

## Chapter 13: Hints and Selected Solutions

### Section 13.1 (page 346)

**13.2** Hint (fill in the supports):

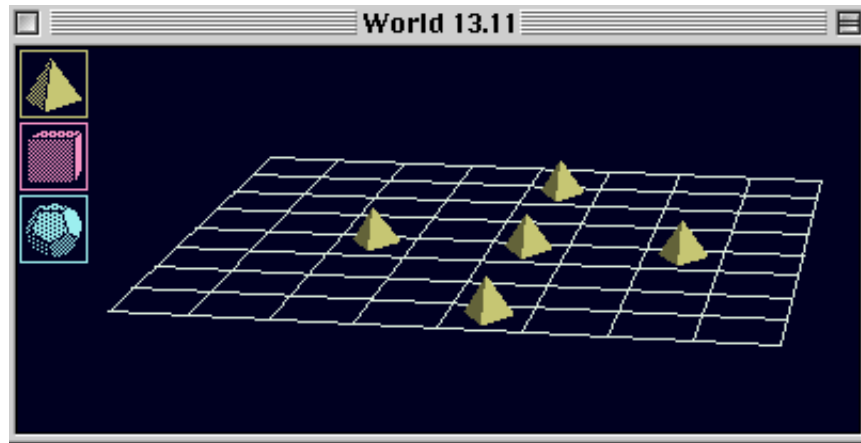
- |    |                                                |                                            |
|----|------------------------------------------------|--------------------------------------------|
| 1. | $\forall x (Cube(x) \leftrightarrow Small(x))$ |                                            |
| 2. | $\forall x Cube(x)$                            |                                            |
| 3. | <b>c</b> ▼                                     |                                            |
| 4. | $Cube(c)$                                      | ▼ <b><math>\forall</math>Elim:</b>         |
| 5. | $Cube(c) \leftrightarrow Small(c)$             | ▼ <b><math>\forall</math>Elim:</b>         |
| 6. | $Small(c)$                                     | ▼ <b><math>\leftrightarrow</math>Elim:</b> |
| 7. | $\forall x Small(x)$                           | ▼ <b><math>\forall</math>Intro:</b>        |

**13.7** Hint:

- |    |                                                                             |                                     |
|----|-----------------------------------------------------------------------------|-------------------------------------|
| 1. | $\forall x \forall y ((Cube(x) \wedge Dodec(y)) \rightarrow FrontOf(x, y))$ |                                     |
| 2. | <b>d</b> ▼ $Cube(d)$                                                        |                                     |
| 3. | <b>e</b> ▼ $Dodec(e)$                                                       |                                     |
| 4. |                                                                             | ▼ <b>Rule?:</b>                     |
| 5. | $FrontOf(d, e)$                                                             | ▼ <b>Rule?:</b>                     |
| 6. |                                                                             | ▼ <b><math>\forall</math>Intro:</b> |
| 7. |                                                                             | ▼ <b>Rule?:</b> 2-6                 |

## Section 13.2 (page 350)

13.11 A counterexample:



13.14 Hint:

- ▣  $\forall x (Cube(x) \leftrightarrow Small(x))$
- ▣  $\exists x \neg Cube(x)$
- ▣  $\neg Cube(b)$
- ▣  $Cube(b) \leftrightarrow Small(b)$     ▾  $\forall Elim$
- ▣  $\neg Small(b)$     ▾ **Taut Con**
- ▣    ▾  $\exists Intro$
- ▣    ▾  $\exists Elim$

### Section 13.3 (page 337)

**13.20** The following proof formalizes the informal proof we gave earlier for Exercise 12.1. (C.f. **Hints and Selected Solutions** for Chapter 12.)

