BFE Note

1. Introduction

The CCD pixel are sensitive to their environment: the variance of flatfields does not rise linearly with illumination, but the rise flattens out, departing from exact Poisson distribution. The paper shows that a variety of CCD sensors deliver stellar images that broaden with increasing flux to some level. This “brighter-fatter” effect complicates the direct use of stars as PSF models.

This paper shows the correlations between neighbour pixels and the brighter-fatter effect can be explained by alterations to the drift field caused by charges already collected in the potential wells of the CCD. The size of the induced distortions and how they decay with distance from the source both depend on manufacturing details of the CCD.

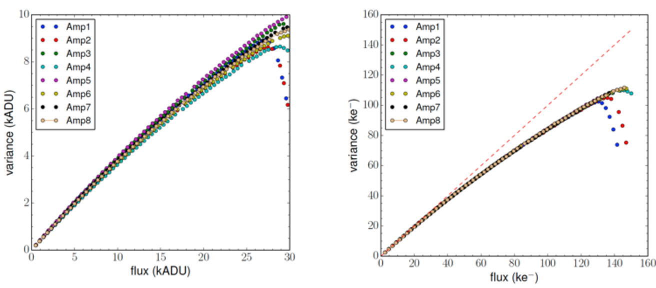
The paper uses three instruments to establish the hypothesis: MegaCam, DECam and the LSST CCD E2V-250. For each instrument, the data set is constituted by both point source illumination exposures and by uniform illuminations.

2. Flat Field

The broadening of spot with increasing flux presented in the previous section can be depicted as a reduction of image contrast. In this section, a similar contrast reduction also appears in flatfield images, manifesting itself as a non-linearity of the PTC. Considering only Poisson noise, the relation between two observables is expected to be linear. However, we see that a significative departure from linearity is actually observed and that it is associated with linearly increasing pixel correlations.

- **PTC non-linearity**

The non-linearity of the photon transfer curve is illustrated in the figure below using eight different segments of the CCD E2V-250. The variance measured at high-flux is significantly lower than expected from extrapolating the variance of low-flux flatfields according to Poisson law: the discrepancy is as high as 20% in the case of the CCD E2V-250.



(a) PTC in ADU (b) PTC in e^-

Fig. 4. (a) Measured photon transfer curve of amplifiers #1 to #8 of the CCD E2V-250 expressed in ADU. (b) PTCs are converted in e^- using the readout gain measured as the inverse of the slope at the origin (see 4.3). Because departure from linearity of the PTCs is similar for different amplifiers this indicates that the cause of the effect is to be found in the CCD itself rather than in the electronic readout.

- **Pixel Spatial Correlations**

Masking: The hot, dark columns and CCD defects are masked when being detected. Correlations that cannot be attributed to any specific feature in the image can be otherwise detected from a non-zero intercept when fitting a given correlation coefficient vs. flux.

Differencing: The correlation coefficients are computed from the difference of two flatfield images that have received the same overall illumination. This is done to remove apparent correlations due to non-uniformity of the flatfield image typically due to pixel size variations, QE variations or spatial variations of the illumination.

Spatial Correlation: The spatial correlation between pixels are evaluated using covariances normalized by variance $R_{k,l}$ refers to the correlation coefficient between pixels separated by k columns and l rows. The statistical precision on any correlation coefficient is $1/\sqrt{N}$ where N is the number of pixels. The statistics is doubled for the off-axis correlations $(k, l \neq 0)$ by combining the measurements of two quadrants ($R_{k,l}$ and $R_{k,-l}$ for instance). The statistical precision is further increased by using many pairs of flatfield, the PTC from the CCD E2V-250 contains ≈ 100 points, which allows us to improve the improve precision on correlation measurement down to 1×10^{-4}.

High Flux Level: Correlation features appear when the pixel contents are approaching full well. These are seen as blooming effects that increase up to saturation. The correlation shows a quite linear behaviour up to full well. In the next section, we focus our analysis of pixel spatial correlations on the dynamic range below this flux level.

