pyecsca

[pietska]

Reverse-engineering black-box elliptic curve cryptography via side-channel analysis

Jan Jancar, Vojtech Suchanek, Petr Svenda,
Vladimir Sedlacek, **Łukasz Chmielewski**

Who we are

Jan Jancar

- PhD candidate
- Finishing soon, on the job market

Łukasz Chmielewski

- Assistant professor

Centre for Research on
Cryptography and Security

Outline

- Status check
- Why? reverse-engineer ECC
  - Elliptic Curve Cryptography
  - Why Is Hardware Security of ECC Implementations Important?
  - Side-Channel Attacks: Assumptions & Reality
- What? are we reverse-engineering
  - 👉 ECC implementations
- How? to reverse-engineer ECC
  - 👉 RPA-RE
  - 👉 ZVP-RE and others

Status check

https://github.com/J08nY/pyecsca-tutorial-ches2024

- Setup done?
  - Installed packages and running Jupyter Lab, or
  - Using launch binder, or
  - Using docker available
- Quickstart done?
  - `notebooks/start.ipynb`
- Knowledge of Python?
- Knowledge of Jupyter?
- Knowledge of ECC?
- Knowledge of SCA?

`setup.md`

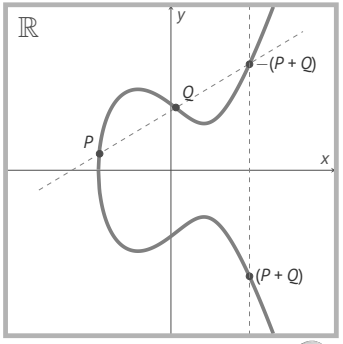
`git pull` now please

Why?

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**: $y^2 \equiv x^3 + ax + b$
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$

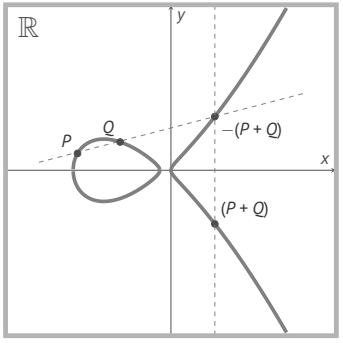


Short Weierstrass

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**: $by^2 \equiv x^3 + ax^2 + x$
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$

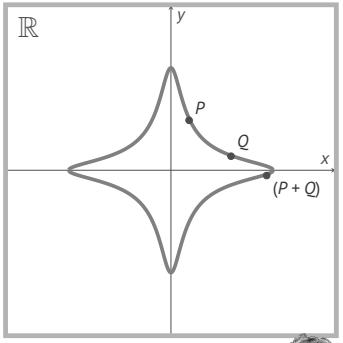


Montgomery

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**: $x^2 + y^2 \equiv c^2(1 + dx^2y^2)$
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$

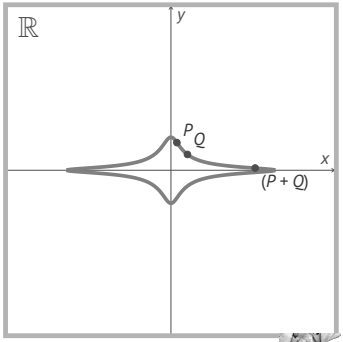


Edwards

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**: $ax^2 + y^2 \equiv 1 + dx^2y^2$
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$



Twisted Edwards

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**:
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$
- **ECDH**: Diffie-Hellman on $E(\mathbb{F}_p)$
  - Scalar multiplication + hash -> Shared secret

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**:
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$
- **ECDH**: Diffie-Hellman on $E(\mathbb{F}_p)$
  - Scalar multiplication + hash -> Shared secret
- **ECDSA**: Digital Signature Algorithm on $E(\mathbb{F}_p)$
  - Random sample + scalar multiplication + hash -> Signature

Why?

Elliptic Curve Cryptography

- **Elliptic Curve**:
  - Points $(x, y) \in E(\mathbb{K})$ form an abelian group
  - Scalar multiplication
    $[n] : E(\mathbb{K}) \to E(\mathbb{K})$
    $P \mapsto [n]P = \underbrace{P + P + \ldots + P}_{n \text{ times}}$
  - Discrete logarithm is hard when $\mathbb{K} = \mathbb{F}_p$
    ECDLP: Find x given $[x]G$ and $G \in E(\mathbb{F}_p)$
- **ECDH**: Diffie-Hellman on $E(\mathbb{F}_p)$
  - Scalar multiplication + hash -> Shared secret
- **ECDSA**: Digital Signature Algorithm on $E(\mathbb{F}_p)$
  - Random sample + scalar multiplication + hash -> Signature
- **XDH**, **EdDSA**, **...**

Why?

Elliptic Curve Cryptography

- Many implementation possibilities

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
    - $y^2 \equiv x^3 + ax + b$

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
    - $y^2 \equiv x^3 + ax + b$
    - $x^2 + y^2 \equiv c^2(1 + dx^2y^2)$

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
    - $y^2 \equiv x^3 + ax + b$
    - $x^2 + y^2 \equiv c^2(1 + dx^2y^2)$
    - $ax^2 + y^2 \equiv 1 + dx^2y^2$

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
    - $y^2 \equiv x^3 + ax + b$
    - $x^2 + y^2 \equiv c^2(1 + dx^2y^2)$
    - $ax^2 + y^2 \equiv 1 + dx^2y^2$
    - $by^2 \equiv x^3 + ax^2 + x$

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
    - (X, Y)

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
    - (X, Y)
    - (X, Y, Z)

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
    - (X, Y)
    - (X, Y, Z)
    - (X, Y, Z, ZZ) …

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas

Why?

Elliptic Curve Cryptography

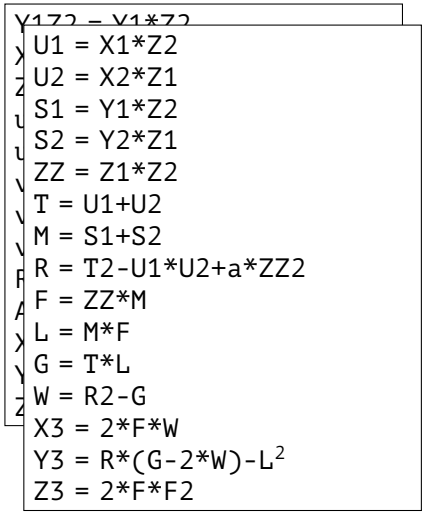
- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas

```
Y1Z2 = Y1*Z2
X1Z2 = X1*Z2
Z1Z2 = Z1*Z2
u = Y2*Z1-Y1Z2
uu = u2
v = X2*Z1-X1Z2
vv = v2
vvv = v*vv
R = vv*X1Z2
A = uu*Z1Z2-vvv-2*R
X3 = v*A
Y3 = u*(R-A)-vvv*Y1Z2
Z3 = vvv*Z1Z2
```

Why?

