Pooled testing with penalized regression models

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Joint work with
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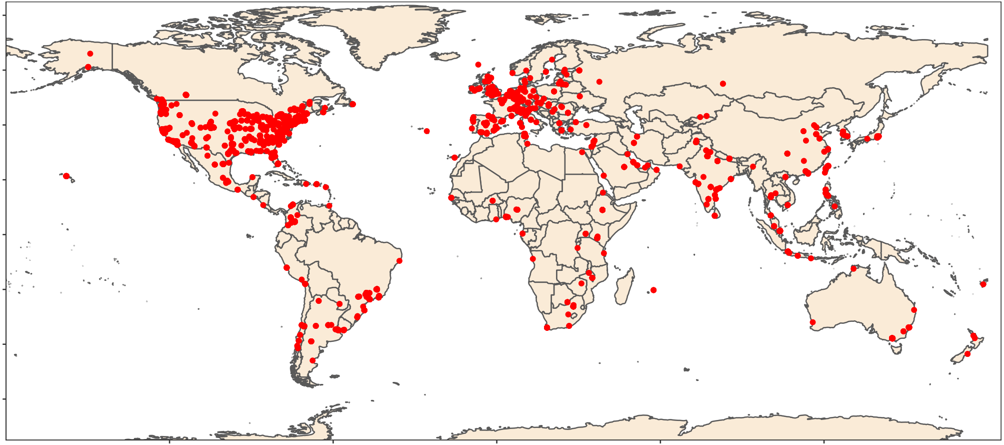
  - Decrease number of tests, increase testing capacity

- Widely used during pandemic
  - A Shiny App for Pooled Testing
  - 91 countries during 6 months of 2020



- Australia
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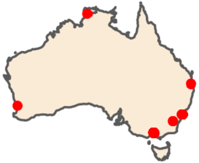

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- Widely used elsewhere: Blood donations, sexually transmitted infections, congenital infections, animal infections, food safety surveillance, computer networks assessments, flower infection levels

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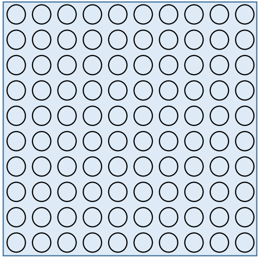
- Purpose: Examine use of viral load response and linear model prediction with "array testing" algorithm

- Array testing



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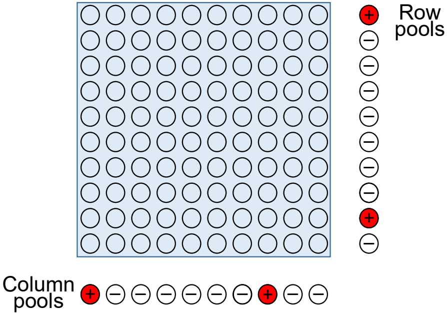

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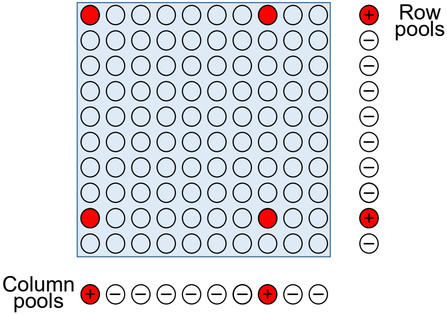

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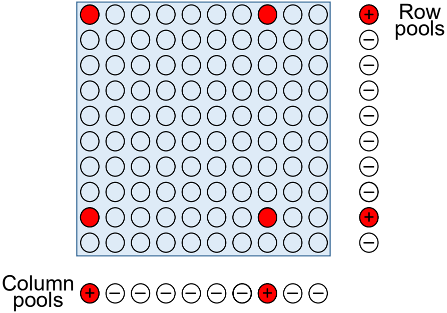

- Key aspect: Test in multiple pools (groups) during first stage to reduce the number of retests in a second stage

