Outlier Detection

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Summary

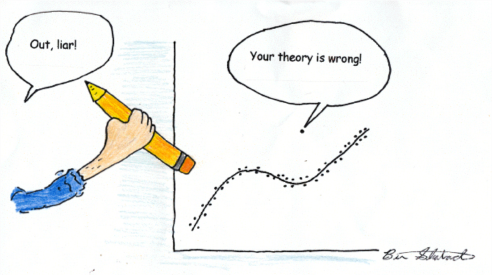
Basic Concepts

Motivation

- Most of data mining tasks focus on creating a model of the “normal” patterns in the data, extracting knowledge from what is common (e.g. frequent patterns).
- Still, rare patterns can also give us some import insights about data.
- Depending on the goal, those insights can be even more interesting/critical than the “normal” patterns.

What is an Outlier?

- *"An outlier is an observation which deviates so much from the other observations as to arouse suspicions that it was generated by a different mechanism"* (Hawkins, 1980)



What is an Outlier? (cont.)

- Outliers represent patterns in data that do not conform to a well defined notion of normal.

- Initially, outliers were considered errors and their identification had data cleaning purposes.

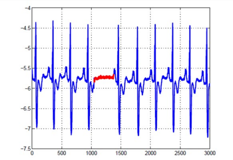
- However, they can represent truthful deviation of data.
- For some applications, they represent critical information, which can trigger preventive or corrective actions.

Outliers and Anomalies

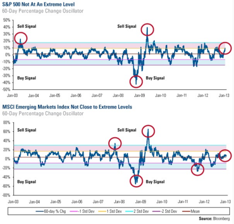
- Outlier and Anomaly detection are roughly related.
- Outliers can have a negative connotation being associated with data noise.
- Anomalies are often associated with unusual data that should be further investigated to identify the cause of occurrence.
- Anomaly can be considered as an outlier.
- But an outlier is not necessarily an anomaly.
- The following outlier detection application and methods involve outliers that can be seen as anomalies, i.e. meaningful outliers.

Where can Outliers occur?

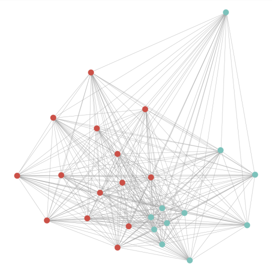
Medical Analysis



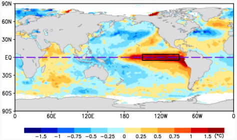
Financial Markets



Social Network Analysis

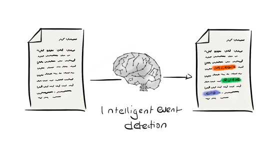


Anomalous Weather Patterns



Fraud Detection

Event Detection in Text/Social Media



Applications of Outlier Detection

- Quality Control and Fault Detection Applications
  - Quality Control
  - Fault Detection and Systems Diagnosis
  - Structure Defect Detection
- Financial Applications
  - Credit Card Fraud
  - Insurance Claim Fraud
  - Stock Market Anomalies
  - Financial Interaction Networks
- Intrusion and Security Applications
  - Host-based Intrusions
  - Network Intrusion Detection
- Web Log Analytics
  - Web Log Anomalies

Applications of Outlier Detection (cont.)

- Market Basket Analysis
  - Outlier transactions in association patterns
- Medical Applications
  - Medical Sensor Diagnostics
  - Medical Imaging Diagnostics
- Text and Social Media Applications
  - Event Detection in Text and Social Media
  - Spam Email
  - Noisy and Spam Links
  - Anomalous Activity in Social Networks
- Earth Science Applications
  - Sea Surface Temperature Anomalies
  - Land Cover Anomalies
  - Harmful Algae Blooms

Challenges of Outlier Detection

- Define every possible “normal” behaviour is hard.
- The boundary between normal and a outlying behaviour is often not precise.
- There is no general outlier definition; it depends on the application domain.
- It is difficult to distinguish real meaningful outliers from simple random noise in data.
- The outlier behaviour may evolve with time.
- Malicious actions adapt themselves to appear as normal.
- Inherent lack of known labeled outliers for training/validation of models.

