

# Multivariate meta-analysis models for high-dimensional data

Alysha De Livera (joint work with Ms Jayamini Liyanage, Prof Luke Prendergast, and Prof Robert Staudte)

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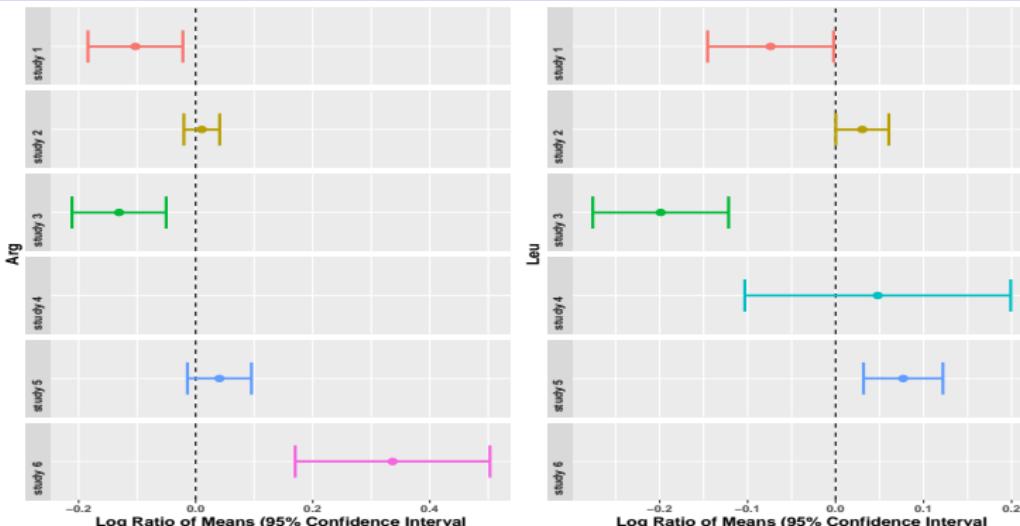
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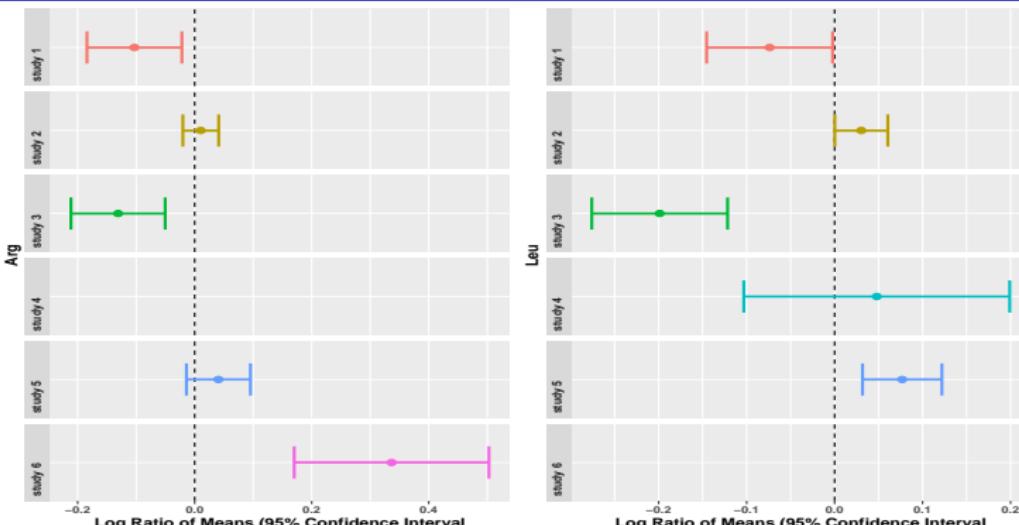
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- This project was motivated by evidence synthesis in metabolomics studies.
- We explored multivariate meta analysis models in the context of metabolomics and other high-dimensional data where we routinely have more variables than the number of studies.

