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Generation of synthetic seismograms with layer reduction

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SUMMARY

The surface motion in a plane multilayered half-space has been considered. Layer matrices of SH and P–SV motions are written as sums of matrices. Using these it is seen that when shear wave velocity of a layer, say the lth, is greater than the phase velocity, and layer thickness is large compared to wavelength, the surface motion can be written with the lth layer as the terminating half-space. Thus in computation of surface motion, the depth of the terminating half-space in a given structure depends on phase velocity and frequency. Further, if the source is below the terminating half-space, it has no contribution to surface motion at that phase velocity and frequency for which the terminating half-space has been obtained. This layer reduction formulation is applied in the generation of synthetic seismograms. As this generation requires computation over large ranges of phase velocity and frequency, the present method is important in avoiding excess computation time for unnecessary layers and in avoiding overflow due to the above mentioned lth layer. The method is most useful in low phase velocity and high frequency ranges.

Key words: layer reduction, surface motion, synthetic seismograms.

1 INTRODUCTION

Synthetic seismograms are now widely used to study the source and the medium. Surface waves have been found to be useful as well as body waves. Here we shall consider the generation of synthetic seismograms in a layered half-space, each layer being homogeneous (Hudson 1969a,b). Synthetic seismograms in such a medium are generally obtained either by normal mode summation (Harvey 1981; Schwab *et al.* 1984; Panza & Suhadolc 1987) or by numerical integration (Kind 1979). A disadvantage of these methods is long computation time due to a large number of layers. Further, during computation in such a medium, overflow and loss of precision are encountered (Press, Harkrider & Seafeldt 1961; Schwab & Knopoff 1970). Loss of precision is reduced by using a compound layer matrix (Dunkin 1965; Thrower 1965; Schwab & Knopoff 1970). However, overflow occurs due to a deep layer where shear wave velocity is greater than phase velocity and the layer thickness is large compared to the wavelength. Schwab *et al.* (1984) controlled the overflow due to such a layer by simple normalization. Bhattacharya (1987) has shown that this layer can be taken as a terminating half-space in surface wave computation.

Since the time of Press *et al.* (1961) it is noted that structure minimization is a critical point regarding the efficiency and the accuracy of computation and a procedure for structure minimization has been proposed. In computing surface wave dispersion, Schwab & Knopoff (1970) proposed that the computed dispersion is correct to a specific accuracy, say σ significant figures, if $(\sigma + 1)/4$ wavelengths of layered structure is retained above a homogeneous half-space. In order to economize on computer time Kerry (1981) also proposed a structure reduction procedure. However, these methods are based on emperical relations. Panza & Suhadolc (1987) computed the displacement–depth function E_m just after evaluating the phase velocity for a given mode–frequency pair; if the minimum value of E_m is noted at the jth interface, the parameters of the $(j + 1)$ layer can be used as a terminating half-space for refinement of phase velocity and evaluating eigenfunctions. However, special care is required when low velocity layers are present in the structural model. Following Bhattacharya (1987), structural reduction is obtained even during evaluation of the phase velocity for a given mode–frequency pair. Here we extend the work of Bhattacharya (1987) and show that the expression of surface motion in a multilayered medium becomes an expression with the above mentioned deep layer as the terminating half-space. We use the notations of Hudson (1969a,b) and Wang & Herrmann (1980).

2 BASIC EQUATIONS

We consider $(N-1)$ homogeneous layers over a half-space (the Nth layer). The source is in the mth layer. From Hudson (1969a, equation 2.19) the Fourier-transformed surface displacement components in cylindrical coordinates are

$$\bar{u}_r(r, \phi, 0, \omega) = \sum_{n=0}^{\infty} \int_0^{\infty} \left\{ [U_r^c(0) \cos n\phi + U_r^s(0) \sin n\phi] \frac{\partial J_n(kr)}{\partial r} - \frac{n}{r} (U_\phi^c(0) \cos n\phi + U_\phi^s(0) \sin n\phi) J_n(kr) \right\} dk,$$

$$\bar{u}_\phi(r, \phi, 0, \omega) = \sum_{n=0}^{\infty} \int_0^{\infty} \left\{ \frac{n}{r} [U_r^s(0) \cos n\phi - U_r^c(0) \sin n\phi] J_n(kr) - (U_\phi^s(0) \cos n\phi - U_\phi^c(0) \sin n\phi) \frac{\partial J_n(kr)}{\partial r} \right\} dk,$$

$$\bar{u}_z(r, \phi, 0, \omega) = \sum_{n=0}^{\infty} \int_0^{\infty} [U_z^c(0) \cos n\phi + U_z^s(0) \sin n\phi] J_n(kr)\, dk. \tag{1}$$

In equation (1) $U_\phi^{c,s}(0)$ are obtained from

$$U_\phi^{c,s}(0) = -\frac{x_{11} s_1^{c,s} + x_{12} s_2^{c,s}}{r_{11}} \tag{2}$$

(Wang & Herrmann 1980), where

$$\mathbf{x} = \mathbf{f}\mathbf{e}_N^{-1}\mathbf{a}_{N-1} \cdots \mathbf{a}_{m+1}\mathbf{a}_{m2},$$

$$\mathbf{r} = \mathbf{f}\mathbf{e}_N^{-1}\mathbf{a}_{N-1} \cdots \mathbf{a}_1, \tag{3}$$

$$\mathbf{a}_{m2} = \mathbf{a}_m(d_m - h_m), \quad \mathbf{a}_{m1} = \mathbf{a}_m(h_m),$$

$$\mathbf{f} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} 1/(\rho v_\beta) & 0 \\ 0 & -\beta^2 \end{pmatrix},$$

$$v_\beta = \sqrt{k^2 - \omega^2/\beta^2}, \quad Q = v_\beta d, \tag{4}$$

$$\mathbf{h} = \begin{pmatrix} \cosh Q & -\sinh Q \\ -\sinh Q & \cosh Q \end{pmatrix},$$

$$\mathbf{a} = \mathbf{a}(d) = \mathbf{e}\mathbf{h}\mathbf{e}^{-1}. \tag{5}$$

Here $\mathbf{a}$ is layer matrix for SH motion and $s_1^{c,s}$, $s_2^{c,s}$ are due to the source which is in the mth layer at a depth h_m beneath $m-1$ interface. $U_r^{c,s}(0)$ and $U_z^{c,s}(0)$ are obtained from

$$\mathbf{R}\mathbf{B}_0^{c,s} = \mathbf{F}\mathbf{K}_N - \mathbf{X}\mathcal{S}^{c,s} \tag{6a}$$

$$= \mathbf{F}\mathbf{K}_N - \mathbf{R}\mathbf{Z}^{-1}\mathcal{S}^{c,s} \tag{6b}$$

