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Performance of transportation network under perturbations: Reliability, vulnerability, and resilience

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ABSTRACT

We review recent studies on transportation network performance under perturbations. Three representative concepts relating to network performance are covered: reliability, vulnerability, and resilience. With an overview of the definitions and the quantitative indices of these three concepts, we analyse and compare their similarities and differences in the context of transportation. These concepts differ from each other in terms of focus, measurement, and application scenario. Numerical examples are conducted to assess these concepts under different perturbation scenarios. The results indicate their rationale in reflecting network performance under perturbations, yet their outputs differ. Moreover, the relationship among the three concepts is intuitively illustrated by the analysis results.

1. Introduction

The urban transportation network is an essential component to the robust operation of megacities. With the rapidly growing demand for transporting people and goods in urban areas, both passenger and freight transportation networks are of great importance to socio-economic development. However, urban transportation networks are often disturbed by recurrent and non-recurrent perturbations, which result in various socio-economic consequences, *e.g.*, blocked supply chain, increased individual travel costs, loss of life, *etc*. Recurrent perturbations such as traffic jams occur periodically in transportation networks and cumulatively cause decline in service level of transportation network by degrading network components. Non-recurrent perturbations are rare and extreme disturbances such as earthquakes, tsunamis, terrorist attacks, *etc*., which cause failures of network components or interrupt network operations. It is therefore necessary for researchers to consider following questions in the planning and operation of transportation systems: (1) what concepts can be used to measure transportation network performance under perturbations; (2) how to quantify these concepts in practice such as appraisal and optimisation of network infrastructure; and (3) what are the similarities and differences between different concepts?

Regarding question (1), risk and resilience are two main concerns for describing the ability of infrastructure systems under recurrent and non-recurrent perturbations (Aven, 2011; Bier and Gutfraind, 2019). Risk has multiple interpretations but is necessarily associated with uncertainty about perturbations and their consequences. Therefore, risk can be separated into two components:

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probability and consequence. In the literature, many studies used alternative concepts, such as reliability, vulnerability, robustness, and survivability, to describe each component of risk. Reliability focuses on the probability component, and is defined as the probability that the transportation network remains satisfactory in terms of service level provision under perturbation. Vulnerability is defined as the susceptibility to perturbations, which focuses on the consequence component of risk (Taylor, 2017a). Robustness and survivability are two other concepts related to the consequence component. Robustness denotes the performance that can be retained under perturbation, which is often considered to be the inverse concept of vulnerability (Mattsson and Jenelius, 2015; Reggiani et al., 2015; Oliveira et al., 2016). Survivability is defined as the ability of a transportation network to withstand perturbations at the original demand level, thus can be considered as a concept comparable to robustness/vulnerability (Faturechi and Miller-Hooks, 2015).

In addition to the risk, increasing attention has been paid to the recovery of network after perturbation, which promotes the development of resilience in the literature (Holling, 1973; Pimm, 1984; Cox et al., 2011; Reggiani, 2013; Wan et al., 2018). The concept of resilience is widely used in the literature, which is defined as the ability of network to resist, absorb, adapt to, and recover from negative impacts of perturbations. Some other studies use alternative concepts such as redundancy or flexibility related to resilience. Redundancy depicts the substitutability of each transportation network component and is generally considered as a part of resilience (Bruneau et al., 2003; Hosseini et al., 2016; Twumasi-Boakye and Sobanjo, 2018; Xu et al., 2018b). Flexibility is similar to resilience, which is widely used to describe the ability of the transportation network to adapt to perturbation (Morlok and Chang 2004; Chen and Kasikitwiwat, 2011). The difference between flexibility and resilience is that resilience-related studies always consider changes on the supply side in addition to changes on the demand side.

From above discussions, it can be seen that reliability, vulnerability and resilience are three important concepts which can representatively describe the performance of transportation network during and after perturbation. In recent years, the three concepts are discussed from different perspectives in a wide range of studies (e.g., Bruneau et al., 2003; Cox et al., 2011; Faturechi and Miller-Hooks, 2015; Mattsson and Jenelius, 2015; Oliveira et al., 2016; Reggiani, 2013; Reggiani et al., 2015; Taylor, 2013; Taylor, 2017a; Wan et al., 2018). However, few studies have considered all three concepts at once and answered questions (2) and (3) in the context of transportation network.

To address questions (2)–(3) and provide insights for the evaluation, operation and planning of transportation networks with consideration of perturbations, this study attempts to explore features of and relationships among reliability, vulnerability and resilience of transportation networks. The contributions of this study are twofold: (1) to clarify the widely used concepts of reliability, vulnerability, and resilience in the context of transportation network, this study exclusively devoted to explicitly illustrating the definitions, quantitative indices and assessment methods of these three concepts; (2) to distinguish the three concepts and understand their applications, in-depth analyses and numerical tests are conducted to investigate the similarities and differences among the three concepts.

In the rest of this paper, we first introduce the definitions, quantitative indices, and analyses of reliability, vulnerability, and resilience, followed by remarks on the three concepts in the transportation network. Then, numerical examples under different perturbation scenarios are presented to intuitively demonstrate the analysis and application of each concept. Finally, conclusions are drawn. The notations used are listed in Table 1.

2. Reliability

Transportation reliability can be interpreted as the probability of travellers successfully completing their trips between certain OD pairs. Unpredictable perturbations in the transportation network lead to uncertainties in both transport supply and travel demand, hence influence reliability. Therefore, transportation reliability not only accounts for the operation of infrastructures, but also considers the choice behaviours of travellers following changes in transport infrastructures. In this regard, reliability can be further divided into three sub-concepts: connectivity reliability, travel time reliability, and capacity reliability.

2.1. Connectivity reliability

Connectivity reliability indicates the probability of certain OD pairs remaining connected during the predetermined time period (Iida and Wakabayashi, 1989). From this perspective, connectivity reliability can also be termed “terminal reliability”. An OD pair is said to be connected if there is at least one connected (or not heavily congested) path therein, while a path is said to be connected only if every link on it remains connected. Under perturbation, the connectivity states of links are likely to change. Let x_a and $E(x_a)$ denote the connectivity state and the expected connectivity of link a. When link a is connected, $x_a = 1$; otherwise, $x_a = 0$. Assume that the connectivity state of each link is independent, then the connectivity reliability of certain OD pair w, $R_{con,w}(\mathbf{x})$, can be expressed as (Mine and Kawai, 1982):

$$R_{con,w}(\mathbf{x}) = E\{1 - \prod_{k \in K_w} [1 - \prod_{a \in K_A} x_a]\} = 1 - \prod_{k \in K_w} [1 - \prod_{a \in K_A} E(x_a)] \tag{1}$$

Connectivity reliability is analysed based on the topology of the transportation network, therefore, it is applicable in perturbation scenarios where links are closed and temporarily removed from the network. However, the connectivity reliability measure mentioned above has received criticism: firstly, it is assumed that link connectivity states are independent under perturbation. This assumption implies that the functionality of one link is unrelated to the states of other links, however, that seems unrealistic in degradable transportation systems (Nicholson and Du, 1997). Secondly, only two connectivity states are considered in the analysis of

Table 1
Notations.

A	Set of links in the network
a	A link in the network
W	Set of ODs in the network
$\|W\|$	Number of ODs in the network
w	An OD in the network
K_w	Set of paths between OD w
k	A path between certain OD
k_A	Set of links on path k
$\mathbf{x}$	Vector of connectivity states of all links in the network
x_a	Connectivity state of link a.
$R_{con,w}(\mathbf{x})$	Connectivity reliability of OD w given link connectivity states
$\bar{T}$	Mean travel time
θ	Parameter indicating required service level after perturbation
T_w	Travel time between OD w before perturbation
T_w'	Travel time between OD w after perturbation
$TTR_w(\theta)$	Travel time reliability between OD w at desired service levelθ
T_k	Travel time on path k
σ_{T_k}	Standard deviation of travel time on path k
λ	Parameter related to the punctuality requirement
b_k	Travel time budget on path k
U	Travel disutility considering both the value of travel time and the value of travel time reliability
P_L	Probability of being late
γ_i	Parameters denoting the influences of different terms on the value of travel time reliability
σ_T	Standard deviation of travel time
$\tilde{H}(.)$	Function of schedule delay penalties
ψ	Extent to which the travel time distribution depends on departure time
μ	Reserve capacity, *i.e.* ratio of capacity after perturbation to travel demand before perturbation
$CR(\theta)$	Capacity reliability at the post-perturbation service levelθ
v_a	Traffic flow volume on link a
C_a	Capacity constraint of link a
$\mathbf{q}$	Matrix of travel demands between all ODs in the network
N	Set of nodes in the network
$\|N\|$	Number of nodes in the network
d_{ij}	Shortest distance between node i and node j before perturbation
d_{ij}'	Shortest distance between node i and node j after perturbation
q_w	Travel demand between OD w before perturbation
q_w'	Travel demand between OD w after perturbation
U_w	Travel disutility on OD w before perturbation
U_w'	Travel disutility on OD w after perturbation
VI_i	Vulnerability indices
$F(m)$	Ratio of the network performance at time m after perturbation to the equilibrium network performance before perturbation
m_i	Parameters denoting time in life-cycle phases of perturbations
RL	Resilience of transportation network
$\bar{T}_w$	Mean travel time between OD w
σ_{T_W}	Standard deviation of travel time between OD w;
t_a	Travel time on link a
t_a^0	Free flow travel time on link a

connectivity reliability, *i.e.*, whether the link is connected with full capacity or unconnected with no capacity (Chen et al., 2002). Hence, connectivity reliability is inapplicable to everyday traffic conditions in which perturbed links remain operable at reduced capacities. Moreover, the effects of varying traffic flow volume are often neglected in connectivity reliability analyses. To fill the gaps in connectivity reliability analysis, the concepts of travel time reliability and capacity reliability are proposed.