# A motivating example: metabolomics



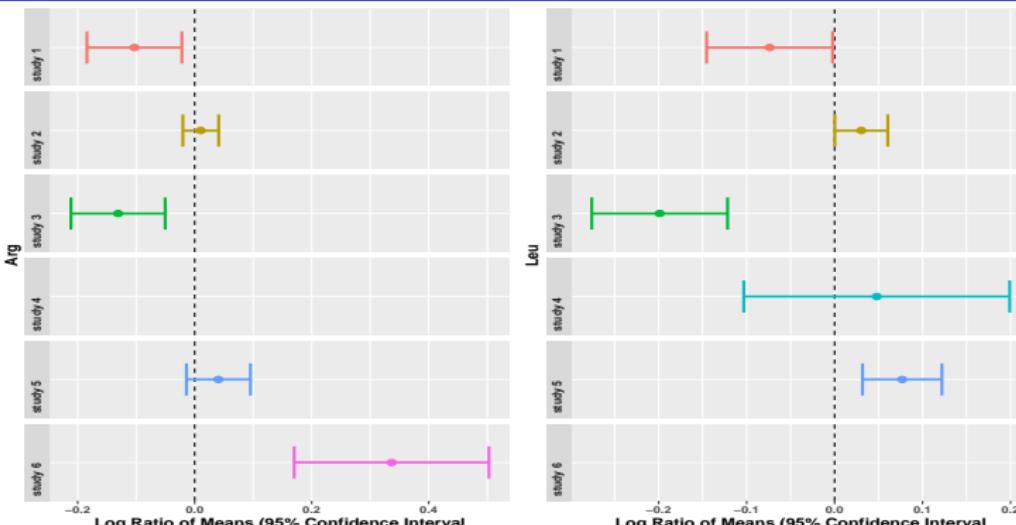
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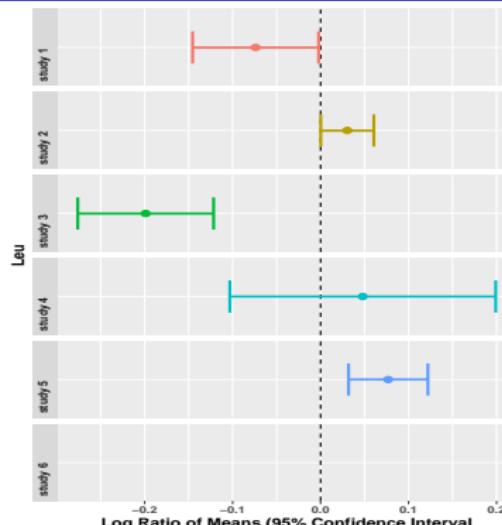
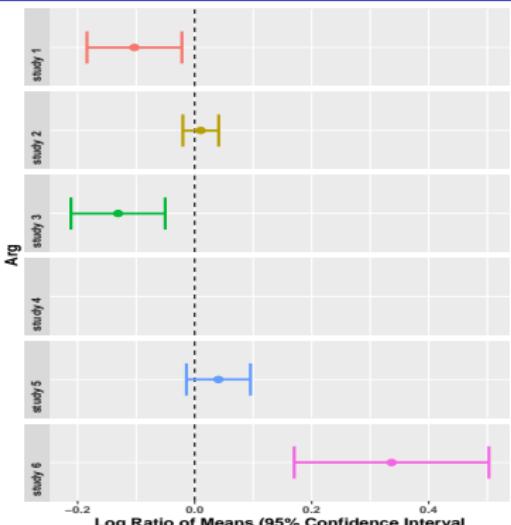
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- Estimates for the means and standard deviations of the *treated* and *control* groups were available.
- 21 metabolites in 6 studies were available.
- Not all metabolites were present in all 6 studies, creating some missing values.

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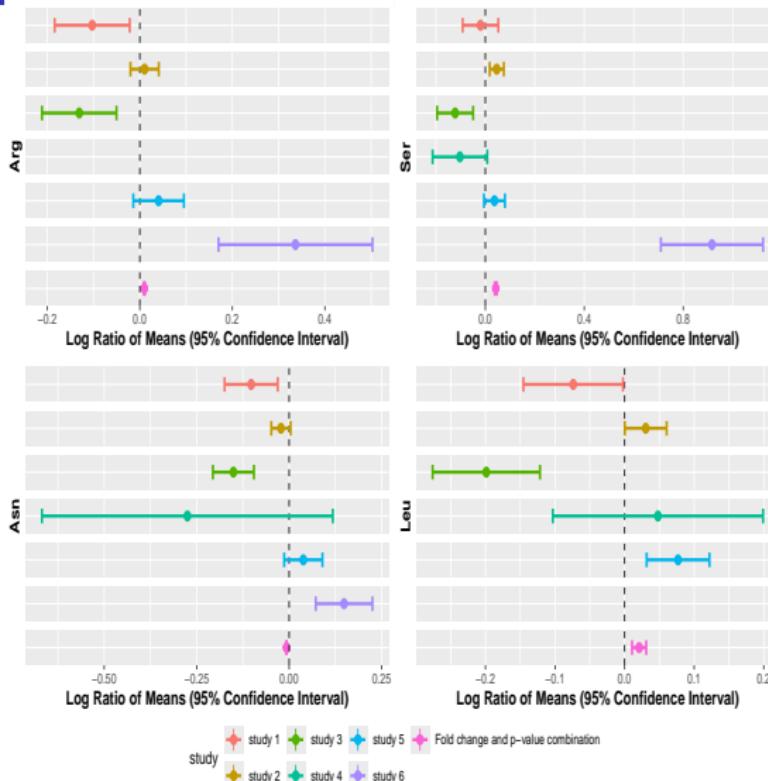
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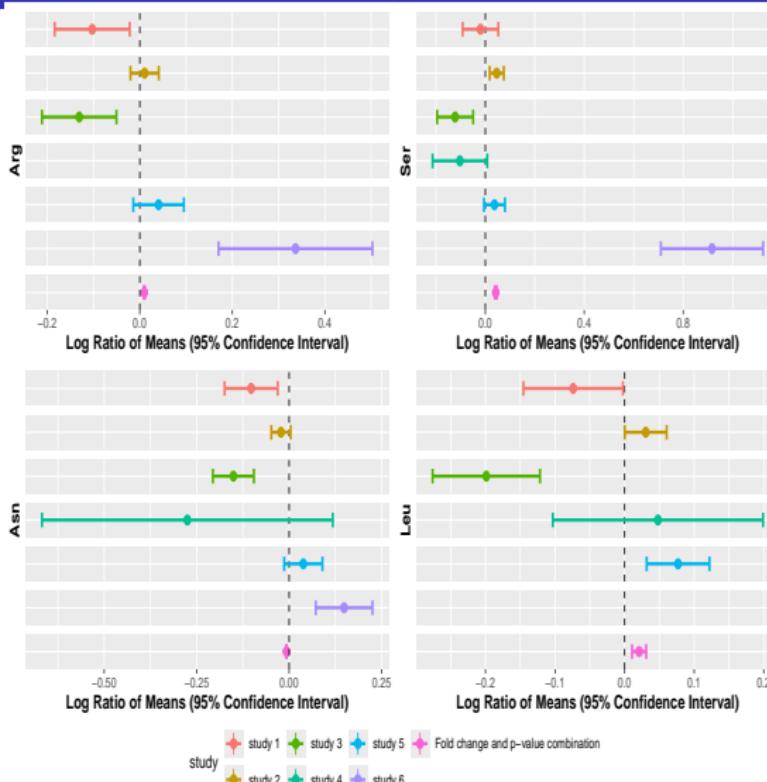
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  - Other: **Combining p-values** not using effect size information, **Using either fixed or random based on a heterogeneity statistic**, **Vote counting** Counting and comparing significant vs. non-significant studies

