile periodogram/cross-periodogram (QPER) from QDFT.

Usage

```
qdft2qper(y.qdft)
```

Arguments

`y.qdft`	matrix or array of QDFT from `qdft()`

Value

matrix or array of quantile periodogram/cross-periodogram

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qper <- qdft2qper(y.qdft)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(y.qper[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qper <- qdft2qper(y.qdft)
qfa.plot(ff[sel.f],tau,Re(y.qper[1,1,sel.f,]))
qfa.plot(ff[sel.f],tau,Re(y.qper[1,2,sel.f,]))
```

---

qdft2qser *Quantile Series (QSER)*

Description

This function computes quantile series (QSER) from QDFT.

Usage

```
qdft2qser(y.qdft)
```

Arguments

`y.qdft`	matrix or array of QDFT from `qdft()` or SQDFT from `sqdft()`

Value

matrix or array of quantile series

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[,1],type='l',xlab="TIME",ylab="QSER")
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qser <- qdft2qser(y.qdft)
plot(y.qser[1,,1],type='l',xlab="TIME",ylab="QSER")
```

qfa.plot *Quantile-Frequency Plot*

Description

This function creates an image plot of quantile spectrum.

Usage

```
qfa.plot(
  freq,
  tau,
  rqper,
  rg.qper = range(rqper),
  rg.tau = range(tau),
  rg.freq = c(0, 0.5),
  color = colorRamps::matlab.like2(1024),
  ylab = "QUANTILE LEVEL",
  xlab = "FREQUENCY",
  tlab = NULL,
  set.par = TRUE,
  legend.plot = TRUE
)
```

Arguments

`freq`	sequence of frequencies in (0,0.5) at which quantile spectrum is evaluated
`tau`	sequence of quantile levels in (0,1) at which quantile spectrum is evaluated
`rqper`	real-valued matrix of quantile spectrum evaluated on the `freq` x `tau` grid
`rg.qper`	`zlim` for `qper` (default = `range(qper)`)
`rg.tau`	`ylim` for `tau` (default = `range(tau)`)
`rg.freq`	`xlim` for `freq` (default = `c(0,0.5)`)
`color`	colors (default = `colorRamps::matlab.like2(1024)`)
`ylab`	label of y-axis (default = `"QUANTILE LEVEL"`)
`xlab`	label of x-axis (default = `"FREQUENCY"`)
`tlab`	title of plot (default = `NULL`)
`set.par`	if `TRUE`, `par()` is set internally (single image)
`legend.plot`	if `TRUE`, legend plot is added

Value

no return value

qkl.divergence — *Kullback-Leibler Divergence of Quantile Spectral Estimate*

Description

This function computes Kullback-Leibler divergence (KLD) of quantile spectral estimate.

Usage

```
qkl.divergence(y.qper, qspec, sel.f = NULL, sel.tau = NULL)
```

Arguments

`y.qper`	matrix or array of quantile spectral estimate from, e.g., `qspec.lw()`
`qspec`	matrix of array of true quantile spectrum/cross-spectrum (same dimension as `y.qper`)
`sel.f`	index of selected frequencies for computation (default = `NULL`: all frequencies)
`sel.tau`	index of selected quantile levels for computation (default = `NULL`: all quantile levels)

Value

real number of Kullback-Leibler divergence

qper — *Quantile Periodogram and Cross-Periodogram (QPER)*

Description

This function computes quantile periodogram/cross-periodogram (QPER) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qper(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qdft`	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
`n.cores`	number of cores for parallel computing of QDFT if `y.qdft` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

matrix or array of quantile periodogram/cross-periodogram

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qper <- qper(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qper <- qper(y.qdft=y.qdft)
```

---

qper2 *Quantile Periodogram Type II (QPER2)*

Description

This function computes type-II quantile periodogram for univariate time series.

Usage

