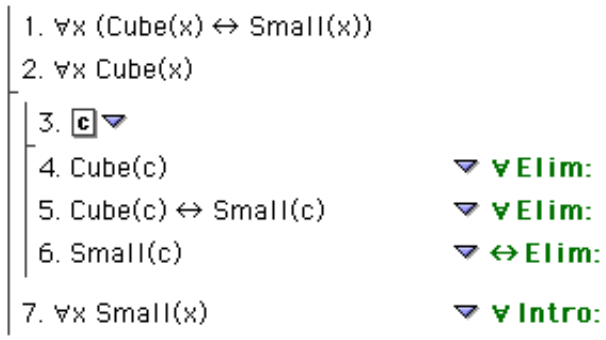
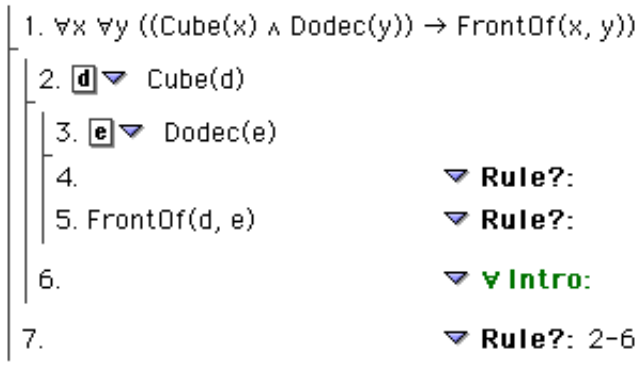
Chapter 13: Hints and Selected Solutions

Section 13.1 (page 346)

13.2 Hint (fill in the supports):

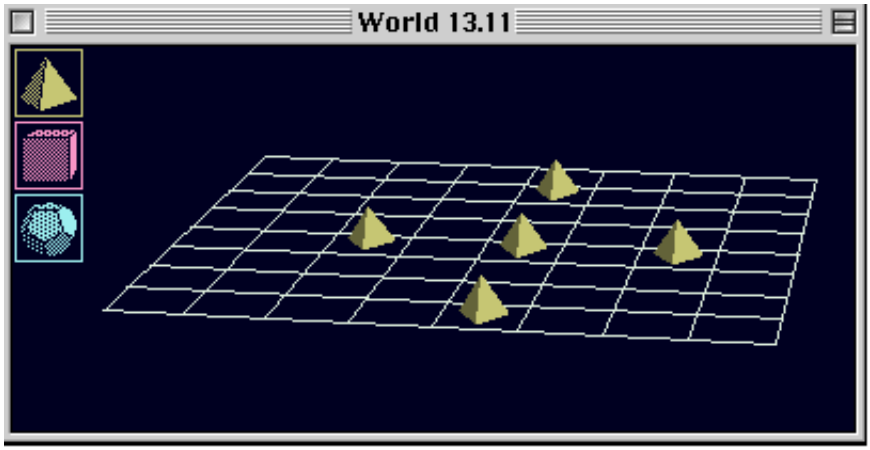


13.7 Hint:

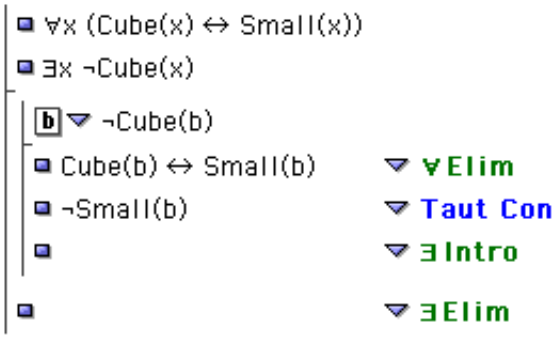


Section 13.2 (page 350)