2.2. Travel time reliability

Travel time is one of the most important trip characteristics, which has long been considered as the major influencing factor of individuals' activity and travel choice behaviour (Fu and Lam, 2018; Liu et al, 2018). In reliability assessment, many studies use travel time to reflect the transportation network's service level from the perspective of travellers. The four main foci for travel time reliability studies suggested by Taylor (2013) are discussed in this section: (1) definitions of reliability; (2) reliability indices; (3) impacts of reliability on travel behaviour; and (4) valuation of travel time reliability.

2.2.1. Definition

The definition of travel time reliability can be interpreted from different perspectives, including the probability of travellers arriving at destinations within predetermined time limits (Asakura and Kashiwadani, 1991), the unpredictable variation in travel times (Chang, 2010), and the maximum travel time within which travellers can reach destinations with a desired probability

(Wakabayashi and Matsumoto, 2012).

2.2.2. Quantitative indices

There are various quantitative indices for travel time reliability. A large body of research measures travel time reliability based on empirical travel time data. van Lint et al. (2008) classified quantitative indices for travel time reliability into four categories: statistical indices, buffer indices, tardy trip indices, and probabilistic indices.

Statistical indices measure travel time reliability based on the distribution of travel time from empirical data. Indices related to standard deviation and coefficient of variation (CV) of travel time are frequently used to measure the variability of travel time (Bates et al., 2001; Sun et al., 2016). Buffer indices can be interpreted as the extra length of travel time required for punctuality because of variation in travel times. The buffer time (difference between the 95th/90th/80th percentile and the average travel time), the buffer time index (ratio of the buffer time to the average or median travel time), and the 95th percentile travel time itself are representative buffer indices (Lam and Small, 2001; van Lint et al., 2008; Lomax et al., 2003; Loon et al., 2011). Tardy trip indices relate to late arrivals and measure the threshold of acceptable lateness. The misery index (MI), which has been adopted in a range of studies (Lomax et al., 2003; van Lint et al., 2008; Taylor, 2017b), measures travel time reliability using the difference between the mean travel time of the 20% trips with the longest travel time and the mean travel time of all trips. Probabilistic indices refer to the proportion of trips whose travel times exceed a predetermined travel time threshold (van Lint et al., 2008).

Besides using indices derived from empirical travel time data, effort has been made to model travel time reliability through approaches with traffic assignment. For instance, probabilistic indices can be modelled using a bi-level programming approach (Chen et al., 2002). The travel time reliability between OD w, $TTR_w(\theta)$, is expressed as the probability that the ratio of OD travel time after perturbation to the OD travel time before perturbation is lower than the required service level θ:

$$TTR_w(\theta) = P\left(\frac{T_w'}{T_w} \leqslant \theta\right) \tag{2}$$

Travel time reliability indices are also applied to investigate the impact of reliability on travel behaviour. The concept of travel time budget, as proposed by Lo et al. (2006), which is similar to the buffer indices discussed above, can be used as the determinants of travel choices in the traffic assignment. Note that the actual travel time fluctuates from time to time. To arrive on time, travellers should reserve more travel time than the expected travel time and depart earlier. The extra time reserved in addition to the expected travel time is termed the travel time margin. The travel time budget between OD w can be expressed as the sum of expected travel time and travel time margin (Lo et al., 2006):

$$b_k = E(T_k) + \lambda\sigma_{T_k}, \quad \forall\ k \in K_w \tag{3}$$

where $E(T_k)$ denotes the expected travel time, $\lambda\sigma_{T_k}$ denotes the travel time margin. The value of parameter λ is associated with the importance of the trip. Travelers require a higher probability of arriving on time for more important trips, which leads to a higher value of λ.

2.2.3. Value of travel time reliability

It has been found that travellers tend to pay for travel time reliability, making the economic valuation of travel time reliability an important consideration in transport policy, investment appraisal, and road pricing, *etc*. Readers can refer to (Li et al., 2010; Carrion and Levinson, 2012) for detailed reviews of the value of travel time reliability.

Various measurements have been developed for value of travel time reliability (Engelson and Fosgerau, 2016). Among them, there are two main types of measures: mean-dispersion measures and measures based on scheduling. The mean-dispersion measures treat travel time variability as a source of disutility. The total travel disutility is composed of the mean and the standard deviation of travel time (Carrion and Levinson, 2012):

$$U = \gamma_1.\ \bar{T} + \gamma_2.\ \sigma_T \tag{4}$$

where γ_i represents the influence of each term; σ_T represents the standard deviation of travel time. Besides standard deviation, the travel time variability on the right-hand side of Eq. (4) can also be represented by the buffer indices introduced in Section 2.2.2.

The other measure is based on travellers' departure time choice and preferred arrival time. Small (1982) considered travel time and schedule delays (early or late arrivals) as sources of travel disutility. Noland and Small (1995) further expressed the expected travel disutility under uncertainty. Bates et al. (2001) assumed the travel time follows an exponential distribution and expressed the expected travel disutility as:

$$E(U) = \gamma_3.\ E(T) + \widetilde{H}(\gamma_3, \gamma_4, \gamma_5, \gamma_6, \psi, \sigma_T).\ \sigma_T + \gamma_6.\ P_L \tag{5}$$

Fosgerau and Karlström (2010) mathematically proved that the optimal expected disutility is linear in the standard deviation of travel time. Their model only assumes the standardised travel time distribution to be fixed, which relaxes the assumption that the travel time distribution is independent of the departure time. Various extensions have been made to their model, including considering scheduling preferences (Engelson and Fosgerau, 2016), validating model assumptions (Fosgerau and Fukuda, 2012), and empirical applications (Abegaz et al., 2017).

2.3. Capacity reliability

As opposed to travel time reliability, which measures reliability from the perspective of travellers, capacity reliability analyses the reliability on the supply-side of a transportation network: capacity reliability was first developed by Chen et al. (1999), who used the concept of reserve capacity to represent the redundancy of network capacity. Chen *et al.* defined capacity reliability as the probability that a transportation network can accommodate a certain travel demand under the desired service level:

$$CR(\mu_r) = P(\mu \geqslant \mu_r) \tag{6}$$

where μ is the reserve capacity parameter developed by Wong and Yang (1997) that can be expressed as the ratio of the accommodated travel demand after perturbation to the travel demand beforehand; μ_r is the travel demand level after perturbation, *i.e.*, the ratio of the travel demand after perturbation to the travel demand beforehand. Capacity reliability is expressed by the probability that the network capacity after perturbation is no less than the travel demand after perturbation.

The method used to derive the reserve capacity parameter, μ, is as shown in Eqs. (7) and (8) (Chen et al., 1999, 2002):

$$\max\ \mu \tag{7}$$

$$s.\ t. \quad v_a(\mu\mathbf{q}) \leqslant C_a, \quad \forall\ a \in A \tag{8}$$

where C_a denotes the capacity constraint of link a; $v_a(\mu\mathbf{q})$ is the traffic flow volume on link a derived by assigning travel demand $\mu\mathbf{q}$ to the post-perturbation transportation network following principles like the user equilibrium principle (Liu et al., 2019a; Sun et al, 2019). The maximum value of multiplier μ satisfying constraint (8) denotes the maximum travel demand that can be accommodated after perturbation. Chen et al. (2002) modelled possible random perturbation scenarios using Monte-Carlo simulation. The reserve capacity parameter μ is iteratively derived under different perturbation scenarios. Extensions to this capacity reliability analysis can be made by adopting different network capacity measures (Chen et al., 2013), and by modelling travel behaviour through different principles (Sumalee and Kurauchi, 2006).

3. Vulnerability

3.1. Definition

Compared to the concept of reliability, vulnerability is a relatively new concept introduced to evaluate transportation networks with diverse interpretations therein. From the perspective of a road transportation network, Berdica (2002) defined vulnerability as the extent to which the transportation network is susceptible to extreme perturbations. Similarly, Husdal (2005) regarded vulnerability as the concept opposite to reliability, depicting the propensity of a transportation network to become inoperable under extreme perturbation. The definition of vulnerability proposed by Berdica and Husdal is similar to that of risk, which intrinsically considers the probability and consequences of perturbations. However, it is difficult to predict the occurrence probabilities of extreme perturbations therefore many studies treat vulnerability as a concept barely focusing on the consequential aspects of perturbation (Faturechi and Miller-Hooks, 2015: Taylor, 2017a).

Jenelius et al. (2006) stated that there are two keys in vulnerability-related studies: to evaluate the reduction of transportation network performance under perturbation (from this perspective, vulnerability is measured through the drop in specific quantitative indices); the other is to identify important components of the transportation network. The importance of a network component is measured through the decrease in network performance indices caused by the removal thereof. It can be concluded that vulnerability analysis is associated with the variation in specific quantitative indices: different indices analyse vulnerability from different perspectives. Quantitative indices and analysis methods for vulnerability are discussed in Section 3.2.

3.2. Quantitative indices

Similar to the classification of reliability, the concept of vulnerability can be further categorised based on the indices used to quantify variation in network performance. In the literature, vulnerability is classified as connectivity vulnerability, accessibility vulnerability, and capacity vulnerability.

Connectivity vulnerability measures the decrease of connectivity in terms of certain topology-based indices (Kurauchi et al., 2009; Demšar et al., 2008). Transportation vulnerability can also be investigated from the perspective of percolation (Berche et al., 2012). As a theory of random graphs, percolation theory is increasingly used to investigate connectivity in a transportation network (Jiang et al., 2018; Li et al., 2015; Talebpour et al., 2017). von Ferber et al. (2012) further illustrated the relevance of the association between percolation theory and transportation vulnerability analysis (both qualitatively and quantitatively).

