Introduction to R

Linear Regression in R

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Variables of the `mtcars` dataset

Help for the `mtcars` dataset:

```
?mtcars
```

- mpg - Miles/(US) gallon
- cyl - Number of cylinders
- disp - Displacement (cu.in.)
- hp - Gross horsepower
- drat - Rear axle ratio
- wt - Weight (1000 lbs)
- qsec - 1/4 mile time
- vs - Engine (0 = V-shaped, 1 = straight)
- am - Transmission (0 = automatic, 1 = manual)
- gear - Number of forward gears
- carb - Number of carburetors

Dataset `mtcars`

mpg	cyl	disp	hp	drat	wt	qsec	vs	am

Mazda RX4 21.0 6 160.0 110 3.90 2.620 16.46 0 1 4 4 Mazda RX4 Wag 21.0 6 160.0 110 3.90 2.875 17.02 0 1 4 4 Datsun 710 22.8 4 108.0 93 3.85 2.320 18.61 1 1 4 1 Hornet 4 Drive 21.4 6 258.0 110 3.08 3.215 19.44 1 0 3 1 Hornet Sportabout 18.7 8 360.0 175 3.15 3.440 17.02 0 0 3 2 Valiant 18.1 6 225.0 105 2.76 3.460 20.22 1 0 3 1 Duster 360 14.3 8 360.0 245 3.21 3.570 15.84 0 0 3 4 Merc 240D 24.4 4 146.7 62 3.69 3.190 20.00 1 0 4 2 Merc 230 22.8 4 140.8 95 3.92 3.150 22.90 1 0 4 2 Merc 280 19.2 6 167.6 123 3.92 3.440 18.30 1 0 4 4 Merc 280C 17.8 6 167.6 123 3.92 3.440 18.90 1 0 4 4 Merc 450SE 16.4 8 275.8 180 3.07 4.070 17.40 0 0 3 3 Merc 450SL 17.3 8 275.8 180 3.07 3.730 17.60 0 0 3 3 Merc 450SLC 15.2 8 275.8 180 3.07 3.780 18.00 0 0 3 3 Cadillac Fleetwood 10.4 8 472.0 205 2.93 5.250 17.98 0 0 3 4 Lincoln Continental 10.4 8 460.0 215 3.00 5.424 17.82 0 0 3 4 Chrysler Imperial 14.7 8 440.0 230 3.23 5.345 17.42 0 0 3 4 Fiat 128 32.4 4 78.7 66 4.08 2.200 19.47 1 1 4 1 Honda Civic 30.4 4 75.7 52 4.93 1.615 18.52 1 1 4 2 Toyota Corolla 33.9 4 71.1 65 4.22 1.835 19.90 1 1 4 1 Toyota Corona 21.5 4 120.1 97 3.70 2.465 20.01 1 0 3 1 Dodge Challenger 15.5 8 318.0 150 2.76 3.520 16.87 0 0 3 2 AMC Javelin 15.2 8 304.0 150 3.15 3.435 17.30 0 0 3 2 Camaro Z28 13.3 8 350.0 245 3.73 3.840 15.41 0 0 3 4 Pontiac Firebird 19.2 8 400.0 175 3.08 3.845 17.05 0 0 3 2 Fiat X1-9 27.3 4 79.0 66 4.08 1.935 18.90 1 1 4

Distributions of two variables of `mtcars`

```
par(mfrow=c(1,2))
plot(density(mtcars$wt)); plot(density(mtcars$mpg))
```

A simple regression model

Dependent variable - miles per gallon (mpg)

Independent variable - weight (wt)

```
m1 <- lm(mpg ~ wt,data=mtcars)
m1
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Coefficients:
## (Intercept)           wt
##      37.285       -5.344
```

Get the model summary

```
summary(m1)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.046 on 30 degrees of freedom
## Multiple R-squared:  0.7528,    Adjusted R-squared:  0.7446
## F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

The model formula

Model without intercept