```
qper2(y, freq, tau, weights = NULL, n.cores = 1, cl = NULL)
```

Arguments

y	univariate time series
freq	sequence of frequencies in [0,1)
tau	sequence of quantile levels in (0,1)
weights	sequence of weights in quantile regression (default = NULL: weights equal to 1)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix of quantile periodogram evaluated on `freq * tau` grid

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper2 <- qper2(y,ff,tau)
qfa.plot(ff[sel.f],tau,Re(y.qper2[sel.f,]))
```

qser *Quantile Series (QSER)*

Description

This function computes quantile series (QSER) from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qser(y, tau, y.qdft = NULL, n.cores = 1, cl = NULL)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
y.qdft	matrix or array of pre-calculated QDFT (default = NULL: compute from y and tau); if y.qdft is supplied, y and tau can be left unspecified
n.cores	number of cores for parallel computing of QDFT if y.qdft = NULL (default = 1)
cl	pre-existing cluster for repeated parallel computing of QDFT (default = NULL)

Value

matrix or array of quantile series

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
# compute from time series
y.qser <- qser(y,tau)
# compute from QDFT
y.qdft <- qdft(y,tau)
y.qser <- qser(y.qdft=y.qdft)
```

qser2ar *Autoregression (AR) Model of Quantile Series*

Description

This function fits an autoregression (AR) model to quantile series (QSER) separately for each quantile level using stats::ar().

Usage

```
qser2ar(y.qser, p = NULL, order.max = NULL)
```

Arguments

`y.qser`	matrix or array of pre-calculated QSER, e.g., using `qser()`
`p`	order of AR model (default = `NULL`: selected by AIC)
`order.max`	maximum order for AIC if `p = NULL` (default = `NULL`: determined by `stats::ar()`)

Value

a list with the following elements:

`A`	matrix or array of AR coefficients
`V`	vector or matrix of residual covariance
`p`	order of AR model
`n`	length of time series
`residuals`	matrix or array of residuals

`qser2sar` *Spline Autoregression (SAR) Model of Quantile Series*

Description

This function fits spline autoregression (SAR) model to quantile series (QSER).

Usage

```
qser2sar(
  y.qser,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("AIC", "BIC", "GCV"),
  weighted = FALSE
)
```

Arguments

`y.qser`	matrix or array of pre-calculated QSER, e.g., using `qser()`
`tau`	sequence of quantile levels where `y.qser` is calculated
`d`	subsampling rate of quantile levels (default = 1)
`p`	order of SAR model (default = `NULL`: automatically selected by AIC)
`order.max`	maximum order for AIC if `p = NULL` (default = `NULL`: determined by `stats::ar()`)
`spar`	penalty parameter alla `smooth.spline` (default = `NULL`: automatically selected)
`method`	criterion for penalty parameter selection: `"AIC"` (default), `"BIC"`, or `"GCV"`
`weighted`	if `TRUE`, penalty function is weighted (default = `FALSE`)

Value

a list with the following elements:

`A`	matrix or array of SAR coefficients
`V`	vector or matrix of SAR residual covariance
`p`	order of SAR model
`spar`	penalty parameter
`tau`	sequence of quantile levels
`n`	length of time series
`d`	subsampling rate of quantile levels
`weighted`	option for weighted penalty function
`fit`	object containing details of SAR fit

`qsmooth.qdft` *Quantile Smoothing of Quantile Discrete Fourier Transform*

Description

This function computes quantile-smoothed version of quantile discrete Fourier transform (QDFT).

Usage

```
qsmooth.qdft(
  y.qdft,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y.qdft`	matrix or array of QDFT from `qdft()`
`method`	smoothing method: `"gamm"` for `mgcv::gamm()` (default), `"sp"` for `stats::smooth.spline()`
`spar`	smoothing parameter in `smooth.spline()` if `method = "sp"` (default = `"GCV"`)
`n.cores`	number of cores for parallel computing (default = 1)
`cl`	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of quantile-smoothed QDFT

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qdft <- qsmooth.qdft(y.qdft,method="sp",spar=0.9)
y.qacf <- qdft2qacf(y.qdft)
y.qper.qslw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.qslw[1,1,sel.f,]))
```

---

qsmooth.qper   *Quantile Smoothing of Quantile Periodogram or Spectral Estimate*

Description

This function computes quantile-smoothed version of quantile periodogram/cross-periodogram (QPER) or other quantile spectral estimate.

Usage

