Association Rules

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Summary

Association Rules in Action

Association Rules: a New Data Mining Task

Data Mining Tasks:

- Predictive
  - Classification
  - Regression
  - ...
- Descriptive
  - Clustering
  - Association Rules
    - find relationships / associations between groups of variables
  - ...

Motivation

Originally stated in the context of Market Basket Analysis

- Data consists of set of items bought by costumers, referred as transactions
- Find unexpected associations between sets of items using the frequency of sets of items
- Discovered sets of items are referred as frequent itemsets or frequent patterns
- Goals
  - Store layout - *Should products A and B be placed together?*
  - Promotions - *If the client is interested in {A,B,C,...}, can we guess other interests?*
  - ...

Actionable Knowledge: shop layout

- Possible actions from rule $\{A1, A4\} \rightarrow \{A6\}$
  - Sell the $A1$, $A4$, $A6$ together (pack)
  - Place article $A6$ next to articles $A1$, $A4$
  - Offer a discount coupon for $A6$ in articles $A1$, $A4$
  - Place a competitor of A6 next to $A1$, $A4$ (brand protection).
- Note
  - These actions must make sense from the business point of view.

Actionable Knowledge: cross selling

- Steps
  - Client puts article A in basket
  - Shop knows rule $A \rightarrow B$
  - Rule has enough confidence (> 20%)
  - Shop tells client he may be interested in B
  - Client decides whether to buy B or not
- Notes
  - Rules are discovered from business records
  - Discovery (mining) can be made off-line
  - Use of rules can be made on-line

Actionable Knowledge: text mining

- Each document is treated as a “bag” of terms and keywords
  - doc1: Student, Teach, School (Education)
  - doc2: Student, School (Education)
  - doc3: Teach, School, City, Game (Education)
  - doc4: Baseball, Basketball (Sport)
  - doc5: Basketball, Player, Spectator (Sport)
  - doc6: Baseball, Coach, Game, Team (Sport)
  - doc7: Basketball, Team, City, Game (Sport)
- Goal: identify co-occurring terms and keywords
- Example:
  - Student, School $\rightarrow$ Education
  - Game $\rightarrow$ Sport

Actionable Knowledge: health

- Rules obtained from the patient's records
- Sooner prevention
- Each patient visits a health unit one or more times
- We record the observations for each visit
    - Symptoms (head ache, temperature)
    - Exam results (blood pressure, sugar level)
- A set of observations may fire a rule
  {Head ache, blood pressure rise} $\rightarrow$ {stroke, immobilization}
- When head ache and blood pressure rise are observed, stroke and immobilization are also expected.
- Not necessarily causal

Actionable Knowledge: web usage analysis

Usage patterns

- Most visited pages
- Frequent page sets
  - Site structure
- Pages associated to users
  - personalization
- Seasonal effects
  - operations, campaigns
- Cross-preferences
  - cross-selling

Association Rules
Basic Concepts

Market Baskets data set

TID	Products
1	A, B, E
2	B, D
3	B, C
4	A, B, D
5	A, C
6	B, C
7	A, C
8	A, B, C, E
9	A, B, C

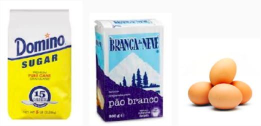
Products are converted in binary flags

→

TID	A	B	C	D	E
1	1	1	0	0	1
2	0	1	0	1	0
3	0	1	1	0	0
4	1	1	0	1	0
5	1	0	1	0	0
6	0	1	1	0	0
7	1	0	1	0	0
8	1	1	1	0	1
9	1	1	1	0	0

Market Basket Analysis: how frequent is an itemset?

- Sugar, Flower and Eggs are sold together



- How important is this set?

- **Support** measures the importance of a set
    - Percentage of transactions *t* containing the set *S*
    - Absolute support: number of transactions *t* containing the set *S*

Market Basket Analysis: how predictive is an itemset?