(Hudson 1969a,b; Wang & Herrmann 1980, equation 6), where

$$\mathbf{B}_0^{c,s} = [U_r^{c,s}(0), U_z^{c,s}(0), 0, 0]^{\mathrm{T}},$$

$$\mathbf{X} = \mathbf{F}\mathbf{E}_N^{-1}\mathbf{A}_{N-1} \cdots \mathbf{A}_{m+1}\mathbf{A}_{m2},$$

$$\mathbf{R} = \mathbf{F}\mathbf{E}_N^{-1}\mathbf{A}_{N-1} \cdots \mathbf{A}_1, \tag{7}$$

$$\mathbf{Z} = \mathbf{A}_{m1}\mathbf{A}_{m-1} \cdots \mathbf{A}_1,$$

$$\mathbf{K}_N = [A', A', B', B']^{\mathrm{T}},$$

$$\mathbf{A}_{m2} = \mathbf{A}_m(d_m - h_m), \quad \mathbf{A}_{m1} = \mathbf{A}_m(h_m),$$

$$\mathbf{F} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix},$$

$$\mathbf{E} = \begin{pmatrix} -1/\rho & 0 & -v_\beta/\rho & 0 \\ 0 & v_\alpha/\rho & 0 & k^2/\rho \\ 1-\gamma & 0 & -\gamma v_\beta & 0 \\ 0 & \gamma v_\alpha/k^2 & 0 & \gamma - 1 \end{pmatrix},$$

$$v_\alpha = \sqrt{k^2 - \omega^2/\alpha^2}, \quad P = v_\alpha d, \quad \gamma = 2\beta^2/c^2,$$

$$\mathbf{H}=\begin{pmatrix}\cosh P & -\sinh P & 0 & 0\\ -\sinh P & \cosh P & 0 & 0\\ 0 & 0 & \cosh Q & -\sinh Q\\ 0 & 0 & -\sinh Q & \cosh Q\end{pmatrix},$$

$$\mathbf{A}=\mathbf{A}(d)=\mathbf{EHE}^{-1}. \tag{8}$$

Here $\mathbf{A}$ is a layer matrix for P–SV motion; $\mathscr{S}^{c,s}$ is a 4×1 matrix and depends on the source; A', B' are coefficients in the downgoing wave solutions for the compressional and shear wave potentials, respectively, in the Nth layer.

Using (6a), Wang & Herrmann (1980) obtained

$$\begin{pmatrix}U_r^{c,s}(0)\\ U_z^{c,s}(0)\end{pmatrix}=\frac{-1}{\bar{R}_{11}}\begin{pmatrix}\mathscr{S}_i^{c,s}X|_{ij}^{12}Z_{j2}\\ -\mathscr{S}_i^{c,s}X|_{ij}^{12}Z_{j1}\end{pmatrix}, \tag{9}$$

where

$$\bar{\mathbf{R}}=\bar{\mathbf{F}}\bar{\mathbf{E}}_N^{-1}\bar{\mathbf{A}}_{N-1}\cdots\bar{\mathbf{A}}_1, \tag{10}$$

and a bar denotes the compound matrix of the corresponding matrix. In (9) the summation convention for subscripts has been used.

In the following we shall omit the superscript c, s in U, $\mathscr{S}$ and s.

3 LAYER REDUCTION FOR U_ϕ (SH MOTION)

Expression (2) for $U_\phi(0)$ is used in the computations. However to prove layer reduction statements we shall obtain another expression for $U_\phi(0)$. We note that

$$\mathbf{x}=\mathbf{rz}^{-1},$$

where $\mathbf{z}=\mathbf{a}_{m1}\mathbf{a}_{m-1}\cdots\mathbf{a}_1$. Using $\mathbf{a}^{-1}(d)=\mathbf{a}(-d)$ and the relations between elements of layer matrix $\mathbf{a}$ we have

$$\mathbf{z}^{-1}=\begin{pmatrix}z_{22} & -z_{12}\\ -z_{21} & z_{11}\end{pmatrix}.$$

Therefore from (2) we get

$$U_\phi(0)=-\left[s_1\left(z_{22}-\frac{r_{12}}{r_{11}}z_{21}\right)+s_2\left(-z_{12}+\frac{r_{12}}{r_{11}}z_{11}\right)\right], \tag{11}$$

where from (3)

$$(r_{11}, r_{12})=(1, -1)\mathbf{e}_N^{-1}\mathbf{a}_{N-1}\cdots\mathbf{a}_1 \tag{12a}$$

$$=(\rho_N\nu_{\beta N}, 1/\beta_N^2)\mathbf{a}_{N-1}\cdots\mathbf{a}_1. \tag{12b}$$

Expressing $\mathbf{h}$ *as a sum of matrices*

We may write $\mathbf{h}$ in equation (5) as

$$\mathbf{h}=\frac{1}{2}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\operatorname{diag}(e^Q, e^{-Q})\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix} \tag{13}$$

$$=\frac{1}{2}\left[e^Q\begin{pmatrix}1\\ -1\end{pmatrix}(1\quad -1)+e^{-Q}\begin{pmatrix}1\\ 1\end{pmatrix}(1\quad 1)\right]. \tag{14}$$

Statement I

At a frequency ω and a phase velocity c, for U_ϕ, a layer l becomes the terminating half-space if $\mathscr{R}e\, Q_l$ is so large that the contribution of the second term of $\mathbf{h}_l$ on the right-hand side of (14) is insignificant compared to that of the first term.