```
qsmooth.qper(
  y.qper,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y.qper`	matrix or array of quantile periodogram/cross-periodogram or spectral estimate
`method`	smoothing method: `"gamm"` for `mgcv::gamm()` (default), `"sp"` for `stats::smooth.spline()`
`spar`	smoothing parameter in `smooth.spline()` if `method = "sp"` (default = `"GCV"`)
`n.cores`	number of cores for parallel computing (default = 1)
`cl`	pre-existing cluster for repeated parallel computing (default = `NULL`)

Value

matrix or array of quantile-smoothed quantile spectral estimate

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
```

```
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[1,1,sel.f,]))
y.qper.lwqs <- qsmooth.qper(y.qper.lw,method="sp",spar=0.9)
qfa.plot(ff[sel.f],tau,Re(y.qper.lwqs[1,1,sel.f,]))
```

---

qspec.ar *Autoregression (AR) Estimator of Quantile Spectrum*

Description

This function computes autoregression (AR) estimate of quantile spectrum/cross-spectrum from time series or quantile series (QSER).

Usage

```
qspec.ar(
  y,
  tau,
  y.qser = NULL,
  p = NULL,
  order.max = NULL,
  freq = NULL,
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qser`	matrix or array of pre-calculated QSER (default = `NULL`: compute from `y` and `tau`); if `y.qser` is supplied, `y` and `tau` can be left unspecified
`p`	order of AR model (default = `NULL`: automatically selected by AIC)
`order.max`	maximum order for AIC if `p` = `NULL` (default = `NULL`: determined by `stats::ar()`)
`freq`	sequence of frequencies in [0,1) (default = `NULL`: all Fourier frequencies)
`n.cores`	number of cores for parallel computing of QDFT if `y.qser` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

a list with the following elements:

`spec`	matrix or array of AR quantile spectrum/cross-spectrum
`freq`	sequence of frequencies
`fit`	object of AR model
`qser`	matrix or array of quantile series if `y.qser` = `NULL`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.ar <- qspec.ar(cbind(y1,y2),tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.ar$spec[1,1,sel.f,]))
```

---

qspec.lw — *Lag-Window (LW) Estimator of Quantile Spectrum*

Description

This function computes lag-window (LW) estimate of quantile spectrum/cross-spectrum from QACF.

Usage

```
qspec.lw(y.qacf, M = NULL)
```

Arguments

`y.qacf`	matrix or array of pre-calculated QACF from `qdft2qacf()`
`M`	bandwidth parameter of lag window (default = `NULL`: quantile periodogram)

Value

A list with the following elements:

`spec`	matrix or array of quantile spectrum/cross-spectrum
`lw`	lag-window sequence

Examples

```
# single time series
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.qdft <- qdft(y1,tau)
y.qacf <- qdft2qacf(y.qdft)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[sel.f,]))
# multiple time series
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y.qdft <- qdft(cbind(y1,y2),tau)
y.qacf <- qdft2qacf(y.qdft)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lw[1,2,sel.f,]))
```

qspec.lwqs *Lag-Window-Quantile-Smoothing (LWQS) Estimator of Quantile Spectrum*

Description

This function computes lag-window-quantile-smoothing (LWQS) estimate of quantile spectrum/cross-spectrum from time series or quantile autocovariance function (QACF).

Usage

```
qspec.lwqs(
  y,
  tau,
  y.qacf = NULL,
  M = NULL,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

y	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
tau	sequence of quantile levels in (0,1)
y.qacf	matrix or array of pre-calculated QACF (default = `NULL`: compute from `y` and `tau`); if `y.qacf` is supplied, `y` and `tau` can be left unspecified
M	bandwidth parameter of lag window (default = `NULL`: quantile periodogram)
method	smoothing method: `"gamm"` for `mgcv::gamm()` (default), `"sp"` for `stats::smooth.spline()`
spar	smoothing parameter in `smooth.spline()` if `method` = `"sp"` (default = `"GCV"`)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = `NULL`)