Elliptic Curve Cryptography

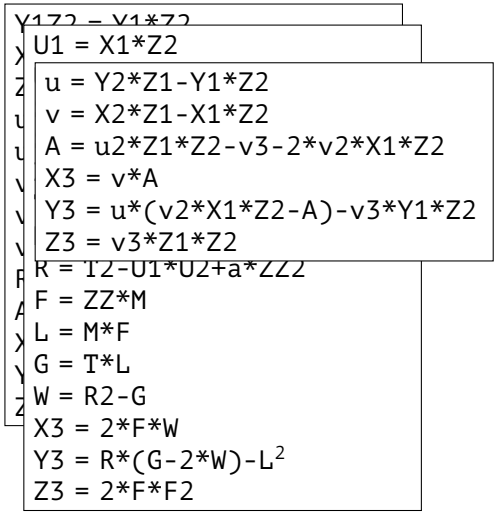
- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas



Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas



Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
    - *fixed-base*, *variale-base*, *multi-scalar*

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
    - *fixed-base*, *variale-base*, *multi-scalar*

Algorithm Left-to-right double-and-add

function LTR($G, k = (k_l, \ldots, k_0)_2$)
  $R = \mathcal{O}$
  for $i = l$ **downto** 0 **do**
    $R = \text{dbl}(R)$
    if $k_i = 1$ **then**
      $R = \text{add}(R, G)$
  return R

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
    - *fixed-base*, *variale-base*, *multi-scalar*

Algorithm Fixed-window scalar multiplier

function Window($G, k = (k_l, \ldots, k_0)_2$)
  $PrecomputedTable = [0 * G, 1 * G, \ldots, 2^w - 1 * G]$
  $\hat{k}$ = recode k to w-bit windows
  $T = \mathcal{O}$
  for $i = 1$ **to** $|\hat{k}|$ **do**
    $T = 2^w T$
    $T = T + PrecomputedTable[\hat{k}_i]$
  return T

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
  - Finite field operations

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
  - Finite field operations
    - Multiplication: *Toom-Cook, Karatsuba, …*

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
  - Finite field operations
    - Multiplication: *Toom-Cook, Karatsuba,* …
    - Squaring: *Toom-Cook, Karatsuba,* …

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
  - Finite field operations
    - Multiplication: *Toom-Cook, Karatsuba, …*
    - Squaring: *Toom-Cook, Karatsuba, …*
    - Reduction: *Barret, Montgomery, …*

Why?

Elliptic Curve Cryptography

- Many implementation possibilities
  - Curve model
  - Coordinates
  - Addition formulas
  - Scalar multiplier
  - Finite field operations
    - Multiplication: *Toom-Cook, Karatsuba, …*
    - Squaring: *Toom-Cook, Karatsuba, …*
    - Reduction: *Barret, Montgomery, …*
    - Inversion: *GCD, Euler*

Why?

Real-world open-source ECC libraries

- Analyzed 18 open-source ECC libraries (📚)
- *BearSSL, BoringSSL, Botan, BouncyCastle, fastecdsa, Go crypto, Intel IPP cryptography, libgcrypt, LibreSSL, libsecp256k1, libtomcrypt, mbedTLS, micro-ecc, Nettle, NSS, OpenSSL, SunEC,* and *SymCrypt*
- Source-code analysis of ECDH, ECDSA, X25519, and Ed25519
- Curve model, Scalar multiplier, Coordinate system, Addition formulas

Why?

Real-world open-source ECC libraries

- **Specific implementations**
  - Curve or architecture-based (10 📄)
  - e.g. $a = -3$ or special prime arithmetic
- **Curve models**
  - Usually outside = inside
  - Montgomery outside, Twisted-Edwards inside (4 📄)
- **Scalar multipliers**
  - *fixed-base* + *variable-base* + *multi-scalar*
  - Comb, fixed-window, wNAF, GLV, …
  - 4 to 7 bit widths
- **Coordinate systems**
  - Usually Jacobian, also homogenous or xz
- **Addition formulas**
  - 112 formula implementations
  - 50 *"standard"*
  - 23 out-of-scope
  - 39 *"non-standard"*

What are the applications of ECC?

- Signatures: ECDSA, EdDSA
- Key Exchange: ECDH, XDH
- Zero-knowledge proofs, …

What are the applications of ECC?

- Signatures: ECDSA, EdDSA
- Key Exchange: ECDH, XDH
- Zero-knowledge proofs, …

- TLS, payments, cryptocurrencies ₿
- Threat: Side-Channel Analysis (SCA)

Passive SCA and Active SCA (Fault Injection)

Passive: analyze device behavior

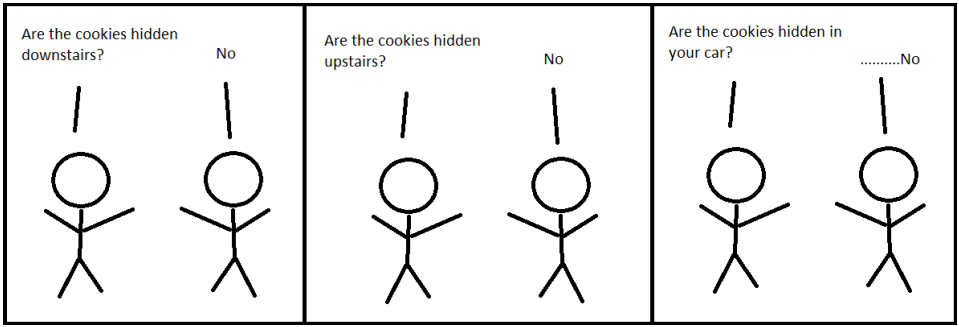
Active: change device behavior

Snow Example: what do you think it is?

Snow Example: what do you think it is? answer.

https://www.independent.co.uk/news/world/europe/melting-snow-being-used-by-police-to-find-cannabis-farms-in-the-netherlands-10036057.html

Cookies Example



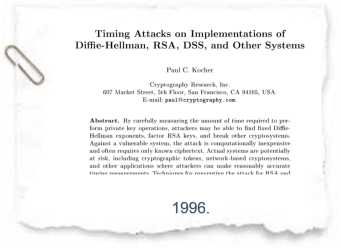
https://www.simplethread.com/great-scott-timing-attack-demo/

Example #1: SPA

```
ScalarMult(P){
 A = ∞
 for ( i = n-1;  i0; i)
        A = DOUBLE(A)
        if (k_i = =1)
               A = ADD(A,P)
        end if
 end for
 Return A = [k]P
}
```

D

A



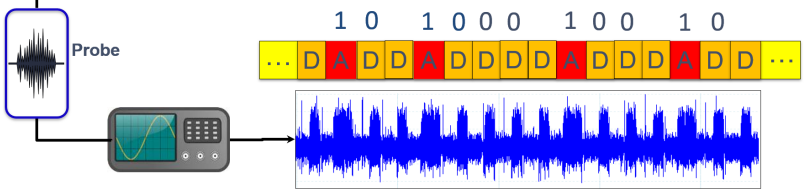
Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems

Paul C. Kocher

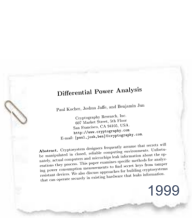
Cryptography Research, Inc.
607 Market Street, 5th Floor, San Francisco, CA 94105, USA.
E-mail: paul@cryptography.com.