13.11 A counterexample:

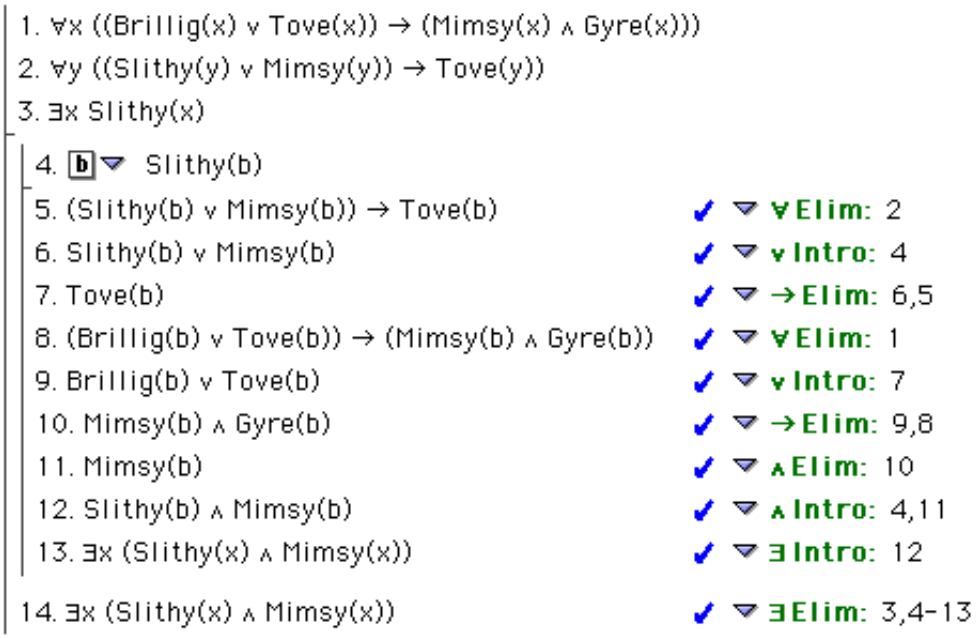


13.14 Hint:

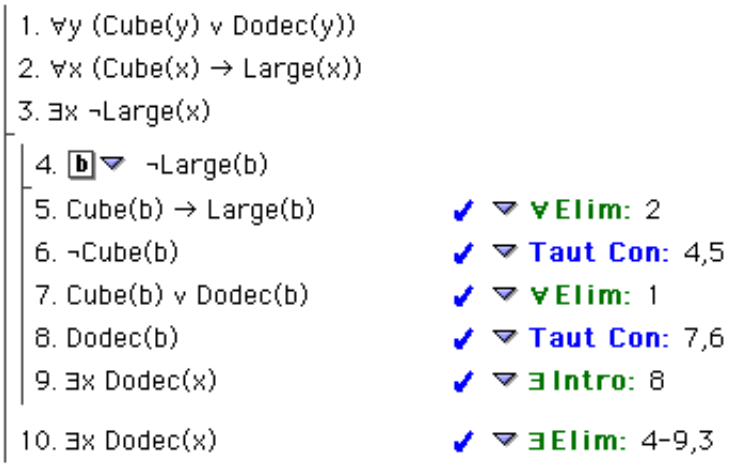


Section 13.3 (page 337)