Key Aspects of Outlier Detection Problem

- Nature of Input Data
- Type of Outliers
- Intended Output
- Learning Task
- Performance Metrics

Nature of Input Data

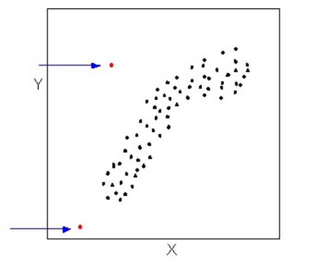
- Each data instance has:
  - One attribute (univariate)
  - Multiple attributes (multivariate)
- Relationship among data instances:
  - None
  - Sequential / Temporal
  - Spatial
  - Spatio-temporal
  - Graph
- Dimensionality of data

Types of Outliers

- Point (or Global) Outlier
- Contextual Outlier
- Collective Outlier

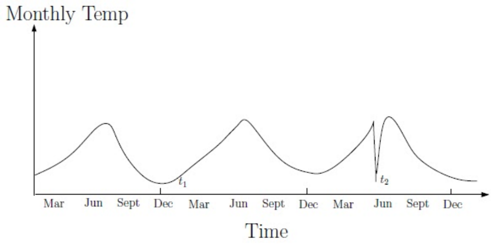
Point Outlier

An instance that individually or in small groups is very different from the rest of the instances.



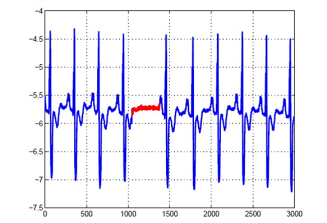
Contextual Outlier

An instance that when considered within a context is very different from the rest of the instances.



Collective Outlier

An instance that, even though individually may not be an outlier, inspected in conjunction with related instances and with respect to the entire data set is an outlier.



Intended Output

- Assign a label/value: identification normal or outlier instance.
- Assign a score: probability of being an outlier.
  - It allows the output to be ranked.
  - Requires the specification of a threshold.

Learning Task

Unsupervised Outlier Detection

- data set has no information on the behaviour of each instance;
- it assumes that instances with normal behaviour are far more frequent;
- most common case in real-life applications.

Semi-supervised Outlier Detection

- data set has a few instances of normal or outlier behaviour;
- some real-life applications, such as fault detection, provide such data.

Supervised Outlier Detection

- data set has instances of both normal and outlier behaviour;
- hard to obtain such data in real-life applications.

Inadequacy of Standard Performance Metrics

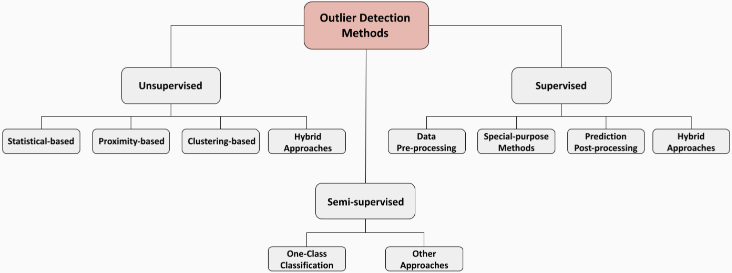
- Standard performance metrics (e.g. *accuracy*, *error rate*) assume that all instances are equally relevant for the model performance.
- These metrics would give a good performance estimation to a model that performs well on normal (frequent) cases and bad on outlier (rare) cases.

  Credit Card Fraud Detection:

  - data set D with only 1% of fraudulent transactions;
  - model M predicts all transactions as non-fraudulent;
  - M has a estimated accuracy of 99%;
  - yet, all the fraudulent transactions were missed!

Outlier Detection Approaches

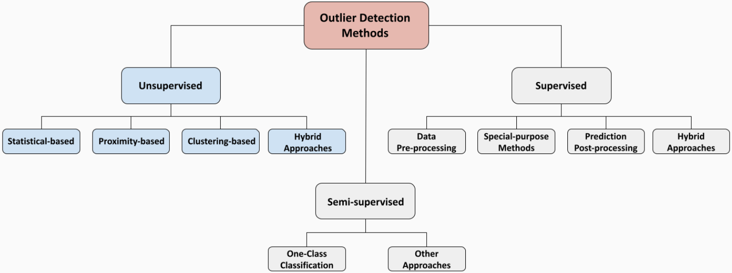
Taxonomy of Outlier Detection Methods



Outlier Detection Approaches

Unsupervised Learning Techniques

Taxonomy of Anomaly Detection Methods



Statistical-based Outlier Detection

Proposal

- All the points that satisfy a statistical discordance test for some statistical model are declared as outliers.