# Current approaches in metabolomics



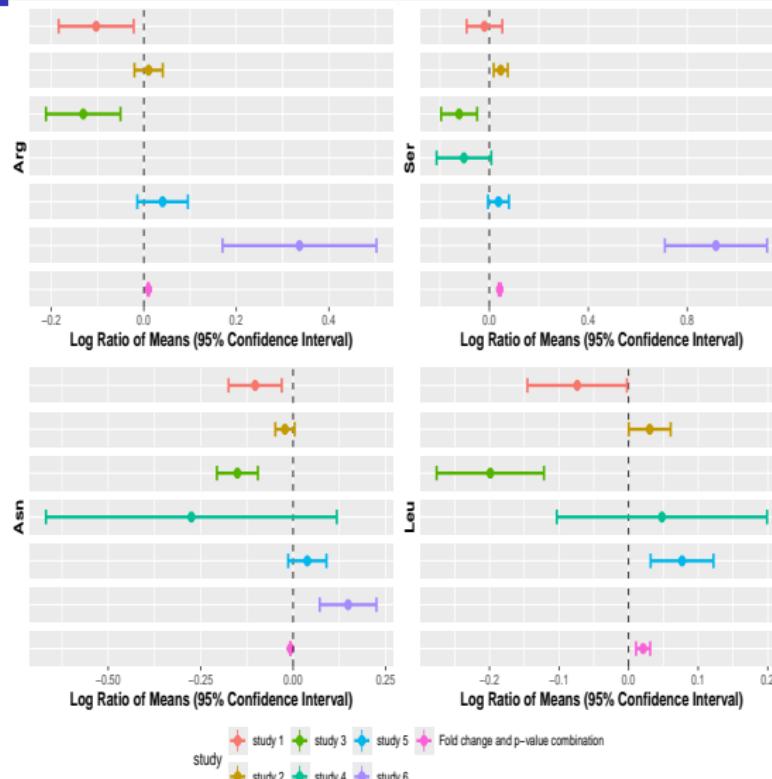
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- Arginine: The combined result appears to be mainly on the 6th study, even though studies 1 and 3 are significantly in the opposite direction. Leucine: Studies 2 and 5 seem to outweigh 1 and 3.
- The standard errors of the combined effect sizes seem to be too small, over-estimating the precision.

## A multivariate meta analysis model

# What we will present?

- We recently published a *multivariate meta-analysis model* and an R package, MetaHD for metabolomics data.

*Liyanage J C, Prendergast L, Staudte R, and De Livera A M, MetaHD: a multivariate meta-analysis model for metabolomics data, Bioinformatics, Volume 40, Issue 7, July 2024, btae470,*  
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- This talk describes this model and some further empirical developments to-date exploring a faster version of the model.

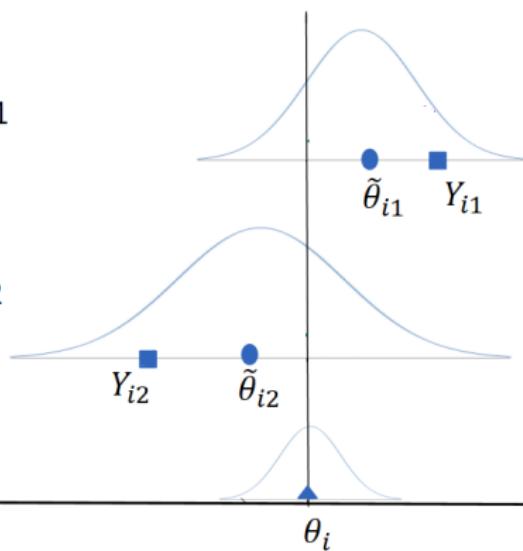


Full Paper

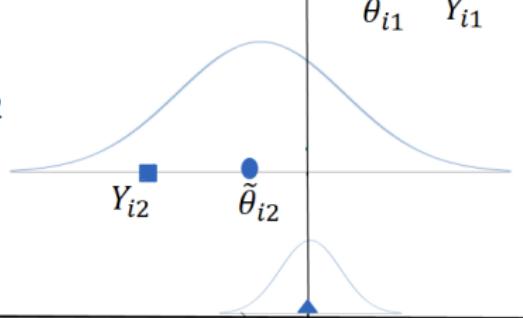


# A multivariate meta analysis model: effect sizes

Study 1



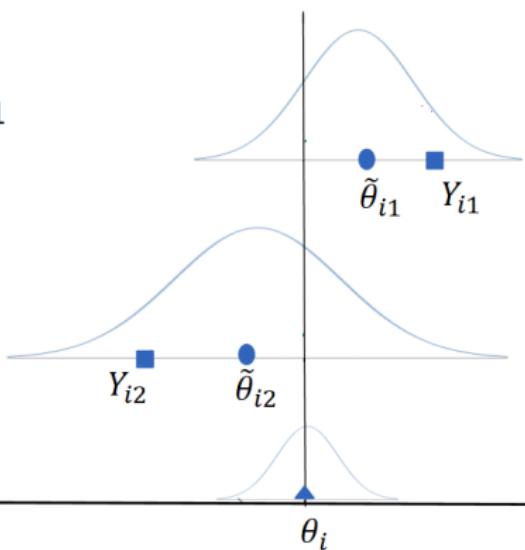
Study 2



- Let  $Y_{ik} = \log\left(\frac{\bar{x}_{k,\text{Treated}}^{(i)}}{\bar{x}_{k,\text{Control}}^{(i)}}\right)$  denote observed effects size for the  $i$ th outcome (metabolite) in the  $k$ th study, for  $k = 1, \dots, K$  and  $i = 1, \dots, N$ . Note that some  $Y_{ik}$ s could be missing due to not being reported in some studies. Let  $\mathbf{Y}_k = [Y_{1k}, Y_{2k}, \dots, Y_{Nk}]'$  be a  $(N \times 1)$  matrix with elements  $Y_{ik}$ .