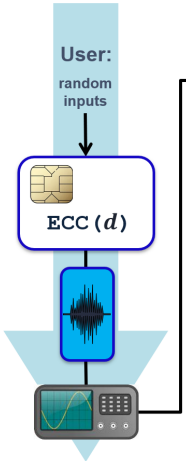
Abstract. By carefully measuring the amount of time required to perform private key operations, attackers may be able to find fixed Diffie-Hellman exponents, factor RSA keys, and break other cryptosystems. Against a vulnerable system, the attack is computationally inexpensive and often requires only known ciphertext. Actual systems are potentially at risk, including cryptographic tokens, network-based cryptosystems, and other applications where attackers can make reasonably accurate timing measurements. Techniques for preventing the attack for RSA and

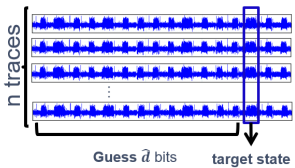
1996.



Example #2: DPA/CPA







Correlation Power Analysis on ECC

f_i = **Selection Function**(random inputs, $\hat{d}$, target state)

- HW of a state in a register
- HD between current and previous register state
- ID model: value of a register
- Most significant bit:

$$f_i = \begin{cases} 0 & if\ HW \leq 16 \\ 1 & if\ HW > 16 \end{cases} \qquad f_i = HW(reg\_state)$$

DPA = Difference of Means

CPA = Pearson correlation

Profiled Attacks

- Attacking a protected imp. is hard and we often do not get traces with the same secret
- Can we attack the key directly?
- So, we use an unprotected device of the same model

Profiled Attacks

- Attacking a protected imp. is hard and we often do not get traces with the same secret
- Can we attack the key directly?
- So, we use an unprotected device of the same model

Figure: protected device (left), unprotected device (right)

Profiled Attacks

- Attacking a protected imp. is hard and we often do not get traces with the same secret
- Can we attack the key directly?
- So, we use an unprotected device of the same model

Figure: protected device (left), unprotected device (right)

- Template Attack Procedure:
  1. Choose a model that describes the power consumption
  2. Profile the unprotected device to create the template (Template Building)
  3. Use the template to break the protected device (Template Matching)
- Neural Networks can be used instead of Template Attacks

Real-World Attacks on ECC Implementations

Minerva, October 3, 2019

Researchers Discover ECDSA
Key Recovery Method

Real-World Attacks on ECC Implementations

TPM-FAIL, November 13, 2019

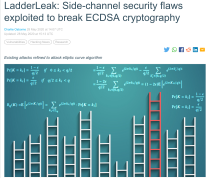
Minerva, October 3, 2019

Researchers Discover ECDSA
Key Recovery Method

Real-World Attacks on ECC Implementations

TPM-FAIL, November 13, 2019

LadderLeak, May 28, 2020



Minerva, October 3, 2019

Real-World Attacks on ECC Implementations

TPM-FAIL, November 13, 2019

LadderLeak, May 28, 2020

SCA Titan: January 7, 2021

Minerva, October 3, 2019

Real-World Attacks on ECC Implementations

TPM-FAIL, November 13, 2019

LadderLeak, May 28, 2020

SCA Titan: January 7, 2021

Minerva, October 3, 2019

TPMScan, March 12, 2024

Real-World Attacks on ECC Implementations

TPM-FAIL, November 13, 2019

LadderLeak, May 28, 2020

SCA Titan: January 7, 2021

Minerva, October 3, 2019

TPMScan, March 12, 2024

EUCLEAK, Yesterday

Why?

Side-Channel Attacks

- Simple Power Analysis
- Differential Power Analysis
- Correlation Power Analysis
- Mutual Information Analysis
- Refined Power Analysis, Zero-value Point Attack, Exceptional Procedure Attack
- Template attacks
- Leakage assessment
- Doubling attack, Collision attacks, Online Template Attack
- …

Why?

Side-Channel Attacks

- Simple Power Analysis
- Differential Power Analysis
- Correlation Power Analysis
- Mutual Information Analysis
- **Refined Power Analysis, Zero-value Point Attack, Exceptional Procedure Attack**
- Template attacks
- Leakage assessment
- Doubling attack, Collision attacks, Online Template Attack
- …

Why?

Assumptions

Why?

Assumptions

$\mathbb{F}_p$ with $p \neq \{2, 3\}$. The algorithm used for the hardware modular multiplication is assumed to be known to the attacker. Moreover, to simplify the attack

[1] Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

Why?

Assumptions

is assumed to be known

[1] Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

Why?

Assumptions

is assumed to be known

[1] Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

Why?

Assumptions

input to s (“fix class”). (This assumes a white-box evaluator that has access to implementation internals.)

is assumed to be known

[2] Oscar Reparaz, Josep Balasch & Ingrid Verbauwhed: Dude, is my code constant time?

Why?

Assumptions

assumes a white-box evaluator

is assumed to be known

[2] Oscar Reparaz, Josep Balasch & Ingrid Verbauwhed: Dude, is my code constant time?

Why?

Assumptions

assumes a white-box evaluator

e 6.1 abstractly depicts a side-channel measurement of such an ex-
tion. For the sake of simplicity, I assume it is a binary exponen-

is assumed to be known

[3] Johann Heyszl: Impact of Localized Electromagnetic Field Measurements on Implementations of Asymmetric Cryptography

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

is assumed to be known

[3] Johann Heyszl: Impact of Localized Electromagnetic Field Measurements on Implementations of Asymmetric Cryptography

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

is assumed to be known

values may be manipulated when working with points P and $2P$. However this idea only works when using the downward routine.

[4] Pierre-Alain Fouque & Frederic Valette: The Doubling Attack – Why Upwards Is Better than Downwards

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

is assumed to be known

only works when using the downward routine.

[4] Pierre-Alain Fouque & Frederic Valette: The Doubling Attack – Why Upwards Is Better than Downwards

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

is assumed to be known

only works when using the downward routine.

[4] Pierre-Alain Fouque & Frederic Valette: The Doubling Attack – Why Upwards Is Better than Downwards

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

a doubling operation from an addition one. This technique, which allows to eventually recover the secret scalar, is applied to three different atomic formulae on elliptic curves,

is assumed to be known

only works when using the downward routine.

[1] Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

Why?

Assumptions

assumes a white-box evaluator

assume it is a binary exp

is applied to three different atomic formulae on elliptic curves,

is assumed to be known

only works when using the downward routine.

[1] Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves

Why?

Assumptions

attack on the Montgomery-López-Dahab ladder algorithm

assumes a white-box evaluator

assume it is a binary exp

is applied to three different atomic formulae on elliptic curves,

is assumed to be known

only works when using the downward routine.

[5] Bo-Yeon Sim & Dong-Guk Han: Key Bit-Dependent Attack on Protected PKC Using a Single Trace

Why?

Assumptions

attack on the Montgomery-López-Dahab ladder algorithm

assumes a white-box evaluator

assume it is a binary exp

is applied to three different atomic formulae on elliptic curves,

is assumed to be known

full knowledge of all algorithms,

only works when using the downward routine.

[6] Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache: A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards

Why?

Assumptions

attack on the Montgomery-López-Dahab ladder algorithm

assumes a white-box evaluator

assume it is a binary exp

is applied to three different atomic formulae on elliptic curves,

knowledge of the ECSM and the elliptic

is assumed to be known

full knowledge of all algorithms,

only works when using the downward routine.

[6] Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache: A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards

Why?