D'este and Taylor (2003) defined vulnerability as a concept associated with accessibility. Accessibility can be interpreted as the potential for reaching spatially distributed opportunities. The variation in accessibility is used to measure vulnerability. Taylor and D'este (2007) further divided accessibility indices used in vulnerability analysis into two groups: (1) indices associated with travel costs (*e.g.* Jenelius et al., 2006; Jenelius and Mattsson, 2012), and (2) indices associated with the Hansen integral accessibility index (*e.g.* Taylor et al., 2006; Chen et al., 2007).

Recently, Bell et al. (2017) proposed a method with which to evaluate vulnerability based on the capacity weighted spectral network partitioning approach. They identified critical links in the network as capacity bottlenecks, which are boundaries of the

Table 2
Examples of quantitative indices for vulnerability.

Category of indices	Example indices	Computation formula
Topology-based indices	Closeness	$VI_1 = \frac{1}{\lvert N \rvert (\lvert N \rvert - 1)} \cdot \frac{\sum_{i \neq j \in N} (d_{ij}' - d_{ij})}{\sum_{i \neq j \in N} d_{ij}}$
	Efficiency	$VI_2 = \frac{1}{\lvert N \rvert (\lvert N \rvert - 1)} \cdot \frac{\sum_{i \neq j \in N} \left(\frac{1}{d_{ij}} - \frac{1}{d_{ij}'}\right)}{\sum_{i \neq j \in N} \frac{1}{d_{ij}}}$
System-based indices	System-based closeness measure	$VI_1' = \frac{1}{\lvert W \rvert} \cdot \frac{\sum_{w \in W} (q_w' U_w' - q_w U_w)}{\sum_{w \in W} q_w U_w}$
	System-based efficiency measure	$VI_2' = \frac{1}{\lvert W \rvert} \cdot \frac{\sum_{w \in W} (q_w / U_w - q_w' / U_w')}{\sum_{w \in W} q_w / U_w}$

network cut with the least capacity.

Based on whether considering the congestion effect caused by traffic flows, Mattsson and Jenelius (2015) further classified vulnerability analyses into topology-based analysis (*e.g.*, connectivity vulnerability and capacity vulnerability) and system-based analysis (*e.g.*, accessibility vulnerability). Some examples of topology-based and system-based vulnerability indices are listed in Table 2.

Topology-based indices, *e.g.*, closeness and network efficiency, investigate the transportation network merely based on network topology without considering the congestion effect. The closeness index VI_1 in Table 2 measures the relative change in the average shortest distances between all nodes. The efficiency index VI_2 replaces the average distance in VI_1 by the average reciprocal distance. As the reciprocal distance between an unconnected OD pair is zero, the efficiency measure is applicable to networks where distances between a few OD pairs are extremely long. Higher value of VI_2 indicates higher efficiency of the network to transport people and goods.

System-based vulnerability indices can be modified from topology-based indices to further consider issues associated with travel cost (Jenelius et al., 2006). For example, instead of the shortest travel distance in VI_1, the system-based index VI_1' uses the travel cost derived from traffic assignment to represent network performance. Qiang and Nagurney (2008) developed the "unified network performance measure" (*i.e.*, VI_2' in Table 2) by using the reciprocal travel cost to replace the reciprocal shortest distance in VI_2. Hence, the proposed index can be interpreted as the travel demand that can be accommodated by a network per unit travel cost. System-based indices can be derived based on either the static traffic assignment approach which depicts the distribution of traffic flow in the long-term, or the dynamic traffic assignment approach that can be used to analyse the evolution of network performance over different time periods.

4. Resilience

4.1. Definition

Compared to reliability and vulnerability, resilience is a more comprehensive concept used in evaluating system performance under extreme perturbations, which was first used in ecological studies (Holling, 1973; Pimm, 1984). Two components are inherently accounted in the concept of resilience: (1) the ability to resist the effects of perturbation; and (2) the ability to recover from perturbation. However, resilience still lacks a commonly accepted definition and has different interpretations when applied to transportation networks. One of the widely adopted interpretations was made by Bruneau et al. (2003), who stated that resilience contains "the 4Rs" in its definition, namely robustness, redundancy, resourcefulness, and rapidity. Robustness and redundancy are to evaluate and enhance the network's resistance, while rapidity and resourcefulness are related to the recovery ability.

4.2. Quantitative indices

Many studies have been conducted to evaluate transportation network resilience on a qualitative basis, while little effort has been devoted to the quantitative analysis thereof. Some studies developed specific indices with which to evaluate different aspects of network resilience, but failed to depict the concept of resilience as a whole. Additionally, these indices are designed based on different traffic conditions in different regions, thus cannot be used to evaluate network resilience in other areas. Moreover, some indices emphasize the resistance ability while the recovery ability implied in the definition of resilience cannot be fully captured.

Similar to the classification of reliability and vulnerability indices, quantitative indices for resilience can also be categorised based on whether considering congestion effect or not. It has been found that network topology influences network resilience in terms of resistance and recovery abilities (Zhang et al., 2015). Connectivity-based indices are widely used in resilience-related studies (*e.g.*, Reggiani, 2013; O'kelly, 2015). Berche et al. (2009) investigated resilience of public transportation networks using a variety of graph

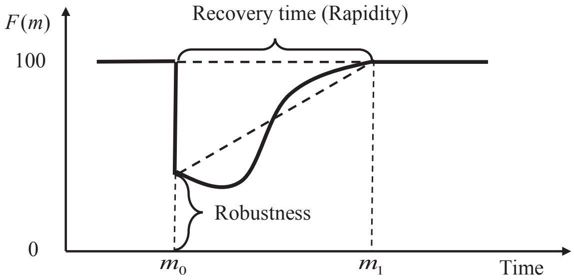

Fig. 1. Illustration of the resilience triangle. Adapted from Bruneau et al. (2003).

theory-based indicators, such as node degree, betweenness centrality, closeness centrality, *etc*. Dunn and Wilkinson (2016) adopted connectivity measures to evaluate resilience strategies in an air transportation network.

Indices considering the congestion effect are also extensively applied in resilience-related studies. For instance, the throughput of logistics network or the traffic volume of passenger network is used in a wide range of studies (Cox et al., 2011; Miller-Hooks et al., 2012). Ip and Wang (2011) derived the resilience of each node through weighted traffic volume and then expressed the network resilience as the sum of weighted node resilience. Omer et al. (2011) used the variation in travel time under perturbed conditions to measure resilience.

Besides the measures focusing on resistance, indices related to recoverability are proposed for quantification of resilience, *e.g.*, the recovery time and the restored performance after certain recovery time, and the ratio of recovered performance to performance loss (Henry and Ramirez-Marquez, 2012). In addition to transportation-related indices, economic indices can also be applied to depict network performance in resilience analysis. Cox et al. (2011) measured transportation network resilience using the ratio of the avoided economic loss to the maximum potential economic loss caused by perturbation.

Quantitative indices of resilience are similar to those of vulnerability. The reason for this similarity is that vulnerability analysis aims to assess the ability of a network to resist the effects of perturbations, which is one of the components enshrined in the concept of resilience. However, the other group of resilience indices, such as recovery speed and recovery quotient of network performance, focus on the ability of a network to recover after perturbation, thus they differ from vulnerability indices.

Bruneau et al. (2003) proposed the "resilience triangle" to evaluate resilience (Fig. 1). The evolution of network performance, $F(m)$, is illustrated in Fig. 1: m_0 and m_1 separately denote the time of occurrence of a perturbation and the time when network recovers its equilibrium. The resilience triangle is enclosed by the performance curves of the network during the life cycle of perturbation. The resulting shape is approximated to a right triangle enclosed by the dotted lines. The size of the resilience triangle simultaneously indicates robustness and rapidity of recovery, and thus can be used to quantify the resilience *RL* (Bruneau et al., 2003):

$$RL = \int_{m_0}^{m_1} [100 - F(m)]dm \tag{9}$$

It can be found that mitigating performance degradation under perturbation and improving recovery efficiency after perturbation can improve network resilience. The resilience of a transportation network relies on both the intrinsic attributes of the network and the optimisation of resilience strategies. Berche et al. (2009) compared the resilience analysis results from assessments of public transportation networks to the analytical results predicted by percolation theory for complex networks. Their comparison results implied an inherent correlation between the resilience and the structure of public transportation networks. Regarding network intrinsic attributes, quantitative resilience analysis can be conducted to appraise, and plan for, investment in transportation infrastructure (Wan et al., 2018).

As for the optimisation of resilience strategies, resilience studies can focus on designing optimal preparation and recovery actions considering both the amount of investment and the benefit reflected by the speed and extent of recovery (Chen et al., 2017; Zhang and Miller-Hooks, 2015; Zhalechian et al., 2018). The effects of actions taken in different phases and their interactions are often considered in resilience optimisation (Miller-Hooks et al., 2012).

5. Remarks

5.1. Features of transportation network

The concepts reviewed in this study, i.e., reliability, vulnerability, and resilience, have been investigated in non-transportation infrastructure networks such as power network, communication network, water distribution network, *etc*. Similar to other infrastructure systems, the transportation system can be represented by a network structure of links and nodes with flows traversing through them, which is operated to fulfil certain demands of the society. Hence, reliability, vulnerability and resilience of both transportation and non-transportation networks can be measured based on either the changes of network topology such as closeness, efficiency, *etc*., or negative impacts on the society such as the demand cannot be accommodated, direct and indirect monetary lost, *etc*.

The differences between transportation and non-transportation networks lie on both the supply side and the demand side. On the

Table 3
Comparison of performances of transportation and non-transportation networks.

