

Dynamics of Vegetation and Runoff Erosion

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Most models of landscape evolution emphasize physical rather than biological processes, yet the biosphere, and vegetation in particular, clearly plays an important role in landscape evolution. Vegetation influences physical erosion both directly, by increasing surface resistance to wash erosion, and indirectly, by influencing infiltration, runoff, and evapotranspiration. For example, the data of Melton (1958) show an inverse correlation between drainage density and humidity, a finding that has been attributed to the stabilizing effects of vegetation (e.g., Moglen et al., 1998). By imposing a significant threshold for runoff erosion, vegetation cover may effectively impose an upper limit to channel network extent (e.g., Horton, 1945; Montgomery and Dietrich, 1989; Dietrich et al., 1993; Kirkby, 1994). Similarly, vegetation is widely believed to contribute to the observed relationship between climate and sediment yield. A number of data sets show a decrease in sediment yield with increasing mean annual rainfall in semi-arid to humid climates, despite the presumed increase in erosive energy associated with more humid climates (Langbein and Schumm, 1958; Douglas, 1967; Wilson, 1973; Moglen et al., 1998). Finally, vegetation is potentially important in governing landscape responses to environmental change (e.g., Tucker and Slingerland, 1997; Howard, 1996).

In fully vegetated landscapes, there is a dynamic competition between vegetation growth and the disruption of vegetation by runoff erosion. Within well-established channels, runoff erosion clearly “wins,” while near drainage divides it is the vegetation that “wins.” The interesting part is what happens in between these two extremes. The outcome of the competition between erosion and vegetation growth in the vicinity of first order channels may have a significant effect on drainage density (in some cases, it may be the determining factor), and often also on sediment

yield. In this chapter, we develop a simple theory to describe the dynamic interaction between vegetation and erosion, and use that theory to explore the nature of that interaction on “geomorphic” time scales (by which we mean time scales relevant to morphologic development as opposed to, say, the time scale for significant erosion of an agricultural field). In the first part, we present the theory and explore some simple outcomes for slope profile development. In the second part, we present several different numerical examples using the CHILD model Version 1.0 (1997 version).

Model of Vegetation and Erosion

Conceptual model

In the context of drainage network development, one of the most important issues is the nature of the hillslope-channel transition. In humid or semi-humid landscapes, hillslopes are typically vegetation-covered, while low-order ephemeral channels contain sparse, intermittent vegetation (clumps of grass, bushes, etc.), and higher-order channels with year-round flow are free of vegetation except along the banks and bars. In deriving a model to describe vegetation-erosion dynamics, we focus on the physical interaction of vegetation and runoff erosion within rills and channels. The most important aspects of that interaction are (1) the effect of vegetation cover on soil/sediment erodibility, (2) the disruption of vegetation due to erosive overland flow, and (3) the rate at which vegetation regrows after being disrupted. Below we attempt to quantify these three interacting processes.

Modeling approach

Vegetation reduces erodibility

There are a number of ways in which the effects of vegetation on surface erodibility might be modeled. A simple but physically plausible approach is to assume that vegetation increases the both the effective shear stress for runoff erosion and the critical shear stress for particle entrainment. Many sediment transport equations have the form

$$q_s = k(\tau - \tau_c)^p, \quad (1)$$

where q_s is sediment transport capacity per unit flow width, τ is fluid shear stress, and τ_c is a threshold for particle entrainment. By binding the soil with roots and leaf cover (grasses, for example), vegetation effectively increases τ_c , an idea consistent with the threshold channel initiation hypothesis (e.g., Horton, 1945; Willgoose et al., 1990; Montgomery and Dietrich, 1989) and also with Foster's (1982) soil erosion model, which forms part of the WEPP agricultural model (Foster et al., 1995).

Vegetation can be considered in terms of a fractional ground cover, V , which ranges from zero to one (or more generally from zero to $V_{max} \leq 1$, where V_{max} is the maximum percent cover that can be supported in a given environment). Few data sets exist with which to constrain the relationship between τ_c and V (Foster 1982 provides some field-calibrated values; Dietrich et al., 1993 give estimates of τ_c at channel heads, based on DEM analysis). In the absence of better information, we assume a linear relationship, recognizing that this may be an oversimplification:

$$\tau_c(V) = \tau_{cs} + V\tau_{cv}, \quad (2)$$

where τ_{cs} is the critical shear stress for an unvegetated surface (primarily a function of grain size)

and τ_{cv} is the added critical shear stress under 100% vegetation cover. Note that this approach assumes, via equation (1), that transport capacity depends on vegetation cover.