Advantages

- If the assumptions of the statistical model hold true, these techniques provide a justifiable solution for outlier detection.
- The outlier score is associated with a confidence interval.

Techniques

- Parametric
- Non-parametric

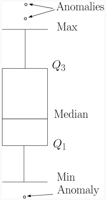
Assume one of the known probability distribution functions.

- *Grubbs' Test* (Grubbs, 1950)
  A statistical test used to detect outliers in a **univariate** data set assumed to come from a normally distributed population.

- Boxplot (Tukey, 1977)
  It assumes a near-normal distribution of the values in a **univariate** data set, and identifies as outlier any value outside the interval

$$[Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR]$$

where Q_1 (Q_3) is the 1st (3rd) quartile and *IQR* is the interquartile range.

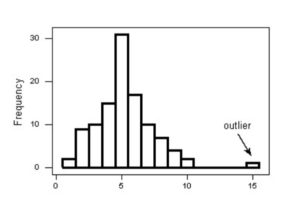


Statistical-based Outlier Detection: Parametric Techniques (cont.)

- *Mahalanobis* distance (Mahalanobis, 1936)
  - It assumes a multivariate normal distribution of data.
  - Incorporates dependencies between attributes by the covariance matrix.
  - Transforms a **multivariate** outlier detection task into a univariate outlier detection problem.
  - All the points with a large *Mahalanobis* distance are indicated as outliers.
- Mixture of parametric distributions
- etc.

The probability distribution function is not assumed, but estimated from data.

- Histograms
  - Used for both univariate and multivariate data. For the later, the attribute-wise histograms are constructed and an aggregated score is obtained.
  - Hard to choose the appropriate bin size.



- Kernel functions
  - Adopt a kernel density estimation to estimate the probability density distribution of the data.
  - Ouliers are in regions of low density.

Disadvantages

- The data does not always follows a statistical model.
- Choosing the best hypothesis test statistics is not straightforward.
- Capture interactions between attributes is not always possible.
- Estimating the parameters for some statistical models is hard.

Proposal

- Normal instances occur in dense neighbourhoods, while outliers occur far from their closest neighbours.

Advantages

- Purely data driven technique
- Does not make any assumptions regarding the underlying distribution of data.

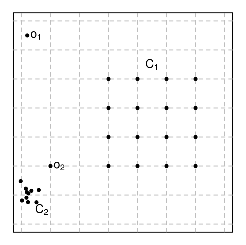
Some Techniques

- Distance-based
- Density-based

A case c is an outlier if less than k cases are within a distance λ of c [Knorr and Ng, 1998]

- Outliers are points far away from other points, thus given a distance metric there should not be a lot of other points in their neighborhood.
- Define proper distance metric (e.g euclidean distance)
  - The notion of distance between cases with many variables may be distorted by different scales, different importance, different types (numerical, nominal)
- Define a “reasonable” neighborhood (λ)
- Define what is “a lot of other points” (k)

- Major cost: for each point is calculated its distance to all the other points.
- The use of **global distance** measures poses difficulties in detecting outliers in data sets with different density regions.
- Example:

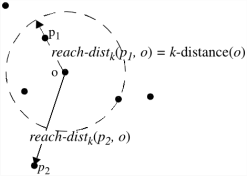


- o_1 and o_2 are outliers
- but, for the point o_2 to be identified as an outlier, all the points in C_1 would have to be identified as outliers too.

- Concept of outliers should be **locally** inspected.
- Compare points to their local neighborhood, instead of the global data distribution
- The density around an outlier is significantly different from the density around its neighbours.
- Use the relative density of a point against its neighbours as the indicator of the degree of the point being an outlier.
- Outliers are points in lower local density areas with respect to the density of its local neighbourhood.

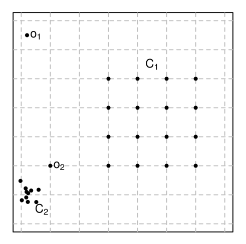
Proximity-based Outlier Detection: Density-based Techniques (cont.)