Network	Infrastructures	Flow	Typical indicators for reliability, vulnerability and resilience analyses
Transportation (road network as an example)	Roads, intersections, parking slots, *etc*.	Automobiles	Travel time, connectivity, capacity, accommodated demand
Non-transportation (Internet network as an example)	Fibre cable, sites, exchange points, *etc*.	Data	Delay, connectivity, goodput, packet delivery ratio

supply side, links and nodes in different networks refer to different infrastructures, which are likely to be exposed to different perturbations and receive different degrees of physical damage. On the demand side, other than providing or distributing products to customers outside the system, the transportation network provides services to the users (*i.e.*, travellers) on it. In this regard, the performance of transportation network under perturbation can be measured from the perspective of travellers. Flows traversing non-transportation networks are materials such as electricity or water, whose movement can be estimated based on physical laws. However, movement of travellers causes the congestion effect, which leads to the adjustment of choice behaviours and the self-organization of traffic flow, hence reversely influences the performance of transportation network (Xu et al., 2018a). Such features of transportation network trigger the development of the measures derived through traffic assignment approaches, such as the system-based indices mentioned in vulnerability and resilience analyses, which are not applicable in non-transportation networks. Table 3 compares the characteristics of performances of transportation and non-transportation networks.

5.2. Congestion effect in reliability, vulnerability, and resilience analysis

As discussed in Section 5.1, the congestion effect is an important feature of the transportation network which is commonly addressed by the traffic assignment approach. However, the introduction of traffic assignment leads to additional model complexity and computational burden. For example, many system-based vulnerability analyses enumerate possible perturbation scenarios by iteratively removing network components, which requires rapidly growing computational cost with increasing network scale. Various methods are proposed to overcome this problem, such as approaches based on sensitivity analysis and network partition (Luathep et al., 2011; Chen et al., 2012). In addition to single-link failure scenarios, multiple-link failure scenarios have received increasing attention but are more computationally inefficient to model (Wang et al., 2016). Effort has also been devoted to reducing the computational burden of vulnerability analysis under multiple-link failure scenarios: for instance, Jenelius and Mattsson (2012) developed a grid-based method with which to measure the importance of each cell with multiple links therein. Xu et al. (2018d) developed a binary integer bi-level program to find the upper and lower bounds of network vulnerability, thus avoiding enumeration of all perturbation scenarios.

The traffic assignment approach is also widely used in resilience analyses but is adjusted to emphasize the recoverability of transportation network. Faturechi and Miller-Hooks (2014b) investigated individual travel choices after perturbation based on the partial user equilibrium model and derived the travel time resilience accordingly. As the recovery after perturbation is a dynamic process with changing traffic conditions, the dynamic traffic assignment is appropriate for resilience analysis. Wang et al. (2015) and Ye and Ukkusuri (2015) considered the evolution of travel behaviours after perturbation and integrated the day-to-day traffic assignment approach to model network recovery speeds after taking certain actions.

As for transportation reliability, recent studies mainly focus on travel time reliability (Palma and Picard, 2005; Li et al., 2008; Chen et al., 2011; Fu et al., 2014; Xu et al., 2018c; Fu and Gu, 2018). Instead of using outcomes of traffic assignment to assess network performance under perturbation, attempts have been made to inherently model the influence of travel time variability on travel behaviour in the traffic assignment (Fu and Lam, 2014; Li et al., 2015; Shao et al., 2006; Sun et al., 2019). However, existing algorithms for the reliable shortest path problem are either computationally intensive or merely applicable to cases with fixed link travel times. This limitation hinders the application of reliability-based traffic assignment in large-scale networks. Furthermore, considering heterogenous travellers requiring distinct on-time arrival probabilities for the same trip purpose could further increase the complexity of deriving a travel time budget.

5.3. Similarities and differences between reliability, vulnerability, and resilience

Reliability, vulnerability, and resilience measure the transportation network performance under perturbations from different perspectives. The definitions, measurements and typical applicable scenarios of the three concepts are presented in Table 4. Various similarities and differences can be found among the three concepts.

Firstly, reliability, vulnerability, and resilience are overlapped in terms of definition in the literature. Reliability and vulnerability both depict the ability of a network to remain satisfactory, focusing on the network's response to perturbation. Vulnerability and resilience both consider the decrease in network performance under perturbations. Furthermore, reliability and vulnerability can be considered as important components included in the concept of resilience (Faturechi and Miller-Hooks, 2014a; Hosseini et al., 2016). In many studies, decrease of vulnerability and improvement of reliability are considered as the objective of resilience optimisation (*e.g.*, Ip and Wang, 2011; Mattsson and Jenelius, 2015).

Secondly, reliability, vulnerability, and resilience have different foci. Reliability and vulnerability are concepts related to risk,

Table 4
Definitions and measurements of reliability, vulnerability, and resilience.

	Reliability	Vulnerability	Resilience
Definition	Probability of network to remain satisfactory under perturbation	Susceptibility of network to perturbations	Ability of network to resist, absorb, adapt to, and recover from perturbations
Measurement	Probability of remaining certain level of network performance under perturbation	Decrease of network performance under perturbation	Remained network performance under perturbation and the extent of restored network performance after perturbation
Applicable scenarios	Both recurrent and non-recurrent perturbation	Specific non-recurrent perturbations	Unspecific non-recurrent perturbations

while they emphasize different components of risk. Reliability focuses on the probability of providing a certain level of performance, while vulnerability is often used to measure the consequence (change in network performance) without accounting for the probability component. Resilience additionally emphasizes the recovery of network after perturbation, hence accounts for the evolution of post-perturbation performance.

Thirdly, although the three concepts can be analysed by similar indices either through graph theory or based on traffic flow, their distinct foci lead to different analysis approaches. Static approaches are widely used in reliability and vulnerability analyses. Reliability analysis often adopts probabilistic measures associated with the performance level, while vulnerability analysis tends to reveal the deterministic decrease in performance indices. Dynamic approaches are widely used in resilience analyses to measure the evolution of post-perturbation performance considering the response of travellers to perturbation as well as preparing and recovery actions. The actions include both direct actions (*e.g.*, retrofit of existing infrastructures, construction of additional infrastructures, preparation of emergency resources; restoration of perturbed infrastructures, *etc.*), and indirect actions such as congestion pricing schemes (Wang et al., 2015; Cheng et al., 2019) and roadside guidance (Kaviani et al., 2017).

Fourthly, reliability, vulnerability, and resilience are applicable to different scenarios and provide information from different perspectives. Reliability can either help planners and operators understand connectivity, capacity and service level of the network under non-recurrent perturbations causing serious damages, or provide individuals trip information associated with travel time under recurrent perturbations with mild consequences. Both vulnerability and resilience are useful for planners and operators to understand the performance of transportation network in non-recurrent perturbations. The difference is that vulnerability is often applied in specific perturbations to illustrate how the network reacts to certain events, while resilience is not specifically considered under certain perturbation and is more associated with the inherent ability of network. Resilience considers the interactions among the transportation network, the selected set of perturbations and the actions taken at different stages of perturbation.

To intuitively compare the three concepts, Fig. 2 gives illustrations about different levels of reliability, vulnerability and resilience. Reliability is measured by the probability that network performance satisfies the required service level, where high probability refers to high reliability; vulnerability is measured by decrease of network performance under non-recurrent perturbation, where small decrease refers to low vulnerability; resilience is measured by the size of resilience triangle as introduced in Section 4.2, where small triangle refers to high resilience. It can be seen from Fig. 2 that high reliability does not necessarily lead to low vulnerability as

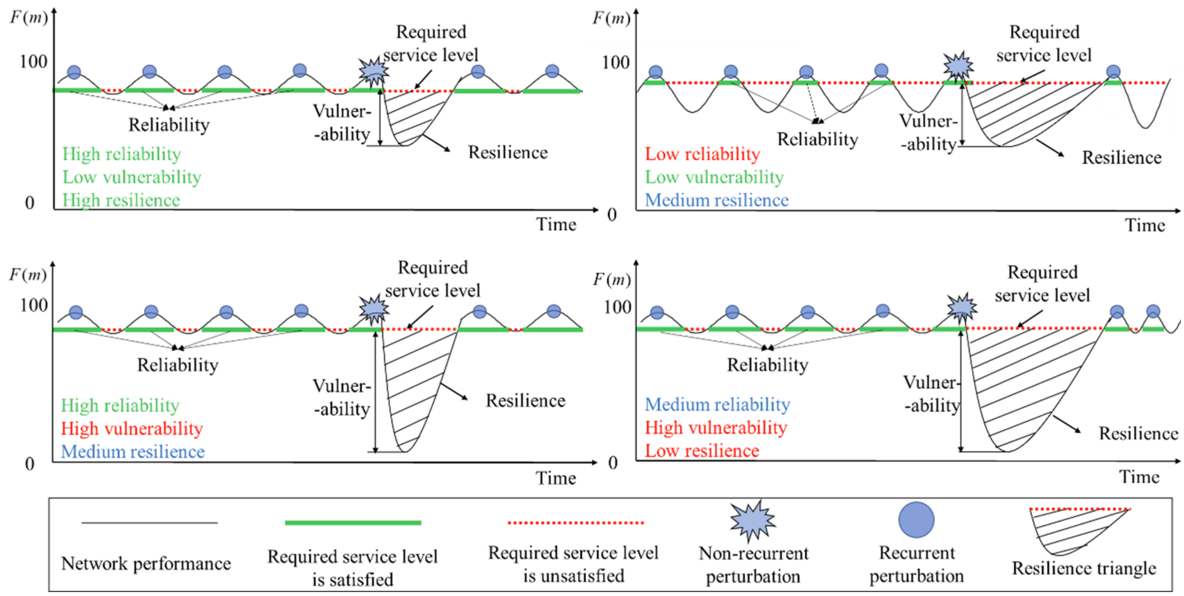

Fig. 2. Illustrations of different levels of reliability, vulnerability, and resilience.