We account for changes in the total shear stress due to increased roughness and for changes in the fraction of shear stress applied to the soil surface using the approach discussed by Foster (1982). The total shear stress and effective shear stress τ_f applied to the soil are assumed to be related by

$$\tau_f = \frac{f_s}{f_t} \tau, \quad (3)$$

with f_s/f_t being the ratio of the friction factor produced by the soil alone to the total friction factor (including vegetation cover). The ratio $R_f = f_s/f_t$ is parameterized as a function of V , again assuming a linear relationship,

$$R_f = k_{rv} V, \quad k_{rv} \leq 1. \quad (4)$$

Runoff erosion disrupts vegetation

Surface runoff clearly disrupts vegetation when it becomes strong enough, but only recently have there been attempts to model the process quantitatively (Thornes, 1990; Kirkby, 1995; Kirkby and Cox, 1995). We speculate that the rate of vegetation destruction by rill or channel runoff is proportional to excess shear stress and to the fractional vegetation cover remaining:

$$\frac{dV}{dt}(\text{erosion}) = -k_{vd} V (\tau - \tau_c)^\eta, \quad (5)$$

with k_{vd} being the rate of vegetation loss per unit excess shear stress (or stress to a power) at 100% vegetation cover. This has the potential for a self-enhancing feedback: removal of vegetation

decreases τ_c , which in turn increases the rate of vegetation loss (but which is also compensated for by reduction in V). Solutions to (5) are sketched as a function of the dimensionless parameter $G = \tau/\tau_{cv}$ (ratio of applied shear stress to vegetation-added critical shear stress) in Figure 1. The

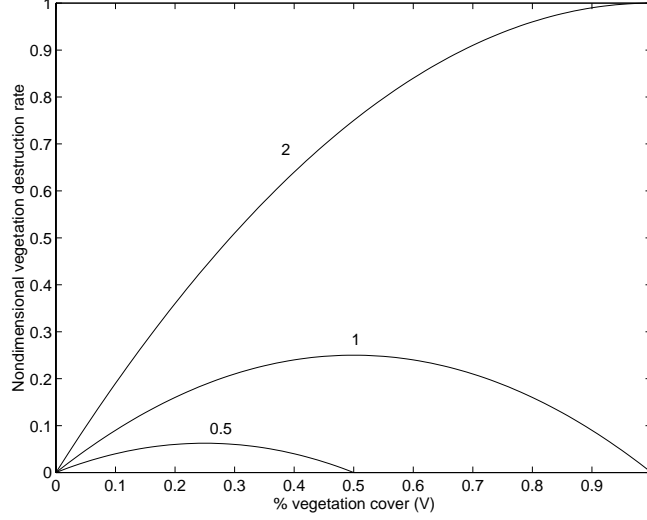


FIGURE 1. Vegetation destruction rate as a function of percent vegetation cover, for different values of G (nondimensional shear stress).

“humped” curves reflect the dual influence of vegetation cover on critical shear stress and on vegetation erodibility; when cover is sparse, there is little vegetation to erode, but when cover is dense, the erosion potential is reduced due to the increase in τ_c .

Vegetation regrowth

After vegetation is disrupted (for example, during a storm) it will begin to regrow. The rate of regrowth of vegetation biomass V_b is sometimes modeled using a logistic growth equation, in which the rate of growth approaches zero as the biomass becomes large, and also approaches zero if the biomass is very small (in which case there are few or no organisms to reproduce) (e.g., Thornes, 1990). For the present purposes, however, it is reasonable to assume that there will always be a ready supply of seeds and colonizing roots near a devegetated area (recall that V represents vegetation cover within a rill or channel; the surrounding hillsides are assumed to have $V=V_{max}$). In view of this, a simpler linear approach seems warranted, with the rate of regrowth

being proportional to the existing cover:

$$\frac{dV}{dt}(\text{growth}) = k_{vg}(1 - V). \quad (6)$$

The parameter k_{vg} is the rate of regrowth on a bare surface, and would be a function of solar radiation, soil moisture, nutrients, and so on. Note that this approach is consistent with the crop growth model used in the WEPP agriculture model (Arnold et al., 1995), which assumes a linear rate of biomass growth and an exponential relationship between plant cover and biomass. The cover-biomass relationship in WEPP is