- **Spatial Correlations vs. BSS**

Pixel spatial correlations are detected on three instruments up to a distance of 4 pixels. For the LSST and DECam, the correlation $R_{0,1}$ is about three times larger than $R_{1,0}$. This anisotropy vanishes at larger distances. At a separation of 4 pixels, all correlations of all CCDs are as low as few 10^{-4} which approaches the limit of sensitivity of the instruments.

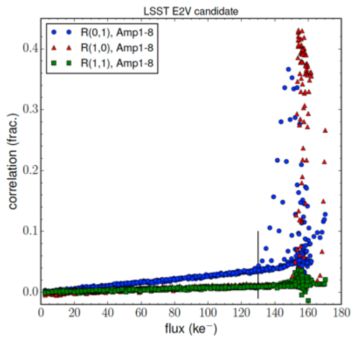


Fig. 6. Superposition of the evolution with flux for nearest pixel correlations (coefficients $R_{0,1}$ $R_{1,1}$ $R_{1,0}$) for read out channels 1 to 8 of CCD E2V-250. The whole dynamical range shows a threshold where correlations strongly increase: First $R_{0,1}$, a correlation in Y direction, and near full well, $R_{1,0}$, a correlation in X direction. The diagonal correlation $R_{1,1}$ does not exhibit any threashold. This is expected since neither transfer nor read out could contribute to correlate pixels in this direction. The vertical dark line indicates the early threshold of $R_{0,1}$.

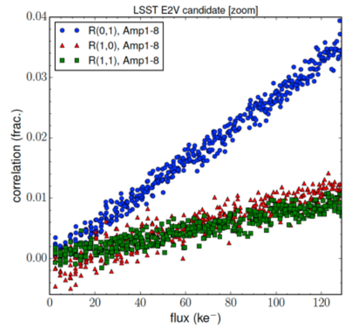


Fig. 9. Superposition of evolution with respect to flux of coefficients $R_{0,1}$ $R_{1,1}$, and $R_{1,0}$ on the dynamical range below the threshold, as indicated by the vertical black line of figure 6. On this interval, all the linearly increasing correlations that are discussed in this section shows a monotonic behavior. On this CCD E2V-250, most channels exhibit a small ($\approx$ - 0.003), but significant anti-correlation pedestal in $R_{1,0}$ (Y-intercept), an offset that is not seen with the others coefficients nor with the other sensors. We attribute it to the electronic chain used to collect this data, and we subtract it to the actual measurements.

The pixel correlation maps are found to scale with fluxes for all detectors. There is no evidence for chromaticity dependence for this trend.

- **Spatial Correlations vs. Flux**

It has been shown in previous work that increasing the parallel clocking voltage (CV) decreases the level of the correlation $R_{0,1}$ while keeping all other coefficient unchanged. In this paper, we complete the study of the impact of varying pixel's backside voltage. The correlation $R_{0,1}$ increases as the BSS is decreased down to $10-20\mathrm{V}$, below this level, the correlation starts decreasing. On the same interval, the correlation coefficient with next pixel in the serial direction $R_{1,0}$ decreases and shifts to negative values below $10-20\ \mathrm{V}$. The other correlation coefficient monotonously increases as BSS decreases. The diffusion mechanism cannot explain the evolution with BSS and CV, the prediction from simple electrostatic simulation are compatible with the observations.

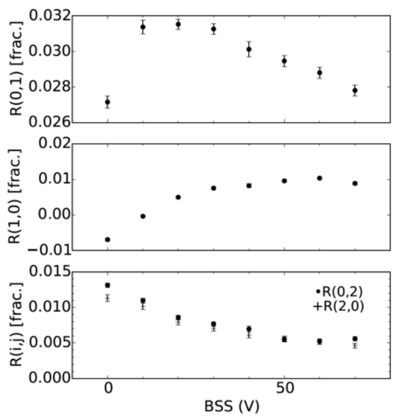


Fig. 8. Variation of correlation coefficients (measured at 100 ke⁻) with respect to BSS measured on the CCD E2V-250. Top panel: $R_{0,1}$. Middle panel: $R_{1,0}$. Bottom panel: $R_{0,2}$ and $R_{2,0}$. The other long range correlations behave like $R_{0,2}$ and $R_{2,0}$: they decrease as the BSS voltage increases.