Value

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
spec.lw	matrix or array of quantile spectrum/cross-spectrum without quantile smoothing
lw	lag-window sequence
qacf	matrix or array of quantile autocovariance function if `y.qacf` = `NULL`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.lwqs <- qspec.lwqs(cbind(y1,y2),tau,M=5,method="sp",spar=0.9)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.lwqs[1,1,sel.f,]))
```

---

qspec.qslw — *Quantile-Smoothing-Lag-Window (QSLW) Estimator of Quantile Spectrum*

Description

This function computes quantie-smoothing-lag-window (QSLW estimate of quantile spectrum/cross-spectrum from time series or quantile discrete Fourier transform (QDFT).

Usage

```
qspec.qslw(
  y,
  tau,
  y.qdft = NULL,
  M = NULL,
  method = c("gamm", "sp"),
  spar = "GCV",
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`tau`	sequence of quantile levels in (0,1)
`y.qdft`	matrix or array of pre-calculated QDFT (default = `NULL`: compute from `y` and `tau`); if `y.qdft` is supplied, `y` and `tau` can be left unspecified
`M`	bandwidth parameter of lag window (default = `NULL`: quantile periodogram)
`method`	smoothing method: `"gamm"` for `mgcv::gamm()` (default), `"sp"` for `stats::smooth.spline()`
`spar`	smoothing parameter in `smooth.spline()` if `method = 'sp'` (default = `"GCV"`)
`n.cores`	number of cores for parallel computing (default = 1)
`cl`	pre-existing cluster for repeated parallel computing (default = `NULL`)

Value

A list with the following elements:

`spec`	matrix or array of quantile spectrum/cross-spectrum
`lw`	lag-window sequence
`qdft`	matrix or array of quantile discrete Fourier transform if `y.qdft = NULL`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.qslw <- qspec.qslw(cbind(y1,y2),tau,M=5,method="sp",spar=0.9)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.qslw[1,1,sel.f,]))
```

---

qspec.sar *Spline Autoregression (SAR) Estimator of Quantile Spectrum*

Description

This function computes spline autoregression (SAR) estimate of quantile spectrum/cross-spectrum.

Usage

```
qspec.sar(
  y,
  y.qser = NULL,
  tau,
  d = 1,
  p = NULL,
  order.max = NULL,
  spar = NULL,
  method = c("AIC", "BIC", "GCV"),
  weighted = FALSE,
  freq = NULL,
  n.cores = 1,
  cl = NULL
)
```

Arguments

`y`	vector or matrix of time series (if matrix, `nrow(y)` = length of time series)
`y.qser`	matrix or array of pre-calculated QSER (default = `NULL`: compute from `y` and `tau`); if `y.qser` is supplied, `y` can be left unspecified
`tau`	sequence of quantile levels in (0,1)
`d`	subsampling rate of quantile levels (default = 1)
`p`	order of SAR model (default = `NULL`: automatically selected by AIC)
`order.max`	maximum order for AIC if `p` = `NULL` (default = `NULL`: determined by `stats::ar()`)
`spar`	penalty parameter alla `smooth.spline` (default = `NULL`: automatically selected)
`method`	criterion for penalty parameter selection: `"AIC"` (default), `"BIC"`, or `"GCV"`
`weighted`	if `TRUE`, penalty function is weighted (default = `FALSE`)
`freq`	sequence of frequencies in [0,1) (default = `NULL`: all Fourier frequencies)
`n.cores`	number of cores for parallel computing of QDFT if `y.qser` = `NULL` (default = 1)
`cl`	pre-existing cluster for repeated parallel computing of QDFT (default = `NULL`)

Value

a list with the following elements:

`spec`	matrix or array of SAR quantile spectrum
`freq`	sequence of frequencies
`fit`	object of SAR model
`qser`	matrix or array of quantile series if `y.qser` = `NULL`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
# compute from time series
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))
# compute from quantile series
y.qser <- qser(cbind(y1,y2),tau)
y.sar <- qspec.sar(y.qser=y.qser,tau=tau,p=1)
qfa.plot(ff[sel.f],tau,Re(y.sar$spec[1,1,sel.f,]))
```

`qspec.sqrlw` *Spline-Quantile-Regression-Lag-Window (SQRLW) Estimator of Quantile Spectrum*

Description

This function computes spline-quantile-regression-lag-window (SQRLW) estimate of quantile spectrum/cross-spectrum from time series or spline quantile discrete Fourier transform (SQDFT).

