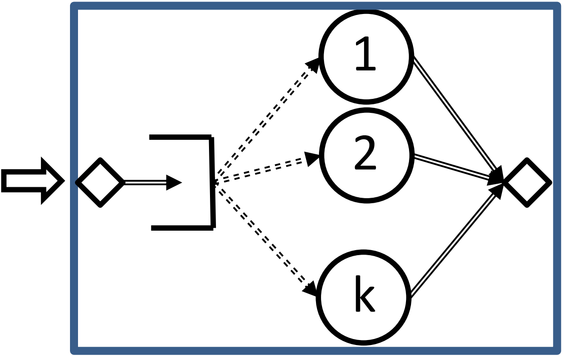
Queueing Theory for Single Workstations

(Source: Hopp & Spearman, Factory Physics, ed. 2)



M/M/1 $[WIP, WIP_q, CT, CT_q, TH, U] = f(\mu_p, \mu_{ia})$

$$U = \frac{\mu_p}{\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$WIP = \frac{U}{1-U} \quad \text{(from Markovian assumptions)}$$
$$TH = \frac{1}{\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$CT = \frac{WIP}{TH} \quad \text{(Little's Law)}$$
$$CT_q = CT - \mu_p \quad \text{(any single workstation)}$$
$$WIP_q = TH * CT_q \quad \text{(Little's Law)}$$

M/M/k $[\,] \approx f(\mu_p, \mu_{ia}, k)$

$$U = \frac{\mu_p}{k\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$CT_q \approx \frac{U^{\sqrt{2(k+1)}-1}}{k(1-U)}\mu_p \quad \text{(Sakasegawa, 1977)}$$
$$\ldots$$
$$CT = CT_q + \mu_p \quad \text{(any single workstation)}$$
$$TH = \frac{1}{\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$WIP_q = TH * CT_q \quad \text{(Little's Law)}$$
$$WIP = TH * CT \quad \text{(Little's Law)}$$

G/G/1 $[\,] \approx f(\mu_p, \sigma_p^2, \mu_{ia}, \sigma_{ia}^2)$ with $c_{ia} = \frac{\sigma_{ia}}{\mu_{ia}}$ and $c_p = \frac{\sigma_p}{\mu_p}$

$$U = \frac{\mu_p}{\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$CT_q \approx \left(\frac{c_{ia}^2 + c_p^2}{2}\right)\left(\frac{U}{1-U}\right)\mu_p \quad \text{(Kingman} \equiv \text{VUT)}$$

Continue as with M/M/k ...

G/G/k $[\,] \approx f(\mu_p, \sigma_p^2, \mu_{ia}, \sigma_{ia}^2, k)$

$$U = \frac{\mu_p}{k\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$CT_q \approx \left(\frac{c_{ia}^2 + c_p^2}{2}\right)\left(\frac{U^{\sqrt{2(k+1)}-1}}{k(1-U)}\right)\mu_p \quad \text{(VUT, Sakasegawa)}$$

Continue as with M/M/k ...

If **Preemptive Outages**
- Time between outages has average M_{TTF}, $CV \approx 1$
- Outages have mean and variance $(M_{TTR}, \sigma_{TTR}^2)$

Then correct (μ_p, σ_p^2) to effective (μ_e, σ_e^2):

$$A = \frac{M_{TTF}}{M_{TTF} + M_{TTR}}$$
$$\mu_e = \frac{\mu_p}{A}$$
$$\sigma_e^2 = \left(\frac{\sigma_p}{A}\right)^2 + \frac{(M_{TTR} + \sigma_{TTR}^2)(1-A)\mu_p}{A \cdot M_{TTR}}$$

If **Nonpreemptive Outages**
- Number of jobs between outages has average N, $CV \approx 1$
- Outages have mean and variance (μ_s, σ_s^2)

Then correct (μ_p, σ_p^2) to effective (μ_e, σ_e^2):

$$\mu_e = \mu_p + \frac{\mu_s}{N}$$
$$\sigma_e^2 = \sigma_p^2 + \frac{\sigma_s^2}{N} + \frac{N-1}{N^2}\mu_s^2$$

If both **Preemptive and Nonpreemptive Outages**
Then apply the formulas consecutively.

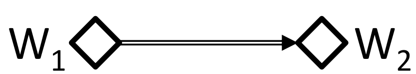
If **Process Batching, Parallel Processing**
- G/G/1 queue with batch size b
- Whole-batch processing time μ_p

$$U = \frac{\mu_p}{b\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$CT_q \approx \left(\frac{\frac{c_{ia}^2}{b} + c_p^2}{2}\right)\left(\frac{U}{1-U}\right)\mu_p \quad \text{(Kingman)}$$
$$CT = \frac{(b-1)\mu_{ia}}{2}\mu_p + CT_q + \mu_p \quad \text{(add wait-to-batch)}$$
$$\ldots$$
$$TH = \frac{1}{\mu_{ia}} \quad \text{(any stable single workstation)}$$
$$WIP_q = TH * CT_q \quad \text{(Little's Law)}$$
$$WIP = TH * CT \quad \text{(Little's Law)}$$

If **Process Batching, Serial Processing**
- G/G/1 queue with batch size b
- Per-element processing time μ_p
- Setup time between batches s
- SCV for batch process & setup time c_e^2
- Assume $c_e^2 = 0.5$ and $c_{ia}^2 = 1$:

$$U = \frac{\mu_p + s/b}{\mu_{ia}}$$
$$CT_q \approx \left(\frac{c_{ia}^2 + c_p^2}{2}\right)\left(\frac{U}{1-U}\right)\mu_p \quad \text{(Kingman)}$$
$$WIBT = \frac{b-1}{2}\mu_p \text{ (lot splitting)}, \quad (b-1)\mu_p \text{ (no splitting)}$$
$$CT = CT_q + s + WIBT + \mu_p$$

Continue as with Parallel Processing ...

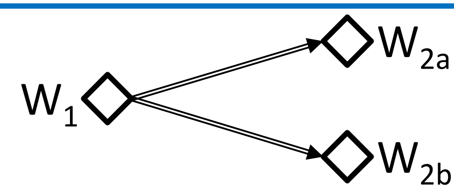


For **serially-arranged** workstations,
the departure process from W_1 is the arrival process at W_2.
$(\mu_d, c_d^2) = f(c_{ia}^2, c_p^2, k, U, TH)$:

$$\mu_d = \frac{1}{TH}$$
$$c_d^2 = 1 + (1-U^2)(c_{ia}^2 - 1) + \frac{U^2}{\sqrt{k}}(c_p^2 - 1)$$

which for $k = 1$ reduces to:

$$c_d^2 = U^2 c_p^2 + (1-U^2)c_{ia}^2$$



For a workstation with **parallel downstream branches**,
If the branch's routing control logic is probabilistic
(prob p_1 of routing to W_{2a}, prob p_2 of routing to W_{2b}, ...)
and if W_1's departure process has $(\mu_d, c_d^2 \approx 1)$
Then results for "splitting" a Poisson process apply.
- Arrivals at W_{2a} will have $(p_1\mu_d, c_d^2 \approx 1)$
- Arrivals at W_{2b} will have $(p_2\mu_d, c_d^2 \approx 1)$, etc.