- LOF: Local Outlier Factor [Breunig et al., 2000]
  - *k-distance*: distance between p and its k-th nearest neighbour
  - *k-distance neighborhood*: all the points whose distance from p is not greater than the k-distance.
  - *reachability-distance* of p with respect to o: the maximum between their k-distance and their actual distance.



  - intuition: high values of reachability-distance between two given points indicates that they may not be in the same cluster

- LOF: Local Outlier Factor [Breunig et al., 2000] (cont.)
  - *local reachability-density* of a point is inversely proportional to the average reachability-distance of its k neighbourhood.
  - *LOF* assigns high values to the points that have a much lower *local reachability-density* in comparison to its *k*-neighbourhood.
  - Example:



- o_2 is assigned an higher LOF compared to the LOF values assigned to the points of C_1 and C_2

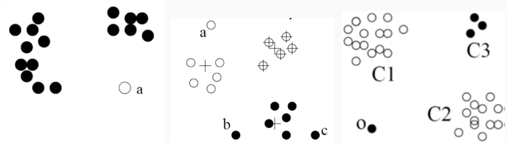
  - This captures a local outlier whose local density is relatively low comparing to the local densities of its *k*-neighbourhood.

Disadvantages

- True outliers and noisy regions of low density may be hard to distinguish.
- These methods need to combine global and local analysis.
- In high dimensional data, the contrast in the distances is lost.
- Computational complexity of the test phase.

Proposal

- Normal instances belong to large and dense clusters, while outlier instances are instances that:
  - do not belong to any of the clusters;
  - are far from its closest cluster;
  - form very small or low density clusters.

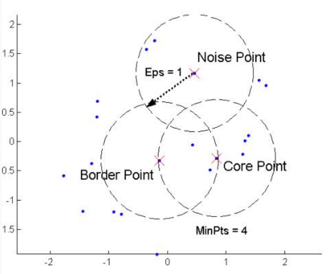


Advantages

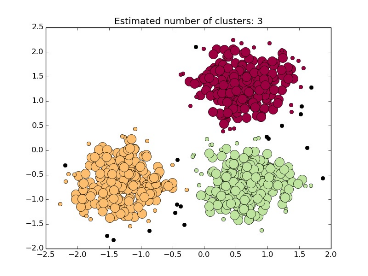
- Easily adaptable to on-line/incremental mode.
- Test phase is fast.

- DBSCAN [Ester et al., 1996]
  - Clustering method based on the notion of “density” of the points
  - The density of a point is estimated by the number of points that are within a certain radius.
  - Based on this idea, points can be classified as:

- *core points*: if the number of points within its radius are above a threshold
- *border points*: if the number of points within its radius are not above a threshold, but they are within a radius of a *core point*
- *noise points*: if do not have enough points within their radius, nor are sufficiently close to any *core point*.



- DBSCAN [Ester et al., 1996] (cont.)
  - *noise points* are removed for the formation of clusters
  - all *core points* that are within a certain distance of each other are allocated to the same cluster
  - each *border point* is allocated to the cluster of the nearest *core points*
  - *noise points* are identified as outliers.



Clustering-based Outlier Detection: Techniques (cont.)

- FindCBLOF [He et al., 2003]
  - To each point, assign a *cluster-based local outlier factor* (CBLOF)
  - The CBLOF score of a point p is determined by the size of the cluster to which p belongs, and the distance between p and
    - its cluster centroid, if p belongs to a large cluster
    - its closest large cluster centroid, if p belongs to a small cluster.
- OR_H [Torgo, 2007]
  - Obtain an agglomerative hierarchical clustering of the data set
  - Use the information on the "path" of each point through the dendogram as a form to determine its degree of outlyingness

Disadvantages

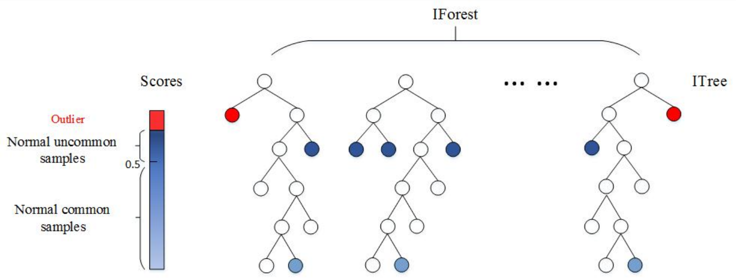
- Computationally expensive in the training phase.
- If normal points do not create any clusters, this technique may fail.
- In high dimensional spaces, clustering algorithms may not give any meaningful clusters.
- Some techniques detect outliers as a byproduct, i.e. they are not optimized to find outliers, their main aim is to find clusters.