Assumptions

attack on the Montgomery-López-Dahab ladder algorithm

assumes a white-box evaluator

assume it is a binary exp

is applied to three different atomic formulae on elliptic curves,

knowledge of the ECSM and the elliptic

is assumed to be known

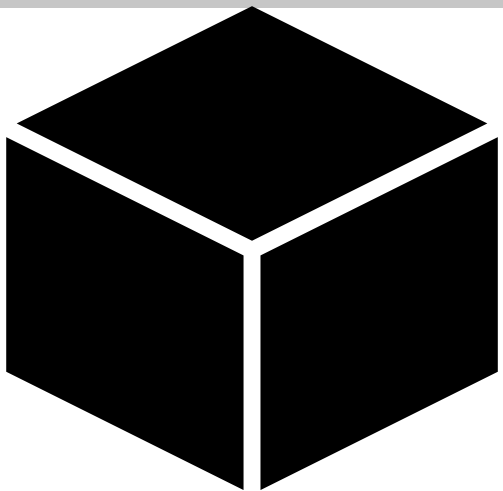
full knowledge of all algorithms,

only works when using the downward routine.

Lots of assumptions you've got there!

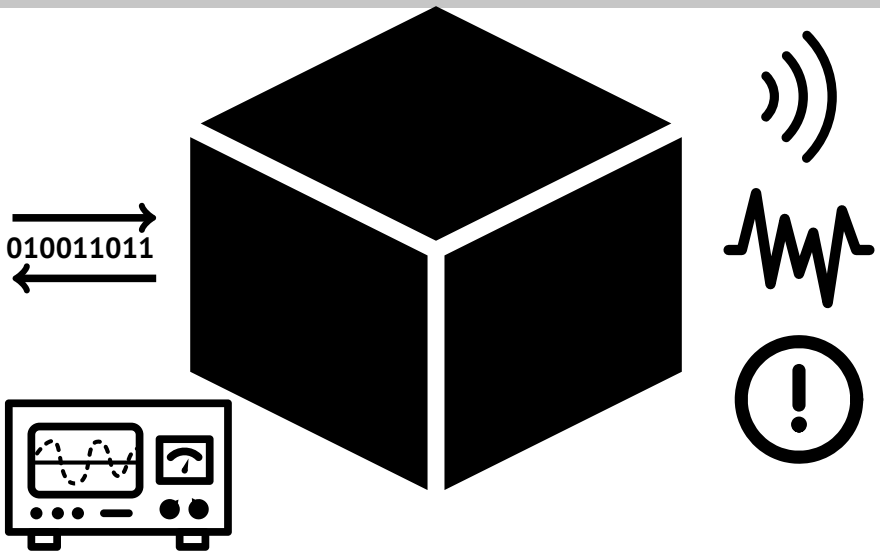
Why?

Reality



Why?

Reality



Why?

- There are a lot of ways in which ECC can be implemented.

Why?

- There are a lot of ways in which ECC can be implemented.
- Most of the known attacks need to know the details of the implementation to succeed.

Why?

- There are a lot of ways in which ECC can be implemented.
- Most of the known attacks need to know the details of the implementation to succeed.
- However, most of the real-world embedded implementations are black-box, and the attacker does not know the implementation.

Why?

- There are a lot of ways in which ECC can be implemented.
- Most of the known attacks need to know the details of the implementation to succeed.
- However, most of the real-world embedded implementations are black-box, and the attacker does not know the implementation.
- How to learn the implementation details?
- Can side channels help?

What?

What?

Implementations

- The object of our study
- Scalar multiplication algorithms
- Real-world open-source ECC
- Countermeasures

Scalar multiplication algorithms

Naive Algorithm #1

Algorithm Left-to-right double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ down to 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad$ **if** $k_i = 1$ **then**
7: $\quad\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
8: **return** $\boldsymbol{R}_0$

Scalar multiplication algorithms

Naive Algorithm #1

Algorithm Left-to-right double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ down to 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad$ **if** $k_i = 1$ **then**
7: $\quad\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm?

Scalar multiplication algorithms

Naive Algorithm #1

Algorithm Left-to-right double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ down to 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad$ **if** $k_i = 1$ **then**
7: $\quad\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm?
- What is wrong?

Scalar multiplication algorithms

Naive Algorithm #2

Algorithm Right-to-left double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: $\boldsymbol{Q} \leftarrow \boldsymbol{O}$
5: **for** $i \leftarrow 0$ **to** $n - 1$ **do**
6: **if** $k_i = 1$ **then**
7: $\boldsymbol{Q} \leftarrow \boldsymbol{Q} + \boldsymbol{R}_0$
8: $\boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
9: **return** $\boldsymbol{Q}$

Scalar multiplication algorithms

Naive Algorithm #2

Algorithm Right-to-left double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: $\boldsymbol{Q} \leftarrow \boldsymbol{O}$
5: **for** $i \leftarrow 0$ **to** $n - 1$ **do**
6: **if** $k_i = 1$ **then**
7: $\boldsymbol{Q} \leftarrow \boldsymbol{Q} + \boldsymbol{R}_0$
8: $\boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
9: **return** $\boldsymbol{Q}$

- What is good about this algorithm?

Scalar multiplication algorithms

Naive Algorithm #2

Algorithm Right-to-left double-and-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: $\boldsymbol{Q} \leftarrow \boldsymbol{O}$
5: **for** $i \leftarrow 0$ **to** $n - 1$ **do**
6: **if** $k_i = 1$ **then**
7: $\boldsymbol{Q} \leftarrow \boldsymbol{Q} + \boldsymbol{R}_0$
8: $\boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
9: **return** $\boldsymbol{Q}$

- What is good about this algorithm?
- What is wrong?

Scalar multiplication algorithms

Let's improve a bit

Algorithm Left-to-right double-and-always-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$, $\boldsymbol{R}_1 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_{1-k_i} \leftarrow \boldsymbol{R}_{1-k_i} + \boldsymbol{P}$
7: **return** $\boldsymbol{R}_0$

Scalar multiplication algorithms

Let's improve a bit

Algorithm Left-to-right double-and-always-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$, $\boldsymbol{R}_1 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_{1-k_i} \leftarrow \boldsymbol{R}_{1-k_i} + \boldsymbol{P}$
7: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Is it more SCA secure?

Scalar multiplication algorithms

Let's improve a bit

Algorithm Left-to-right double-and-always-add algorithm

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{O}$, $\boldsymbol{R}_1 \leftarrow \boldsymbol{O}$
4: **for** $i \leftarrow n - 1$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_{1-k_i} \leftarrow \boldsymbol{R}_{1-k_i} + \boldsymbol{P}$
7: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Is it more SCA secure?
- What is wrong? What about fault attacks?

Scalar multiplication algorithms

Let's improve a bit... more

Algorithm Left-to-right double-and-add-always algorithm [Coron99]

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: **for** $i \leftarrow n - 2$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_1 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
7: $\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_{k_i}$
8: **return** $\boldsymbol{R}_0$

Scalar multiplication algorithms

Let's improve a bit... more

Algorithm Left-to-right double-and-add-always algorithm [Coron99]

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: **for** $i \leftarrow n - 2$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_1 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
7: $\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_{k_i}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm?

Scalar multiplication algorithms

Let's improve a bit... more

Algorithm Left-to-right double-and-add-always algorithm [Coron99]

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: **for** $i \leftarrow n - 2$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_1 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
7: $\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_{k_i}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Elimination of if-statements, even dummy.