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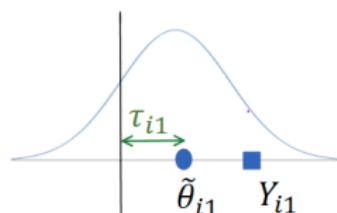


Study 2

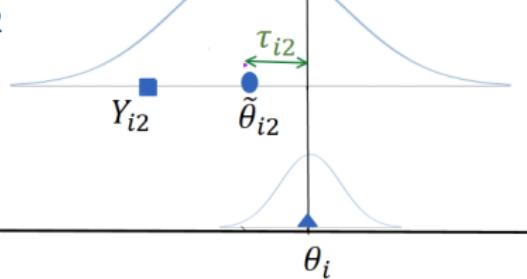
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- For the  $i$ th outcome, assume that the *population effect size*  $\tilde{\theta}_{i,k}$  in the  $k$ th study, is drawn from a distribution of *population effect sizes* with *true mean* across the studies  $\theta_i$  and variance  $\sigma_{\theta_i}^2$ . The size of  $\sigma_{\theta_i}^2$  indicates the degree of heterogeneity in the population effect sizes for the  $i$ th outcome, and  $\theta_i$  describes their central tendency. Let  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_N]'$  be a  $(N \times 1)$  matrix with elements  $\theta_i$ .

# A multivariate meta analysis model: heterogeneity

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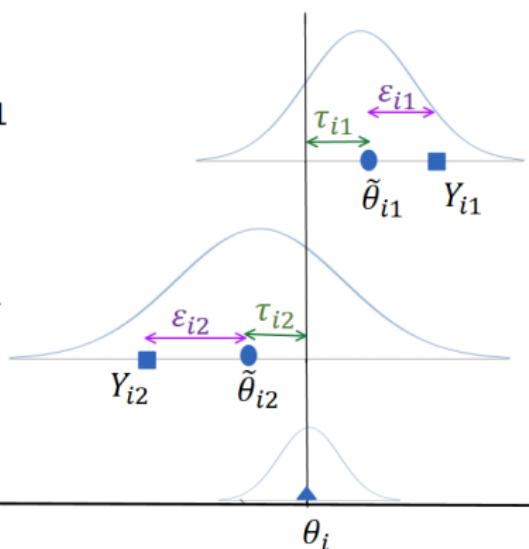
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- Let  $\tau_{i,k}$  be an error term by which the population effect size  $\tilde{\theta}_{i,k}$  differs from the mean  $\theta_i$ , representing *true* heterogeneity in effect sizes due to random population effects in the  $k$ th study. Let  $\tau_k = [\tau_{1k}, \tau_{2k}, \dots, \tau_{Nk}]'$  be a  $(N \times 1)$  matrix with elements  $\tau_{ik}$ .

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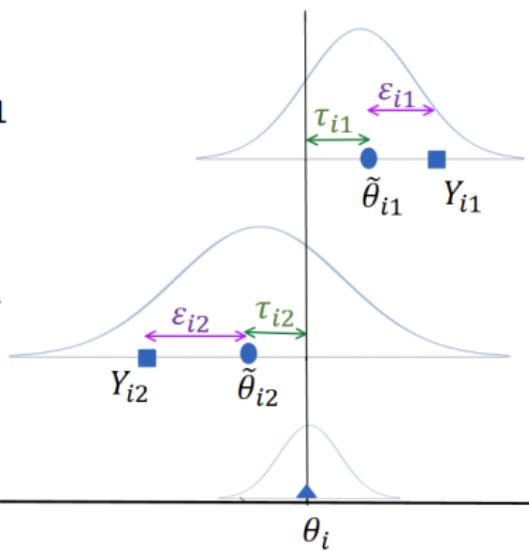


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- Let  $\epsilon_{i,k}$  represent an error term by which the observed effect size  $Y_{ik}$  differs from  $\tilde{\theta}_{i,k}$ , representing the sampling error in the  $k$ th study. Let  $\epsilon_k = [\epsilon_{1k}, \epsilon_{2k}, \dots, \epsilon_{Nk}]'$  be a  $(N \times 1)$  matrix with elements  $\epsilon_{ik}$ .