```
m2 <- lm(mpg ~ - 1 + wt,data=mtcars)
summary(m2)$coefficients
```

```
##    Estimate Std. Error  t value     Pr(>|t|)
## wt 5.291624  0.5931801 8.920771 4.55314e-10
```

Adding further variables

```
m3 <- lm(mpg ~ wt + cyl,data=mtcars)
summary(m3)$coefficients
```

```
##              Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 39.686261  1.7149840 23.140893 3.043182e-20
## wt          -3.190972  0.7569065 -4.215808 2.220200e-04
## cyl         -1.507795  0.4146883 -3.635972 1.064282e-03
```

Estimation based on a subsample

```r
lm(mpg~wt+disp, data=mtcars, subset=(wt>3))
```

```
##
## Call:
## lm(formula = mpg ~ wt + disp, data = mtcars, subset = (wt > 3))
##
## Coefficients:
## (Intercept)           wt         disp
##    28.40497     -1.46360     -0.02016
```

- where only cars heavier than 3000 lb are considered.

The command `as.formula`

Creating a formula object

```
?as.formula
```

```
fo <- mpg ~ wt + cyl
```

```
class(fo)
```

```
## [1] "formula"
```

```
# The formula object can be used in the regression:
m3 <- lm(fo,data=mtcars)
```

Further possibilities to specify the formula

Take all available predictors

```
m3_a<-lm(mpg~.,data=mtcars)
```

Interaction effect

```
# effect of cyl and interaction effect:
m3a<-lm(mpg~wt*cyl,data=mtcars)

# only interaction effect:
m3b<-lm(mpg~wt:cyl,data=mtcars)
```

Take the logarithm

```
m3d<-lm(mpg~log(wt),data=mtcars)
```

Further transformations

Further transformations:

Tranformations of variables are directly included with the $I()$ function:

```
fo2 <- I(log(mpg))~wt+I(wt^2)+disp
lm(fo2, data=mtcars)
```

```
##
## Call:
## lm(formula = fo2, data = mtcars)
##
## Coefficients:
## (Intercept)           wt      I(wt^2)         disp
##   4.0000825   -0.3499056    0.0275548   -0.0009865
```

The command `setdiff`

- We can use the command to create a dataset with only the features, without the dependent variable

```
names(mtcars)
```

```
##  [1] "mpg"  "cyl"  "disp" "hp"   "drat" "wt"   "qsec" "vs"   "am"   "gear"
```

```
features <- setdiff(names(mtcars), "mpg")
features
```

```
##  [1] "cyl"  "disp" "hp"   "drat" "wt"   "qsec" "vs"   "am"   "gear" "carb"
```

```
featdat <- mtcars[,features]
```

The command `model.matrix`

- With `model.matrix` the qualitative variables are automatically dummy encoded

```
?model.matrix
```

```
model.matrix(m3d)
```

```
##                     (Intercept)   log(wt)
## Mazda RX4                     1 0.9631743
## Mazda RX4 Wag                 1 1.0560527
## Datsun 710                    1 0.8415672
## Hornet 4 Drive                1 1.1678274
## Hornet Sportabout             1 1.2354715
## Valiant                       1 1.2412686
## Duster 360                    1 1.2725656
## Merc 240D                     1 1.1600209
## Merc 230                      1 1.1474025
## Merc 280                      1 1.2354715
## Merc 280C                     1 1.2354715
## Merc 450SE                    1 1.4036430
## Merc 450SL                    1 1.3164082
```

Model matrix (II)

- We can also create a model matrix directly from the formula and data arguments
- See `Matrix::sparse.model.matrix` for increased efficiency on large dimension data.