- Frequent itemsets are used to generate association rules.
- If you buy sugar and flower, you also buy eggs.
- How strong is this rule?
- **Confidence** measures the strength of the rule
  - Percentage of transactions *t* that having sugar and flower also have eggs

Association Rules: Basic Concepts

- Consider a set of items I
- A transaction t is a subset of items, i.e. $t \subseteq I$
- Given a data set of transactions $D = \{t_i\}_{i=1}^{N}$
- An association rule is defined as an implication $X \rightarrow Y$, where
  - X and Y are itemsets, i.e. $X, Y \subseteq I$
  - $X \neq \emptyset$, $Y \neq \emptyset$ and $X \cap Y = \emptyset$
- $sup(X)$ is the proportion of transactions in D that include the itemset X
- support: $sup(X \rightarrow Y) = sup(X \cup Y)$
- confidence: $conf(X \rightarrow Y) = sup(X \cup Y)/sup(X)$

Association Rules: an example

Given the data

Transactions ID	Items Bought
100	A, B, C
200	A, C
150	A, D
500	B, E, F

$\rightarrow$

TID	A	B	C	D	E	F
100	1	1	1	0	0	0
200	1	0	1	0	0	0
150	1	0	0	1	0	0
500	0	1	0	0	1	1

- The itemsets with a minimum support of 50%

Frequent Itemsets	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

- Rules with minimum support of 50% and minimum confidence of 50%
  - $A \rightarrow C$
    - $sup(A \rightarrow C) = sup(\{A, C\}) = 50\%$
    - $conf(A \rightarrow C) = sup(\{A, C\})/sup(\{A\}) = 66.6\%$
  - $C \rightarrow A$
    - $sup(C \rightarrow A) = sup(\{A, C\}) = 50\%$
    - $conf(C \rightarrow A) = sup(\{A, C\})/sup(\{C\}) = 100\%$

Mining Association Rules

Problem Definition

- Given:
  - data set of transactions *D*
  - minimal support *minsup*
  - minimal confidence *minconf*
- Obtain:
  - **all** association rules

    $$X \rightarrow Y \quad (s = Sup, c = Conf)$$

    such that

    $$Sup \geq minsup \text{ and } Conf \geq minconf$$

Apriori Algorithm

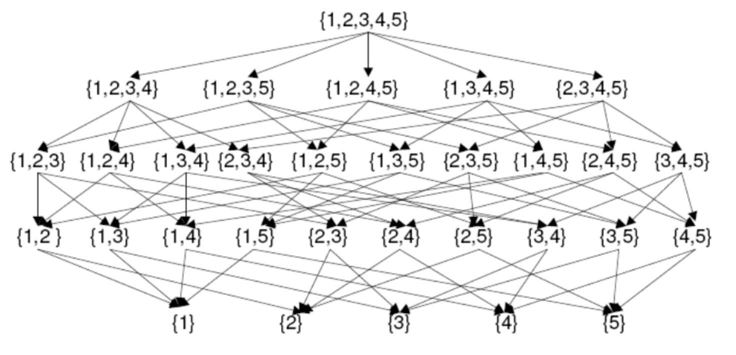
The **Apriori Algorithm** [Agrawal and Srikant, 1994] works in two steps:

1. **Frequent itemset generation**
   - itemsets with *support* $\geq$ *minsup*
2. **Rule generation**
   - generate all confident association rules from the frequent itemsets, i.e. rules with *confidence* $\geq$ *minconf*

Apriori Algorithm (cont.)

- Problem:
  - there is a very large number of candidate frequent itemsets!
    - for transactions with k items, there are $2^k - 1$ distinct subsets.
- Downward Closure Property
  - every subset of a frequent itemset must also be frequent.
    - ex: if $\{A1, A2, A4\}$ is frequent, so is $\{A1, A2\}$ because every transaction containing $\{A1, A2, A4\}$ also contains $\{A1, A2\}$.
  - thus, every superset of an infrequent itemset is also infrequent.
    - ex: if $\{A1, A2\}$ is infrequent, so is $\{A1, A2, A4\}$.
- Apriori Pruning Principle:
  - if an itemset is below the minimal support, discard all its supersets.

Search Space for 5 items



Example - 1 (cont.)