Proof. We shall assume that the source is above the lth layer. (The case of the source below the lth layer is dealt with in statement II.) Neglecting the second term in (14) for $\mathbf{h}_l$ we write (12a) as

$$(r_{11}, r_{12})=(1, -1)\mathbf{e}_N^{-1}\mathbf{a}_{N-1}\cdots\mathbf{a}_{l+1}\tfrac{1}{2}\mathbf{e}_l e^{Q_l}\begin{pmatrix}1\\ -1\end{pmatrix}(1, -1)\mathbf{e}_l^{-1}\mathbf{a}_{l-1}\cdots\mathbf{a}_1$$

$$=\eta(1, -1)\mathbf{e}_l^{-1}\mathbf{a}_{l-1}\cdots\mathbf{a}_1.$$

We find the scalar factor η is common to r_{11} and r_{12}. So for this case (r_{11}, r_{12}) in (11) is given by

$$(r_{11}, r_{12}) = (1, -1)\mathbf{e}_l^{-1}\mathbf{a}_{l-1} \cdots \mathbf{a}_1, \tag{15}$$

which when compared with (12a) shows that we are considering layer l as the terminating half-space.

We note that

$$Q = \nu_\beta d = kd\sqrt{1 - \frac{c^2}{\beta^2}} = \frac{2\pi d}{\lambda}\sqrt{1 - \frac{c^2}{\beta^2}},$$

where λ is the wavelength. Thus $\mathcal{R}e\, Q$ is large when $c > \beta$ and d/λ is large.

We further note that Schwab *et al.* (1984) removed the second term on the right-hand side of (14) in case Q is real and large but did not consider it as the terminating half-space.

Statement II

If the source is below the terminating half-space, the source has no contribution to surface motion at that frequency and phase velocity for which terminating half-space has been obtained.

Proof. We suppose that layer l is the terminating half-space for $U_\phi(0)$, whose expression is given in (11); however (r_{11}, r_{12}) is given by (15). We have

$$\begin{aligned}\begin{pmatrix} z_{11} \\ z_{21} \end{pmatrix} &= \mathbf{a}_{m1}\mathbf{a}_{m-1} \cdots \mathbf{a}_{l+1}\mathbf{e}_l \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{Q_l}(1, -1)\mathbf{e}_l^{-1}\mathbf{a}_{l-1} \cdots \mathbf{a}_2 \begin{pmatrix} a_{1(11)} \\ a_{1(21)} \end{pmatrix} \\ &= \mathring{\mathbf{a}} e^{Q_l} r_{11},\end{aligned} \tag{16}$$

where r_{11} is given by (15) and

$$\mathring{\mathbf{a}} = \mathbf{a}_{m1}\mathbf{a}_{m-1} \cdots \mathbf{a}_{l+1}\mathbf{e}_l \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{17}$$

From (16)

$$\begin{aligned}\frac{r_{12}}{r_{11}}\begin{pmatrix} z_{11} \\ z_{21} \end{pmatrix} &= \mathring{\mathbf{a}} e^{Q_l} r_{12} \\ &= \mathring{\mathbf{a}} e^{Q_l}(1, -1)\mathbf{e}_l^{-1}\mathbf{a}_{l-1} \cdots \mathbf{a}_2 \begin{pmatrix} a_{1(12)} \\ a_{1(22)} \end{pmatrix} \\ &= \begin{pmatrix} z_{12} \\ z_{22} \end{pmatrix}.\end{aligned}$$

Thus

$$z_{12} = \frac{r_{12}}{r_{11}} z_{11} \quad \text{and} \quad z_{22} = \frac{r_{12}}{r_{11}} z_{21},$$

and substituting these in (11) we find $U_\phi(0) = 0$.

4 LAYER REDUCTION FOR U_r, U_z (P–SV MOTION)

Although for $U_r(0)$, $U_z(0)$ expression (9) is useful for computational purpose, we obtain another expression which will be used to prove the layer reduction statements. Using (6b) we obtain

$$\begin{pmatrix} U_r(0) \\ U_z(0) \end{pmatrix} = -\frac{1}{\bar{R}_{11}}\begin{pmatrix} \bar{R}_{11} & 0 & -\bar{R}_{14} & -\bar{R}_{15} \\ 0 & \bar{R}_{11} & \bar{R}_{12} & \bar{R}_{13} \end{pmatrix}\mathbf{Z}^{-1}\mathcal{S}, \tag{18}$$

where using the first row of (10) we have

$$(\bar{R}_{11} \quad \bar{R}_{12} \quad \bar{R}_{13} \quad \bar{R}_{14} \quad \bar{R}_{15} \quad \bar{R}_{16}) = (0 \quad -1 \quad 1 \quad 1 \quad -1 \quad 0)\bar{\mathbf{E}}_N^{-1}\bar{\mathbf{A}}_{N-1} \cdots \bar{\mathbf{A}}_1. \tag{19}$$

Using $\mathbf{A}^{-1}(d) = \mathbf{A}(-d)$ and relations between the elements of layer matrix (Haskell 1964) we have

$$\mathbf{Z}^{-1} = \begin{pmatrix} Z_{44} & Z_{34}/k^2 & -Z_{24}/k^2 & -Z_{14} \\ k^2 Z_{43} & Z_{33} & -Z_{23} & -k^2 Z_{13} \\ -k^2 Z_{42} & -Z_{32} & Z_{22} & k^2 Z_{12} \\ -Z_{41} & -Z_{31}/k^2 & Z_{21}/k^2 & Z_{11} \end{pmatrix}. \tag{20}$$