Usage

```
qspec.sqrlw(
  y,
  tau,
  y.sqdft = NULL,
  M = NULL,
  c0 = 0.02,
  d = 4,
  weighted = FALSE,
  n.cores = 1,
  cl = NULL
)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
y.sqdft	matrix or array of pre-calculated SQDFT (default = NULL: compute from y and tau); if y.sqdft is supplied, y and tau can be left unspecified
M	bandwidth parameter of lag window (default = NULL: quantile periodogram)
c0	penalty parameter for SQDFT
d	subsampling rate of quantile levels for SQDFT (default = 1)
weighted	if TRUE, SQR penalty function is weighted (default = FALSE)
n.cores	number of cores for parallel computing of SQDFT (default = 1)
cl	pre-existing cluster for repeated parallel computing of SQDFT (default = NULL)

Value

A list with the following elements:

spec	matrix or array of quantile spectrum/cross-spectrum
lw	lag-window sequence
sqdft	matrix or array of spline quantile discrete Fourier transform if y.sqdft = NULL

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qper.sqrlw <- qspec.sqrlw(cbind(y1,y2),tau,M=5,c0=0.02,d=4)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.sqrlw[1,1,sel.f,]))
```

---

qspec2qcoh *Quantile Coherence Spectrum*

Description

This function computes quantile coherence spectrum (QCOH) from quantile spectrum and cross-spectrum of multiple time series.

Usage

```
qspec2qcoh(qspec, k = 1, kk = 2)
```

Arguments

qspec	array of quantile spectrum/cross-spectrum
k	index of first series (default = 1)
kk	index of second series (default = 2)

Value

matrix of quantile coherence evaluated at Fourier frequencies in (0,0.5)

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
n <- length(y1)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qacf(cbind(y1,y2),tau)
y.qper.lw <- qspec.lw(y.qacf,M=5)$spec
y.qcoh <- qspec2qcoh(y.qper.lw,k=1,kk=2)
qfa.plot(ff[sel.f],tau,y.qcoh)
```

`sar.eq.bootstrap` *Bootstrap Simulation of SAR Coefficients for Testing Equality of Granger-Causality in Two Samples*

Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for testing equality of Granger-causality in two samples based on their SAR models under H0: effect in each sample equals the average effect.

Usage

```
sar.eq.bootstrap(
  y.qser,
  fit,
  fit2,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)
```

Arguments

`y.qser`	matrix or array of QSER from `qser()` or `qspec.sar()$qser`
`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`fit2`	object of SAR model for the other sample
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)
`nsim`	number of bootstrap samples (default = 1000)
`method`	method of residual calculation: `"ar"` (default) or `"sar"`
`n.cores`	number of cores for parallel computing (default = 1)

`mthreads`	if `TRUE`, multithread BLAS is enabled when available (default = `FALSE`, required for parallel computing)
`seed`	seed for random sampling (default = `1234567`)

Value

array of simulated bootstrap samples of selected SAR coefficients

Examples

```
y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21),tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22),tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)
```

`sar.eq.test` *Wald Test and Confidence Band for Equality of SAR-Based Granger-Causality in Two Samples*

Description

This function computes Wald test and confidence band for equality of Granger-causality in two samples using bootstrap samples generated by `sar.eq.bootstrap()` based on the spline autoregression (SAR) models of quantile series (QSER).

Usage

```
sar.eq.test(A1, A1.sim, A2, A2.sim, sel.lag = NULL, sel.tau = NULL)
```

Arguments

`A1`	matrix of selected SAR coefficients for sample 1
`A1.sim`	simulated bootstrap samples from `sar.eq.bootstrap()` for sample 1
`A2`	matrix of selected SAR coefficients for sample 2
`A2.sim`	simulated bootstrap samples from `sar.eq.bootstrap()` for sample 2
`sel.lag`	indices of time lags for Wald test (default = `NULL`: all lags)
`sel.tau`	indices of quantile levels for Wald test (default = `NULL`: all quantiles)

Value

a list with the following elements:

`test`	list of Wald test result containing `wald` and `p.value`
`D.u`	matrix of upper limits of 95% confidence band for `A1 - A2`
`D.l`	matrix of lower limits of 95% confidence band for `A1 - A2`

Examples