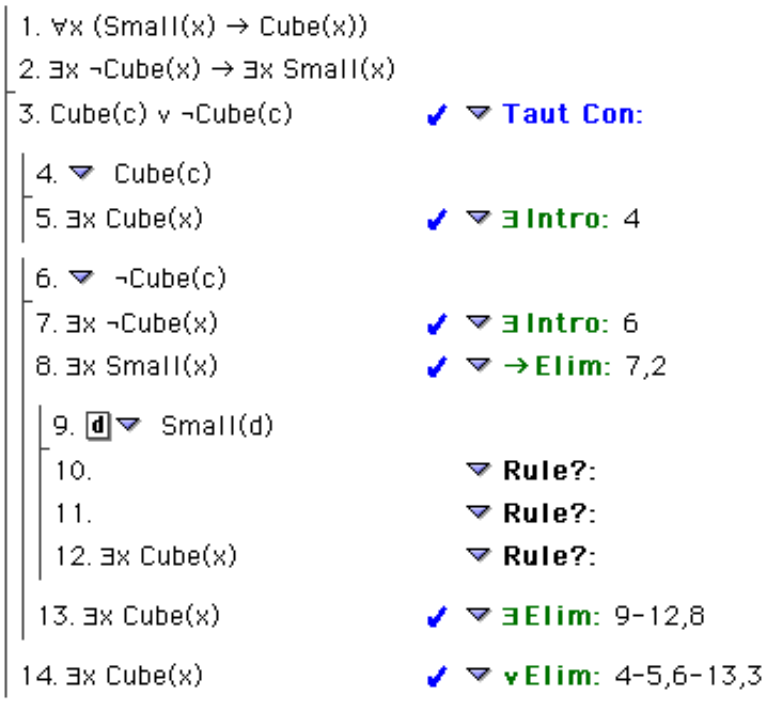
13.20 The following proof formalizes the informal proof we gave earlier for Exercise 12.1. (C.f. **Hints and Selected Solutions** for Chapter 12.)



13.23 The following formalizes the informal proof we gave for Exercise 12.4.



13.29 Remember that FOL assumes that claims are made about non-empty domains. You will need to use a name in giving this proof, even though no names appear in the original argument. Here is a nearly completed proof. You should fill in the details that are missing.



13.32


1. ¬∃x (Tet(x) ∧ Small(x))
2. a Tet(a)
3. ¬(Large(a) ∨ Medium(a))
4. ¬Small(a)
5. ¬Medium(a) ✓ Taut Con: 3
6. ¬Large(a) ✓ Taut Con: 3
7. ⊥ ✓ Ana Con: 6,5,4
8. Small(a) ✓ ¬ Intro: 4-7
9. Tet(a) ∧ Small(a) ✓ ∧ Intro: 2,8
10. ∃x (Tet(x) ∧ Small(x)) ✓ ∃ Intro: 9
11. ⊥ ✓ ⊥ Intro: 10,1
12. Large(a) ∨ Medium(a) ✓ ¬ Intro: 3-11
13. ∀x (Tet(x) → (Large(x) ∨ Medium(x))) ✓ ∀ Intro: 2-12

13.33

1. ¬∃x (Tet(x) ∧ Small(x))
2. ∀y (Small(y) ∨ Medium(y) ∨ Large(y))
   - 3. [a] Tet(a)
   - 4. Small(a) ∨ Medium(a) ∨ Large(a) ✓ ∀ Elim: 2
     - 5. Small(a)
     - 6. Tet(a) ∧ Small(a) ✓ ∧ Intro: 3,5
     - 7. ∃x (Tet(x) ∧ Small(x)) ✓ ∃ Intro: 6
     - 8. ⊥ ✓ ⊥ Intro: 7,1
   - 9. ¬Small(a) ✓ ¬ Intro: 5-8
   - 10. Large(a) ∨ Medium(a) ✓ Taut Con: 9,4
   - 11. ∀x (Tet(x) → (Large(x) ∨ Medium(x))) ✓ ∀ Intro: 3-10

Section 13.5 (page 361)

13.43

1. ¬∀x Cube(x)
   - 2. ¬∃x ¬Cube(x)
     - 3. [c]
       - 4. ¬Cube(c)
       - 5. ∃x ¬Cube(x) ✓ ∃ Intro: 4
       - 6. ⊥ ✓ ⊥ Intro: 5,2
     - 7. Cube(c) ✓ ¬ Intro: 4-6
     - 8. ∀x Cube(x) ✓ ∀ Intro: 3-7
     - 9. ⊥ ✓ ⊥ Intro: 1,8
   - 10. ∃x ¬Cube(x) ✓ ¬ Intro: 2-9

13.51 An sparse outline of a proof is shown below. It shows the main trick needed but leaves out the details.