1.	$\forall x ((\text{Brillig}(x) \vee \text{Tove}(x)) \rightarrow (\text{Mimsy}(x) \wedge \text{Gyre}(x)))$	
2.	$\forall y ((\text{Slithy}(y) \vee \text{Mimsy}(y)) \rightarrow \text{Tove}(y))$	
3.	$\exists x \text{Slithy}(x)$	
4.	$\boxed{b} \nabla \text{Slithy}(b)$	
5.	$(\text{Slithy}(b) \vee \text{Mimsy}(b)) \rightarrow \text{Tove}(b)$	✓ ▽ <b>VElim:</b> 2
6.	$\text{Slithy}(b) \vee \text{Mimsy}(b)$	✓ ▽ <b>vIntro:</b> 4
7.	$\text{Tove}(b)$	✓ ▽ <b>→Elim:</b> 6,5
8.	$(\text{Brillig}(b) \vee \text{Tove}(b)) \rightarrow (\text{Mimsy}(b) \wedge \text{Gyre}(b))$	✓ ▽ <b>VElim:</b> 1
9.	$\text{Brillig}(b) \vee \text{Tove}(b)$	✓ ▽ <b>vIntro:</b> 7
10.	$\text{Mimsy}(b) \wedge \text{Gyre}(b)$	✓ ▽ <b>→Elim:</b> 9,8
11.	$\text{Mimsy}(b)$	✓ ▽ <b>∧Elim:</b> 10
12.	$\text{Slithy}(b) \wedge \text{Mimsy}(b)$	✓ ▽ <b>∧Intro:</b> 4,11
13.	$\exists x (\text{Slithy}(x) \wedge \text{Mimsy}(x))$	✓ ▽ <b>∃Intro:</b> 12
14.	$\exists x (\text{Slithy}(x) \wedge \text{Mimsy}(x))$	✓ ▽ <b>∃Elim:</b> 3,4-13

**13.23** The following formalizes the informal proof we gave for Exercise 12.4.

1.	$\forall y (\text{Cube}(y) \vee \text{Dodec}(y))$	
2.	$\forall x (\text{Cube}(x) \rightarrow \text{Large}(x))$	
3.	$\exists x \neg \text{Large}(x)$	
4.	$\boxed{b} \nabla \neg \text{Large}(b)$	
5.	$\text{Cube}(b) \rightarrow \text{Large}(b)$	✓ ▽ <b>VElim:</b> 2
6.	$\neg \text{Cube}(b)$	✓ ▽ <b>Taut Con:</b> 4,5
7.	$\text{Cube}(b) \vee \text{Dodec}(b)$	✓ ▽ <b>VElim:</b> 1
8.	$\text{Dodec}(b)$	✓ ▽ <b>Taut Con:</b> 7,6
9.	$\exists x \text{Dodec}(x)$	✓ ▽ <b>∃Intro:</b> 8
10.	$\exists x \text{Dodec}(x)$	✓ ▽ <b>∃Elim:</b> 4-9,3

**13.29** Remember that FOL assumes that claims are made about non-empty domains. You will need to use a name in giving this proof, even though no names appear in the original argument. Here is a nearly completed proof. You should fill in the details that are missing.

1.	$\forall x (Small(x) \rightarrow Cube(x))$	
2.	$\exists x \neg Cube(x) \rightarrow \exists x Small(x)$	
3.	$Cube(c) \vee \neg Cube(c)$	✓ ▼ Taut Con:
4.	▼ $Cube(c)$	
5.	$\exists x Cube(x)$	✓ ▼ $\exists$ Intro: 4
6.	▼ $\neg Cube(c)$	
7.	$\exists x \neg Cube(x)$	✓ ▼ $\exists$ Intro: 6
8.	$\exists x Small(x)$	✓ ▼ $\rightarrow$ Elim: 7,2
9.	$\boxed{d}$ ▼ $Small(d)$	
10.		▼ Rule?:
11.		▼ Rule?:
12.	$\exists x Cube(x)$	▼ Rule?:
13.	$\exists x Cube(x)$	✓ ▼ $\exists$ Elim: 9-12,8
14.	$\exists x Cube(x)$	✓ ▼ $\vee$ Elim: 4-5,6-13,3