- Apriori enumerates and counts the support of patterns with increasing length.
- Starts looking for frequent itemsets of size 1 (F_1), assuming $minsup = 50\%$ (2 transactions)
- $C_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

TID	ITEM-SET
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

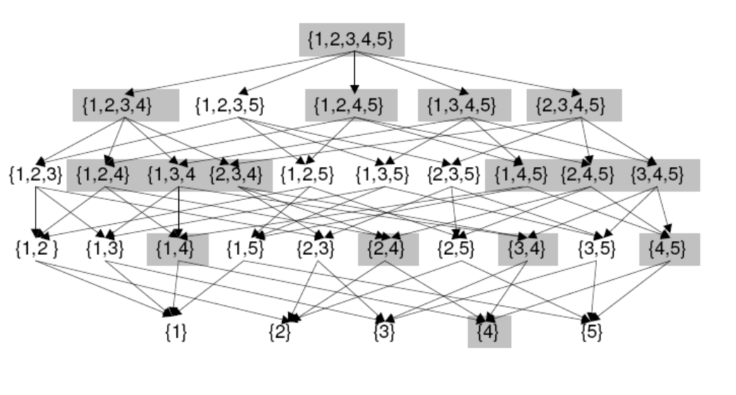
→

ITEM-SET	Support
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

- $F_1 = \{\{1\}, \{2\}, \{3\}, \{5\}\}$

Example - 1 (cont.)

- Filtered Search Space for 5 items (after removing item “4”)



Example - 1 (cont.)

- Looks for frequent itemsets of size 2 (F_2) from frequent itemsets of size 1 (F_1)
- Candidates $C_2 = \{\{a, b\} | \{a\} \in F_1 \wedge \{b\} \in F_1\}$
- $C_2 = \{\{1, 2\}, \{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 5\}, \{3, 5\}\}$

ITEM-SET	Support
{1,2}	1
{1,3}	2
{1,5}	1
{2,3}	2
{2,5}	3
{3,5}	2

- $F_2 = \{\{1, 3\}, \{2, 3\}, \{2, 5\}, \{3, 5\}\}$

Example - 1 (cont.)

- Looks for frequent itemsets of size 3 (F_3) from frequent itemsets of size 2 (F_2)
- Generation:
  $C0_3 = \{\{a, b, c\} | \{a, b\} \in F_2 \wedge \{a, c\} \in F_2\}$
- Filter:
  $C_3 = \{\{a, b, c\} | \{a, b, c\} \in C0_3 \wedge \forall x \in \{a, b, c\}\ S - \{x\} \in F_2\}$
- $C_3 = \{\{2, 3, 5\}\}$

ITEM-SET	Suporte
{2,3,5}	2

- $F_3 = \{\{2, 3, 5\}\}$
- There are no frequent itemsets of size 4

Step 1 - Identifying Frequent Itemsets

- Candidate generation (Self-Join step)
  - generates new candidate k-itemsets based on the frequent (k-1)-itemsets found in the previous iteration.
- Candidate pruning (Prune step)
  - eliminates some of the candidate k-itemsets using the support-based pruning strategy.

Step 1 - Identifying Frequent Itemsets (cont.)

- Self-Join Example:

  Given the size k candidates
  $\{A, B, C\}$
  $\{A, B, D\}$
  $\{A, C, D\}$
  $\{B, C, D\}$
  $\{A, B, E\}$
  $\{B, C, E\}$
  and assuming that in each itemset the items are lexicographically sorted

- Which are the candidates of size $k + 1$?

- What is the most efficient way of finding them (without repetitions)?

- Look for pairs of sets with the same prefix of size $k-1$
  $\{A, B, C\}$ and $\{A, B, D\}$
- Combine both, keeping the prefix
  $\{A, B, C, D\}$
- This way
  - No frequent set is unnoticed
  - No candidate is generated more than once

Step 1 - Identifying Frequent Itemsets (cont.)

- Prune Example:

  $F_3 = \{\{A, B, C\}, \{A, B, D\}, \{A, C, D\}, \{A, C, E\}, \{B, C, D\}\}$

  $C_4 = \{\{A, B, C, D\}, \{A, C, D, E\}\}$

  but $\{A, C, D, E\}$ can be pruned away

  because $\{A, D, E\} \notin F_3$

- Note:
  - Prune maintains the completeness of the process

Step 2 - Rule Generation

- Given a frequent set $\{A, B, C, D\}$
- Which are the possible rules?
  - $\{A, B, C\} \rightarrow \{D\}$
  - $\{A, B, D\} \rightarrow \{C\}$
  - $\{A, B\} \rightarrow \{C, D\}$
- How to generate them systematically?
- How to reduce the search space?

Step 2 - Rule Generation (cont.)