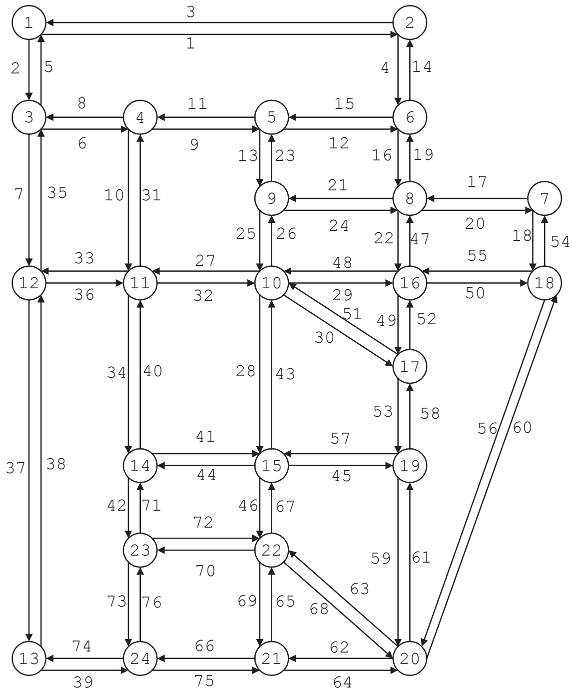

Fig. 3. The Sioux-Falls network.

the two concepts are often considered under different types of perturbations with distinct measurements. Furthermore, both the vulnerability and the recovery speed contribute to resilience. Thus, instead of lowering vulnerability to achieve higher resilience, efforts should also be devoted to shortening the duration in which the perturbation exerts negative impact on the network.

6. Numerical studies

Numerical examples are conducted in this section to illustrate the definitions, quantitative indices and analyses of the three concepts related to network performance, i.e., reliability, vulnerability, and resilience. The Sioux-Falls (S-F) network (as shown in Fig. 3), which is commonly used in network modelling studies, is selected as a typical transportation network from which to capture the performance of an urban transportation network under perturbations. The reliability, vulnerability, and resilience of the S-F network are analysed under different perturbation scenarios. The criticality of each network component is analysed using different indices of different concepts. Comparisons are conducted between the analytical results of different concepts and quantitative indices to demonstrate the similarities and differences between the three concepts. There are 24 nodes and 76 links in the S-F network. The link travel time is derived by the BPR function:

$$t_a = t_a^0\left[1 + 0.15\left(\frac{v_a}{C_a}\right)^4\right], \forall\, a \in A \tag{10}$$

where v_a, C_a, t_a, and $t_a{}^0$ denote the traffic flow volume, capacity, travel time, and free flow travel time on link a respectively.

The measures of reliability, vulnerability, and resilience used in numerical studies are listed in Table 5.

6.1. Network performance evaluation

In this section, the three concepts (i.e., reliability, vulnerability and resilience) are applied to evaluate the network performance under perturbations.

6.1.1. Experiment setup

In addition to the S-F network, the Nguyen-Dupuis (N-D) network (Nguyen and Dupuis, 1984) is used in this section as a case of less congested network to better illustrate the three concepts. All the links in both networks are simultaneously perturbed. We iteratively degrade the expected capacities of all links in the same ratio to show the network performance using different concepts. In addition, link capacities may not remain at certain levels in an uncertain world, but fluctuate over time. At each capacity ratio, the analyses are conducted in 100 random fluctuation scenarios. In this section, two commonly used distributions, the normal distribution and the uniform distribution are used to model the variation of link capacity. The standard deviation of link capacity is

Table 5
Concepts and indices used in numerical studies.

Concepts	Indices	Computation formula
Reliability	Capacity reliability (CR)	$CR(\mu_r) = P(\mu \geqslant \mu_r)$
	Travel time reliability (TTR)	$TTR_{CV} = \frac{1}{\lvert W \rvert} \cdot \sum_{w \in W} \frac{\sigma_{T_w}}{T_w}$
Vulnerability	System-based vulnerability (SV)	$VI_2' = \frac{1}{\lvert W \rvert} \cdot \frac{\sum_{w \in W} (q_w / U_w - q_w' / U_w')}{\sum_{w \in W} q_w / U_w} \cdot 100\%$
		$VI_2'' = \frac{1}{\lvert W \rvert} \cdot \frac{\sum_{w \in W} q_w' / U_w'}{\sum_{w \in W} q_w / U_w} \cdot 100\%$
	Topology-based vulnerability (TV)	$VI_2 = \frac{1}{\lvert N \rvert (\lvert N \rvert - 1)} \cdot \frac{\sum_{i \neq j \in N} (\frac{1}{d_{ij}} - \frac{1}{d_{ij}'})}{\sum_{i \neq j \in N} \frac{1}{d_{ij}}} \cdot 100\%$
Resilience	System-based resilience (SR)	$RL' = \sum_{m_0}^{m_1} VI_2'(m)$
	Topology-based resilience (TR)	$RL = \sum_{m_0}^{m_1} VI_2(m)$

derived by multiplying the expected capacity and the CV (set as 0.4) of link capacity.

The CR shown in Table 5 is adopted to represent the reliability. Note that under the perturbation scenario described in Section 6.1.1, the performance (capacity) of the whole network decreases, however, all links remain physically connected until the capacity ratio drops to 0%, which prevents the use of connectivity reliability analysis and topology-based vulnerability/resilience analyses. Thus, in this section, network vulnerability and resilience are obtained using system-based performance indices (i.e., SV and SR) rather than topology-based indices. We use the average value of the VI_2'' obtained in 100 fluctuation scenarios to represent SV at each capacity ratio. It is assumed that recovery actions are taken immediately after perturbation, restoring 10% of the original capacity per time unit, then, we can obtain SR as Table 5 shows.

6.1.2. Comparison between different concepts

The reliability, vulnerability, and resilience of the S-F network and the N-D network at different capacity ratios are shown in Fig. 4. A low capacity ratio indicates a severe perturbation. As is consistent with the comparative study done in the road network of the Rio de Janeiro Metropolitan Region (Oliveira et al., 2016), the three concepts show different sensitivities to the severity of perturbation. The network reliability remains almost unchanged at high capacity ratios, but rapidly decreases when the capacity ratio reaches a threshold (i.e., 0.7 for the S-F network and 0.4 for the N-D network). In contrast, vulnerability and resilience decrease more steadily. Thus, it can be concluded that the transportation network tends to experience a sudden decline of capacity reliability when the perturbation reaches a certain level, while network vulnerability and resilience are not as sensitive to this level. The network resilience shown in Fig. 4 is obtained from the size of resilience triangle, which is related not only to the network vulnerability but also to the recovery speed. For example, the resilience triangle of S-F network at 0% capacity ratio is demonstrated in Fig. 5.

In the absence of real capacity degradation data, two types of distribution are assumed in this numerical example to model the variation of link capacity under perturbation. Comparing Fig. 4(a) to 4(c) or comparing Fig. 4(b) to 4(d), it can be seen that the link capacity distribution has little influence on analysis results. The differences between the network performance of S-F network and that of N-D network can also be found in Fig. 4. Compared to the S-F network, the N-D network has fewer congested links under slight perturbation. Therefore, the N-D network shows higher reliability, vulnerability and resilience than the S-F network when the capacity ratio is high. Under severe perturbations where capacity ratios are low, however, it is found that different networks show similar performance.

To illustrate the effects of using different quantitative indices for the same concept, Fig. 6 shows the numerical results of travel time reliability compared to capacity reliability in the S-F network. From Fig. 6, it can be found that the capacity reliability is more sensitive to perturbations with moderate consequences (i.e., where capacity ratios are from 0.5 to 0.7) than to those with severe or slight damages. The changes in travel time reliability, however, can be observed at both high and low levels of severity. Therefore, capacity reliability is applicable to moderate perturbations, while travel time reliability can be used in both severe and slight perturbations. It is of significance to choose different indices for evaluating network performance under different levels of perturbations.

6.2. Component criticality evaluation

The other main objective of reliability, vulnerability, and resilience analysis is to search for the critical component in a transportation network. In this section, the criticality of each link in the S-F network is iteratively investigated based on changes in different quantitative indices of the three concepts.