Scalar multiplication algorithms

Let's improve a bit... more

Algorithm Left-to-right double-and-add-always algorithm [Coron99]

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: **for** $i \leftarrow n - 2$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_1 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
7: $\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_{k_i}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Elimination of if-statements, even dummy.
- What is different?

Scalar multiplication algorithms

Let's improve a bit... more

Algorithm Left-to-right double-and-add-always algorithm [Coron99]

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \boldsymbol{P}$
4: **for** $i \leftarrow n - 2$ **down to** 0 **do**
5: $\quad \boldsymbol{R}_0 \leftarrow 2\boldsymbol{R}_0$
6: $\quad \boldsymbol{R}_1 \leftarrow \boldsymbol{R}_0 + \boldsymbol{P}$
7: $\quad \boldsymbol{R}_0 \leftarrow \boldsymbol{R}_{k_i}$
8: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Elimination of if-statements, even dummy.
- What is different? Initialization, one vs. two secret dependent accesses.
- Are safe-error attacks possible?

Scalar multiplication algorithms

What is the most common algorithm [High-Level]?

Algorithm The Montgomery Ladder

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \mathcal{O}$
4: $\boldsymbol{R}_1 \leftarrow \boldsymbol{P}$
5: **for** $i \leftarrow n - 1$ **down to** 0 **do**
6: $\quad b \leftarrow 1 - k_i$
7: $\quad \boldsymbol{R}_b \leftarrow \boldsymbol{R}_0 + \boldsymbol{R}_1$
8: $\quad \boldsymbol{R}_{k_i} \leftarrow 2 \cdot \boldsymbol{R}_{k_i}$
9: **return** $\boldsymbol{R}_0$

Scalar multiplication algorithms

What is the most common algorithm [High-Level]?

Algorithm The Montgomery Ladder

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \mathcal{O}$
4: $\boldsymbol{R}_1 \leftarrow \boldsymbol{P}$
5: **for** $i \leftarrow n - 1$ **down to** 0 **do**
6: $\quad b \leftarrow 1 - k_i$
7: $\quad \boldsymbol{R}_b \leftarrow \boldsymbol{R}_0 + \boldsymbol{R}_1$
8: $\quad \boldsymbol{R}_{k_i} \leftarrow 2 \cdot \boldsymbol{R}_{k_i}$
9: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Similarly to the previous one: Regularity...

Scalar multiplication algorithms

What is the most common algorithm [High-Level]?

Algorithm The Montgomery Ladder

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \mathcal{O}$
4: $\boldsymbol{R}_1 \leftarrow \boldsymbol{P}$
5: **for** $i \leftarrow n - 1$ **down to** 0 **do**
6: $\quad b \leftarrow 1 - k_i$
7: $\quad \boldsymbol{R}_b \leftarrow \boldsymbol{R}_0 + \boldsymbol{R}_1$
8: $\quad \boldsymbol{R}_{k_i} \leftarrow 2 \cdot \boldsymbol{R}_{k_i}$
9: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Similarly to the previous one: Regularity...
- Is it constant-time?

Scalar multiplication algorithms

What is the most common algorithm [High-Level]?

Algorithm The Montgomery Ladder

1: **Input:** $\boldsymbol{P}$, $k = (k_{n-1}, k_{n-2}, \ldots, k_0)_2$
2: **Output:** $\boldsymbol{Q} = [k]\boldsymbol{P}$
3: $\boldsymbol{R}_0 \leftarrow \mathcal{O}$
4: $\boldsymbol{R}_1 \leftarrow \boldsymbol{P}$
5: **for** $i \leftarrow n - 1$ **down to** 0 **do**
6: $\quad b \leftarrow 1 - k_i$
7: $\quad \boldsymbol{R}_b \leftarrow \boldsymbol{R}_0 + \boldsymbol{R}_1$
8: $\quad \boldsymbol{R}_{k_i} \leftarrow 2 \cdot \boldsymbol{R}_{k_i}$
9: **return** $\boldsymbol{R}_0$

- What is good about this algorithm? Similarly to the previous one: Regularity...
- Is it constant-time? Caching

Scalar multiplication algorithms

What is the most common algorithm [Low-Level]? Is this really constant time?

Algorithm The Montgomery ladder for x-coordinate-based X25519 scalar multiplication on $E : y^2 = x^3 + 486662x^2 + x$

Input: $k \in \{0, \ldots, 2^{255} - 1\}$ and the x-coordinate x_P of a point P
Output: $x_{[k]P}$, the x-coordinate of $[k]P$
$X_1 \leftarrow 1, Z_1 \leftarrow 0, X_2 \leftarrow x_P, Z_2 \leftarrow 1$
$p \leftarrow 0$
for $i \leftarrow 254$ **down to** 0 **do**
    $c \leftarrow k_i \oplus p, p \leftarrow k_i$
    $(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, c), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, c)$
    $(X_1, Z_1, X_2, Z_2) \leftarrow \text{ladderstep}(x_P, X_1, Z_1, X_2, Z_2)$
$(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, p), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, p)$
return (X_1, Z_1)

Scalar multiplication algorithms

What is the most common algorithm [Low-Level]? Is this really constant time?

Algorithm The Montgomery ladder for x-coordinate-based X25519 scalar multiplication on $E : y^2 = x^3 + 486662x^2 + x$

Input: $k \in \{0, \ldots, 2^{255} - 1\}$ and the x-coordinate x_P of a point P
Output: $x_{[k]P}$, the x-coordinate of $[k]P$
$X_1 \leftarrow 1, Z_1 \leftarrow 0, X_2 \leftarrow x_P, Z_2 \leftarrow 1$
$p \leftarrow 0$
for $i \leftarrow 254$ **down to** 0 **do**
    $c \leftarrow k_i \oplus p, p \leftarrow k_i$
    $(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, c), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, c)$
    $(X_1, Z_1, X_2, Z_2) \leftarrow \text{ladderstep}(x_P, X_1, Z_1, X_2, Z_2)$
$(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, p), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, p)$
return (X_1, Z_1)

- There are several optimizations here: cswap, p, and ladderstep - let me explain.

Scalar multiplication algorithms

What is the most common algorithm [Low-Level]? Is this really constant time?

Algorithm The Montgomery ladder for x-coordinate-based X25519 scalar multiplication on $E : y^2 = x^3 + 486662x^2 + x$

Input: $k \in \{0, \ldots, 2^{255} - 1\}$ and the x-coordinate x_P of a point P
Output: $x_{[k]P}$, the x-coordinate of $[k]P$
$X_1 \leftarrow 1, Z_1 \leftarrow 0, X_2 \leftarrow x_P, Z_2 \leftarrow 1$
$p \leftarrow 0$
for $i \leftarrow 254$ **down to** 0 **do**
    $c \leftarrow k_i \oplus p, p \leftarrow k_i$
    $(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, c), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, c)$
    $(X_1, Z_1, X_2, Z_2) \leftarrow \text{ladderstep}(x_P, X_1, Z_1, X_2, Z_2)$
$(X_1, X_2) \leftarrow \text{cswap}(X_1, X_2, p), (Z_1, Z_2) \leftarrow \text{cswap}(Z_1, Z_2, p)$
return (X_1, Z_1)

- There are several optimizations here: cswap, p, and ladderstep - let me explain. But is it constant time?