```
y11 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y21 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
y12 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y22 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y1.sar <- qspec.sar(cbind(y11,y21),tau=tau,p=1)
y2.sar <- qspec.sar(cbind(y12,y22),tau=tau,p=1)
A1.sim <- sar.eq.bootstrap(y1.sar$qser,y1.sar$fit,y2.sar$fit,index=c(1,2),nsim=5)
A2.sim <- sar.eq.bootstrap(y2.sar$qser,y2.sar$fit,y1.sar$fit,index=c(1,2),nsim=5)
A1 <- sar.gc.coef(y1.sar$fit,index=c(1,2))
A2 <- sar.gc.coef(y2.sar$fit,index=c(1,2))
test <- sar.eq.test(A1,A1.sim,A2,A2.sim,sel.lag=NULL,sel.tau=NULL)
```

---

sar.gc.bootstrap — *Bootstrap Simulation of SAR Coefficients for Granger-Causality Analysis*

Description

This function simulates bootstrap samples of selected spline autoregression (SAR) coefficients for Granger-causality analysis based on the SAR model of quantile series (QSER) under H0: (a) for multiple time series, the second series specified in `index` is not causal for the first series specified in `index`; (b) for single time series, the series is not causal at the lags specified in `index`.

Usage

```
sar.gc.bootstrap(
  y.qser,
  fit,
  index = c(1, 2),
  nsim = 1000,
  method = c("ar", "sar"),
  n.cores = 1,
  mthreads = FALSE,
  seed = 1234567
)
```

Arguments

`y.qser`	matrix or array of QSER from `qser()` or `qspec.sar()$qser`
`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)
`nsim`	number of bootstrap samples (default = 1000)
`method`	method of residual calculation: `"ar"` (default) or `"sar"`
`n.cores`	number of cores for parallel computing (default = 1)
`mthreads`	if `TRUE`, multithread BLAS is enabled when available (default = `FALSE`, required for parallel computing)
`seed`	seed for random sampling (default = `1234567`)

Value

array of simulated bootstrap samples of selected SAR coefficients

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)
```

---

sar.gc.coef *Extraction of SAR Coefficients for Granger-Causality Analysis*

Description

This function extracts the spline autoregression (SAR) coefficients from an SAR model for Granger-causality analysis. See `sar.gc.bootstrap` for more details regarding the use of `index`.

Usage

```
sar.gc.coef(fit, index = c(1, 2))
```

Arguments

`fit`	object of SAR model from `qser2sar()` or `qspec.sar()$fit`
`index`	a pair of component indices for multiple time series or a sequence of lags for single time series (default = `c(1,2)`)

Value

matrix of selected SAR coefficients (number of lags by number of quantiles)

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))
```

sar.gc.test — *Wald Test and Confidence Band for SAR-Based Granger-Causality Analysis*

Description

This function computes Wald test and confidence band for Granger-causality using bootstrap samples generated by `sar.gc.bootstrap()` based the spline autoregression (SAR) model of quantile series (QSER).

Usage

```
sar.gc.test(A, A.sim, sel.lag = NULL, sel.tau = NULL)
```

Arguments

`A`	matrix of selected SAR coefficients
`A.sim`	simulated bootstrap samples from `sar.gc.bootstrap()`
`sel.lag`	indices of time lags for Wald test (default = `NULL`: all lags)
`sel.tau`	indices of quantile levels for Wald test (default = `NULL`: all quantiles)

Value

a list with the following elements:

`test`	list of Wald test result containing `wald` and `p.value`
`A.u`	matrix of upper limits of 95% confidence band of `A`
`A.l`	matrix of lower limits of 95% confidence band of `A`

Examples