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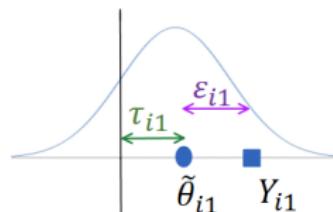


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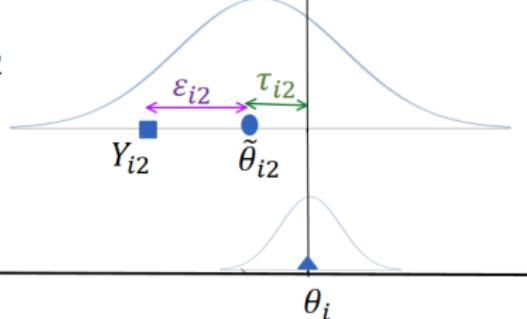
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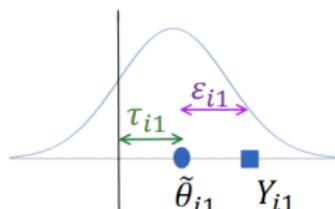
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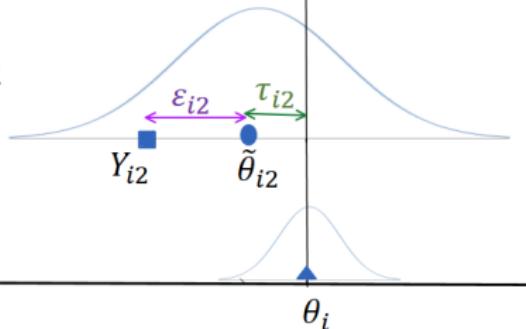
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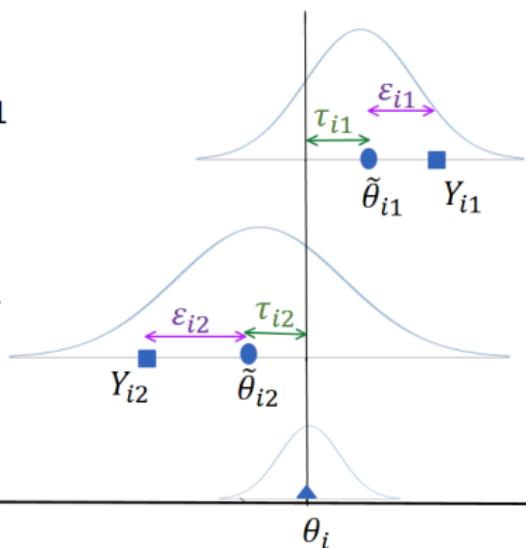
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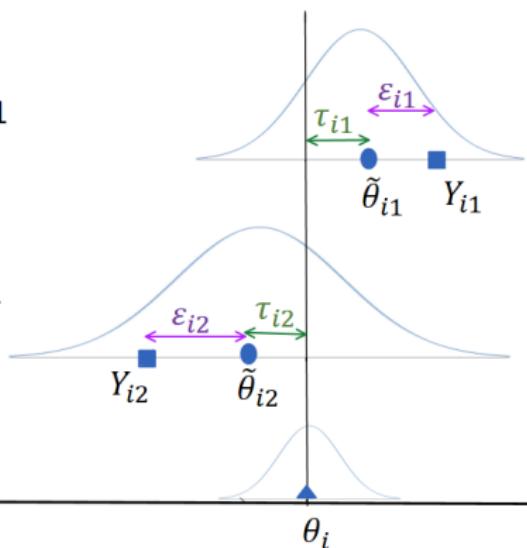


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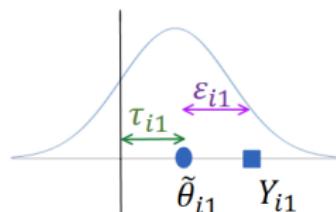


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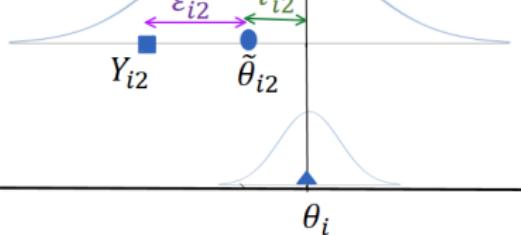
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- **The off-diagonals of the between-study covariance matrix  $\Psi$  reflect the correlation arising when the same outcomes are also measured by other studies.**

# A multivariate meta analysis model: special cases

Study 1



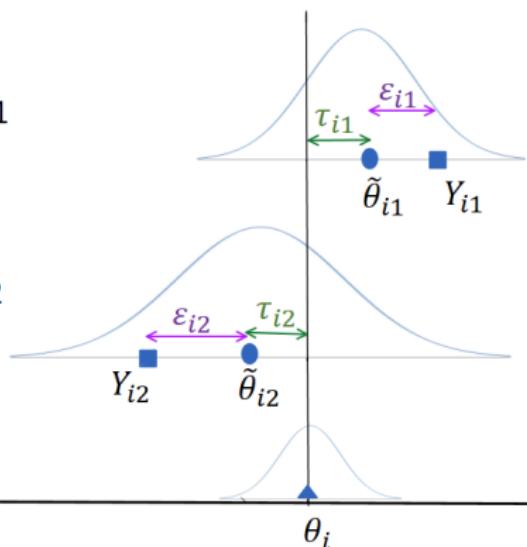
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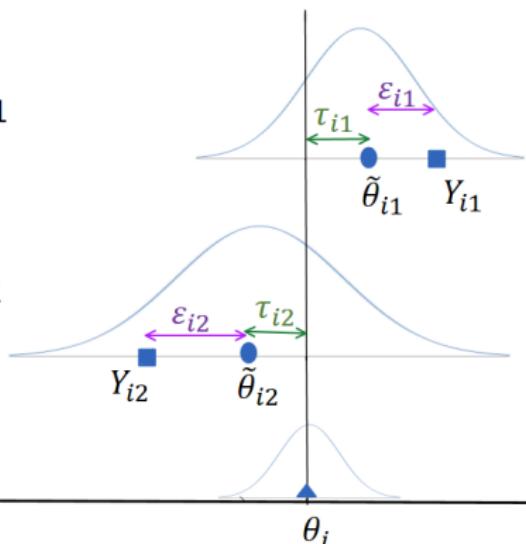
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Study 2

- For estimation of the model parameters, we incorporated a shrinkage approach to large-scale covariance estimation.
- When the within-study (and between-study) correlations are all zero, the model is equivalent to several separate univariate random-effects models.
- In addition to the above, if the between-study variances are set to zero, the model is equivalent to several separate univariate fixed-effects models.