- A 3×3 array

	Column1	Column 2	Column 3
Row 1	1	2	3
Row 2	4	5	6
Row 3	7	8	9

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- Alternative form

	Specimens									Regular
Pools	1	2	3	4	5	6	7	8	9	Array
1	1	1	1	0	0	0	0	0	0	Row 1
2	0	0	0	1	1	1	0	0	0	Row 2
3	0	0	0	0	0	0	1	1	1	Row 3
4	1	0	0	1	0	0	1	0	0	Col. 1
5	0	1	0	0	1	0	0	1	0	Col. 2
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5	0	1	0	0	1	0	0	1	0	Col. 2
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- Pooling matrix: $X_{6\times 9}$ is a matrix of 0's and 1's

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  - R = Number of rows of array, C = Number of columns of array
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  - Penalized regression model: non-negative LASSO

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    - #2: Same as #1 but
        - Retest specimens in a second stage for estimates in an indeterminate range
        - Indeterminate range: $0 < \hat{\beta}_i < c$

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- Estimate efficiency and PPA
  - No closed form expressions for linear model-based algorithms
  - Use Monte Carlo simulation

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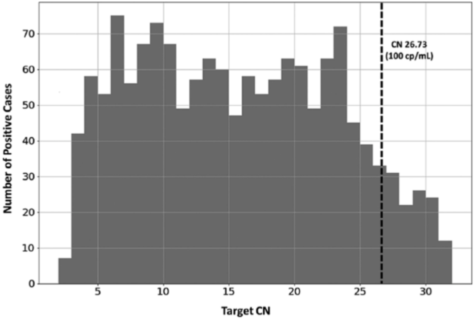

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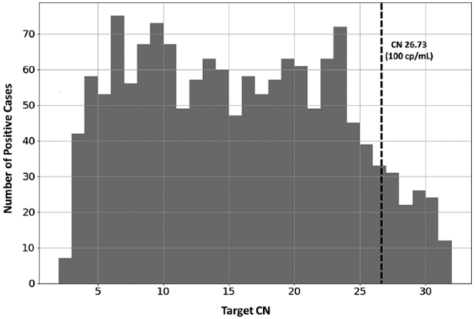

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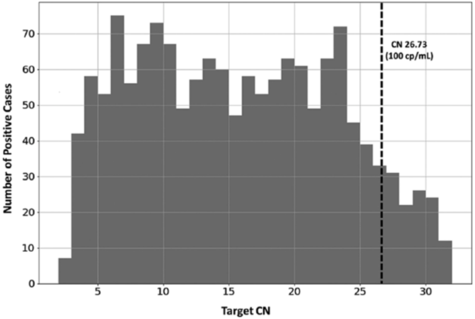

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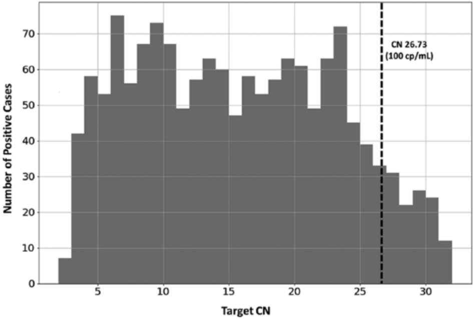