*Expressing **H** and $\bar{\boldsymbol{H}}$ as a sum of matrices*

We may write **H** in equation (8) as

$$\mathbf{H}=\frac{1}{2}\begin{pmatrix}1&0&1&0\\-1&0&1&0\\0&-1&0&1\\0&1&0&1\end{pmatrix}\operatorname{diag}(e^{P},e^{Q},e^{-P},e^{-Q})\begin{pmatrix}1&-1&0&0\\0&0&-1&1\\1&1&0&0\\0&0&1&1\end{pmatrix}$$

$$=\frac{1}{2}\left[e^{P}\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}(1\;\;-1\;\;0\;\;0)+e^{Q}\begin{pmatrix}0\\0\\-1\\1\end{pmatrix}(0\;\;0\;\;-1\;\;1)+e^{-P}\begin{pmatrix}1\\1\\0\\0\end{pmatrix}(1\;\;1\;\;0\;\;0)+e^{-Q}\begin{pmatrix}0\\0\\1\\1\end{pmatrix}(0\;\;0\;\;1\;\;1)\right]. \tag{21}$$

Thus

$\bar{\mathbf{A}}=\bar{\mathbf{E}}\bar{\mathbf{H}}\bar{\mathbf{E}}^{-1}$,

where

$$\bar{\mathbf{H}}=\frac{1}{4}\begin{pmatrix}0&2&0&0&0&0\\-1&0&1&1&0&1\\1&0&1&-1&0&1\\1&0&-1&1&0&1\\-1&0&-1&-1&0&1\\0&0&0&0&-2&0\end{pmatrix}\operatorname{diag}(e^{P+Q},1,e^{P-Q},e^{Q-P},1,e^{-P-Q})\begin{pmatrix}0&-1&1&1&-1&0\\2&0&0&0&0&0\\0&1&1&-1&-1&0\\0&1&-1&1&-1&0\\0&0&0&0&0&-2\\0&1&1&1&1&0\end{pmatrix}$$

$$=\frac{1}{4}\left[e^{P+Q}\begin{pmatrix}0\\-1\\1\\1\\-1\\0\end{pmatrix}(0\;\;-1\;\;1\;\;1\;\;-1\;\;0)+\begin{pmatrix}2\\0\\0\\0\\0\\0\end{pmatrix}(2\;\;0\;\;0\;\;0\;\;0\;\;0)\right.$$

$$+e^{P-Q}\begin{pmatrix}0\\1\\1\\-1\\-1\\0\end{pmatrix}(0\;\;1\;\;1\;\;-1\;\;-1\;\;0)+e^{Q-P}\begin{pmatrix}0\\1\\-1\\1\\-1\\0\end{pmatrix}(0\;\;1\;\;-1\;\;1\;\;-1\;\;0)$$

$$\left.+\begin{pmatrix}0\\0\\0\\0\\0\\-2\end{pmatrix}(0\;\;0\;\;0\;\;0\;\;0\;\;-2)+e^{-P-Q}\begin{pmatrix}0\\1\\1\\1\\1\\0\end{pmatrix}(0\;\;1\;\;1\;\;1\;\;1\;\;0)\right]. \tag{22}$$

Statement III

At a frequency ω and a phase velocity c, for $U_r(0)$ and $U_z(0)$, a layer l becomes the terminating half-space if $\mathcal{R}e\, P_l$ and $\mathcal{R}e\, Q_l$ are so large that the contribution of last five terms of $\bar{\mathbf{H}}_l$ on the right-hand side of (22) is insignificant compared to that of the first term.

Proof. We shall assume that the source is above the lth layer. Retaining only the first term of $\bar{\mathbf{H}}_l$ in (22) we have from (19)

$$(\bar{R}_{11}\quad \bar{R}_{12}\quad \bar{R}_{13}\quad \bar{R}_{14}\quad \bar{R}_{15}\quad \bar{R}_{16}) = (0\quad -1\quad 1\quad 1\quad -1\quad 0)\bar{\mathbf{E}}_N^{-1}\bar{\mathbf{A}}_{N-1}\cdots\bar{\mathbf{A}}_{l+1}\frac{1}{4}e^{P_l+Q_l}\bar{\mathbf{E}}_l$$

$$\times\begin{pmatrix}0\\-1\\1\\1\\-1\\0\end{pmatrix}(0\quad -1\quad 1\quad 1\quad -1\quad 0)\bar{\mathbf{E}}_l^{-1}\bar{\mathbf{A}}_{l-1}\cdots\bar{\mathbf{A}}_1$$

$$=\zeta(0\quad -1\quad 1\quad 1\quad -1\quad 0)\bar{\mathbf{E}}_l^{-1}\bar{\mathbf{A}}_{l-1}\cdots\bar{\mathbf{A}}_1.$$

We find the scalar factor ζ is common to all $\bar{R}_{1i}$ $(i=1,\ldots,6)$. So for this case in (18) we can take

$$(\bar{R}_{11}\quad \bar{R}_{12}\quad \bar{R}_{13}\quad \bar{R}_{14}\quad \bar{R}_{15}\quad \bar{R}_{16}) = (0\quad -1\quad 1\quad 1\quad -1\quad 0)\bar{\mathbf{E}}_l^{-1}\bar{\mathbf{A}}_{l-1}\cdots\bar{\mathbf{A}}_1, \tag{23}$$

which means that we are considering lth layer as terminating half-space.

Proof of statement II for $U_r(0)$ and $U_z(0)$. We suppose that the layer l is the terminating half-space. $U_r(0)$ and $U_z(0)$ are given by (18) but $\bar{R}_{1i}$ $(i=1,\ldots,6)$ are given by (23). In this case $\mathbf{H}_l$ is given by the first two terms of (21). Thus

$$\mathbf{Z}=\mathbf{A}_{m1}\mathbf{A}_{m-1}\cdots\mathbf{A}_{l+1}\mathbf{E}_l\frac{1}{2}\begin{pmatrix}1&0\\-1&0\\0&-1\\0&1\end{pmatrix}\operatorname{diag}(e^{P_l},e^{Q_l})\begin{pmatrix}1&-1&0&0\\0&0&-1&1\end{pmatrix}\mathbf{E}_l^{-1}\mathbf{A}_{l-1}\cdots\mathbf{A}_1$$

$$=\mathring{\mathbf{A}}\operatorname{diag}(e^{P_l},e^{Q_l})\begin{pmatrix}R_{11}&R_{12}&R_{13}&R_{14}\\R_{21}&R_{22}&R_{23}&R_{24}\end{pmatrix},$$

where $\mathbf{R}$ is now (compare with equation 7)

$$\mathbf{R}=\mathbf{F}\mathbf{E}_l^{-1}\mathbf{A}_{l-1}\cdots\mathbf{A}_1, \tag{24}$$

and

$$\mathring{\mathbf{A}}=\mathbf{A}_{m1}\mathbf{A}_{m-1}\cdots\mathbf{A}_{l+1}\mathbf{E}_l\frac{1}{2}\begin{pmatrix}1&0\\-1&0\\0&-1\\0&1\end{pmatrix}.$$