```
ff <- mpg ~ log(wt):cyl
m <- model.frame(ff, mtcars)
```

```
(mat <- model.matrix(ff, m))
```

```
##                     (Intercept) log(wt):cyl
## Mazda RX4                     1    5.779046
## Mazda RX4 Wag                 1    6.336316
## Datsun 710                    1    3.366269
## Hornet 4 Drive                1    7.006964
## Hornet Sportabout             1    9.883772
## Valiant                       1    7.447612
## Duster 360                    1   10.180525
## Merc 240D                     1    4.640084
## Merc 230                      1    4.589610
## Merc 280                      1    7.412829
## Merc 280C                     1    7.412829
```

A model with interaction effect

```
# disp    -  Displacement (cu.in.)
m3d<-lm(mpg~wt*disp,data=mtcars)
m3dsum <- summary(m3d)
m3dsum$coefficients
```

```
##                Estimate  Std. Error   t value     Pr(>|t|)
## (Intercept) 44.08199770 3.123062627 14.114990 2.955567e-14
## wt          -6.49567966 1.313382622 -4.945763 3.216705e-05
## disp        -0.05635816 0.013238696 -4.257078 2.101721e-04
## wt:disp      0.01170542 0.003255102  3.596022 1.226988e-03
```

Residual plot - model assumptions violated?

- We have model assumptions violated if points deviate with a pattern from the line

```
plot(m3,1)
```

Residual plot

```
plot(m3,2)
```

Another example for object orientation

- `m3` is now a special regression object
- Various functions can be applied to this object

```
predict(m3) # Prediction
resid(m3) # Residuals
```

```
##          Mazda RX4     Mazda RX4 Wag        Datsun 710    Hornet 4 Drive
##           22.27914          21.46545          26.25203          20.38052
## Hornet Sportabout           Valiant
##           16.64696          19.59873

##          Mazda RX4     Mazda RX4 Wag        Datsun 710    Hornet 4 Drive
##         -1.2791447        -0.4654468        -3.4520262         1.0194838
## Hornet Sportabout           Valiant
##          2.0530424        -1.4987281
```

Make model prediction

```
pre <- predict(m1)
head(mtcars$mpg)
```

```
## [1] 21.0 21.0 22.8 21.4 18.7 18.1
```

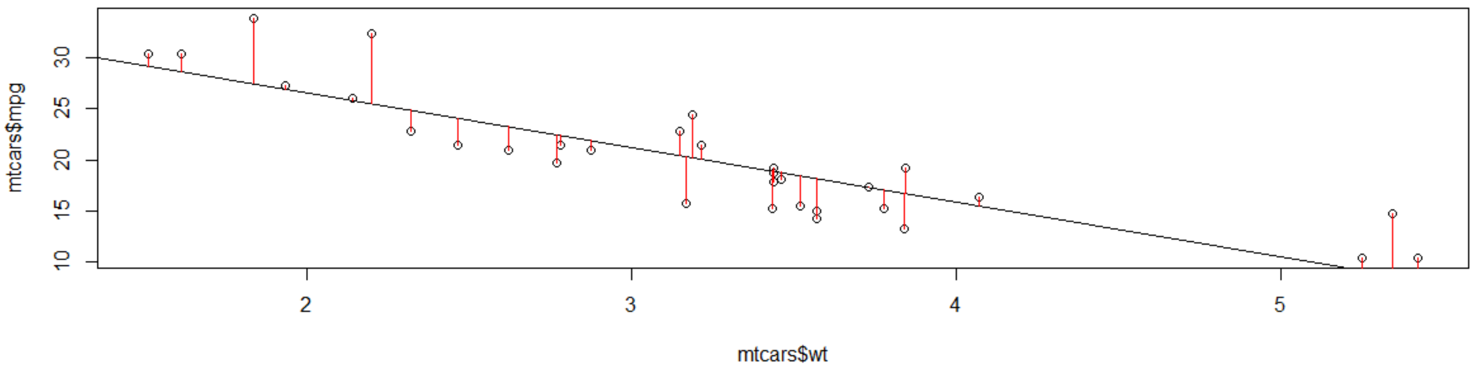
```
head(pre)
```

```
##         Mazda RX4     Mazda RX4 Wag        Datsun 710    Hornet 4 Drive
##          23.28261          21.91977          24.88595          20.10265
## Hornet Sportabout           Valiant
##          18.90014          18.79325
```

Regression diagnostic with base-R

Visualizing residuals