13.32

1. $\neg \exists x (Tet(x) \wedge Small(x))$	
2. $\boxed{a} \triangleright Tet(a)$	
3. $\triangleright \neg (Large(a) \vee Medium(a))$	
4. $\triangleright \neg Small(a)$	
5. $\neg Medium(a)$	✓ $\triangleright$ Taut Con: 3
6. $\neg Large(a)$	✓ $\triangleright$ Taut Con: 3
7. $\perp$	✓ $\triangleright$ Ana Con: 6,5,4
8. $Small(a)$	✓ $\triangleright$ $\neg$ Intro: 4-7
9. $Tet(a) \wedge Small(a)$	✓ $\triangleright$ $\wedge$ Intro: 2,8
10. $\exists x (Tet(x) \wedge Small(x))$	✓ $\triangleright$ $\exists$ Intro: 9
11. $\perp$	✓ $\triangleright$ $\perp$ Intro: 10,1
12. $Large(a) \vee Medium(a)$	✓ $\triangleright$ $\neg$ Intro: 3-11
13. $\forall x (Tet(x) \rightarrow (Large(x) \vee Medium(x)))$	✓ $\triangleright$ $\forall$ Intro: 2-12

13.33	1. $\neg \exists x (\text{Tet}(x) \wedge \text{Small}(x))$	
	2. $\forall y (\text{Small}(y) \vee \text{Medium}(y) \vee \text{Large}(y))$	
	3. <b>a</b> $\nabla$ Tet(a)	
	4. $\text{Small}(a) \vee \text{Medium}(a) \vee \text{Large}(a)$	✓ $\nabla$ <b><math>\forall</math> Elim:</b> 2
	5. $\nabla$ Small(a)	
	6. $\text{Tet}(a) \wedge \text{Small}(a)$	✓ $\nabla$ <b><math>\wedge</math> Intro:</b> 3,5
	7. $\exists x (\text{Tet}(x) \wedge \text{Small}(x))$	✓ $\nabla$ <b><math>\exists</math> Intro:</b> 6
	8. $\perp$	✓ $\nabla$ <b><math>\perp</math> Intro:</b> 7,1
	9. $\neg \text{Small}(a)$	✓ $\nabla$ <b><math>\neg</math> Intro:</b> 5-8
	10. $\text{Large}(a) \vee \text{Medium}(a)$	✓ $\nabla$ <b>Taut Con:</b> 9,4
	11. $\forall x (\text{Tet}(x) \rightarrow (\text{Large}(x) \vee \text{Medium}(x)))$	✓ $\nabla$ <b><math>\forall</math> Intro:</b> 3-10

### Section 13.5 (page 361)

13.43	1. $\neg \forall x \text{Cube}(x)$	
	2. $\nabla$ $\neg \exists x \neg \text{Cube}(x)$	
	3. <b>c</b> $\nabla$	
	4. $\nabla$ $\neg \text{Cube}(c)$	
	5. $\exists x \neg \text{Cube}(x)$	✓ $\nabla$ <b><math>\exists</math> Intro:</b> 4
	6. $\perp$	✓ $\nabla$ <b><math>\perp</math> Intro:</b> 5,2
	7. $\text{Cube}(c)$	✓ $\nabla$ <b><math>\neg</math> Intro:</b> 4-6
	8. $\forall x \text{Cube}(x)$	✓ $\nabla$ <b><math>\forall</math> Intro:</b> 3-7
	9. $\perp$	✓ $\nabla$ <b><math>\perp</math> Intro:</b> 1,8
	10. $\exists x \neg \text{Cube}(x)$	✓ $\nabla$ <b><math>\neg</math> Intro:</b> 2-9

**13.51** An sparse outline of a proof is shown below. It shows the main trick needed but leaves out the details.

1.	
2. $\exists x \neg P(x) \vee \neg \exists x \neg P(x)$	▼ <b>Taut Con:</b>
3. ▼ $\exists x \neg P(x)$	
4.	▼ <b>Rule?:</b>
5. $\exists x (P(x) \rightarrow \forall y P(y))$	▼ <b>Rule?:</b>
6. ▼ $\neg \exists x \neg P(x)$	
7.	▼ <b>Rule?:</b>
8. $\exists x (P(x) \rightarrow \forall y P(y))$	▼ <b>Rule?:</b>
9. $\exists x (P(x) \rightarrow \forall y P(y))$	✓ ▼ <b>vElim:</b> 3-5,6-8,2