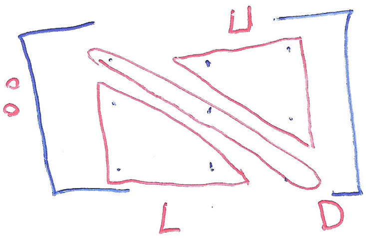
Iterative Methods:

* Jacobi Method:

[checking]

- Check if we can do Jacobi

The diagonal entries must be nonzero

- A : 

For Jacobi to converge,

$M_J = -D^{-1}[L+U]$, $\|M_J\|_\infty < 1$

or $a_{ii} > \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| \ \forall i$

Note (Norms):

vector norms:

- $\|V\|_1 = |V_1| + |V_2| + \dots + |V_n|$
- $\|V\|_\infty = \max\{|V_1|, |V_2|, \dots, |V_n|\}$
- $\|V\|_2 = (V_1^2 + V_2^2 + \dots + V_n^2)^{\frac{1}{2}}$

matrix norms:

- $\|A\|_1 = \max_{1\leq j\leq n} \left\{ \sum_{i=1}^{n} |a_{ij}| \right\}$ col-sum
- $\|A\|_\infty = \max_{1\leq i\leq n} \left\{ \sum_{j=1}^{n} |a_{ij}| \right\}$ row-sum
- $\|A\|_E = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2 \right]^{\frac{1}{2}}$

[solving]

- $Ax = b$

$[D+L+U]x = b$

$Dx^{(1)} = -[L+U]x^{(0)} + b$

$$\boxed{x^{(n)} = D^{-1}\left(-[L+U]x^{(n-1)} + b\right)}$$

to solve faster:

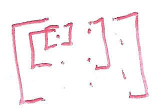
$$x_i^{(n)} = \frac{1}{a_{ii}}\left(-\sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij} x_j^{(n-1)} + b_i\right)$$

* Gauss-Seidel Method:

Similar to Jacobi except:

[checking]

$M_{GS} = -[D+L]^{-1}U$, $\|M_{GS}\|_\infty < 1$



or $a_{ii} > \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}| \ \forall i$ or $A^T = A$ & all determinants of A positive

[solving]

$$\boxed{x^{(n)} = [D+L]^{-1}\left(-Ux^{(n-1)} + b\right)}$$

$$x_i^{(n)} = \frac{1}{a_{ii}}\left(-\sum_{j=1}^{i-1} a_{ij} x_j^{(n)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(n+1)} + b_i\right)$$