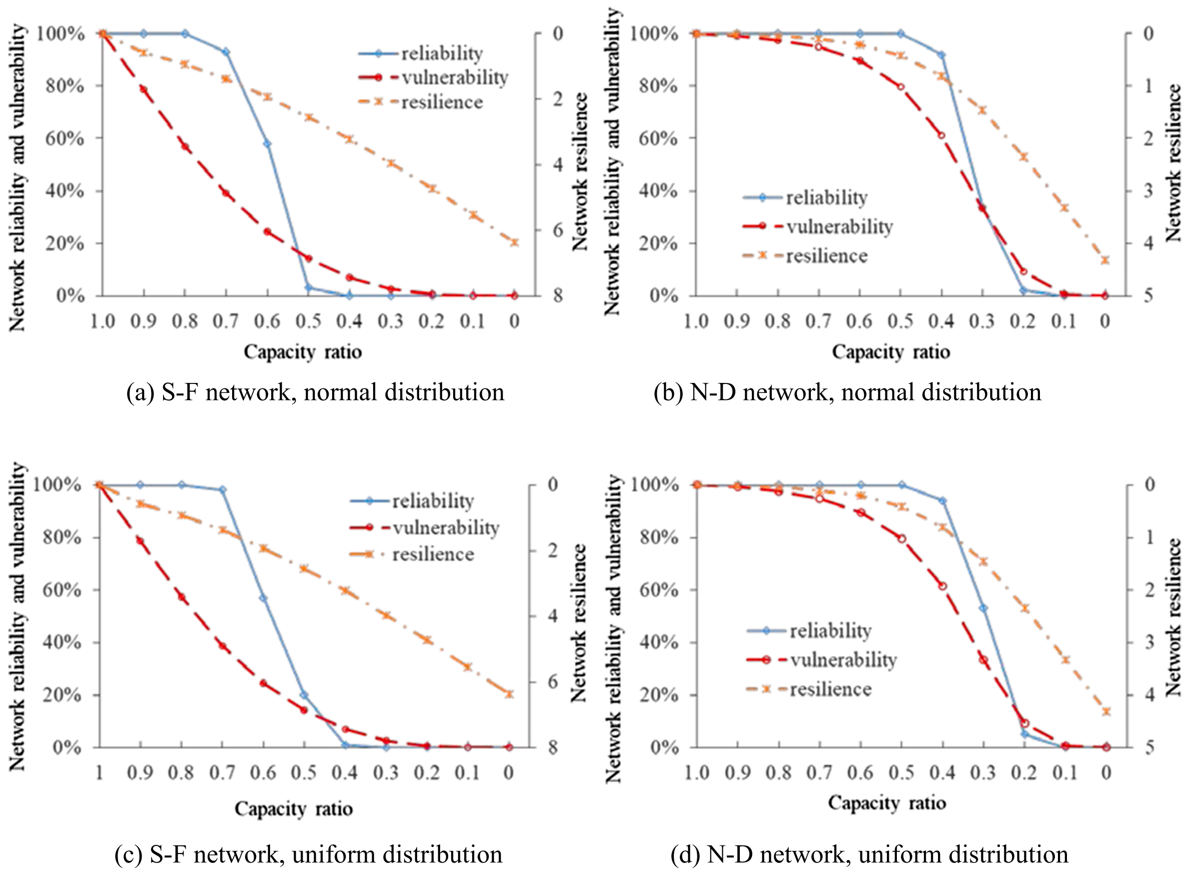

Fig. 4. Analytical results of network reliability, vulnerability, and resilience.

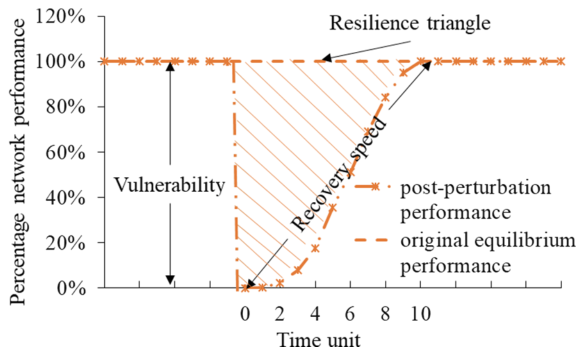

Fig. 5. Analytical results of network resilience.

6.2.1. Experiment setup

In this section it is hypothesised that a perturbation could cause failure of a single link in the network. To measure link criticality, each link is iteratively removed from the network, while the expected capacities of unremoved links remain unchanged. The capacities of unremoved links follow normal distributions. 100 stochastic scenarios are generated in each iteration to model probable capacity fluctuations after link removal.

The measures used in capacity reliability analysis, travel time reliability analysis, and system-based resilience analysis are consistent with those described in Section 6.1.1. In the system-based vulnerability analysis, VI_2' is adopted to quantify vulnerability. For comparison with system-based analyses, the topology-based vulnerability index *TV* and the resilience index derived therefrom, *TR*, are utilised in topology-based analyses.

During the recovery phase in the resilience analyses, the rate of recovery of each link is fixed and equal to one length unit per unit time. The restored link capacity is proportional to the elapsed time. Taking Link 1 as an example, the total recovery time for Link 1 is six time units, and 16.7% of its original capacity is restored per time unit.

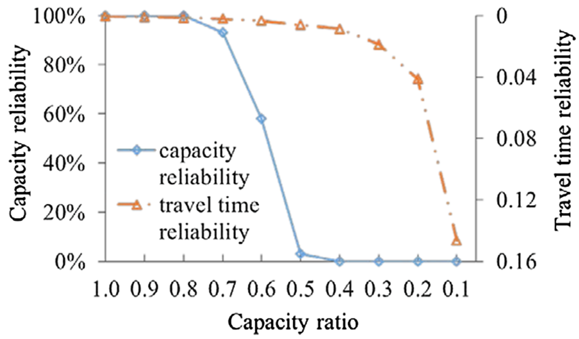

Fig. 6. Analytical results of network reliability.

6.2.2. Comparison between different concepts and indices

The criticality of each link is ranked according to the magnitude of network reliability, vulnerability, or resilience indices resulting from the removal of this link. The top-ten most critical links analysed by different concepts and different indices are summarised in Table 6. A low value of capacity reliability indicates that link removal makes the network unreliable, implying high link criticality. In contrast, higher values of other indices indicate a higher criticality of the link.

It can be found from Table 6 that different concepts lead to distinct rankings of link criticality. No link is simultaneously regarded as being in the top-ten most critical links in these six rankings. Using different indices might generate even greater differences than using different concepts. As also discussed in (Mattsson and Jenelius, 2015), the reason can be attributed to involvement of traffic assignment in system-based analyses. System-based indices consider the impact of traffic flow on transportation supply, thus can provide more network information under perturbation and may generate more reasonable results when applied in congested road networks.

The results shown in Table 6 are derived on the basis of a congested scenario where traffic on some links (*e.g.*, links 28 and 43) are heavy. To reveal the effect of road congestion and investigate applicable scenarios for different concepts and indices, two tests were conducted based on scenarios where demand levels are relatively low (1% and 10% of the original demand, separately). The rankings of link criticality are compared to those derived on the basis of the original demand level in Table 7.

Note that the topology-based indices (TV and TR) are not influenced by a change in link flow, and the capacity reliability (CR) remains 100% in all single-link failure scenarios when demand levels are relatively low, therefore, TV, TR, and CR are not included in Table 7. At low demand levels, SV and SR analyses produce similar results. This is because vulnerability is the inverse of robustness, thus can be considered to be a part of resilience (Faturechi and Miller-Hooks, 2014a, 2015; Reggiani et al., 2015). The effect of removing one link is negligible when traffic is light, making the network rapidly recover to its original level of performance, thus eliminating the difference between vulnerability and resilience. In addition, analytical results from topology-based and system-based indices are similar at lower demand levels. Without accounting for behaviours of road users, the topology-based analyses are more efficient when evaluating the performance of networks where the impact of traffic flow is negligible, *e.g.*, uncongested road networks, public transportation networks (Liu et al., 2017), aviation networks (O'kelly, 2015), *etc.*

Differing from SV and SR analyses, the TTR analyses can still provide distinct results even when the demand level decreases to 1%. Only four links simultaneously rank among the top-ten at the 1% and 10% demand levels. This makes travel time reliability applicable to the evaluation of slight variations in network performance.

Table 6
Criticality ranking of links based on different analyses.

Ranking	Reliability				Vulnerability				Resilience			
	Link ID	CR (%)	Link ID	TTR (10^{-3})	Link ID	SV (%)	Link ID	TV (%)	Link ID	SR (10^{-1})	Link ID	TR (10^{-3})
1	43	0	43	5.5	43	23	16	2.9	51	10.0	38	7.3
2	28	0	28	4.9	25	21	19	2.9	30	8.7	73	7.3
3	16	2	16	4.3	28	20	9	2.2	28	8.1	11	7.0
4	19	3	19	4.2	37	20	11	2.2	21	7.3	14	7.0
5	58	12	45	3.7	27	19	49	2.0	43	6.6	15	6.8
6	60	13	69	3.6	32	19	52	2.0	24	4.1	18	6.8
7	56	16	57	3.4	26	18	53	1.9	27	3.8	3	6.6
8	48	17	15	3.3	56	16	58	1.9	23	3.6	13	6.6
9	53	21	60	3.2	60	16	18	1.8	32	3.5	6	5.9
10	29	22	39	3.2	23	15	54	1.8	37	3.1	34	5.9

Indices: CR – capacity reliability; TTR – travel time reliability; SV – system-based vulnerability; TV – topology-based vulnerability; SR – system-based resilience; TR – topology-based resilience.

Table 7
Criticality ranking of links (shown with ID) under different demand levels.

Ranking	Demand level								
	1%			10%			100%		
	TTR	SV	SR	TTR	SV	SR	TTR	SV	SR
1	60	49	49	43	49	49	43	43	51
2	28	52	52	74	52	52	28	25	30
3	48	53	53	19	53	48	16	28	28
4	56	58	58	16	58	29	19	37	21
5	67	48	48	17	48	53	45	27	43
6	16	29	29	40	29	58	69	32	24
7	19	26	26	54	67	67	57	26	27
8	45	25	25	56	46	46	15	56	23
9	32	46	46	60	26	26	60	60	32
10	58	67	67	46	25	25	39	23	37

Indices: TTR – travel time reliability; SV – system-based vulnerability; SR – system-based resilience.

7. Conclusions

In this study, the definitions and quantitative indices of three concepts related to network performance under perturbations, i.e., reliability, vulnerability and resilience, are reviewed and compared in the context of transportation network. The three concepts are analysed and proved to be related but have significant differences in terms of focus, quantitative index and analysis, and application scenario. Numerical examples are presented to intuitively demonstrate and compare the concepts of reliability, vulnerability, and resilience under different perturbations.

It is found that the three concepts are able to evaluate transportation network performance under perturbation with different foci. There are many similarities and distinctions between the three concepts. Different concepts show various sensitivities to the severity of perturbation. High reliability does not necessarily lead to low vulnerability. Network resilience is related to but not entirely determined by vulnerability. It is of significance to choose proper concepts and indices for evaluating network performance under different levels of perturbations. In addition to performance evaluation, the three concepts are also found to have different applicability in scenarios with varying congestion levels in the identification of critical network components.