$$V = 1 - \exp(-V_b/B) \quad (7)$$

and the biomass growth rate is

$$\frac{dV_b}{dt} = \frac{d}{dt}(-B \ln(1 - V)) = k, \quad (8)$$

which implies that

$$\frac{dV}{dt} = \frac{k}{B}(1 - V). \quad (9)$$

Equations (8) and (9) say that although the biomass can continue to grow indefinitely, it eventually reaches a point where additional biomass growth does not significantly increase the fractional vegetation cover on the surface.

Combining the terms for vegetation growth and erosion,

$$\frac{dV}{dt} = k_{vg}(1 - V) - k_{vd}V(R_f\tau - \tau_c). \quad (10)$$

In the next section, we explore analytical solutions to this equation. In doing so, we make the further simplifications $R_f=1$ (i.e., vegetation does not affect total shear stress), $\eta=1$, and $\tau_{cs}=0$.

Characteristic Form Profiles

Nondimensionalization

It is interesting to consider what a vegetation profile across a steady-state (characteristic form) hillslope might look like, and how the presence of vegetation influences the shape of the hillslope. We can obtain a steady-state vegetation profile by solving equation (10) for the case $dV/dt=0$, and solving equation (1) for the case $q_s = Ex$, where E is an erosion rate that is constant along the profile. To facilitate the analysis, we introduce the following nondimensionalization:

$$\text{Vegetation growth time scale} \quad T_v = 1/k_{vg} \quad (11)$$

$$\text{Nondimensional time} \quad t' = t/T_v \quad (12)$$

$$\text{Nondimensional shear stress} \quad \tau' = \tau/\tau_{cv} \quad (13)$$

$$\text{Nondimensional distance along slope profile} \quad x' = x/L, \quad L = \text{slope length} \quad (14)$$

$$\text{Vegetation number} \quad N_v = \frac{k_{vd}\tau_{cv}}{k_{vg}} \quad (15)$$

The vegetation number represents the efficiency of vegetation growth relative to destruction; low values represent fast-growing and/or destruction-resistant vegetation, and vice-versa. Introducing these dimensionless numbers into equation (10), we have

$$\frac{dV}{dt'} = N_v V^2 - (1 + N\tau')V + 1, \quad (16)$$

which is a quadratic equation that can be solved if dV/dt is constant or zero.

Wash Profile

The equilibrium vegetation profile depends in part on the rate at which it is being disrupted by runoff erosion, which may vary along a slope. First we consider the case of a hillslope profile that is eroded solely by wash (equation (1)), at a uniform rate E such that $q_s = Ex'$. Under that condition, the equilibrium shear stress is

$$\tau' = N_E^{\frac{1}{p}} x'^{\frac{1}{p}} + V, \quad N_E = \left[\frac{LE}{k\tau_{cv}} \right] \quad (17)$$

Substituting this into equation (16) yields

$$V = \frac{1}{1 + N_v N_E^q x'^q}, \quad q = \frac{1}{p}. \quad (18)$$

Similarly, we can solve for equilibrium slope along the profile using the shear stress relationship

$$\tau = k_t L^m x'^m S^n. \quad (19)$$

Solving for the equilibrium condition,

$$S = \left[\frac{N_E^q}{N_t} x'^{q-m} + \frac{1}{N_t x'^m (1 + N_v N_E^q x'^q)} \right]^{1/n}, \quad (20)$$

where $N_t = k_t L^m / \tau_{cv}$. Figure 2 depicts solutions to (18) and (20) for different values of the dimensionless parameters N_v , N_E , and N_t , assuming typical values of $m=2/3$, $n=2/3$, $q=1/p=2/3$ (e.g., the Meyer-Peter and Mueller relation) or $q=1/p=1/3$ (the Einstein-Brown relation). Vegetation cover decreases downslope in response to the increasing shear stress, asymptotically approaching zero as x' becomes large. This vegetation gradient has the effect of increasing the concavity of the hillslope profile, particularly when the erosion number N_E is small relative to the other parameters

(compare Figure 2, top and bottom). The vegetation number N_v controls the downslope rate of reduction in vegetation cover. When N_v is large (Figure 2, middle), there is an abrupt reduction in vegetation cover in the upper part of the profile that corresponds to a sharp concavity in the topography.

