# PHIL12A

# Section questions, 25 April 2011

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# 1 Formal proofs

Decide whether the following arguments are valid or not. If they are, give formal proofs for them; otherwise give counterexamples.

1. (Ex 13.2)

1 
$$\forall x (Cube(x) \leftrightarrow Small(x))$$
  
2  $\forall x Cube(x)$   
3  $\forall x Small(x)$ 

```
1
         \forall x (Cube(x) \leftrightarrow Small(x))
         \forall x Cube(x)
3
            a
                                                     ∀Elim: 2
4
            Cube (a)
5
                                                     ∀Elim: 1
            Cube(a) \leftrightarrow Small(a)
6
            Small(a)
                                                     \leftrightarrowElim: 5, 4
         \forall x Small(x)
                                                     ∀Intro: 3-6
```

### 2. **(Ex 13.3)**

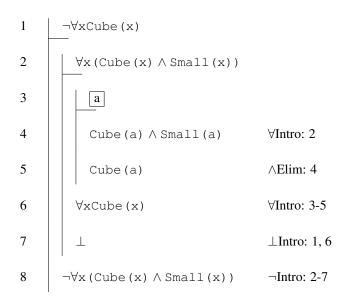
1  $\forall x \text{Cube}(x)$ 2  $\forall x \text{Small}(x)$ 3  $\forall x \text{(Cube}(x) \land \text{Small}(x))$ 

Proof:



### 3. (Ex 13.4)

1 
$$\neg \forall x \text{Cube}(x)$$
  
2  $\neg \forall x \text{(Cube}(x) \land \text{Small}(x))$ 



4.

$$\begin{array}{c|c}
1 & \forall x \text{Cube}(x) \\
\hline
2 & \exists x \text{Cube}(x)
\end{array}$$

Proof:

1 
$$\forall$$
xCube(x)  
2 Cube(e)  $\forall$ Elim: 1  
3  $\exists$ xCube(x)  $\exists$ Intro: 2

5.

1 
$$\exists x \text{Cube}(x)$$
  
2  $\neg \forall x \neg \text{Cube}(x)$ 

### 6. (Ex 13.12)

Proof:

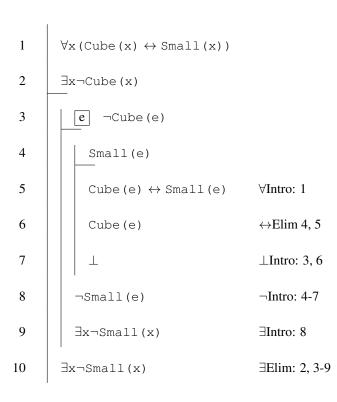
1	$\forall x (Cube(x) \lor Tet(x))$	
2	∃x¬Cube(x)	
3	e ¬Cube(e)	∃Elim: 2
4	Cube(e) V Tet(e)	∀Elim: 1
5	Cube (e)	
6		⊥Intro: 3, 5
7	Tet(e)	⊥Elim: 6
8	Tet(e)	
9	Tet(e)	Reit: 8
10	Tet(e)	∨Elim: 4, 5-7, 8-9
11	∃xTet(x)	∃Intro: 10
12	∃xTet(x)	∃Elim: 2, 3-11

Or:

```
1
        \forall x (Cube(x) \lor Tet(x))
 2
        \exists x \neg Cube(x)
          e ¬Cube(e)
                                        ∃Elim: 2
 3
                                        \forall Elim: 1
           Cube(e) V Tet(e)
 4
 5
             Cube(e)
 6
             \perp
                                        \perpIntro: 3, 5
7
             \exists x Tet(x)
                                        ⊥Elim: 6
 8
             Tet(e)
9
                                        ∃Intro: 8
             \exists x Tet(x)
10
           \exists x Tet(x)
                                        ∨Elim: 4, 5-7, 8-9
11
        ∃xTet(x)
                                        ∃Elim: 2, 3-10
```

### 7. **(Ex 13.14)**

1 
$$\forall x (Cube(x) \leftrightarrow Small(x))$$
  
2  $\exists x \neg Cube(x)$   
3  $\exists x \neg Small(x)$ 



### 2 Informal proofs

Are the following informal proofs correct or not?

#### 1. (Ex 12.11)

There is a number greater than every other number.

**Purported proof:** Let n be an arbitrary number. Then n is less than some other number, n+1 for example. Let m be any such number. Thus  $n \leq m$ . But n is an arbitrary number, so every number is less or equal m. Hence there is a number that is greater than every other number.

No. We pick n arbitrary, find a number bigger than n and existentially generalize, concluding that there is some m such that m is greater than n. Now we cannot use universal generalization to say that everything is smaller than m, since the existence of m depended on n.

#### 2. (Ex 12.15)

```
1 \forall x \forall y \forall z [ (Outgrabe(x,y) \land Outgrabe(y,z)) \rightarrow Outgrabe(x,z) ]
2 \forall x \forall y [Outgrabe(x,y) \rightarrow Outgrabe(y,x) ]
3 \exists x \exists y Outgrabe(x,y)
4 \forall x Outgrabe(x,x)
```

**Purported proof:** Applying existential instantiation to the third premise, let b and c be arbitrary objects in the domain of discourse such that b outgrabes c. By the second premise, we also know that c outgrabes b. Applying the first premise (with x = z = b and y = c) we see that b outgrabes itself. But b was arbitrary. Thus by universal generalization,  $\forall x \text{Outgrabe}(x, x)$ .

No. We introduced b by existential instantiation on premise 3. So b is not completely arbitrary, and we cannot universally generalize on a sentence containing it.