Isolation Forest

- iForest [Liu et al., 2008] detects outliers purely based on the concept of isolation without employing any distance or density measure.
- Isolation: separating an instance from the rest of the instances
- A two-stage process.
  1. The first (training) stage builds an ensemble of data-induced random binary decision trees (isolation trees) using sub-samples of the given training set.
  2. The second (evaluation) stage passes test instances through isolation trees to obtain an outlier score for each instance.
- Parameters: number of trees and subsampling size

Isolation Forest (cont.)

- The score is related to average path length
  - outliers are more likely to be isolated closer to the root
  - normal points are more likely to be isolated at the deeper levels



Source: https://github.com/zmzhang/IOS/blob/master/images/IOS.jpg

Isolation Forest (cont.)

Advantages

- No distance or density measures to detect anomalies;
- Eliminates a major computational cost of distance calculation in all distance-based and density-based methods;
- Scales up to handle extremely large data size and high-dimensional problems with a large number of irrelevant attributes.

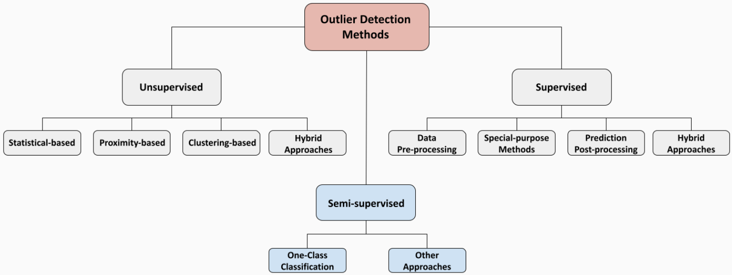
Disadvantages

- Hyperparameters that must be tuned;
- Randomness component: different runs can give different results;
- Large sample sizes may cause masking or swamping.

Outlier Detection Approaches

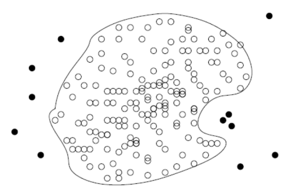
Semi-supervised Learning Techniques

Taxonomy of Outlier Detection Methods



Proposal

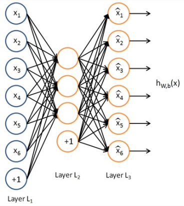
- Build a prediction model to the normal behaviour and classify any deviations from this behaviour as outliers.



Advantages

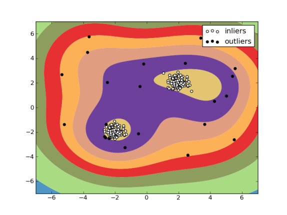
- Models are interpretable.
- Normal behaviour can be accurately learned.
- Can detect new outliers that may not appear close to any outlier points in the training set.

- Auto-associative neural networks [Japkowicz et al., 1995]
  - A feed-forward perceptron-based network is trained with normal data only.
  - The network has the same number of input and output nodes and a decreased number of hidden nodes to induce a bottleneck.
  - This bottleneck reduces the redundancies and focus on the key attributes of data.
  - After training, the output nodes recreate the example given as input nodes.
  - The network will successfully recreate normal data but will generate a high-recreation error for outlier data.



One Class Classification: Techniques (cont.)

- One-class SVM [Tax and Duin, 2004]
  - It obtains a spherical boundary, in the feature space, around the normal data. The volume of this hypersphere is minimized, to minimize the effect of incorporating outliers in the solution.
  - The resulting hypersphere is characterized by a centre **c** and a radius R.
  - The optimization problem consists of minimizing the volume of the hypersphere, so that includes all the training points.
  - Every point lying outside this hypersphere is an outlier.



Disadvantages

- Requires previous labeled instances for normal behaviour.
- Possible high false alarm rate - previously unseen normal data may be identified as an outlier.

Outlier Detection Approaches

Advanced Topics

Proposal

- If a data instance is an outlier in a specific context (but not otherwise), then it is considered as a contextual outlier.
- Each data instance is defined using two sets of attributes:
  - Contextual attributes used to determine the context (or neighbourhood) for that instance.
    - Sequential Context: position, time.
    - Spatial Context: latitude, longitude.
    - Graph Context: weights, edges.
  - Behavioural attributes which define the non-contextual characteristics of an instance.
- The outlier behaviour is determined using the values for the behavioural attributes within a specific context.