Diffusive Profile

The wash model ignores the effect of creep-related (slope-dependent) processes that would tend to produce convex-upward slopes near a drainage divide (e.g., Gilbert, 1909). The profile shape under the action of both creep and wash processes cannot be solved analytically, but we can obtain some idea of what a vegetation profile might look like on a diffusion-dominated slope by solving for the characteristic-form slope as if diffusion were the sole process. Essentially, we assume that wash controls the amount of vegetation but does not contribute significantly to shaping the topography.

Assuming that the transport rate by creep is given by $q_s = k_d S$, the characteristic form profile is

$$S = \frac{UL}{k_d} x' = N_d x'. \quad (21)$$

Substituting into equation (19), the shear stress is given by

$$\tau' = \frac{k_t L^m N_d^n}{\tau_{cv}} x'^{m+n} = N_{dt} x'^{m+n}. \quad (22)$$

Combining with (16) (for $dV/dt'=0$) gives the vegetation profile

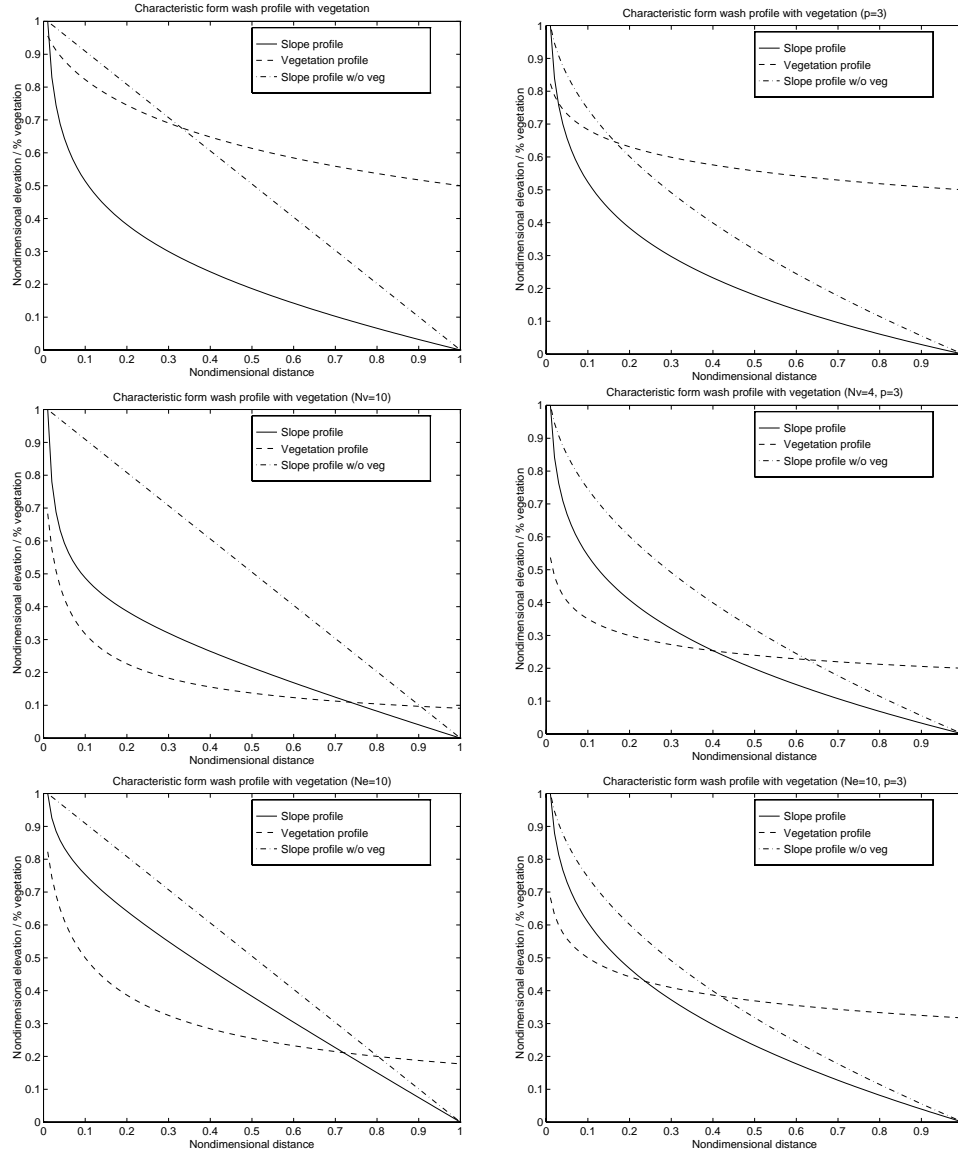


FIGURE 2. Solutions to equations (18) and (20) for wash-dominated hillslope profiles, showing the effect of vegetation on profile shape.

$$N_v V^2 - (1 + N_v N_{dt} x^{m+n}) V + 1 = 0. \quad (23)$$

Example solutions to (21) and (23) are shown in Figure 3. Note that equation (23) has two roots.

In the example shown in the figure, one of the roots implies full vegetation cover along the profile, while the other implies decreasing cover. This example of bi-stability is similar to the bistable landscape states explored by Howard (1996), as well as the multiple phase-states in the soil ero-