```
plot(mtcars$wt,mtcars$mpg)
abline(m1)
segments(mtcars$wt, mtcars$mpg, mtcars$wt, pre, col="red")
```



The bias-variance tradeoff (I)

The bias–variance tradeoff is the property of a set of predictive models whereby models with a lower bias in parameter estimation have a higher variance of the parameter estimates across samples, and vice versa.

The bias error

… is an error from erroneous assumptions in the learning algorithm. High bias can cause an algorithm to miss the relevant relations between features and target outputs (underfitting).

The variance

… is an error from sensitivity to small fluctuations in the training set. High variance can cause an algorithm to model the random noise in the training data, rather than the intended outputs (overfitting).

The bias-variance tradeoff (II)

The mean squared error (mse)

- The **MSE** measures the average of the squares of the errors
- **The lower the better**

```
(mse5 <- mean((mtcars$mpg -  pre)^2)) # model 5
```

```
## [1] 8.697561
```

```
(mse3 <- mean((mtcars$mpg -  predict(m3))^2))
```

```
## [1] 5.974124
```

Package `Metrics` to compute mse

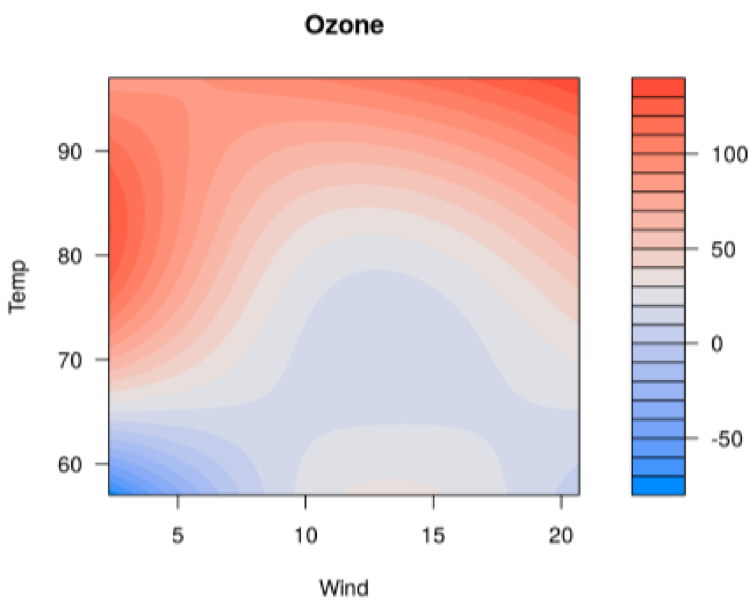
```
library(Metrics)
mse(mtcars$mpg,predict(m3))
```

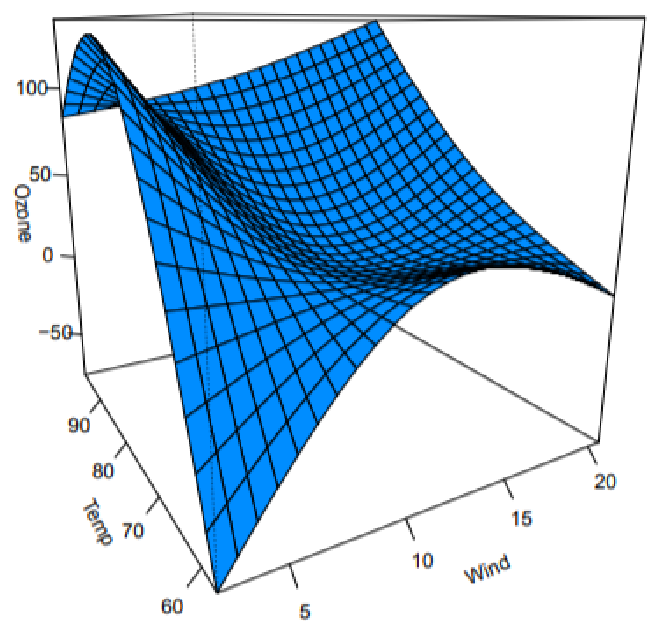
```
## [1] 5.974124
```

The `visreg`-package