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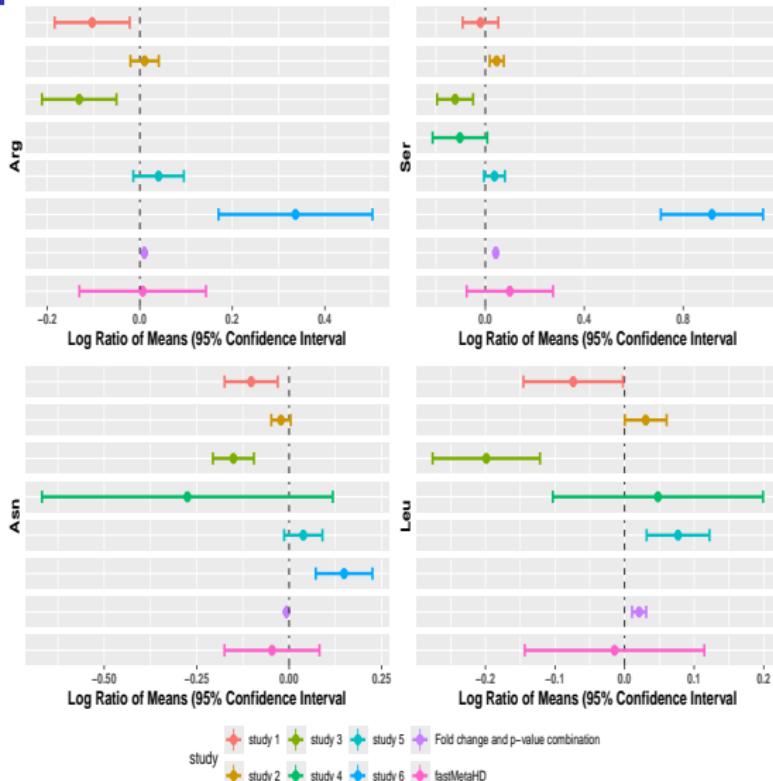
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  - **Step 1:** The model first fitted assuming zero within and between correlations (i.e., univariate random effects model). Variables are then ranked by p-values and organised into smaller sub-groups.
  - **Step 2:** The multivariate model is then re-fitted (without restrictions on correlations) within each sub-group, and the results are combined.

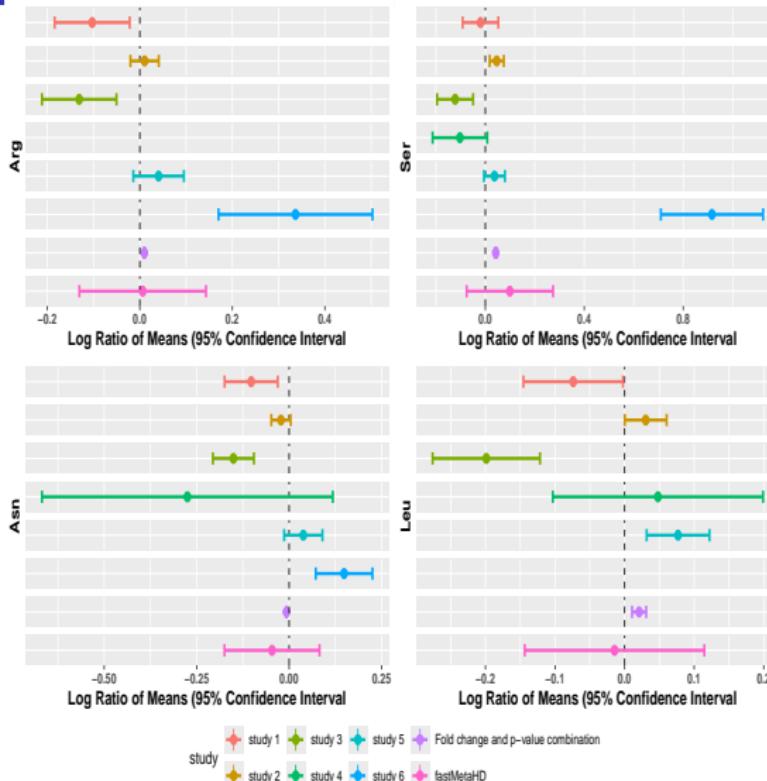
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# Real data: Example I (described previously)