Thus

$$\mathbf{Z}=e^{P_l}\begin{pmatrix}\mathring{A}_{11}\\\mathring{A}_{21}\\\mathring{A}_{31}\\\mathring{A}_{41}\end{pmatrix}(R_{11}\quad R_{12}\quad R_{13}\quad R_{14})+e^{Q_l}\begin{pmatrix}\mathring{A}_{12}\\\mathring{A}_{22}\\\mathring{A}_{32}\\\mathring{A}_{42}\end{pmatrix}(R_{21}\quad R_{22}\quad R_{23}\quad R_{24}). \tag{25}$$

From (25)

$$Z_{ij}=e^{P_l}\mathring{A}_{i1}R_{1j}+e^{Q_l}\mathring{A}_{i2}R_{2j}, \tag{26}$$

$i=1,\ldots,4$ and $j=1,\ldots,4$. From (18) and (20)

$$U_r(0)=-\frac{1}{\bar{R}_{11}}\Big[(\bar{R}_{11}Z_{44}+\bar{R}_{14}k^2Z_{42}+\bar{R}_{15}Z_{41})\mathscr{S}_1+\frac{1}{k^2}(\bar{R}_{11}Z_{34}+\bar{R}_{14}k^2Z_{32}+\bar{R}_{15}Z_{31})\mathscr{S}_2$$

$$-\frac{1}{k^2}(\bar{R}_{11}Z_{24}+\bar{R}_{14}k^2Z_{22}+\bar{R}_{15}Z_{21})\mathscr{S}_3-(\bar{R}_{11}Z_{14}+\bar{R}_{14}k^2Z_{12}+\bar{R}_{15}Z_{11})\mathscr{S}_4\Big]. \tag{27}$$

The right-hand side of (27) in parenthesis contains

$$\bar{R}_{11}Z_{i4}+\bar{R}_{14}k^2Z_{i2}+\bar{R}_{i5}Z_{i1}, \tag{28}$$

$i=1,\ldots,4$. Using (26) we note

$$\bar{R}_{11}Z_{i4}+\bar{R}_{15}Z_{i1}=e^{P_l}\mathring{A}_{i1}(\bar{R}_{11}R_{14}+\bar{R}_{15}R_{11})+e^{Q_l}\mathring{A}_{i2}(\bar{R}_{11}R_{24}+\bar{R}_{15}R_{21}). \tag{29}$$

Substituting

$$\bar{R}_{11}=R_{11}R_{22}-R_{12}R_{21} \quad \text{and} \quad \bar{R}_{15}=R_{12}R_{24}-R_{22}R_{14}$$

in (29) we have

$$\begin{aligned}\bar{R}_{11}Z_{i4}+\bar{R}_{15}Z_{i1}&=e^{P_i}\mathring{A}_{i1}R_{12}(R_{11}R_{24}-R_{14}R_{21})+e^{Q_i}\mathring{A}_{i2}R_{22}(R_{11}R_{24}-R_{14}R_{21})\\&=(e^{P_i}\mathring{A}_{i1}R_{12}+e^{Q_i}\mathring{A}_{i2}R_{22})\bar{R}_{13} \quad \text{since } \bar{R}_{13}=R_{11}R_{24}-R_{14}R_{21}\\&=\bar{R}_{13}Z_{i2} \quad \text{using (26)}\\&=-k^2\bar{R}_{14}Z_{i2},\end{aligned} \tag{30}$$

using $\bar{R}_{13}=-k^2\bar{R}_{14}$ (Wang & Herrmann 1980). Substituting (30) we find that the expression (28) becomes zero and thus from (27), $U_r(0)=0$.

Similarly we can prove $U_z(0)=0$.

5 APPLICATION

In order to apply the layer reduction formulation we have considered the Gutenberg–Bullen A (GBA) model (Alterman, Zarosch & Pekeris 1961; Kanamori & Abe 1968). The model has 24 layers down to 1×10^6 m. Below this depth we have taken the layer as a homogeneous half-space in the initial model. The model is shown in Fig. 1. As mentioned earlier, we divide a source layer into upper and lower portions of the source, i.e. m_1 and m_2 layers respectively.

We shall consider far field; so that the transverse component is U_ϕ and radial and vertical components are U_r and U_z respectively. As described below, two programs have been prepared: one for the transverse component, and other for the vertical and radial components. Integration is done numerically over $p=1/c$ and ω. The procedure follows from Kind & Odom (1983) and Müller (1985). The results of the programs are verified by comparison with the synthetic seismograms given by Wang & Herrman (1980) and Müller (1985).

We obtain synthetic seismograms at an epicentral distance 1.50×10^6 m from a source with focal depths 1.50×10^4, 5.50×10^4 and 2.75×10^5 m. We consider the source time function of Wang & Herrmann (1980) with rise time $=4\tau=8$ s. The