Scalar multiplication algorithms

Example of cswap in software

```
void fe25519_cswap(fe25519* in1, fe25519* in2, int condition) {
    int32_t mask = condition;
    mask = -mask;

    for (uint32_t ctr = 0; ctr < 8; ctr++) {
        uint32_t val1 = in1->as_uint32_t[ctr];
        uint32_t val2 = in2->as_uint32_t[ctr];
        uint32_t temp = val1;

        val1 ^= mask & (val2 ^ val1);
        val2 ^= mask & (val2 ^ temp);

        in1->as_uint32_t[ctr] = val1;
        in2->as_uint32_t[ctr] = val2;
    }
}
```

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones
- Scalar randomization (next slide)

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones
- Scalar randomization (next slide)
- Coordinate (re-)randomization, point blinding
  - Projective coordinates: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r \cdot X, r \cdot Y, r \cdot Z)$;
  - Jacobian representation: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r^2 \cdot X, r^3 \cdot Y, r \cdot Z)$;

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones
- Scalar randomization (next slide)
- Coordinate (re-)randomization, point blinding
  - Projective coordinates: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r \cdot X, r \cdot Y, r \cdot Z)$;
  - Jacobian representation: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r^2 \cdot X, r^3 \cdot Y, r \cdot Z)$;
- The whole ECDSA and ECDH need protection
  - In ECDSA: final modular multiplication, among others.
  - In ECDH: coordinate conversions; we will look at traces from: `https://github.com/sca-secure-library-sca25519/sca25519`

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones
- Scalar randomization (next slide)
- Coordinate (re-)randomization, point blinding
  - Projective coordinates: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r \cdot X, r \cdot Y, r \cdot Z)$;
  - Jacobian representation: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r^2 \cdot X, r^3 \cdot Y, r \cdot Z)$;
- The whole ECDSA and ECDH need protection
  - In ECDSA: final modular multiplication, among others.
  - In ECDH: coordinate conversions; we will look at traces from: `https://github.com/sca-secure-library-sca25519/sca25519`
- Generic fault injection countermeasures:
  - a flow-counter and fault counter
  - random output in case of error
  - protecting against invalid curve attacks: duplication/detection

Countermeasures

Besides what we saw above, like regularity, what other countermeasures are possible?

- Algorithmic vs. Generic ones
- Scalar randomization (next slide)
- Coordinate (re-)randomization, point blinding
  - Projective coordinates: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r \cdot X, r \cdot Y, r \cdot Z)$;
  - Jacobian representation: for $r \in_R \mathbb{F}_p$ compute: $(X, Y, Z) \mapsto (r^2 \cdot X, r^3 \cdot Y, r \cdot Z)$;
- The whole ECDSA and ECDH need protection
  - In ECDSA: final modular multiplication, among others.
  - In ECDH: coordinate conversions; we will look at traces from: `https://github.com/sca-secure-library-sca25519/sca25519`
- Generic fault injection countermeasures:
  - a flow-counter and fault counter
  - random output in case of error
  - protecting against invalid curve attacks: duplication/detection
- Specialized countermeasures: against template attacks, address-bit attacks...

Countermeasures

Scalar randomization techniques

Group scalar randomization

function Mult(G, k)

$r \xleftarrow{\$} \{0, 1, \ldots, 2^{32}\}$

return $[k + rn]G$

Additive splitting

function Mult(G, k)

$r \xleftarrow{\$} \mathbb{Z}_n^*$

return $[k - r]G + [r]G$

Euclidean splitting

function Mult(G, k)

$r \xleftarrow{\$} \{0, 1, \ldots, 2^{\lfloor \log_2(n)/2 \rfloor}\}$

$S \leftarrow [r]G$

$k_1 \leftarrow k \mod r$

$k_2 \leftarrow \lfloor \frac{k}{r} \rfloor$

return $[k_1]G + [k_2]S$

Multiplicative splitting

function Mult(G, k)

$r \xleftarrow{\$} \{0, 1, \ldots, 2^{32}\}$

$S \leftarrow [r]G$

return $[kr^{-1} \mod n]S$

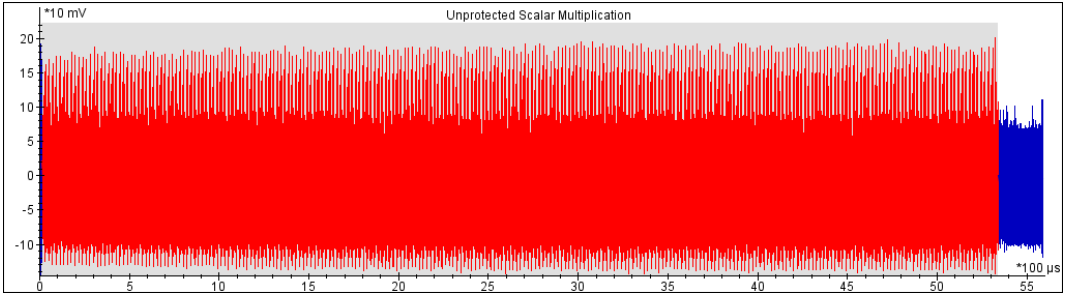
Countermeasures

Advanced Example: Protected Ephemeral X25519

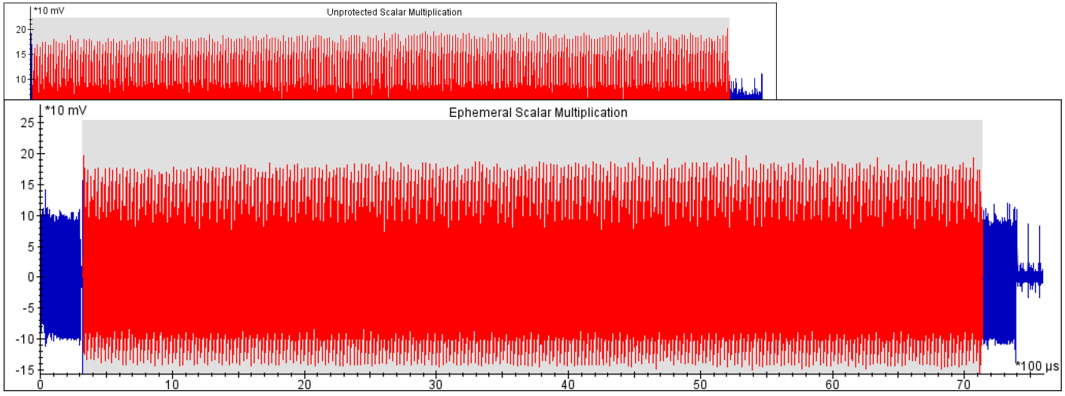
Input: A 255-bit scalar k and the x-coordinate x_P of some point P. **Output:** $x_{[k]P}$.