Based on this study, future directions for research into transportation network performance under perturbations could include:

(1) Investigate reliability, vulnerability, and resilience in multi-modal transportation networks. The characteristics and interdependency of different transport modes, as well as travellers' attitudes towards different modes should be specifically addressed in the evaluation of network performance under perturbation.
(2) Based on the characteristics of different types of transportation networks, *e.g.*, passenger transportation and logistics networks, devise specific quantitative indices and methods for analysing network performance under perturbation.
(3) Consider the interdependency among different network components in the analyses of reliability, vulnerability, and resilience. Whether, how, and to what extent do interactions between network components influence the choice behaviour of travellers thus the performances of these three concepts should be investigated.
(4) Investigate the interaction among reliability-, vulnerability- and resilience-oriented managements in the design and operation of transportation network. In the investment appraisal and the policy evaluation, it is necessary to understand how strategies aiming to optimise different concepts influence network performance under uncertain future perturbations.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.tre.2019.11.003.

References

Abegaz, D., Hjorth, K., Rich, J., 2017. Testing the slope model of scheduling preferences on stated preference data. Transport. Res. Part B: Methodol. 104, 409–436.
Asakura, Y., Kashiwadani, M., 1991. Road network reliability caused by daily fluctuation of traffic flow. Eur. Transp. Highway Plan. 19, 73–84.
Aven, T., 2011. On some recent definitions and analysis frameworks for risk, vulnerability, and resilience. Risk Anal. 31 (4), 515–522.
Bates, J., Polak, J., Jones, P., Cook, A., 2001. The valuation of reliability for personal travel. Transport. Res. Part E Log. Transport. Rev. 37 (2–3), 191–229.

Bell, M.G.H., Kurauchi, F., Perera, S., Wong, W., 2017. Investigating transport network vulnerability by capacity weighted spectral analysis. Transport. Res. Part B: Methodol. 46 (4), 251–266.
Berdica, K., 2002. An introduction to road vulnerability: what has been done, is done and should be done. Transp. Policy 9, 117–127.
Berche, B., von Ferber, C., Holovatch, T., Holovatch, Y., 2009. Resilience of public transport networks against attacks. Phys. Condens. Matter. 71 (1), 125–137.
Berche, B., von Ferber, C., Holovatch, T., Holovatch, Y., 2012. Transportation network stability: a case study of city transit. Adv. Complex Syst. 15 (1), 1250063.
Bier, V., Gutfraind, A., 2019. Risk analysis beyond vulnerability and resilience – characterizing the defensibility of critical systems. Eur. J. Oper. Res. 276, 626–636.
Bruneau, M., Chang, S.E., Eguchi, R.T., Lee, G.C., O'Rourke, T.D., Reinhorn, A.M., Shinozuka, M., Tierney, K., Wallace, W.A., Von Winterfeldt, D., 2003. A framework to quantitatively assess and enhance seismic resilience of communities. Earthquake Spectra. 19, 733–752.
Carrion, C., Levinson, D., 2012. Value of travel time reliability: a review of current evidence. Transport. Res. Part A Policy Pract. 46 (4), 720–741.
Chang, J.S., 2010. Assessing travel time reliability in transport appraisal. J. Transp. Geogr. 18 (3), 419–425.
Chen, A., Kasikitwiwat, P., 2011. Modeling capacity flexibility of transportation networks. Transport. Res. Part A: Policy Pract. 45 (2), 105–117.
Chen, A., Kasikitwiwat, P., Yang, C., 2013. Alternate capacity reliability measures for transportation networks. J. Adv. Transport. 47, 79–104.
Chen, A., Yang, C., Kongsomsaksakul, S., Lee, M., 2007. Network-based accessibility measures for vulnerability analysis of degradable transportation networks. Netw. Spatial Econ. 7, 241–256.
Chen, A., Yang, H., Lo, H.K., Tang, W.H., 1999. Capacity related reliability for transportation networks. J. Adv. Transport. 33, 183–200.
Chen, A., Yang, H., Lo, H.K., Tang, W.H., 2002. Capacity reliability of a road network: an assessment methodology and numerical results. Transport. Res. Part B Methodol. 36, 225–252.
Chen, B.Y., Lam, W.H.K., Sumalee, A., Shao, H., 2011. An efficient solution algorithm for solving multi-class reliability-based traffic assignment problem. Math. Comput. Modell. 54 (5), 1428–1439.
Chen, B.Y., Lam, W.H.K., Sumalee, A., Li, Q., Li, Z.-C., 2012. Vulnerability analysis for large-scale and congested road networks with demand uncertainty. Transport. Res. Part A Policy Pract. 46, 501–516.
Chen, H., Cullinane, K., Liu, N., 2017. Developing a model for measuring the resilience of a port-hinterland container transportation network. Transport. Res. Part E Log. Transport. Rev. 97, 282–301.
Cheng, Q., Liu, Z., Szeto, W.Y., 2019. A cell-based dynamic congestion pricing scheme considering travel distance and time delay. Transportmetrica B: Transp. Dyn. 7 (1), 1286–1304.
Cox, A., Prager, F., Rose, A., 2011. Transportation security and the role of resilience: a foundation for operational indexes. Transp. Policy 18, 307–317.
Demšar, U., Špatenková, O., Virrantaus, K., 2008. Identifying critical locations in a spatial network with graph theory. Trans. GIS 12 (1), 61–82.
D'este, G.M., Taylor, M.A.P., 2003. Network vulnerability: an approach to reliability analysis at the level of national strategic transport networks. In: Iida, Y., Bell, M.G.H. (Eds.), The Network Reliability of Transport. Elsevier, Oxford, pp. 23–44.
Dunn, S., Wilkinson, S.M., 2016. Increasing the resilience of air traffic networks using a network graph theory approach. Transport. Res. Part E Log. Transport. Rev. 90, 39–50.
Engelson, L., Fosgerau, M., 2016. The cost of travel time variability: three measures with properties. Transport. Res. Part B: Methodol. 91, 555–564.
Faturechi, R., Miller-Hooks, E., 2014a. A mathematical framework for quantifying and optimizing protective actions for civil infrastructure systems. Comput.-Aided Civ. Infrastruct. Eng. 29 (8), 572–589.
Faturechi, R., Miller-Hooks, E., 2014b. Travel time resilience of roadway networks under disaster. Transport. Res. Part B Methodol. 70, 47–64.
Faturechi, R., Miller-Hooks, E., 2015. Measuring the performance of transportation infrastructure systems in disasters: a comprehensive review. J. Infrastruct. Syst. 21, 1–15.
von Ferber, C., Berche, B., Holovatch, T., Holovatch, Y., 2012. A tale of two cities: vulnerabilities of the London and Paris transit networks. J. Transport. Secur. 5 (3), 199–216.
Fosgerau, M., Fukuda, D., 2012. Valuing travel time variability: characteristics of the travel time distribution on an urban road. Transport. Res. Part C Emerg. Technol. 24, 83–101.
Fosgerau, M., Karlström, A., 2010. The value of reliability. Transport. Res. Part B Methodol. 44 (1), 38–49.
Fu, X., Gu, Y., 2018. Impact of a new metro line: analysis of metro passenger flow and travel time based on smart card data. J. Adv. Transport. 2018, 1–13.
Fu, X., Lam, W.H.K., 2014. A network equilibrium approach for modelling activity-travel pattern scheduling problems in multi-modal transit networks with uncertainty. Transportation 41, 37–55.
Fu, X., Lam, W.H.K., 2018. Modelling joint activity-travel pattern scheduling problem in multi-modal transit networks. Transportation 45, 23–49.
Fu, X., Lam, W.H.K., Chen, B.Y., 2014. A reliability-based traffic assignment model for multi-modal transport network under demand uncertainty. J. Adv. Transport. 48, 66–85.
Henry, D., Ramirez-Marquez, J.E., 2012. Generic indexes and quantitative approaches for system resilience as a function of time. Reliab. Eng. Syst. Saf. 99, 114–122.
Holling, C.S., 1973. Resilience and stability of ecological systems. Annu. Rev. Ecol. Syst. 4, 1–23.
Hosseini, S., Barker, K., Ramirez-Marquez, J.E., 2016. A review of definitions and measures of system resilience. Reliab. Eng. Syst. Saf. 145, 47–61.
Husdal, J., 2005. The vulnerability of road networks in a cost-benefit perspective. In: Paper presented at the 84th Transportation Research Board Annual Meeting, Washington, DC.
Iida, Y., Wakabayashi, H., 1989. An approximation method of terminal reliability of road network using partial minimal path and cut set. In: Proceedings of the Fifth World Conference on Transport Research (WCTR), Yokohama, pp. 367–380.
Ip, W.H., Wang, D., 2011. Resilience and friability of transportation networks: evaluation, analysis and optimization. IEEE Syst. J. 5, 189–198.
Jenelius, E., Petersen, T., Mattsson, L.G., 2006. Importance and exposure in road network vulnerability analysis. Transport. Res. Part A Policy Pract. 40, 537–560.
Jenelius, E., Mattsson, L.-G., 2012. Road network vulnerability analysis of area-covering disruptions: a grid-based approach with case study. Transport. Res. Part A Policy Pract. 46, 746–760.
Jiang, Y., Timmermans, H.J.P., Yu, B., 2018. Relocation of manufacturing industry from the perspective of transport accessibility – An application of percolation theory. Transp. Policy 63, 10–29.
Kaviani, A., Thompson, R.G., Rajabifard, A., 2017. Improving regional road network resilience by optimised traffic guidance. Transportmetrica A Transp. Sci. 13 (9), 794–828.
Kurauchi, F., Uno, N., Sumalee, A., Seto, Y., 2009. Network evaluation based on connectivity vulnerability. In: Lam, W.H.K., Wong, S.C., Lo, H.K. (Eds.), Transportation and Traffic Theory 2009: Golden Jubilee. Springer, pp. 637–649.
Lam, T.C., Small, K.A., 2001. The value of time and reliability: measurement from a value pricing experiment. Transport. Res. Part E Log. Transport. Rev. 37 (2–3), 231–251.
Li, D., Fu, B., Wang, Y., Lu, G., Berezin, Y., Stanley, H.E., Havlin, S., 2015a. Percolation transition in dynamical traffic network with evolving critical bottlenecks. Proc. Natl. Acad. Sci. U.S.A. 112 (3), 669–672.
Li, X., Lam, W.H.K., Shao, H., Gao, Z., 2015b. Dynamic modelling of traffic incident impacts on network reliability. Transportmetrica A Transp. Sci. 11 (9), 856–872.
Li, Z., Hensher, D.A., Rose, J.M., 2010. Willingness to pay for travel time reliability in passenger transport: a review and some new empirical evidence. Transport. Res. Part E Log. Transport. Rev. 46 (3), 384–403.
Li, Z.C., Lam, W.H.K., Wong, S.C., Huang, H.-J., Zhu, D.-L., 2008. Reliability evaluation for stochastic and time-dependent networks with multiple parking facilities. Netw. Spatial Econ. 8 (4), 355–381.
van Lint, J.W.C., van Zuylen, H.J., Tu, H., 2008. Travel time unreliability on freeways: why measures based on variance tell only half the story. Transport. Res. Part A Policy Pract. 42 (1), 258–277.
Liu, Z., Chen, X., Meng, Q., Kim, I., 2018. Remote park-and-ride network equilibrium model and its applications. Transp. Res. Part B 117, 37–62.
Liu, Z., Liu, Y., Meng, Q., Cheng, Q., 2019. A tailored machine learning approach for urban transport network flow estimation. Transp. Res. Part C 108, 130–150.
Liu, J., Lu, H., Ma, H., Liu, W., 2017. Network vulnerability analysis of rail transit plans in Beijng-Tianjin-Hebei region considering connectivity reliability.