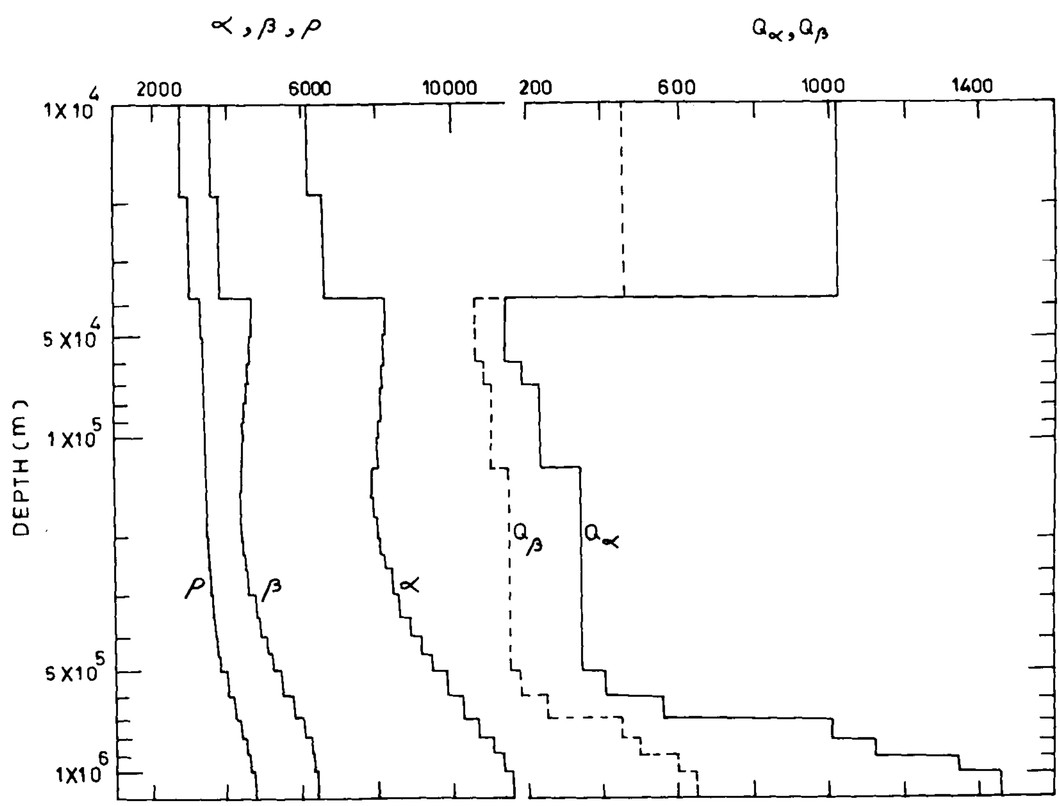

Figure 1. Gutenberg–Bullen A model (Alterman *et al.* 1961; Kanamori & Abe 1968); ρ in kg m^{-3}, α in m s^{-1}, β in m s^{-1}. The quality factors Q_α, Q_β are from Ben-Menahem & Singh (1981).

range of phase velocity c is from 2200 to 9000 m s^{-1} and the range of frequency ($f = \omega/2\pi$) is from 0 to 0.5 Hz. Since the present method is beneficial in the range of low values of c, we have obtained synthetics from the time a little before S-wave arrivals.

5.1 Transverse component

We evaluate U_ϕ according to equation (2). Denoting

$$\mathbf{b}_l = \mathbf{a}_l \mathbf{a}_{l-1} \cdots \mathbf{a}_1,$$

we note

$$\begin{aligned}\mathbf{b}_l &= \mathbf{a}_l \mathbf{b}_{l-1}\\ &= \begin{pmatrix} 1/(\rho v_\beta) & 0 \\ 0 & -\beta^2 \end{pmatrix}_l \mathbf{h}_l \begin{pmatrix} \rho v_\beta & 0 \\ 0 & -1/\beta^2 \end{pmatrix}_l \mathbf{b}_{l-1}\\ &= \frac{1}{2}\left[\begin{pmatrix} 1/(\rho v_\beta) \\ \beta^2 \end{pmatrix}(\rho v_\beta \quad 1/\beta^2)e^Q + \begin{pmatrix} 1/(\rho v_\beta) \\ -\beta^2 \end{pmatrix}(\rho v_\beta \quad -1/\beta^2)e^{-Q}\right]_l \mathbf{b}_{l-1},\end{aligned}$$

using (14). Thus

$$(b_{1i})_l = X_i + Y_i \qquad (i = 1, 2),$$

where

$$X_i = \frac{e^{Q_l}}{2\rho_l v_{\beta l}}(\rho v_\beta \quad 1/\beta^2)_l \begin{pmatrix} b_{1i} \\ b_{2i} \end{pmatrix}_{l-1},$$

$$Y_i = \frac{e^{-Q_l}}{2\rho_l v_{\beta l}}(\rho v_\beta \quad -1/\beta^2)_l \begin{pmatrix} b_{1i} \\ b_{2i} \end{pmatrix}_{l-1}.$$