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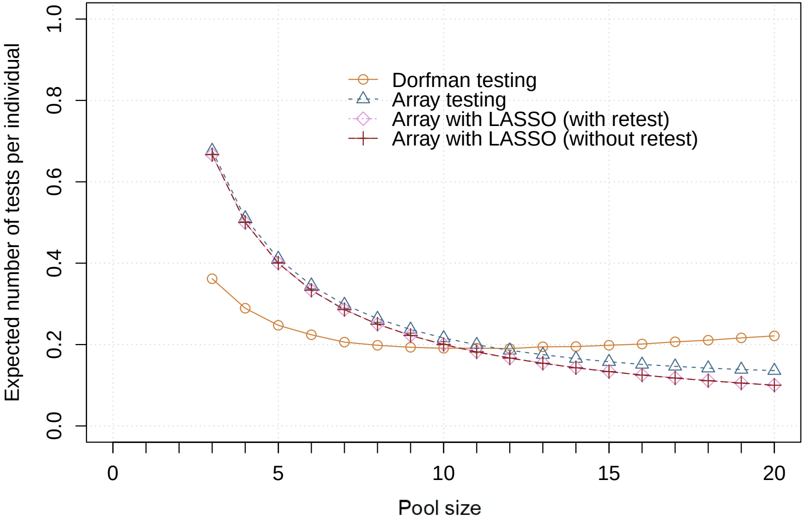
  - One setting: $p = 0.01$, pool sizes 3 to 20


Dorfman testing
Array testing
Array with LASSO (with retest)
Array with LASSO (without retest)
Expected number of tests per individual
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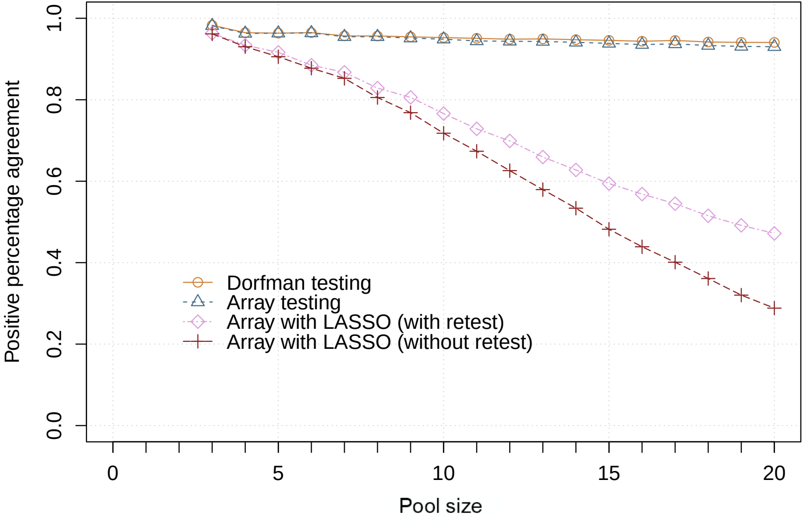
0.2
0.4
0.6
0.8
1.0
Pool size
0
5
10
15
20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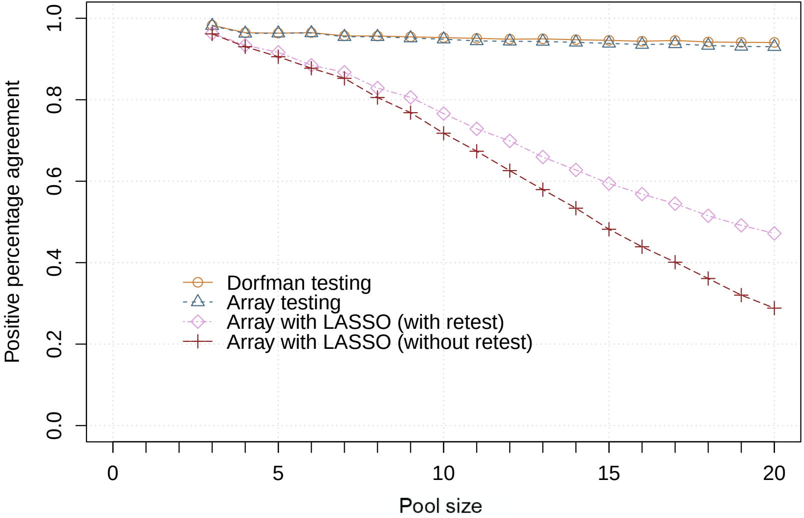

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- Incorporate statistical inference - $H_0 : \beta_i = 0$ vs. $H_a : \beta_i > 0$

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Joint work with
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