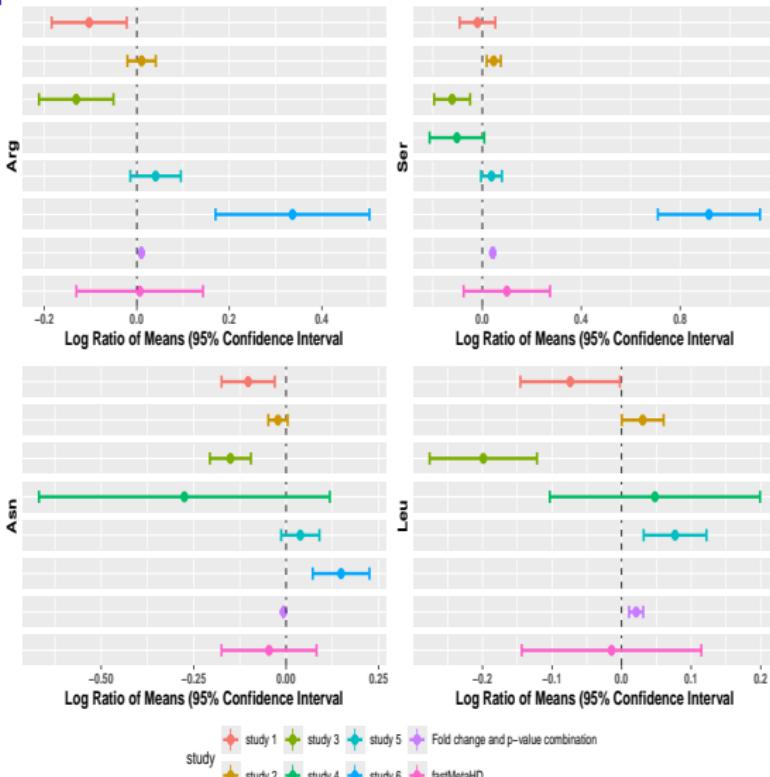
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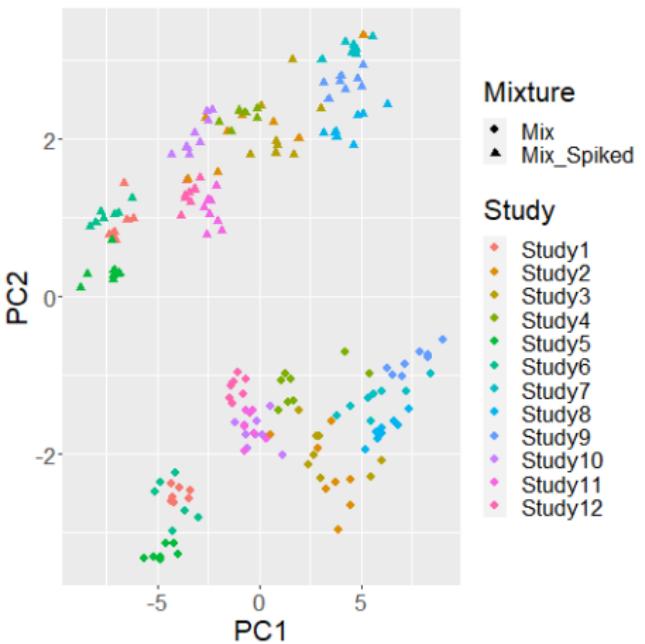
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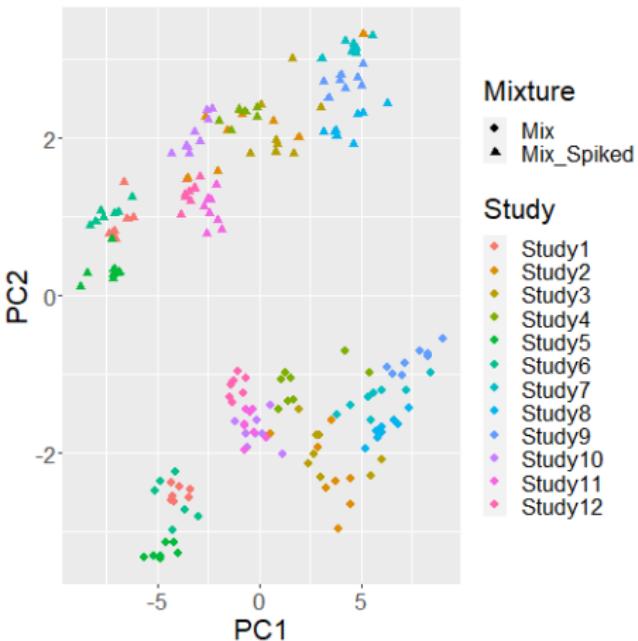
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- No ground truth with this real data set.
- To evaluate the multivariate meta analysis approach, we used multiple real-datasets that had some known differentially-expressed variables and simulated data.

# Real data: Example II



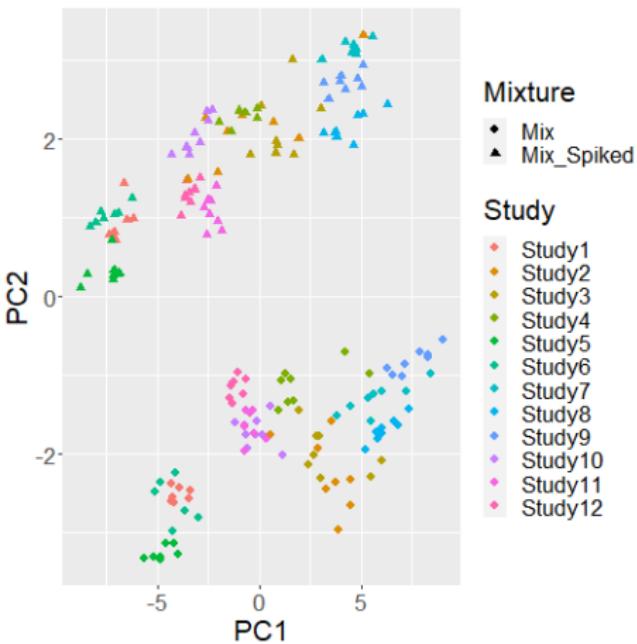
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- 33 metabolites were detected across all studies, in 185 samples. 11 metabolites were present in MIX-SPIKED in 3-fold amounts, one was present in a 5-fold amount compared to MIX, and the other metabolites remained unchanged.

## Real data: Example II cont

**Table 1:** RMSE values are on log scale. Values are in hundreds and the smallest value in each fold-change category is shown in boldface type.