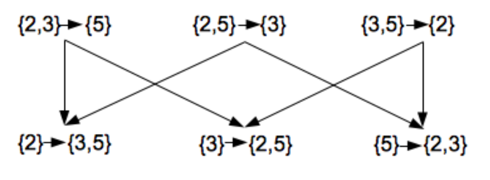
- The rules are generated as follows:
  - generates all non-empty subsets s of each frequent itemset l
  - for each subset s computes the confidence of the rule $(l - s) \rightarrow s$
  - selects the rules whose confidence is higher than *minconf*

Consider again

Cliente (TID)	Itens (Item-set)
100	1, 3, 4
200	2, 3, 5,
300	1, 2, 3, 5,
400	2, 5,

and $I = \{2,3,5\} (= F_3)$

- Rules generated from the frequent itemset $\{2,3,5\}$



- Select rules $(I - a) \rightarrow a$, where $a \subseteq I$, with *minconf* $= 1$

$$conf((I-a) \rightarrow a) = \frac{sup(I)}{sup(I-a)}$$

Step 2 - Rule Generation (cont.)

- Rules with 1 consequent

$\{2,3\} \rightarrow \{5\}$ (conf= 2/2)
$\{2,5\} \rightarrow \{3\}$ (conf= 2/3) eliminated because *minconf* $= 1$
$\{3,5\} \rightarrow \{2\}$ (conf= 2/2)

- Rules with 2 consequents

$\{3\} \rightarrow \{2,5\}$ (conf= 2/3) eliminated because *minconf* $= 1$

- we don't need to worry about rules with item 3 in the consequent, because any rule obtained from $\{2,5\} \rightarrow \{3\}$ will have a *conf* $< 2/3$

> Moving items from the antecedent to the consequent never changes support and never increases confidence.

Number of DB scans

- 1 to count frequencies of C_1
- C_2 built in memory
- 2 to count frequencies of C_2
- ...
- n to count frequencies of C_n
- Rule generation does not need to scan DB
- Number of scans is n
  - if the size of the largest frequent set is n or $n-1$

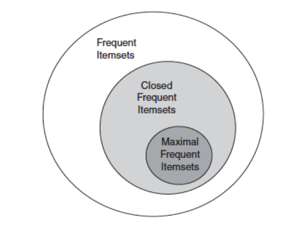
Complexity factors

- Number of items
- Number of transactions
- Minimal support
- Average size of transactions
- Number of frequent sets
- Average size of a frequent size
- Number of DB scans
  - k or $k+1$, where k is the size of the largest frequent set

Compact Representation of Itemsets

- The number of frequent itemsets produced from a transaction data set can be very large.
- It is useful to identify a small representative set of itemsets from which all other frequent itemsets can be derived.

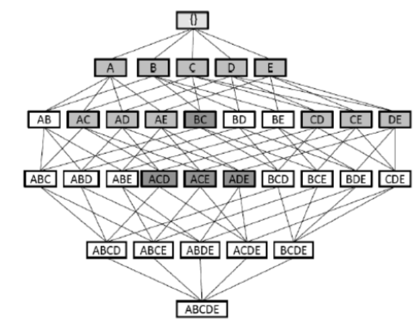
- Two such representations are:
  - maximal
  - closed



Compact Representation of Itemsets (cont.)

- s is a **closed frequent itemset** if it is a frequent itemset that has no frequent supersets with the same support.
- Example: find closed frequent itemsets with *minsup* $= 30\%$

TID	Itemset
1	A D E
2	B C D
3	A C E
4	A C D E
5	A E
6	A C D
7	B C
8	A C D E
9	B C E
10	A D E

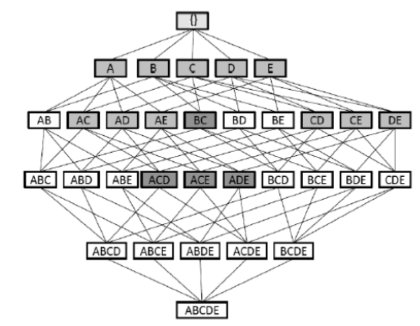


closed frequent itemsets are:
$\{A\}, \{C\}, \{D\}, \{E\}, \{A, C\}, \{A, D\}, \{A, E\},$
$\{B, C\}, \{C, D\}, \{C, E\}, \{A, C, D\}, \{A, C, E\}, \{A, D, E\}$

Compact Representation of Itemsets (cont.)