- **Total Flux**

The connection between non-linearity of the PTC and the linearly increasing correlations can be illustrated by summing all the correlations and adding them to the PTC. It can be verified that the process that correlates the pixels also conserves charges. This is also straightforward to see from a linear fit of flatfield mean flux versus exposure time.

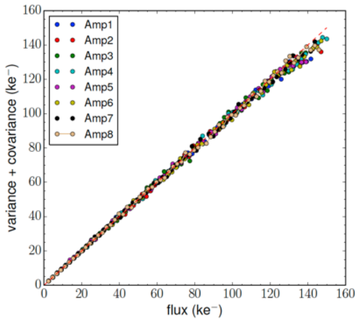


Fig. 10. Comparison for the CCD E2V-250 between the expected Poisson noise (red dashed line) and raw PTCs that are corrected by summing covariances up to 4 pixels distance. For a 100 ke⁻ flux level, these correlations add up to 18% of the variance. The corrected PTCs slopes coincide with Poisson law at ≈ 0.5%, indicating that more than 97% of the correlations are considered by the truncation at a 4 pixels distance of the integral of the correlation function.

- **PTC Non-Linearity vs. Linear Correlation**

The gain function can be modified in the following way, with α being an empirical parameter that is introduced to describe the quadratic behaviour of the PTC, which is expected given a linear rise of the correlations

$$V(N) = -\alpha N^2 + \frac{1}{G} N$$

3. Model of Columbian Force

In this section, we derive a parametrized model of the effects of electric field distortion in a CCD that are induced by charges residing within the CCD during exposure. We model the displacement of effective boundaries of pixel cases by a charge $q_{i,j}$ in a bucket at position (i,j) as $\delta_{i,j}^X/p = a_{i,j}^X Q_{i,j}/2$. In this equation, we have expressed the perturbing electric field due to charge proportional to the charge $Q_{i,j}$. X indexes the four boundaries of the pixel $(0,0)$ and we label each boundary by the coordinates of the pixel that shares it with $(0,0)$. $X \in [(0,0),(1,0),(0,-1),(-1,0)]$.

- **Perturbed Charge**

The difference between charge contents with and without the perturbing electric field is called the *charge transfer*. The *pixel boundary displacement* δ^X induces a charge transfer between pixel $(0,0)$ and pixel X. This charge transfer is proportional to both the pixel boundary displacement and to the *charge density* flowing on this boundary. The charge density drifting on the boundary between $(0,0)$ and pixel X is given as

$$\rho_{00}^{X} = (Q_{00} + Q_X)/2$$

Therefore, the *net charge transfers* due to perturbing electric fields between pixel $(0,0)$ and its neighbour X is given by

$$\delta Q_{00}^{X} = \frac{\delta_X}{p}(Q_{00} + Q_X)/2 \qquad\qquad \delta Q_{00} = \sum_X \delta Q_{00}^{X} \quad \text{and} \quad Q'_{00} = Q_{00} + \delta Q_{00}$$

- **Correlation**

Using this parametrization, the variance between pixels in a uniform exposure of average μ and variance V reads

$$\mathrm{Cov}\left(Q'_{ij}, Q'_{00}\right) = V\mu \sum_X a_{ij}^{X} \qquad\qquad \mathrm{Cov}(Q'_{00}, Q'_{00}) = V + V\mu \sum_X a_{00}^{X}$$

The electrostatic influence from collected charge, induces covariances between pixels in uniform exposures that scale with the average and the variance of pixel contents. If one measures correlation coefficients (ratio of covariance to variance), those are expected to scale with the illumination level of the uniform exposure.