Linear Systems

***Gauss Elimination:** • transform to upper triangular $\left(\begin{array}{c|c} A & b \end{array}\right) \rightarrow \left(\begin{array}{c|c} \; & \; \end{array}\right)$ with forward elimination

• find values of $x_1 \dots x_n$ using Back Substitution.

examples:

pivot →
$$\begin{bmatrix} 1 & 3 & 1 & | & 9 \\ \boxed{1} & 1 & -1 & | & 1 \\ \boxed{3} & 11 & 6 & | & 36 \end{bmatrix} \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 3 & 1 & | & 9 \\ 0 & -2 & -2 & | & -8 \\ 0 & \boxed{2} & 3 & | & 9 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & b \end{array} \\ \begin{bmatrix} 1 & 3 & 1 & | & 9 \\ 0 & -2 & -2 & | & -8 \\ 0 & 0 & \boxed{1} & | & \boxed{1} \end{bmatrix} \end{array} \rightarrow x_3 = 1$$

first we must turn these into zeros

now we repeat the process to turn this 2 into zero, using R2 as a pivot.

$x_2 = \frac{-8+2}{-2} = 3$

$x_1 = 9 - 1 - 9 = -1$

+Partial Pivoting: • formula to choose which row should be the pivot.

• optional and won't affect results; makes our work easier.

$$r = \frac{\text{first entry in row}}{\text{largest absolute entry}}$$

ex:
$$\begin{bmatrix} -4 & -3 & 5 & | & 0 \\ 6 & 7 & -3 & | & 2 \\ 2 & -1 & 1 & | & 6 \end{bmatrix} \quad \begin{array}{l} r_1 = 4/5 \\ r_2 = 6/7 \\ r_3 = 2/2 = 1 \end{array}$$

=> R3 is the pivot

=> interchange R_1 & R_2.

***Gauss-Jordan:** • Variant of Jacobi where we eliminate the unknown from equations above our pivot in addition to those below it.

• Doesn't require back substitution.

examples:

$$\begin{bmatrix} \boxed{1} & 3 & 1 & | & 9 \\ 1 & 1 & -1 & | & 1 \\ 3 & 11 & 6 & | & 36 \end{bmatrix} \xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 3 & 1 & | & 9 \\ 0 & \boxed{-2} & -2 & | & -8 \\ 0 & 2 & 3 & | & 9 \end{bmatrix} \xrightarrow{R_2 = -R_2/2} \begin{bmatrix} 1 & \boxed{3} & 1 & | & 9 \\ 0 & 1 & 1 & | & 4 \\ 0 & \boxed{2} & 3 & | & 9 \end{bmatrix}$$

first element of pivot is 1 => no need to normalize

must be normalized

$$\xrightarrow[R_1 = R_1 - 3R_2]{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & 0 & \boxed{-2} & | & -3 \\ 0 & 1 & \boxed{1} & | & 4 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \xrightarrow[R_1 = R_1 + 2R_3]{R_2 = R_2 - R_3} \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \end{array} \begin{array}{l} x_1 \\ x_2 \\ x_3 \end{array}$$