```
install.packages("visreg")
install.packages("Metrics")
```

```
library(visreg)
```

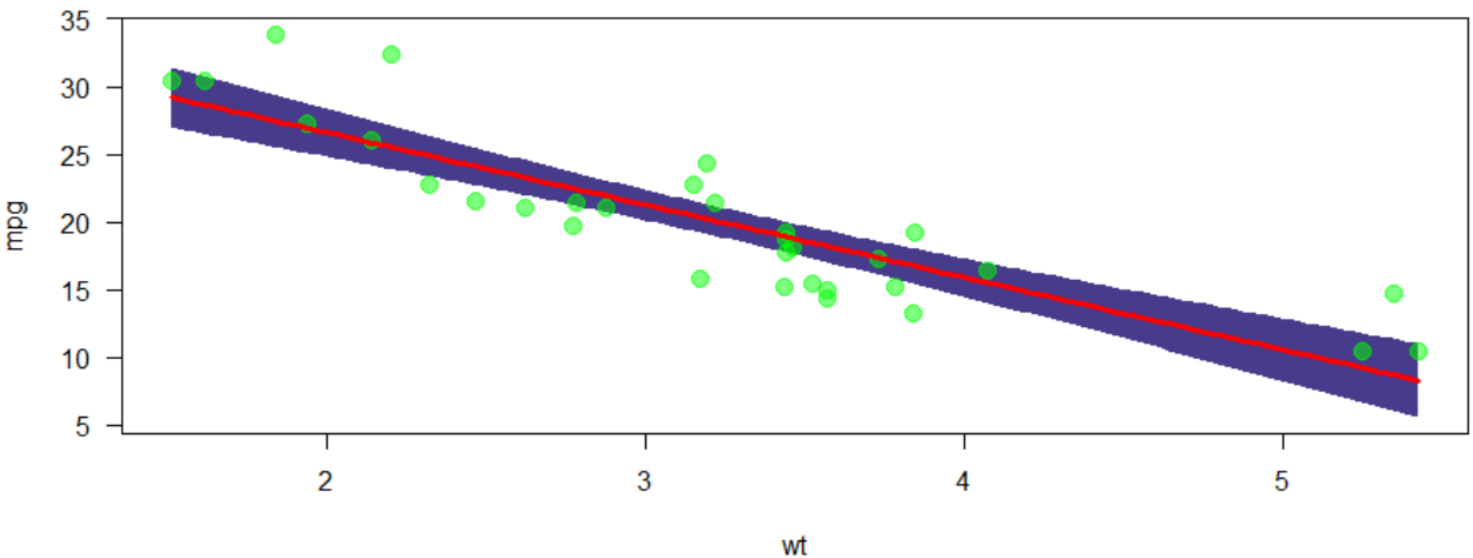




The `visreg`-package

- The default-argument for `type` is `conditional`.
- Scatterplot of `mpg` and `wt` plus regression line and confidence bands