```
y1 <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
y2 <- stats::arima.sim(list(order=c(1,0,0), ar=-0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sar <- qspec.sar(cbind(y1,y2),tau=tau,p=1)
A <- sar.gc.coef(y.sar$fit,index=c(1,2))
A.sim <- sar.gc.bootstrap(y.sar$qser,y.sar$fit,index=c(1,2),nsim=5)
y.gc <- sar.gc.test(A,A.sim)
```

sqdft — *Spline Quantile Discrete Fourier Transform (SQDFT)*

Description

This function computes spline quantile discrete Fourier transform (SQDFT) for univariate or multivariate time series through trigonometric spline quantile regression.

Usage

```
sqdft(y, tau, c0 = 0.02, d = 4, weighted = FALSE, n.cores = 1, cl = NULL)
```

Arguments

y	vector or matrix of time series (if matrix, nrow(y) = length of time series)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
n.cores	number of cores for parallel computing (default = 1)
cl	pre-existing cluster for repeated parallel computing (default = NULL)

Value

matrix or array of the spline quantile discrete Fourier transform of y

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
y.sqdft <- sqdft(y,tau,c0=0.02,d=4)
n <- length(y)
ff <- c(0:(n-1))/n
sel.f <- which(ff > 0 & ff < 0.5)
y.qacf <- qdft2qacf(y.sqdft)
y.qper.sqrlw <- qspec.lw(y.qacf,M=5)$spec
qfa.plot(ff[sel.f],tau,Re(y.qper.sqrlw[sel.f,]))
```

---

sqr.fit *Spline Quantile Regression (SQR)*

Description

This function computes spline quantile regression (SQR) solution from response vector and design matrix. It uses the FORTRAN code rqfnb.f in the "quantreg" package with the kind permission of Dr. R. Koenker.

Usage

```
sqr.fit(y, X, tau, c0, d = 1, weighted = FALSE, mthreads = FALSE)
```

Arguments

y	response vector
X	design matrix (nrow(X) = length(y))
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
mthreads	if TRUE, multithread BLAS is enabled when available (default = FALSE, required for parallel computing)

Value

A list with the following elements:

`coefficients`	matrix of regression coefficients
`nit`	number of iterations

`tqr.fit` *Trigonometric Quantile Regression (TQR)*

Description

This function computes trigonometric quantile regression (TQR) for univariate time series at a single frequency.

Usage

```
tqr.fit(y, f0, tau, prepared = TRUE)
```

Arguments

`y`	vector of time series
`f0`	frequency in [0,1)
`tau`	sequence of quantile levels in (0,1)
`prepared`	if `TRUE`, intercept is removed and coef of cosine is doubled when `f0 = 0.5`

Value

object of `rq()` (coefficients in `$coef`)

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
plot(tau,fit$coef[1,],type='o',pch=0.75,xlab='QUANTILE LEVEL',ylab='TQR COEF')
```

`tsqr.fit` *Trigonometric Spline Quantile Regression (TSQR)*

Description

This function computes trigonometric spline quantile regression (TSQR) for univariate time series at a single frequency.

Usage

```
tsqr.fit(y, f0, tau, c0, d = 1, weighted = FALSE, prepared = TRUE)
```

Arguments

y	vector of time series
f0	frequency in [0,1)
tau	sequence of quantile levels in (0,1)
c0	penalty parameter
d	subsampling rate of quantile levels (default = 1)
weighted	if TRUE, penalty function is weighted (default = FALSE)
prepared	if TRUE, intercept is removed and coef of cosine is doubled when f0 = 0.5

Value

object of sqr.fit() (coefficients in $coef)

Examples

```
y <- stats::arima.sim(list(order=c(1,0,0), ar=0.5), n=64)
tau <- seq(0.1,0.9,0.05)
fit <- tqr.fit(y,f0=0.1,tau=tau)
fit.sqr <- tsqr.fit(y,f0=0.1,tau=tau,c0=0.02,d=4)
plot(tau,fit$coef[1,],type='p',xlab='QUANTILE LEVEL',ylab='TQR COEF')
lines(tau,fit.sqr$coef[1,],type='l')
```

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