sion model of Thornes (1990). Depending on the initial conditions, the system may evolve toward a different equilibrium state. This analytical example of a diffusion-dominated profile is of course rather artificial because it does not incorporate the linkage between slope form and wash erosion. To explore the complete interaction of wash, diffusive transport, and vegetation growth, numerical simulations are needed.

Numerical Examples

Figure 4 shows the topography and percent vegetation cover in a synthetic catchment simulated using the CHILD model. The basin is close to a state of dynamic equilibrium between uplift and erosion. Vegetation cover ranges from 100% on the hillslopes to very low values in the main channels. (The presence of a small vegetation cover within the channels is due to the fact that vegetation always regrows during interstorm periods, regardless of position within the landscape). In general, the downstream transition from full to sparse vegetation cover is a function of two factors: (1) the threshold imposed by the vegetation itself, which retards runoff erosion on upper slopes (as seen in Figure 2), and (2) the rounding of hillslope profiles due to diffusion, which reduces slopes and thus shear stresses (as in the extreme case of Figure 3). In this particular example, the hillslope diffusivity constant k_d is sufficiently high that the latter effect is more

FIGURE 3. Solutions to equation (23) for equilibrium vegetation cover on a diffusion-dominated hillslope on which vegetation growth is limited by wash erosion.

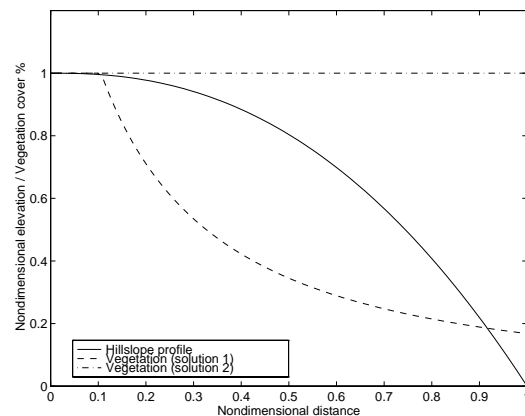
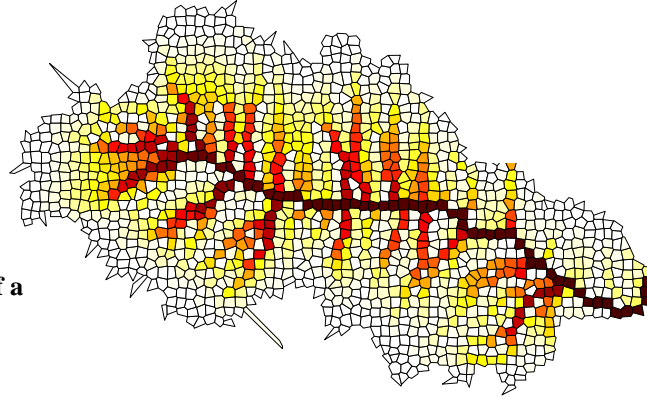


FIGURE 5. Short term-effect of a decrease in τ_{cv} on percent vegetation cover in the synthetic catchment shown in Figure 4.



important.

Figure 5 illustrates the effect of a hypothetical decrease in the maximum sustainable vegetation cover. The decrease in sustainable vegetation maximum is modeled by a tenfold decrease in the threshold parameter τ_{cv} . The decrease in τ_{cv} is accompanied by an increase in the effectiveness of runoff erosion, which leads to an upslope extension of the sparsely-vegetated tributaries.

Vegetation cover typically influences not only surface resistance to erosion, but also the soil infiltration capacity and hence runoff production. Figure 6 shows a hypothetical example in

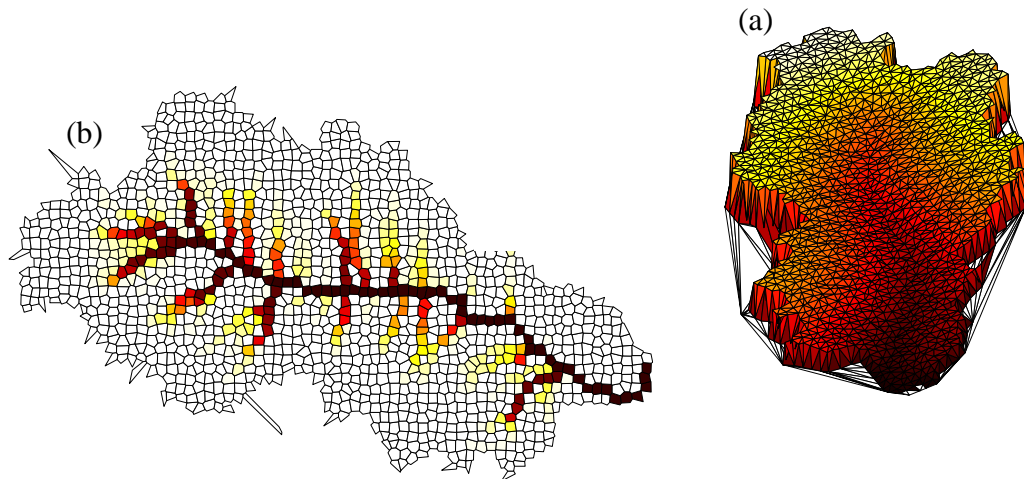


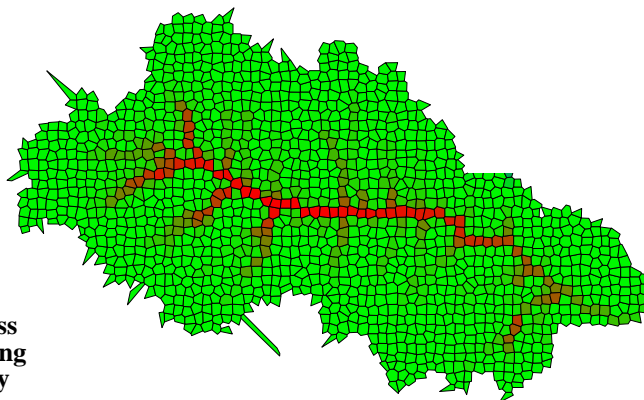
FIGURE 4. Simulated catchment with variable vegetation cover. (a) Topography. (b) Percent vegetation cover. White = 100% cover, black = 0% cover.

which the synthetic catchment shown in Figure 4 is subjected to a complete loss in vegetation accompanied by a reduction in infiltration capacity ($I_c = 0$). The figure depicts areas of erosion (red), deposition (not shown), and little or no change (green). The net effect of vegetation loss in this example is an increase in the rate of scour along the main channels. This result is somewhat contrary to intuition, which would suggest that a loss of vegetation should lead to accelerated erosion on hillslopes, where the initial vegetation cover (and thus the threshold) was greatest (this type of behavior was observed by Tucker and Slingerland (1997) in experiments with a constant-threshold model). The explanation appears to lie in the topography of the catchment and the nature of the hillslope-valley transition. With a relatively high diffusivity constant k_d , the valley network extent in the simulation is limited by hillslope diffusion rather than by the threshold imposed by vegetation cover. Thus, reducing the threshold does not lead to a rapid expansion of the valley network, as it did in the constant-threshold experiments of Tucker and Slingerland (1997). An implication of this result is that the nature of catchment responses may depend to a large extent on the nature of hillslope-valley transitions, and in particular on whether that transition is governed by a process transition or by a vegetation-imposed erosion threshold. There is still much to be learned about what controls the type of response in the model and, by extension, in natural catchments. We anticipate that this question can be fruitfully addressed through a program of systematic model sensitivity experiments and comparison of the model predictions with experimental and observational datasets.

References

- Arnold, J.G., Wertz, M.A., Alberts, E.E., and Flanagan, D.C., 1995, Chapter 8: Plant growth component, in USDA-Water Erosion Prediction Project, National Soil Erosion Research Laboratory Report No. 10, p. 8.1-8.41 (<http://soils.ecn.purdue.edu/~wephtml/wepp/wepptut/ahtml/doc.html>).

FIGURE 6. Patterns of erosion (red=max. erosion, green=no change) following a complete loss of vegetation and a corresponding reduction in infiltration capacity in the synthetic catchment shown in Figure 4.



- Dietrich, W.E., Wilson, C.J., Montgomery, D.R., and McKean, J., 1993, Analysis of erosion thresholds, channel networks, and landscape morphology using a digital terrain model: *Journal of Geology*, v. 101, p. 259-278.
- Douglas, I., 1967, Man, vegetation, and the sediment yield of rivers: *Nature*, v. 215, p. 925-928.
- Foster, G.R., 1982, Modeling the erosion process, in Haan, C.T., Johnson, H.P., and Brakensiek, D.L., eds., *Hydrologic modeling of small watersheds*: St. Joseph, Michigan, American Society of Agricultural Engineers monographs, p. 297-380
- Foster, G.R., Flanagan, D.C., Nearing, M.A., Lane, L.J., Risse, L.M., and Finkner, S.C., 1995, Chapter 11: Hillslope erosion component, in *USDA-Water Erosion Prediction Project*, National Soil Erosion Research Laboratory Report No. 10, p. 11.1-11.12 (<http://soils.ecn.purdue.edu/~wephtml/wep/wepptut/ahtml/doc.html>).
- Gilbert, G.K., 1909, The convexity of hilltops: *Journal of Geology*, v. 17, p. 344-350.
- Horton, R.E., 1945, Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology: *Geological Society of America Bulletin*, v. 56, p. 275-370.
- Howard, A.D., 1996, Thresholds and bistable states in landform evolution models: *EOS Transactions American Geophysical Union*, v. , p. S136 [abstract].
- Kirkby, M.J., 1994, Thresholds and instability in stream head hollows: a model of magnitude and frequency for wash processes, in Kirkby, M.J., ed., *Process Models and Theoretical Geomorphology*, New York, John Wiley and Sons, Inc, p. 295-314.
- Kirkby, M.J., 1995, Modelling the links between vegetation and landforms: *Geomorphology*, v. 13, p. 319-335.
- Kirkby, M.J., and Cox, N.J., 1995, A climatic index for soil erosion potential (CSEP) including seasonal and vegetation factors: *Catena*, v. 25, p. 333-352.
- Langbein, W.B., and Schumm, S.A., 1958, Yield of sediment in relation to mean annual precipitation: *Transactions of the American Geophysical Union*, v. 39, p. 1076-1084.
- Melton, M.A., 1958, Correlation structure of morphometric properties of drainage systems and their controlling agents: *Journal of Geology*, v. 66, p. 442-460.
- Moglen, G.E., Eltahir, E.A.B., and Bras, R.L., 1998, On the sensitivity of drainage density to climate change: *Water Resources Research*, 34, 855-862.

- Montgomery, D.R., and Dietrich, W.E., 1989, Source areas, drainage density, and channel initiation: *Water Resources Research*, v. 25, no. 8, p. 1907-1918.
- Thornes, J.B., 1990, the interaction of erosional and vegetational dynamics in land degradation: spatial outcomes, in Thornes, J.B., ed., *Vegetation and Erosion*, New York, John Wiley and Sons Ltd., p. 41-53.
- Tucker, G.E., and Slingerland, R.L., 1997, Drainage basin response to climate change: *Water Resources Research*, v. 33, no. 8, p. 2031-2047.
- Willgoose, G.R., Bras, R.L., and Rodriguez-Iturbe, I., 1990, A model of river basin evolution: *EOS Transactions American Geophysical Union*, v. 71, p. 1806-1807.
- Wilson, L., 1973, Variations in mean annual sediment yield as a function of mean annual precipitation: *American Journal of Science*, v. 273, p. 335-349.