```
visreg(m1, "wt", type = "conditional")
```



Regression with factors

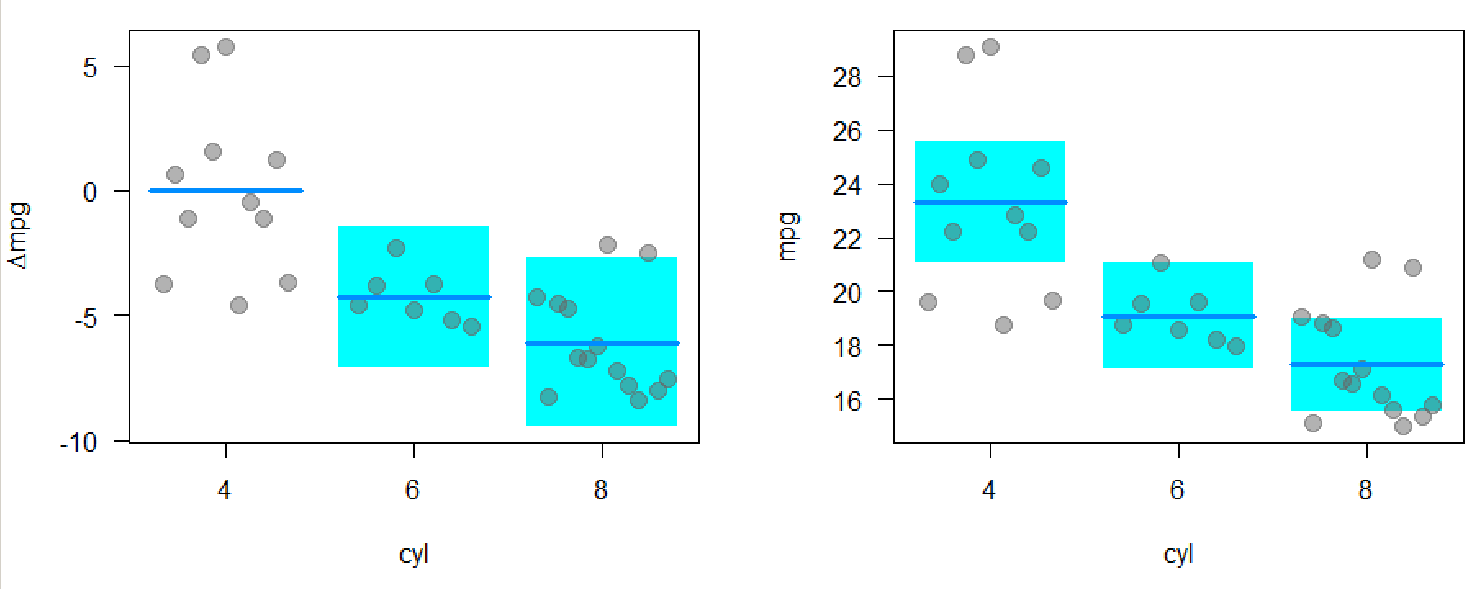
- The effects of factors can also be visualized with `visreg`:

```
mtcars$cyl <- as.factor(mtcars$cyl)
m4 <- lm(mpg ~ cyl + wt, data = mtcars)
# summary(m4)
```

```
##               Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 33.990794  1.8877934 18.005569 6.257246e-17
## cyl6        -4.255582  1.3860728 -3.070244 4.717834e-03
## cyl8        -6.070860  1.6522878 -3.674214 9.991893e-04
## wt          -3.205613  0.7538957 -4.252065 2.130435e-04
```

Effects of factors

```
par(mfrow=c(1,2))
visreg(m4, "cyl", type = "contrast")
visreg(m4, "cyl", type = "conditional")
```



The package `visreg` - Interactions

```
m5 <- lm(mpg ~ cyl*wt, data = mtcars)
# summary(m5)
```

```
##              Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)  39.571196   3.193940 12.3894599 2.058359e-12
## cyl6        -11.162351   9.355346 -1.1931522 2.435843e-01
## cyl8        -15.703167   4.839464 -3.2448150 3.223216e-03
## wt           -5.647025   1.359498 -4.1537586 3.127578e-04
## cyl6:wt       2.866919   3.117330  0.9196716 3.661987e-01
## cyl8:wt       3.454587   1.627261  2.1229458 4.344037e-02
```

Control of the graphic output with `layout`.

```
visreg(m5, "wt", by = "cyl",layout=c(3,1))
```

The package `visreg` - Interactions overlay

```
m6 <- lm(mpg ~ hp + wt * cyl, data = mtcars)
```

```
visreg(m6, "wt", by="cyl", overlay=TRUE, partial=FALSE)
```

The package `visreg` - `visreg2d`

```
visreg2d(m6, "wt", "hp", plot.type = "image")
```

Exercise: regression Ames housing data

1) Install the package `AmesHousing` and create a **processed version** of the Ames housing data with (at least) the variables `Sale_Price`, `Gr_Liv_Area` and `TotRms_AbvGrd` 2) Create a regression model with `Sale_Price` as dependent and `Gr_Liv_Area` and `TotRms_AbvGrd` as independent variables. Then create seperated models for the two independent variables. Compare the results. What do you think?

The Ames Iowa Housing Data