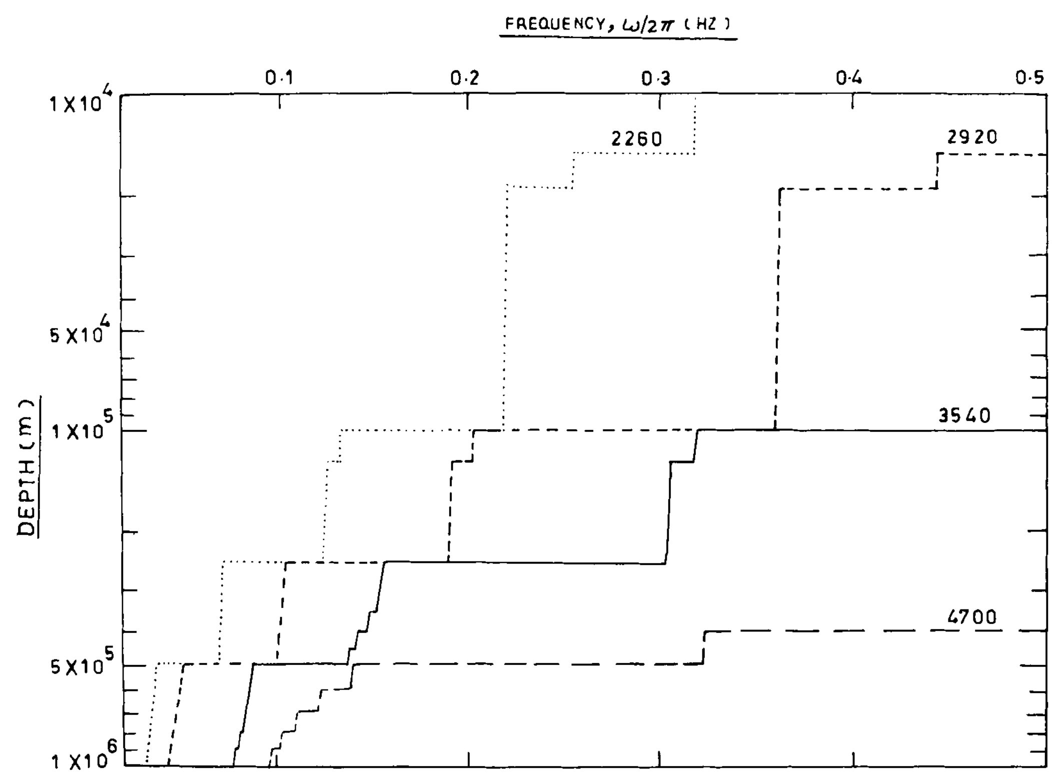

Figure 2. The depth of the top of the terminating half-space versus frequency for selected phase velocities during computation of transverse component of synthetic seismograms for focal depth 1.50×10^4 m with $\epsilon = 0.1 \times 10^{-8}$. Values of c (in m s^{-1}) marked on each curve to which it corresponds.

On the other hand

$(b_{2i})_l = \beta_l^2 \rho_l \nu_{\beta l}(X_i - Y_i).$

For a layer l where $\beta_l > c$ we compute X_i, Y_i before computing $\mathbf{b}_l$. If

$$Y_i/X_i < \epsilon \qquad (i = 1, 2), \tag{31}$$

we consider the lth layer as the terminating half-space. With the lth layer as a half-space $r_{11} = X_1(2\rho_1 \nu_{\beta l}/e^{Q_l})$. In (31), ϵ is a small quantity. It is seen that in model GBA, $\bar{u}_\phi$ with six significant figure accuracy can be achieved with $\epsilon = 0.1 \times 10^{-8}$. Fig. 2

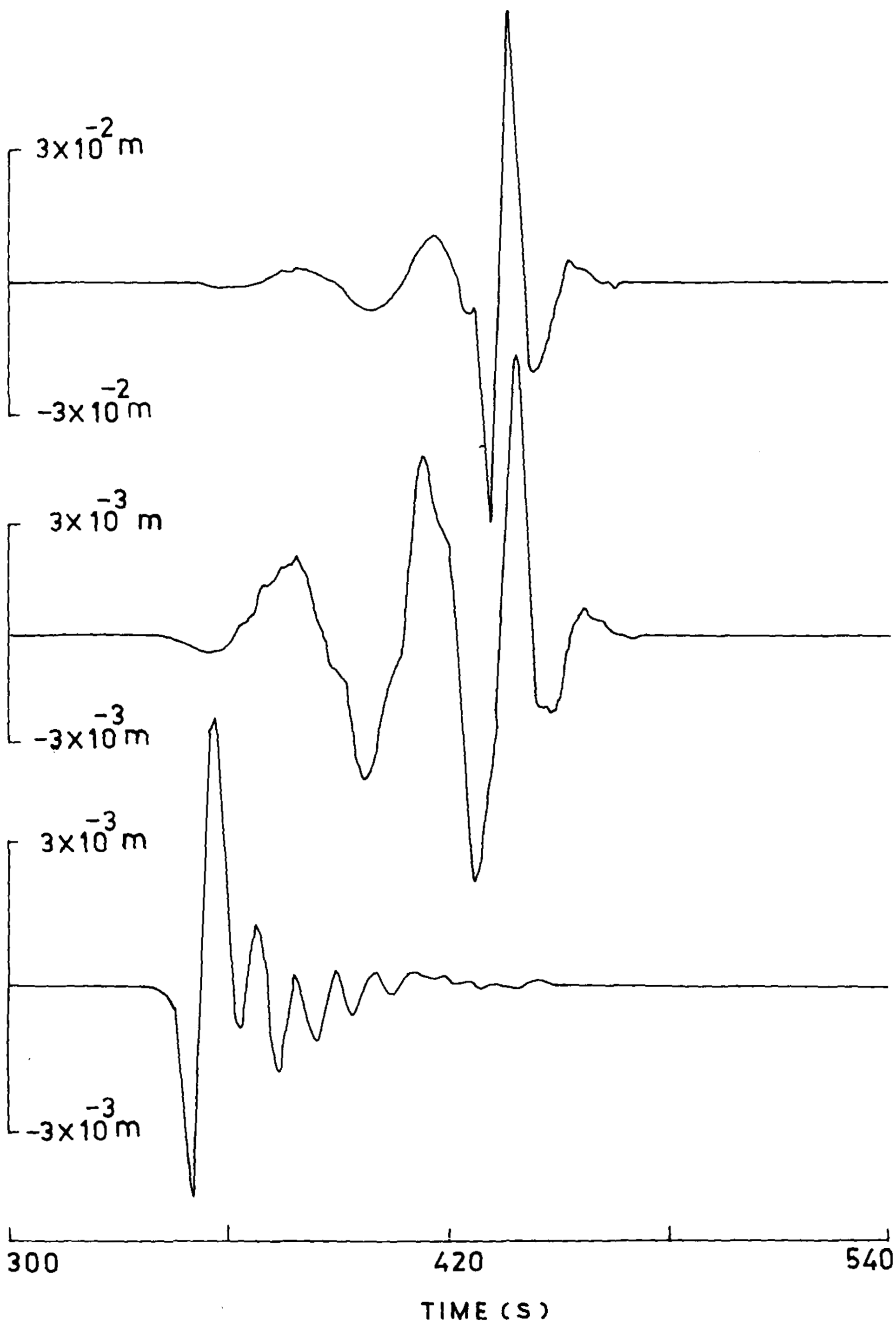

Figure 3. Transverse component of displacement (TSS) at epicentral distance 1.50×10^6 m for vertical strike–slip source with moment 10^{22} dyn m for focal depths 1.50×10^4 (top), 5.50×10^4 (middle) and 2.75×10^5 m (bottom) as recorded by a long period seismograph (15–100 s) with peak magnification 1500.

shows the depth of the top of the terminating half-space with this value of ϵ at different frequencies at a few selected phase velocities. As frequency increases the depth either remains the same or decreases. However, in a model in which a layer above the low velocity zone (LVZ) is thick and the S-wave velocity in the LVZ (β_{LVZ}) is low, it is seen that $c > \beta_{LVZ}$, although for a range of frequencies, a layer above the LVZ becomes a terminating half-space, while at a higher frequency range the terminating half-space can be below the LVZ.

