Derivation

* Reminder: Differential Equation: relation between x and its derivative(s)

- Numerical Methods:

Euler's Method: $x' = f(t, x)$, $x(a) = x_0$, solve in interval $[a, b]$ with k steps

$\therefore h = \frac{b-a}{k}$

$$x_{n+1} = x_n + (h) f(t_n, x_n)$$

$$t_{n+1} = t_n + h$$ ← this t is used in x'

Euler's Midpoint:

step 1: $x_{n+1/2} = x_n + \left(\frac{h}{2}\right) f(t_n, x_n)$

step 2: $x_{n+1} = x_n + h f(t_{n+\frac{1}{2}}, x_{n+1/2})$

$t_{n+\frac{1}{2}} = t_n + \frac{h}{2}$

Runge-Kutta:

Same as before but instead of taking the midpoint take multiple points.

Fourth order Range-kutta Method:

$$x_{n+1} = x_n + \frac{h}{6}(k_0 + 2k_1 + 2k_2 + k_3)$$

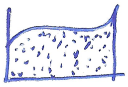
$k_0 = f(t_n, x_n)$

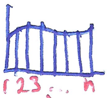
$k_1 = f(t_n + \frac{h}{2}, x_n + \frac{h}{2}(k_0))$

$k_2 = f(t_n + \frac{h}{2}, x_n + \frac{h}{2}(k_1))$

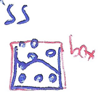
$k_3 = f(t_n + h, x_n + h(k_2))$

Integration

Goal: get surface area 

* FEM: cut it into pieces: 

$\text{area} = \sum_{i=1}^{n} f(t_i)(x_i - x_{i-1})$ where $f(t_i)$ is one of:
- $f(x_{i-1})$
- $f(x_i)$
- $\min(f(x_i), f(x_{i-1}))$
- $\max(f(x_i), f(x_{i-1}))$
- $f\left(\frac{x_i + x_{i-1}}{2}\right)$ (midpoint)
- $\frac{f(x_i) + f(x_{i-1})}{2}$ (trapezoid)

* Monte Carlo: shoot at the area & see if hit or miss 

give x&y
- $\text{area} = \frac{N_{inside}}{N_{total}} \cdot A_{box}$ (N_{inside}: # of hits)

algorithm:
- get random point (x, y)
- if $y \leq f(x)$: hit else miss

give
- MVT way: cut box into N squares & do monte carlo on each: