Data Structures and Algorithms Spring 2023 — Problem Sets

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Week 5. Problem set

The National Health Council wants to improve health care in three of the most underdeveloped regions. Currently it has five medical teams available to allocate among such regions to improve their medical care, health education, and training programs. Therefore, the council needs to determine how many teams (if any) to allocate to each of these regions to maximize the total effectiveness of the five teams. The teams must be kept intact, so the number allocated to each region must be an integer.

The measure of performance being used is additional person-years of life. (For a particular region, this measure equals the increased life expectancy in years times the region's population.) The following table gives the estimated additional person-years of life (in multiples of 1,000) for each region for each possible allocation of medical teams.

1. Describe a general algorithm for any number M of available medical teams, any number R of regions and table `E` with estimates:
   (a) Summarize the idea for a naïve recursive algorithm.
   (b) Identify overlapping subproblems.
   (c) Write down pseudocode for the dynamic programming algorithm that solves the problem (top-down or bottom-up). It is enough to compute maximum performance without specifying team allocation.
2. Provide asymptotic worst-case time complexity with justification
   (a) for the naïve recursive algorithm
   (b) for the dynamic programming algorithm
3. Apply the dynamic programming algorithm to an instance of the problem below. You must provide the table with solutions for subproblems that are computed in the algorithm, as well as give the final answer to the problem.

Number of Teams	Region A	Region B	Region C
0	0	0	0
1	45	50	20
2	70	70	45
3	90	80	75
4	105	100	110
5	120	130	150

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition.* The MIT Press 2022