Method	Non-changing	3-fold	5-fold
fastMetaHD	<b>6.8</b>	<b>45.2</b>	<b>4.6</b>
Univariate fixed	13.8	50.3	17.7
Univariate fixed or random	10.2	47.2	18.6
Univariate random	10.2	47.2	18.6
Fold change	10.2	48.6	19.7

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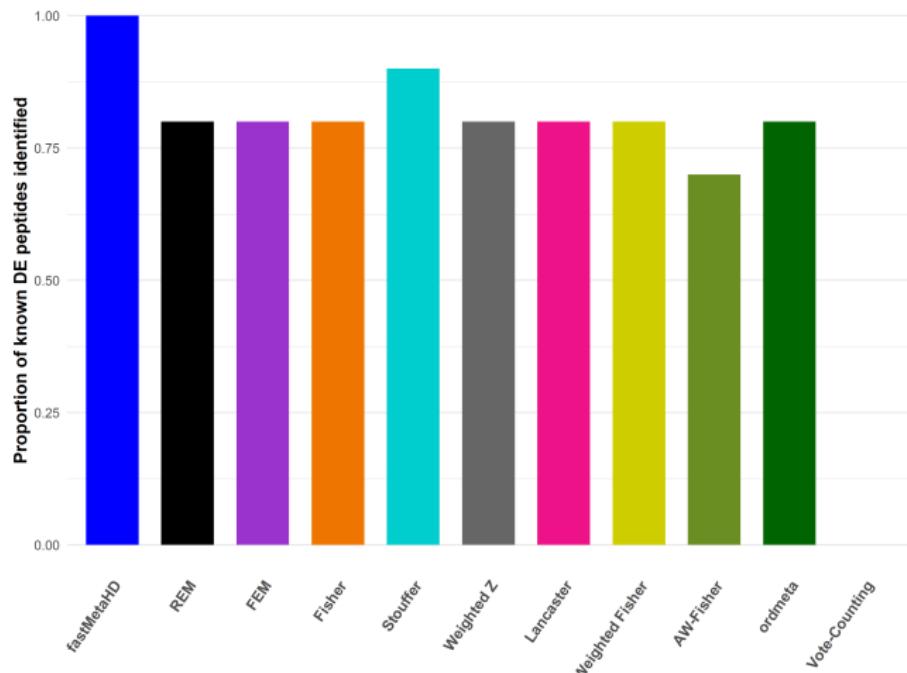
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- There are 366 peptides measured from 233 samples across 4 mass spectrometry batches ('studies')
- We normalised the data within each batch using RUV-2 [7] and obtained summary statistics for each metabolite within each batch (means, variances, p-values).
- The top 10 differentially-expressed peptides identified in a published analysis [13] that included all data was used to calculate the proportion of correctly identified peptides for each method.

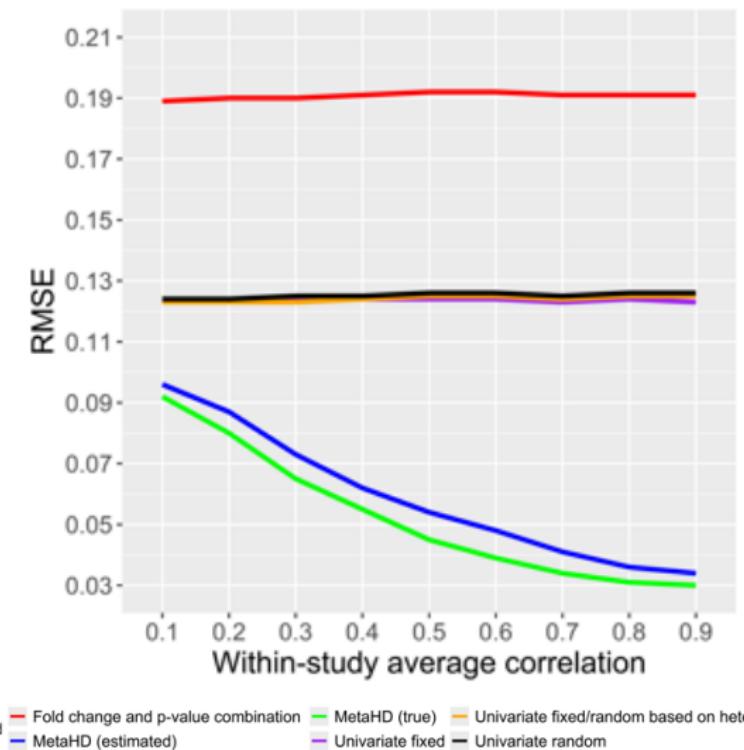
# Mouse proteomic data: Example III cont

**Figure 1:** Bar plots showing the proportion of correctly identified top 10 DE peptides across different meta-analysis methods.



# Simulation study

- Population effect sizes and observed effects were generated from  $\tilde{\theta}_k \sim \text{MVN}(\boldsymbol{\theta}, \boldsymbol{\Psi})$ , and  $\mathbf{Y}_k \sim \text{MVN}(\tilde{\boldsymbol{\theta}}_k, \mathbf{S}_k)$  respectively, with parameters mimicking real-data.
- Root mean square error (RMSE) comparing: Multivariate meta analysis using the known correlation structures, Multivariate meta analysis estimating the unknown correlation structures using observed effects, fixed and random-effects models, and fold-change approach.



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- The gain in RMSE increased as the within-study correlation increased and also when good approximations to the covariance matrices were available.
- Another advantage is that this multivariate meta analysis approach can accommodate missing values.
- Multivariate meta-analysis models (and fixed and random effects models) cannot be used with limited data (e.g., when only the p-values or only the effect sizes are available). In such cases combining p-values and/or fold changes may be the only approaches available.

# Thank you..



Alysha De Livera  
Academic (Statistics)



Research Group Website



CRAN Package



MetaHD



Online Tutorial

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