Завдання 9

Микола Коломієць

17 квітня 2024 р.

Знайти похідні таких (узагальнених) функцій:

1. $D^n|t|\sin t$, для $n = 1, 2, 3$

$$n = 1 : D|t|\sin t = -\int_{\mathbb{R}} |t|\sin(t)\varphi'(t)dt = -\int_0^\infty t\sin(t)\varphi'(t)dt + \int_{-\infty}^0 t\sin(t)\varphi'(t)dt =$$

$$= -(t\sin(t)\varphi(t)|_0^\infty - \int_0^\infty \varphi(t)(t\cos(t) + \sin(t))dt) + t\sin(t)\varphi(t)|_{-\infty}^0 - \int_{-\infty}^0 \varphi(t)(t\cos(t) + \sin(t))dt =$$

$$= \int_0^\infty \varphi(t)(t\cos(t) + \sin(t))dt - \int_{-\infty}^0 \varphi(t)(t\cos(t) + \sin(t))dt =$$

$$= \int_{\mathbb{R}} \varphi(t)(t\cos(t) + \sin(t))sign(t)dt \Rightarrow D|t|\sin t = (t\cos(t) + \sin(t))sign(t)$$

$$n = 2 : D^2|t|\sin t = D(t\cos(t) + \sin(t)) =$$

2. $D\ln|t|$

$$D\ln|t| = -\int_{\mathbb{R}} \ln|t|\varphi'(t)dt = -\int_0^\infty \ln t\varphi'(t)dt + \int_0^\infty \ln(-t)\varphi'(t)dt =$$

$$-(\ln t\varphi(t)|_0^\infty - \int_0^\infty \frac{\varphi(t)}{t}dt) + \ln -t\varphi(t)|_{-\infty}^0 - \int_{-\infty}^0 \frac{\varphi(t)}{-t}dt = \int_0^\infty \frac{\varphi(t)}{t}dt + \int_{-\infty}^0 \frac{\varphi(t)}{t}dt = \int_{\mathbb{R}} \frac{\varphi(t)}{t}dt$$

$$D\ln|t| = \frac{1}{t}$$

3. $D\mathcal{P}\frac{1}{l}$

$$D\mathcal{P}\frac{1}{t} = -\int_{\mathbb{R}} \frac{\varphi'}{t}dt = -\frac{\varphi}{t}|_{\mathbb{R}} - \int_{\mathbb{R}} \frac{\varphi}{t^2} \Rightarrow D\mathcal{P}\frac{1}{t} = -\frac{1}{t^2}$$

4. $D^n \operatorname{sign} t$

$$n = 1 : Dsign(t) = -\int_{\mathbb{R}} sign(t)\varphi'(t)dt = -\int_0^\infty \varphi'(t)dt + \int_{-\infty}^0 \varphi'(t)dt = 2\varphi(0)$$

$$Dsign(t) = 2\delta$$

$$n >= 2 : D^n sign(t) = 2(-1)^{n-1}\varphi^{(n-1)}(0)$$

5. $D^n|t|$

$$n=1: D|t| = -\int_{\mathbb{R}} |t|\varphi'(t)dt = -\int_0^\infty t\varphi'(t)dt + \int_{-\infty}^0 t\varphi'(t)dt = -\int_0^\infty \varphi(t)dt + \int_{-\infty}^0 \varphi(t)dt =$$

$$= -\int_{\mathbb{R}} sign(t)\varphi(t)dt \Rightarrow D|t| = sign(t)$$

$$D^2|t| = 2\delta$$

$$D^n|t| = 2(-1)^n\varphi^{n-2}(0)$$

6. $D^n[t]$

7. Довести рівність:

$$D^2|\sin t| + |\sin t| = 2\sum_{k\in\mathbb{Z}} \delta_{k\pi}$$

$$D|\sin t| = -\int_{\mathbb{R}} |\sin t|\varphi'(t)dt = -\int_0^\infty \sin t\varphi'(t)dt + \int_{-\infty}^0 \sin t\varphi'(t)dt =$$

$$= -\int_0^\infty \varphi(t)\cos tdt + \int_{-\infty}^0 \varphi(t)\cos tdt = -\int_{\mathbb{R}} sign(t)\varphi(t)\cos tdt$$

$$D^2|\sin t| = -\int_{\mathbb{R}} sign(t)\cos(t)\varphi'(t)dt = -\int_0^\infty \cos(t)\varphi'(t)dt + \int_{-\infty}^0 \cos(t)\varphi'(t)dt =$$

$$= -(-\varphi(0) + \int_0^\infty \varphi(t)\sin(t)dt) + \varphi(0) + \int_{-\infty}^0 \varphi(t)\sin(t)dt = 2\varphi(0) - \int_{\mathbb{R}} \varphi(t)\sin(t)dt$$