Sustainability 9, 1479.
Lo, H.K., Luo, X.W., Siu, B.W.Y., 2006. Degradable transport network: travel time budget of travelers with heterogeneous risk aversion. Transport. Res. Part B Methodol. 40, 792–806.
Lomax, T., Schrank, D., Turner, S., Margiotta, R., 2003. Selecting Travel Reliability Measures. Texas Transportation Institute, Austin TX Report TTI-2003-3 http://tti.tamu.edu/documents/474360-1.pdf/.
Loon, R.V., Rietveld, P., Brons, M., 2011. Travel-time reliability impacts on railway passenger demand: a revealed preference analysis. J. Transp. Geogr. 19 (4), 917–925.
Luathep, P., Sumalee, A., Ho, H.W., Kurauchi, F., 2011. Large-scale road network vulnerability analysis: a sensitivity analysis based approach. Transportation 38 (5), 799–817.
Mattsson, L.-G., Jenelius, E., 2015. Vulnerability and resilience of transport systems – a discussion of recent research. Transport. Res. Part A Policy Pract. 81, 16–34.
Miller-Hooks, E., Zhang, X., Faturechi, R., 2012. Measuring and maximizing resilience of freight transportation networks. Comput. Oper. Res. 39, 1633–1643.
Mine, H., Kawai, H., 1982. Mathematics for Reliability and Availability, Asakura-shoten (in Japanese).
Morlok, E.K., Chang, D.J., 2004. Measuring capacity flexibility of a transportation system. Transport. Res. Part A: Policy Pract. 33 (6), 405–420.
Nguyen, S., Dupuis, C., 1984. An efficient method for computing traffic equilibria in networks with asymmetric transportation costs. Transport. Sci. 18 (2), 185–202.
Nicholson, A., Du, Z., 1997. Degradable transportation systems: an integrated equilibrium model. Transport. Res. Part B: Methodol. 31 (3), 209–223.
Noland, R.B., Small, K.A., 1995. Travel-time uncertainty, departure time choice, and the cost of morning commutes. Transport. Res. Rec. J. Transport. Res. Board 1493, 150–158.
O'kelly, M.E., 2015. Network Hub Structure and Resilience. Netw. Spatial Econ. 15, 235–251.
Oliveira, E.L.D., Portugal, L.D.S., Junior, W.P., 2016. Indicators of reliability and vulnerability: similarities and differences in ranking links of a complex road system. Transport. Res. Part A Policy Pract. 88, 195–208.
Omer, M., Mostashari, A., Nilchiani, R., 2011. Measuring the resiliency of the Manhattan points of entry in the face of severe disruption. Am. J. Eng. Appl. Sci. 4 (1), 153–161.
Palma, A.D., Picard, N., 2005. Route choice decision under travel time uncertainty. Transport. Res. Part A Policy Pract. 39, 295–324.
Pimm, S.L., 1984. The complexity and stability of ecosystems. Nature 307, 321–326.
Qiang, Q., Nagurney, A., 2008. A unified network performance measure with importance identification and the ranking of network components. Opt. Lett. 2 (1), 127–142.
Reggiani, A., 2013. Network resilience for transport security: some methodological considerations. Transp. Policy 28, 63–68.
Reggiani, A., Nijkamp, P., Lanzi, D., 2015. Transport resilience and vulnerability: the role of connectivity. Transport. Res. Part A Policy Pract. 81, 4–15.
Shao, H., Lam, W., Meng, Q., Tam, M., 2006. Demand-driven traffic assignment problem based on travel time reliability. Transport. Res. Rec. J. Transport. Res. Board 1985, 220–230.
Small, K.A., 1982. The scheduling of consumer activities: work trips. Am. Econ. Rev. 72 (3), 467–479.
Sumalee, A., Kurauchi, F., 2006. Network capacity reliability analysis considering traffic regulation after a major disaster. Netw. Spatial Econ. 6, 205–219.
Sun, C., Cheng, L., Zhu, S., Han, F., Chu, Z., 2019. Multi-criteria user equilibrium model considering travel time, travel time reliability and distance. Transport. Res. Part D: Transp. Environ. 66, 3–12.
Sun, Y., Shi, J., Schonfeld, P.M., 2016. Identifying passenger flow characteristics and evaluating travel time reliability by visualizing AFC data: a case study of Shanghai Metro. Public Transp. 8, 1–23.
Taylor, M.A.P., 2013. Travel through time: the story of research on travel time reliability. Transportmetrica B Transp. Dyn. 1 (3), 174–219.
Taylor, M.A.P., Sekhar, S.V.C., D'este, G.M., 2006. Application of accessibility based methods for vulnerability analysis of strategic road networks. Netw. Spatial Econ. 6, 267–291.
Taylor, M.A.P., D'este, G.M., 2007. Transport network vulnerability: a method for diagnosis of critical locations in transport infrastructure systems. In: Murray, A.T., Grubesic, T.H. (Eds.), Critical Infrastructure: Reliability and Vulnerability. Springer, Berlin, pp. 9–30.
Taylor, M.A.P., 2017a. Vulnerability Analysis for Transportation Networks. Elsvier, Oxford.
Taylor, M.A.P., 2017b. Fosgerau's travel time reliability ratio and the burr distribution. Transport. Res. Part B: Methodol. 97, 50–63.
Talebpour, A., Mahmassani, H.S., Hamdar, S.H., 2017. Effect of information availability on stability of traffic flow: percolation theory approach. Transp. Res. Procedia 23, 81–100.
Twumasi-Boakye, R., Sobanjo, J.O., 2018. Resilience of regional transportation networks subjected to hazard-induced bridge damages. J. Transport. Eng., Part A: Syst. 144 (10), 1–13.
Wakabayashi, H., Matsumoto, Y., 2012. Comparative study on travel time reliability indexes for highway users and operators. J. Adv. Transport. 46 (4), 318–339.
Wan, C., Yang, Z., Zhang, D., Yan, X., Fan, S., 2018. Resilience in transportation systems: a systematic review and future directions. Transp. Rev. 38 (4), 479–498.
Wang, D.Z.W., Liu, H., Szeto, W.Y., Chow, A.H.F., 2016. Identification of critical combination of vulnerable links in transportation networks – a global optimisation approach. Transportmetrica A Transp. Sci. 12 (4), 346–365.
Wang, Y., Liu, H., Han, K., Friesz, T.L., Yao, T., 2015. Day-to-day congestion pricing and network resilience. Transportmetrica A Transp. Sci. 11 (9), 873–895.
Wong, S.C., Yang, H., 1997. Reserve capacity of a signal-controlled road network. Transport. Res. Part B Methodol. 31 (5), 397–402.
Xu, C., Wang, Y., Liu, P., Wang, W., Bao, J., 2018a. Quantitative risk assessment of freeway crash casualty using high-resolution traffic data. Reliab. Eng. Syst. Saf. 169, 299–311.
Xu, X., Chen, A., Jansuwan, S., Yang, C., Ryu, S., 2018b. Transportation network redundancy: complementary measures and computational methods. Transport. Res. Part B: Methodol. 114, 68–85.
Xu, X., Chen, A., Lo, H.K., Yang, C., 2018c. Modeling the impacts of speed limits on uncertain road networks. Transportmetrica A Transp. Sci. 14 (1–2), 1–19.
Xu, X., Chen, A., Yang, C., 2018d. An optimization approach for deriving upper and lower bounds of transportation network vulnerability under simultaneous disruptions of multiple links. Transport. Res. Part C Emerg. Technol. 94, 338–353.
Ye, Q., Ukkusuri, S.V., 2015. Resilience as an objective in the optimal reconstruction sequence for transportation networks. J. Transport. Safety Sec. 7, 91–105.
Zhalechian, M., Ali Torabib, S., Mohammadi, M., 2018. Hub-and-spoke network design under operational and disruption risks. Transport. Res. Part E Log. Transport. Rev. 109, 20–43.
Zhang, X., Miller-Hooks, E., 2015. Scheduling short-term recovery activities to maximize transportation network resilience. J. Comput. Civil Eng. 29, 1–10.
Zhang, X., Miller-Hooks, E., Denny, K., 2015. Assessing the role of network topology in transportation network resilience. J. Transp. Geogr. 46, 35–45.