```
ames_data <- AmesHousing::make_ames()
```

Some Variables

- `Gr_Liv_Area`: Above grade (ground) living area square feet
- `TotRms_AbvGrd`: Total rooms above grade (does not include bathrooms
- `MS_SubClass`: Identifies the type of dwelling involved in the sale.
- `MS_Zoning`: Identifies the general zoning classification of the sale.
- `Lot_Frontage`: Linear feet of street connected to property
- `Lot_Area`: Lot size in square feet
- `Street`: Type of road access to property
- `Alley`: Type of alley access to property
- `Lot_Shape`: General shape of property
- `Land_Contour`: Flatness of the propert

Multicollinearity

- As p increases we are more likely to capture multiple features that have some multicollinearity.
- When multicollinearity exists, we often see high variability in our coefficient terms.
- E.g. we have a correlation of 0.801 between `Gr_Liv_Area` and `TotRms_AbvGrd`
- Both variables are strongly correlated to the response variable (`Sale_Price`).

```
ames_data <- AmesHousing::make_ames()
cor(ames_data[,c("Sale_Price","Gr_Liv_Area","TotRms_AbvGrd")])
```

```
##               Sale_Price Gr_Liv_Area TotRms_AbvGrd
## Sale_Price     1.0000000   0.7067799     0.4954744
## Gr_Liv_Area    0.7067799   1.0000000     0.8077721
## TotRms_AbvGrd  0.4954744   0.8077721     1.0000000
```

Multicollinearity

```
lm(Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data = ames_data)
```

```
##
## Call:
## lm(formula = Sale_Price ~ Gr_Liv_Area + TotRms_AbvGrd, data = ames_data)
##
## Coefficients:
##   (Intercept)    Gr_Liv_Area  TotRms_AbvGrd
##       42767.6          139.4       -11025.9
```

- When we fit a model with both these variables we get a positive coefficient for `Gr_Liv_Area` but a negative coefficient for `TotRms_AbvGrd`, suggesting one has a positive impact to Sale_Price and the other a negative impact.

Seperated models

- If we refit the model with each variable independently, they both show a positive impact.
- The `Gr_Liv_Area` effect is now smaller and the `TotRms_AbvGrd` is positive with a much larger magnitude.

```
lm(Sale_Price ~ Gr_Liv_Area, data = ames_data)$coefficients
```

```
## (Intercept) Gr_Liv_Area
##   13289.634     111.694
```

```
lm(Sale_Price ~ TotRms_AbvGrd, data = ames_data)$coefficients
```

```
##   (Intercept) TotRms_AbvGrd
##      18665.40      25163.83
```

- This is a common result when collinearity exists.
- Coefficients for correlated features become over-inflated and can fluctuate significantly.

Consequences

- One consequence of these large fluctuations in the coefficient terms is **overfitting**, which means we have high variance in the bias-variance tradeoff space.
- We can use tools such as **variance inflaction factors** (Myers, 1994) to identify and remove those strongly correlated variables, but it is not always clear which variable(s) to remove.
- Nor do we always wish to remove variables as this may be removing signal in our data.

Links - linear regression

- Regression - **r-bloggers**
- The complete book of **Faraway**- very intuitive
- Good introduction on **Quick-R**
- **Multiple regression**
- **15 Types of Regression you should know**
- **`ggeffects` - Create Tidy Data Frames of Marginal Effects for 'ggplot' from Model Outputs**
- **Machine learning iteration**

Nice table output with `stargazer`

```r
library(stargazer)
stargazer(m3, type="html")
```

Example HTML output:

	Dependent variable:
	mpg
wt	-3.125***
	(0.911)
cyl	-1.510***
	(0.422)

Shiny App - Diagnostics for linear regression

- Shiny App - **Simple Linear Regression**
- Shiny App - **Multicollinearity in multiple regression**