- s is a **maximal frequent itemset** if it is a frequent itemset for which none of its supersets is frequent.
- Example: find maximal frequent itemsets with $minsup = 30\%$

TID	Itemset
1	A D E
2	B C D
3	A C E
4	A C D E
5	A E
6	A C D
7	B C
8	A C D E
9	B C E
10	A D E



maximal frequent itemsets are:

$\{B, C\}, \{A, C, D\}, \{A, C, E\}, \{A, D, E\}$

Compact Representation of Itemsets (cont.)

- From the maximal itemsets is possible to derive all frequent itemsets (not their support) by computing all non-empty intersections.
  - subsets of the maximal frequent itemset $\{A, C, D\}$ are frequent itemsets
  - $\{A\}$, $\{C\}$, $\{D\}$, $\{A, C\}$, $\{A, D\}$, $\{C, D\}$
- The set of all closed itemsets preserves the knowledge about the support values of all frequent itemsets.
  - $\{D, E\}$ is a non closed frequent itemset. What is its support?
  - As it is not closed, its support must be equal to one of its immediate supersets.
  - look for the most frequent closed itemset that contains $\{D, E\}$: $\{A, D, E\}$
  - $sup(\{D, E\}) = sup(\{A, D, E\})$
- There are algorithms that take advantage of this compact representation of frequent itemsets.

Too many rules ...

- The association rule algorithms tend to generate an excessive number of rules (for some problems, there can be thousands).
- Too many rules leads to model's interpretability lack.
- How can we reduce this number?
  - Changing the parameters: *minsup*, *minconf*
  - Restrictions on items: which items are relevant?
  - Summarization techniques: can we represent subsets of rules by a single representative rule?
  - Filter rules: improvement, measures of interest, ...

How to measure the improvement of a rule?

Improvement [Bayardo and Ag, 2000]

- **Improvement** of a rule is the minimum difference between its confidence and the confidence of any of its immediate simplifications.

  $$improv(A \rightarrow C) = min(\{conf(A \rightarrow C) - conf(As \rightarrow C) \mid As \subseteq A\})$$

- Example:
  - $R_1 : \{eggs, flower, bread\} \rightarrow \{sugar\}(conf = 0.505)$
  - $R_2 : \{eggs, flower\} \rightarrow \{sugar\}(conf = 0.5)$
  - $improv(R_1)$ is at most 0.005
  - with a *minmprov* of 0.01, R_1 is excluded.

Are all the rules interesting?

- Are all the discovered patterns interesting?
- In recent years, several measures have been proposed to extract interesting patterns.
- The idea is to select a subset of rules, that somehow are more relevant.
- Interesting rule (Silberschatz & Tuzhilin,95)
  - Unexpected, surprising to the user
    - Measure of interest: deviation from the expected or from the initial belief
  - Useful, actionable
    - Measure of interest: estimated benefit

How to measure the interest of a rule?

- Subjective measures: based on user's belief in the data (ex: unexpectedness, novelty, actionability, confirm hypothesis user wishes to validate)
  - These measures are hard to incorporate in the pattern discovery task.
- Objective measures: based on facts, statistics and structures of patterns (ex: support and confidence), independent of the domain considered.
  - For instance, patterns that involve mutually independent items or cover very few transactions are considered uninteresting.

How to measure the interest of a rule? (cont.)

Typically

- $A \rightarrow B$ is interesting if A and B are not statistically independent
- if A and B are statistically independent, the occurrence of A does not affect the probability of occurrence of B

$$sup(A \cup B) \approx sup(A) * sup(B)$$

$$conf(A \rightarrow B) \approx conf(\emptyset \rightarrow B)$$

- $A \rightarrow B$ may have high support and confidence and still not be interesting.
  - $\{butter\} \rightarrow \{bread\}(sup = 5\%, conf = 95\%)$
  - it is not unexpected
  - it is not useful

How to measure the interest of a rule? (cont.)

- A measure of interest should evaluate the deviation from independence.
- A rule is unexpected as it deviates from independence.
- There are different approaches to measure this deviation:
  - *lift*
  - *conviction*
  - χ^2
  - *correlation*
  - ...

