RANKABILITY AS A SENSITIVITY MEASURE

Start with unweighted data, i.e., $\mathbf{D}$ is a binary matrix of 0s and 1s with 0s on the diagonal. Later we will do weighted data. Start with small n and later increase.

1. Create the best case rankability situation, which is a perfect dominance graph $\mathbf{D}_{n\times n}$, a strictly upper triangular matrix of ones. Verify that the rankability scores are the best case, perfect values of $k = 0$, $p = 1$, and the lone ranking vector in P is $\mathbf{r} = \begin{pmatrix} 1 & 2 & \dots & n \end{pmatrix}$.

1B. Run the LOP ranking method[1] to get one optimal ranking $\mathbf{r}$. (We know the optimal LOP ranking $\mathbf{r} = \begin{pmatrix} 1 & 2 & \dots & n \end{pmatrix}$ for the perfect dominance graph. Other ranking methods such as Massey and Colley should also get this same ranking vector $\mathbf{r} = \begin{pmatrix} 1 & 2 & \dots & n \end{pmatrix}$.)

2. Create the perturbed matrix $\tilde{\mathbf{D}} = \mathbf{D} + 5\%$noise. Create the Dtilde matrix by randomly identifying 5% of the $n^2 - n$ off-diagonal locations and flipping their values so that for these locations 0s becomes 1s and 1s become 0s.

2B. Run Hillside Count on $\tilde{\mathbf{D}}$ to compute the rankability scores $\tilde{k}$ and $\tilde{p}$ for the perturbed matrix.

2C. Run the LOP ranking method on $\tilde{\mathbf{D}}$ to get the ranking vector $\tilde{\mathbf{r}}$ for the perturbed matrix.

3. Compare the original ranking vector $\mathbf{r}$ and the perturbed ranking vector $\tilde{\mathbf{r}}$ with the (weighted) Kendall tau τ or Spearman footrule measure. We want to measure how far apart these two rankings are. We hypothesize that rankable data $\mathbf{D}$ is insensitive with respect to the ranking to noise so that $\tau(\mathbf{r}, \tilde{\mathbf{r}})$ should be small.

4. Do Steps 2-4 one hundred times, creating a new $\tilde{\mathbf{D}}$ with different 5% noise one hundred times and measuring the distance $\tau(\mathbf{r}, \tilde{\mathbf{r}})$ one hundred times.

5. Report the average of these 100 $\tau(\mathbf{r}, \tilde{\mathbf{r}})$ scores.

6. On one plot, with the x-axis as the iteration numbers 1 through 100, use the y-axis to plot three values at each iteration: $\tilde{k}$, $\tilde{p}$, and $\tau(\mathbf{r}, \tilde{\mathbf{r}})$. So there should be three graphs on one figure. We want to use these graphs to see the trends for the relationship between rankability and τ.

7. Repeat Steps 2-6 for increasing levels of noise, up to, say, 50%.

8. Repeat Steps 1-7 but replace Step 1 with the worst case rankability situation, the empty graph $\mathbf{D}$ of all zeros. Verify that the rankability scores k and p are worst case values. Run the LOP ranking method (and/or Massey, Colley, etc.) to get the ranking vector $\mathbf{r}$. We hypothesize that unrankable data is very sensitive with respect to ranking to noise so $\tau(\mathbf{r}, \tilde{\mathbf{r}})$ should be large.

9. Repeat Steps 1-7 but replace Step 1 with a real data instance from LOLIB. Try several instances, picking some that are rankable and some that are unrankable. If the LOLIB instance

[1] In order to generalize this work, we can run several ranking methods, e.g., LOP, Massey, Colley, PageRank, etc.

has a weighted $\mathbf{D}$, simply transform it to an unweighted instance by $\mathbf{D} = \mathbf{D} > \mathbf{0}$.