1: $ctr \leftarrow 0$ ▷ Initialize iteration counter
2: $x_P \xleftarrow{\$} \{0, \ldots, 2^{256} - 1\}$ ▷ Initialize output buffer to random bytes
3: $k \leftarrow \mathsf{clamp}(k)$
4: $k \leftarrow k/8$ ▷ Divide scalar k by 8 to account for initial 3 doublings
5: Increase(ctr)
6: $(X_P, Z_P) \leftarrow \mathsf{montdouble}(\mathsf{montdouble}(\mathsf{montdouble}(x_p, 1)))$ ▷ 3 doublings to multiply by co-factor 8
7: Increase(ctr)
8: **if** $Z_P = 0$ **then**
9: **go to** Line 23 ▷ Early-abort if input point is in order-8 subgroup
10: $x_P \leftarrow X_P \cdot Z_P^{-1}$ ▷ Return to affine x-coordinate
11: $X_1 \leftarrow 1, Z_1 \leftarrow 0$
12: $Z_2 \xleftarrow{\$} \{0, \ldots, 2^{255} - 20\}, X_2 \leftarrow x_P \cdot Z_2$ ▷ Initial randomization of projective representation
13: $k \leftarrow k \oplus 2k$ ▷ Precompute condition bits for cswap
14: Increase(ctr)
15: **for** i from 252 downto 1 **do** ▷ Main scalar-multiplication loop
16: $(X_1, Z_1, X_2, Z_2) \leftarrow \mathsf{cswaprr}\ (X_1, Z_1, X_2, Z_2, k[i])$ ▷ Projective re-randomization merged with cswap
17: $(X_1, Z_1, X_2, Z_2) \leftarrow \mathsf{ladderstep}\ (x_P, X_1, Z_1, X_2, Z_2)$
18: Increase(ctr)
19: $(X_1, Z_1, X_2, Z_2) \leftarrow \mathsf{cswaprr}\ ((X_1, Z_1, X_2, Z_2), k[0])$
20: $x_P \leftarrow X_2 \cdot Z_2^{-1}$
21: Increase(ctr)
22: **if** ! Verify(ctr) **then** ▷ Detected wrong flow, including iteration count
23: $x_P \xleftarrow{\$} \{0, \ldots, 2^{256} - 1\}$ ▷ Set output buffer to random bytes
24: **return** x_P

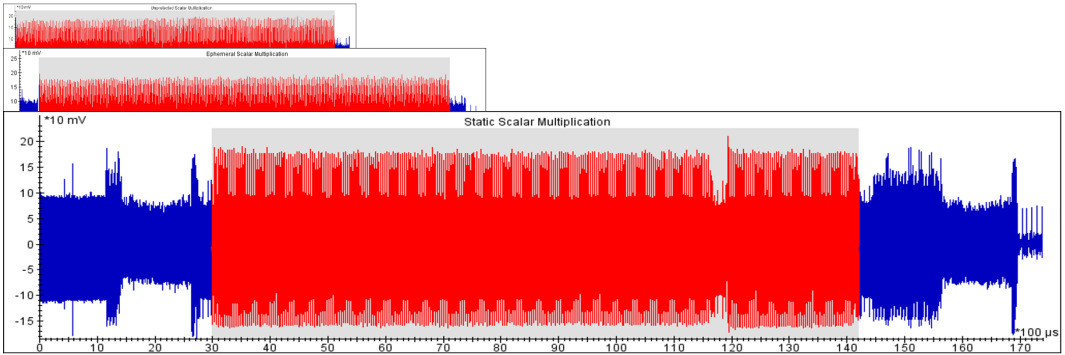
Effect of Countermeasures on Power Traces



Effect of Countermeasures on Power Traces



Effect of Countermeasures on Power Traces



Hands-on work

Goal: Extract implementation details from black-box ECC implementation using side-channels

- Individually or groups of 2-3, knowledge of Python, SCA, ECC
- Work with the **pyecsca** toolkit
- Mostly simulations, some work with captured **traces**
- Three notebooks + one bonus
  - 👉 Quickstart
  - 👉 Implementations
  - 👉 RPA-RE
  - 👉 ZVP-RE

pyecsca

Python **E**lliptic **C**urve **S**ide-**C**hannel **A**nalysis toolkit

- Enumerate ECC implementations
- Simulate ECC with IV granularity
- Generate C implementations of ECC
- Trace acquisition, processing, visualization
- Smart-card support
- Automated reverse engineering of ECC implementations

pyecsca

pyecsca.org

Quickstart

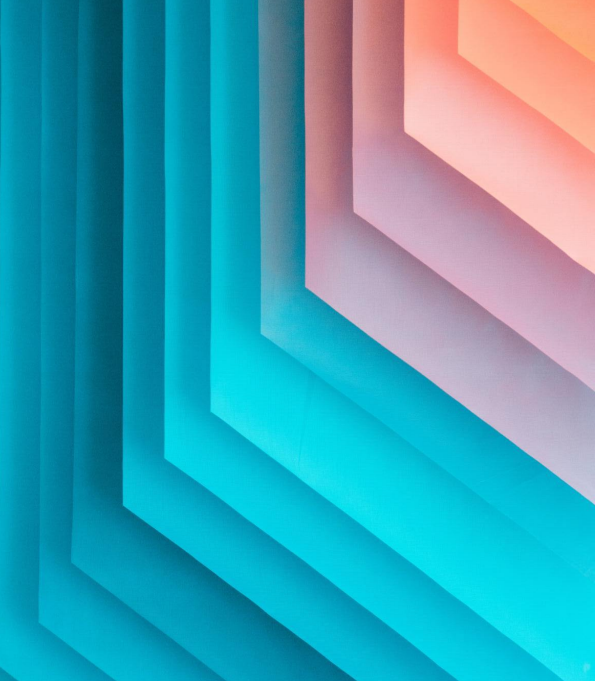
`start.ipynb`
Test the setup and familiarize with the toolkit

1. Start Jupyter notebook
2. Open `notebooks/start.ipynb`
3. Run the prepared cells
4. Solve the exercise

ECC implementations

`implementations.ipynb`
Manually explore ECC implementations

1. We have three implementations (of ECDH). Plot their traces and tell me what you can learn from how they look.
2. Now we concentrate on the implementation 1. Let's count how many iterations there are. Tell me what curve it can be.
3. Can we learn more from the scalar correlation? Is it left-to-right or right-to-left?
4. Let's see what we can learn from CPA.
5. Can we verify that the `ladd-1987-m-3` formula is used? Can we verify the order of intermediates computed?
6. Investigate a number of different possibilities to implement scalar multiplication.



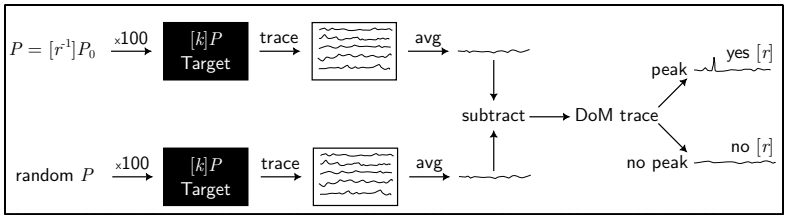
How?