Measures of Interest: limitations of support and confidence

- Assume we are interested in studying the relationship between people who drink tea and coffee.
- We summarize the preferences of 1000 people

	Coffee	$\neg$*Coffee*	
Tea	150	50	200
$\neg$*Tea*	650	150	800
	800	200	1000

- How interesting is the rule $Tea \rightarrow Coffee$?
- $sup = 150/1000 = 15\%$ and $conf = 150/200 = 75\%$
- The confidence of the rule is high, however the likelihood of a person drinking coffee regardless of drinking tea is 80%.
- Knowing that a person drinks tea actually decreases the probability of drinking coffee (from 80% to 75%).
- Thus, the rule is indeed deceitful.
- High confidence rules can be misleading.

Measures of Interest: LIFT

- **lift** is the ratio between confidence of the rule and the support of the itemset appearing in the consequent:

$$lift(A \rightarrow B) = \frac{conf(A \rightarrow B)}{sup(B)} = \frac{sup(A \cup B)}{sup(A)sup(B)}$$

- Measures the influence of A in the presence of B.
- $lift = 1$: A and B are independent ($sup(A \cup B) = sup(A)sup(B)$).
- $lift < 1$: A and B are negatively correlated.
- $lift > 1$: A and B are positively correlated.

- $lift(Tea \rightarrow Coffee) = 0.15/(0.2 * 0.8) = 0.9375$
- negative correlation between tea and coffee drinkers.

Measures of Interest: LIFT (cont.)

- The **lift** is a measure of the deviation from a rule $A \rightarrow B$ regarding the statistical independence between the antecedent A and consequent B.

- Takes values between 0 and infinity:
  - a value close to 1 indicates that A and B often appear together
    - the occurrence of A has no effect on the occurrence of B.
  - a value smaller than 1 indicates that A and B appear less frequently than expected together
    - the occurrence of A has a negative effect on the occurrence of B, i.e. the occurrence of A is likely to lead to the absence of B.
  - a value greater than 1 indicates that A and B appear more often together than expected
    - the occurrence of A has a positive effect on the occurrence of B, i.e. the occurrence of A increases the likelihood of occurrence of B.

Measures of Interest: Conviction

- **lift** measures co-occurrence only (not implication) and is symmetric with respect to antecedent and consequent, i.e. $\mathit{lift}(A \rightarrow B) = \mathit{lift}(B \rightarrow A)$
- **conviction** is a measure proposed to tackle some of the weaknesses of *confidence* and **lift**.
- Unlike **lift**, **conviction** is sensitive to rule direction. It indicates the departure from independence of A and B taking into account the implication direction.
- Is inspired in the logical definition of implication and attempts to measure the degree of implication of a rule.

- **conviction** of a rule $A \rightarrow B$ is the ratio between
  - the expected frequency that A occurs without B, if A and B were independent
  - the observed frequency that the rule makes of incorrect predictions.
- Is the inverse **lift** of the rule $R' = A \rightarrow \neg B$.

$$conviction(A \rightarrow B) = \frac{1 - sup(B)}{1 - conf(A \rightarrow B)} = \frac{sup(A)sup(\neg B)}{sup(A \cup \neg B)}$$

Measures of Interest: Conviction (cont.)

- $conviction(A \to B) = 1$ indicates independence between A and B.
- A high value of **conviction** means that the consequent depends strongly on the antecedent.
- **conviction** increases a lot when *confidence* gets closer to 1.
- Example:
  - $sup(female) = 0.5$, $sup(mother) = 0.2$
  - $conf(mother \to female) = 1$
  - $lift(mother \to female) = 0.2/(0.2 * 0.5) = 2$
  - $conviction(mother \to female) = (1 - 0.5)/(1 - 1) = \infty$

Improving Apriori

- Challenges of Frequent Pattern Mining
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - Reduce number of transaction database scans
  - Shrink number of candidates (*bottleneck* of Apriori)
  - Facilitate support counting of candidates
- Some methods that improve Apriori's efficiency
  - Partitioning [Savasere et al., 1995]
  - Sampling [Toivonen, 1996]
  - Dynamic Itemset Counting [Brin et al., 1997]
  - Frequent Pattern Projection and Growth (FP-Growth) [Han et al., 2004]

Association Rules: Conclusions

- GOAL: Finding associations
- Association rule mining:
  - Frequent itemsets (requires min support)
  - Association rules (requires min confidence)
    - Probabilistic implications
- One of the most used data mining tools
  - Problem: generates too much rules
  - Pattern compression and pattern selection
- Several algorithms:
  - Apriori is the most known algorithm
  - There are variants of Apriori that return exactly the same patterns!
  - Completeness: find all rules.

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