Contextual Outlier Detection (cont.)

Example:

- Detect outlier customers in the context of customer groups
  - Contextual attributes: age group, postal code
  - Behavioural attributes: the number of transactions per year, annual total transaction amount

Advantages

- Allow a natural definition of outlier in many real-life applications.
- Detects outliers that are hard to detect when analyzed in the global perspective.

Contextual Outlier Detection (cont.)

Techniques

- Reduction to point outlier detection
  - Segment data using contextual attributes.
  - Apply a traditional point outlier within each context using behavioural attributes.
  - Model “normal” behaviour with respect to contexts: an object is an outlier if its behavioural attributes significantly deviate from the values predicted by the model.

- Utilizing structure in data
  - Build models from the data using contextual attributes to predict the expected behaviour with respect to a given context.
  - Avoids explicit identification of specific contexts

Disadvantages

- Identifying a set of good contextual attributes.
- It assumes that all normal instances within a context will be similar (in terms of behavioural attributes), while the outliers will be different.

Proposal

- If a collection of related data instances is anomalous with respect to the entire data set, then it is considered a collective outlier.
- The individual data instances in a collective outlier may not be outliers by themselves, but their occurrence together as a collection is anomalous.

Advantages

- Allow a natural definition of outlier in many real-life applications in which data instances are related.

Collective Outlier Detection (cont.)

Techniques

- A collective outlier can also be a contextual outlier if analyzed with respect to a context.
- A collective outlier detection problem can be transformed to a contextual outlier detection problem by incorporating the context information.

Collective Outlier Detection (cont.)

Disadvantages

- Contrary to contextual outliers, the structures are often not explicitly defined, and have to be discovered as part of the outlier detection process.
- Need to extract features by examining the structure of the dataset, i.e. the relationship among data instances for:
  - sequence data to detect anomalous sequences;
  - spatial data to detect anomalous sub-regions;
  - graph data to detect anomalous sub-graphs.
- The exploration of structures in data typically uses heuristics, and thus may be application dependent.
- The computational cost is often high due to the sophisticated mining process.

Challenges

- Interpretation of outliers
  - Detecting outliers without saying why they are outliers is not very useful in high-D due to the many features (or dimensions) involved
  - Identify the subspaces that manifest the outliers
- Data sparsity
  - Data in high-D spaces is often sparse
  - The distance between objects becomes heavily dominated by noise as the dimensionality increases
- Data subspaces
  - Capturing the local behavior of data
- Scalable with respect to dimensionality
  - # of subspaces increases exponentially

Outlier Detection in High Dimensional Data (cont.)

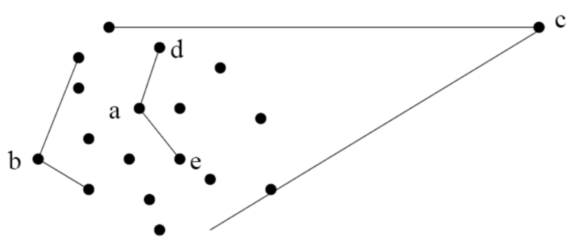
Techniques

- Find distance-based outliers, but use the ranks of distance instead of the absolute distance in outlier detection.
- Dimensionality reduction: the principal components with low variance are preferred because, on such dimensions, normal objects are likely close to each other and outliers often deviate from the majority.
- Project data onto various subspaces to find an area whose density is much lower than average.

Techniques (cont.)

- Develop new models for high-dimensional outliers directly. Avoid proximity measures and adopt new heuristics that do not deteriorate in high-dimensional data.

  E.g. Angle-based outliers.



Summary

Summary

- Outliers are not necessarily random noise. They can represent critical information that can trigger preventive or corrective actions.
- The interpretability of an outlier detection method is extremely important.
- The nature of the outlier detection problem is dependent on the application domain.
- Different approaches to this problem are necessary.
- Contextual and collective outliers are having increasing applicability in several real-world domains.
- Online Outlier Detection and Distributed Outlier Detection are emerging topics.
- There is much space for the development of new techniques in this area.

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