General idea

- **Idea:** Use side-channel attacks and turn them around
  - ~~Assume knowledge of the impl. and target the key~~
  - Assume knowledge of the key and target the impl.
- Concretely "*special-point-based*" attacks: **RPA, ZVP, EPA**
- Can recognize when a special point appears in scalar multiplication
- **Idea:** Behavior of different implementations differs under these attacks

RPA

- **Refined Power Analysis** attack by Goubin
- Zero-coordinate points $P_0 = (x, 0)$ and $P_0 = (0, y)$
- 0 leaks!
- **Idea:** Introduce 0 conditionally on secret key
- $P = [r^{-1} \mod n]P_0$ as input to $[k]P$ computation (ECDH)
  - $[r]P$ computed $\Rightarrow$ 0 ☑
  - $[r]P$ not computed $\Rightarrow$ no 0 ✖



RPA-RE

`rpa.ipynb`
Reverse engineer using RPA

1. Explore scalar multipliers
2. Practice the RPA attack
3. Use RPA for reverse engineering

ZVP-RE

Bonus

`zvp.ipynb`

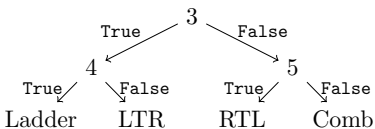
Reverse engineer using ZVP

1. Explore symbolic formula evaluation
2. Construct ZVP points
3. Distinguish formulas using ZVP points

Generalizing

- **Oracle**: Any behavior of the target that differs between impls.
- **Efficiency**: "*Shape*" of the decision table $\rightarrow$ decision tree
  - Oracle arity $\rightarrow$ limit on tree degree
  - Equal rows $\rightarrow$ indistinguishable configurations
  - Equal columns $\rightarrow$ useless inputs (but 1)
- **ZVP-RE**: Extend RPA-RE, as ZVP extends RPA

$\mathcal{I}_{\mathrm{RPA}}$:	$[2^{-1}]P_0$	$[3^{-1}]P_0$	$[4^{-1}]P_0$	$[5^{-1}]P_0$
LTR	`True`	`True`	`False`	`False`
RTL	`True`	`False`	`True`	`True`
Comb	`True`	`False`	`True`	`False`
Ladder	`True`	`True`	`True`	`False`



Generalizing

Methods

- Zero Value Point attack $\rightarrow$ ZVP-RE
  - Forcing a zero coordinate $\rightarrow$ forcing a zero intermediate value
  - Formulas differ in intermediate values $\rightarrow$ we can RE them + coordinates
  - ZVP construction can be hard, need to bound the discrete log between points
- Exceptional Procedure Attack $\rightarrow$ EPA-RE
  - Use curves with composite p (in $\mathbb{F}_p$)
  - Causes lots of *"exceptional points"*
  - Formulas differ in behavior

Method	Curve	Coordinates	Formulas	Multiplier	Scalar	Input point
RPA-RE	chosen	any	any	**target**	known	chosen
ZVP-RE	chosen	**target**	**target**	known	known	chosen
EPA-RE	chosen	**target**	**target**	known	known	chosen

Conclusions

- Implementing elliptic curve cryptography is a non-trivial process requiring a range of implementation choices.
- Most of the attacks require knowledge of the details of the implementation.
- Most of the embedded implementations are black-box, and the attacker does not know the implementation.
- You learned how to
  - semi-manually use side-channel analysis to learn some details of the implementation;
  - to use automatic methods to learn the implementation details (based on Refined Power Analysis).

pyecsca

Thank you for your attendance!

pyecsca.org

Come to our talk: **Saturday 7th 10:45 B2**

pyecsca: Reverse engineering black-box elliptic curve cryptography via side-channel analysis

References

- Jan Jancar, Vojtech Suchanek, Petr Svenda, Vladimir Sedlacek & Łukasz Chmielewski;
  pyecsca: Reverse-engineering black-box elliptic curve cryptography via side-channel analysis

References

Example attacks (and their assumptions)

1. Aurélie Bauer, Eliane Jaulmes, Emmanuel Prouff, Jean-René Reinhard & Justine Wild: Horizontal Collision Correlation Attack on Elliptic Curves
2. Oscar Reparaz, Josep Balasch & Ingrid Verbauwhed: Dude, is my code constant time?
3. Johann Heyszl: Impact of Localized Electromagnetic Field Measurements on Implementations of Asymmetric Cryptography
4. Pierre-Alain Fouque & Frederic Valette: The Doubling Attack – Why Upwards Is Better than Downwards
5. Bo-Yeon Sim & Dong-Guk Han: Key Bit-Dependent Attack on Protected PKC Using a Single Trace
6. Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache: A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards

References

ECC attack and countermeasure surveys

- Junfeng Fan, Xu Guo, Elke De Mulder, Patrick Schaumont, Bart Preneel & Ingrid Verbauwhede; State-of-the-art of secure ECC implementations: A survey on known side-channel attacks and countermeasures
- Junfeng Fan & Ingrid Verbauwhede; An updated survey on secure ECC implementations: Attacks, countermeasures and cost
- Jean-Luc Danger, Sylvain Guilley, Philippe Hoogvorst, Cédric Murdica & David Naccache; A synthesis of side-channel attacks on elliptic curve cryptography in smart-cards
- Rodrigo Abarzúa, Claudio Valencia Cordero & Julio Cesar López-Hernández; Survey on performance and security problems of countermeasures for passive side-channel attacks on ECC

References

Special-point-based attacks

- Louis Goubin;
  A Refined Power-Analysis Attack on Elliptic Curve Cryptosystems
- Toru Akishita & Tsuyoshi Takagi;
  Zero-value point attacks on elliptic curve cryptosystem
- Tetsuya Izu & Tsuyoshi Takagi;
  Exceptional procedure attack on elliptic curve cryptosystems
- Vladimir Sedlacek, Jesús-Javier Chi-Domínguez, Jan Jancar & Billy Bob Brumley;
  A formula for disaster: a unified approach to elliptic curve special-point-based attacks

Related work

Side-channel-based disassembly

JavaCard smartcards, PIC16F, ATMega163, ARM Cortex-M3, …

- Jean-Jacques Quisquater & David Samyde;
  Automatic code recognition for smart cards using a Kohonen neural network
- Dennis Vermoen, Marc F. Witteman & Georgi Gaydadjiev;
  Reverse engineering Java Card applets using power analysis
- Thomas Eisenbarth, Christof Paar & Björn Weghenkel;
  Building a side channel based disassembler
- …and much more (see the paper)

Related work

Side-channel-based reverse engineering

DES, secret ciphers, RSA

- Christophe Clavier;
  Side channel analysis for reverse engineering (SCARE) – an improved attack against a secret A3/A8 GSM algorithm
- Rémy Daudigny, Hervé Ledig, Frédéric Muller & Frédéric Valette;
  SCARE of the DES
- Manuel San Pedro, Mate Soos & Sylvain Guilley;
  FIRE: Fault injection for reverse engineering
- Frederic Amiel, Benoit Feix & Karine Villegas;
  Power analysis for secret recovering and reverse engineering of public key algorithms
- …and some more (see the paper)

Related work

Manual reverse engineering

- Thomas Roche, Victor Lomné, Camille Mutschler & Laurent Imbert;
  A Side Journey to Titan
- Thomas Roche;
  EUCLEAK: Side-Channel Attack on the YubiKey 5 Series

Resources

Icons and photos

Icons from Noun Project & Font Awesome

Photos from Unsplash

- https://unsplash.com/photos/9V5GssdEG-4
- https://unsplash.com/photos/wKc-i5zwfok
- https://unsplash.com/photos/Tk0B3Dfkf_4
- https://unsplash.com/photos/pAyN5zqdqDo