Although initially we choose the maximum frequency $f_{max} = 0.5$ Hz, for a phase velocity the actual highest frequency limit f_{lim} ($\leq f_{max}$) will depend on the source depth as follows. As stated before, at a given phase velocity and frequency, if the source is below the terminating layer l, then U_ϕ is zero. Thus at a given phase velocity, f_{lim} is the frequency above which the terminating layer remains above the source.

Examples of the transverse component of displacement (TSS) computed in this manner for vertical strike–slip sources are shown in Fig. 3. Here, TSS has the same meaning as Herrmann & Wang (1985).

When

$$\mathscr{Re}\, Q_l > \bar{Q} \tag{32}$$

(where $\bar{Q}$ is real and large), the second term of $\mathbf{h}_l$ on the right-hand side of (14) is negligible, and from statement I the layer l becomes the terminating half-space. Therefore a simpler method can be introduced by making a layer l the terminating half-space if (32) is valid. The value of $\bar{Q}$ increases with an increase of accuracy. In the former method based on (31) with $\epsilon = 0.1 \times 10^{-8}$, it is noted that the $\mathscr{Re}\, Q_l$ value, which makes a layer l the terminating half-space, is generally around 9, but can sometimes take a value around 10. When we use the latter method we must fix $\bar{Q}$ as 10 for the same accuracy. Thus, the former method is slightly more efficient than the latter one for reducing deep layers; however the latter method can easily be applied to any available program.

5.2 Vertical and radial components

For U_r and U_z we have applied the latter method. Here for a given phase velocity and frequency, if (32) is valid, the contribution by the sum of the last five terms in $\mathbf{H}_l$ on the right-hand side of (22) is negligible compared to that of the first term, and by statement III the layer l becomes the terminating half-space. Since $\alpha > \beta$, here we have considered $\mathscr{Re}\, P > \mathscr{Re}\, Q$ if $\mathscr{Re}\, Q > 0$.

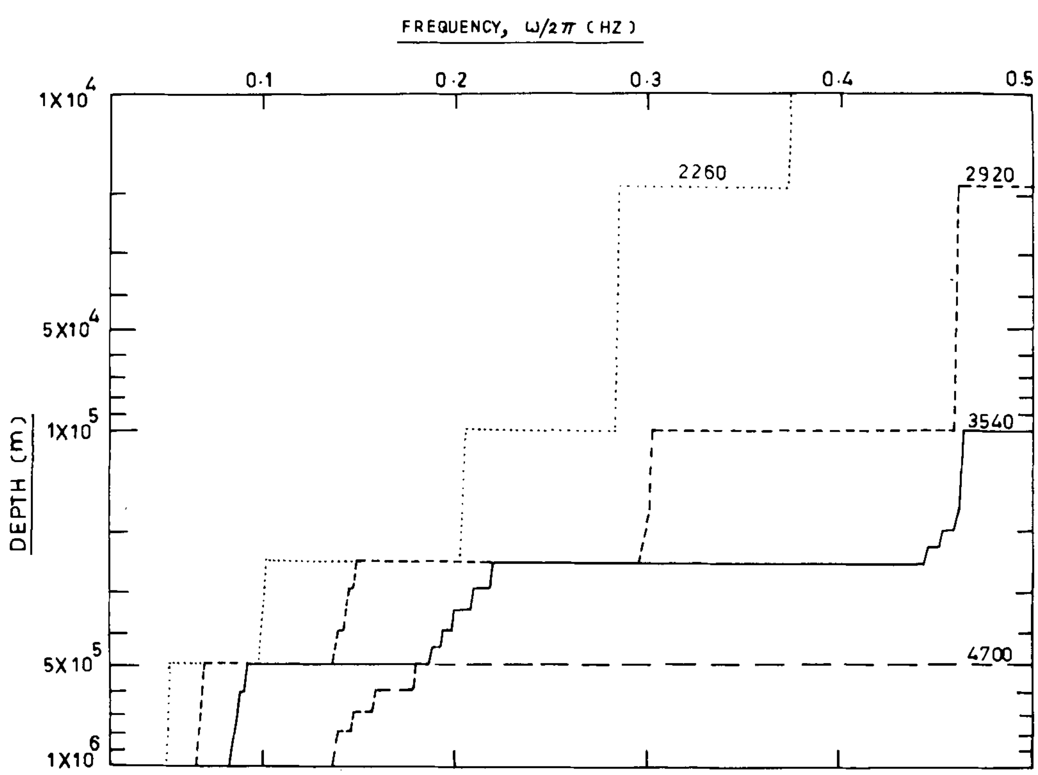

Figure 4. The depth of the top of the terminating half-space versus frequency for selected phase velocities during computation of vertical and radial components of synthetic seismograms for focal depth 1.50×10^4 m with $\bar{Q} = 12$. Values of c (in m s^{-1}) are marked on each curve to which it corresponds.

We modify the program of Kind & Odom (1983) as follows. For each c and for each layer i we obtain the lowest frequency f_i which satisfies (32); however if $f_i > f_{i-1}$ then $f_i = f_{i-1}$, or, if $f_i > f_{\max}$, then $f_i = f_{\max}$. Thus for that c we compute U_r and U_z with the ith layer as the terminating half-space for frequency f with $f_i < f \leq f_{i-1}$ if $f_i < f_{i-1}$; if $f_i = f_{i-1}$, the terminating half-space is below the ith layer for $f \leq f_i$ and above the ith layer for $f > f_i$. We are to compute U_r and U_z only when the terminating half-space is below the source. Thus the actual highest frequency limit for computation is $f_{\lim} = f_{m1}$, where m_1 is the layer just above the source.

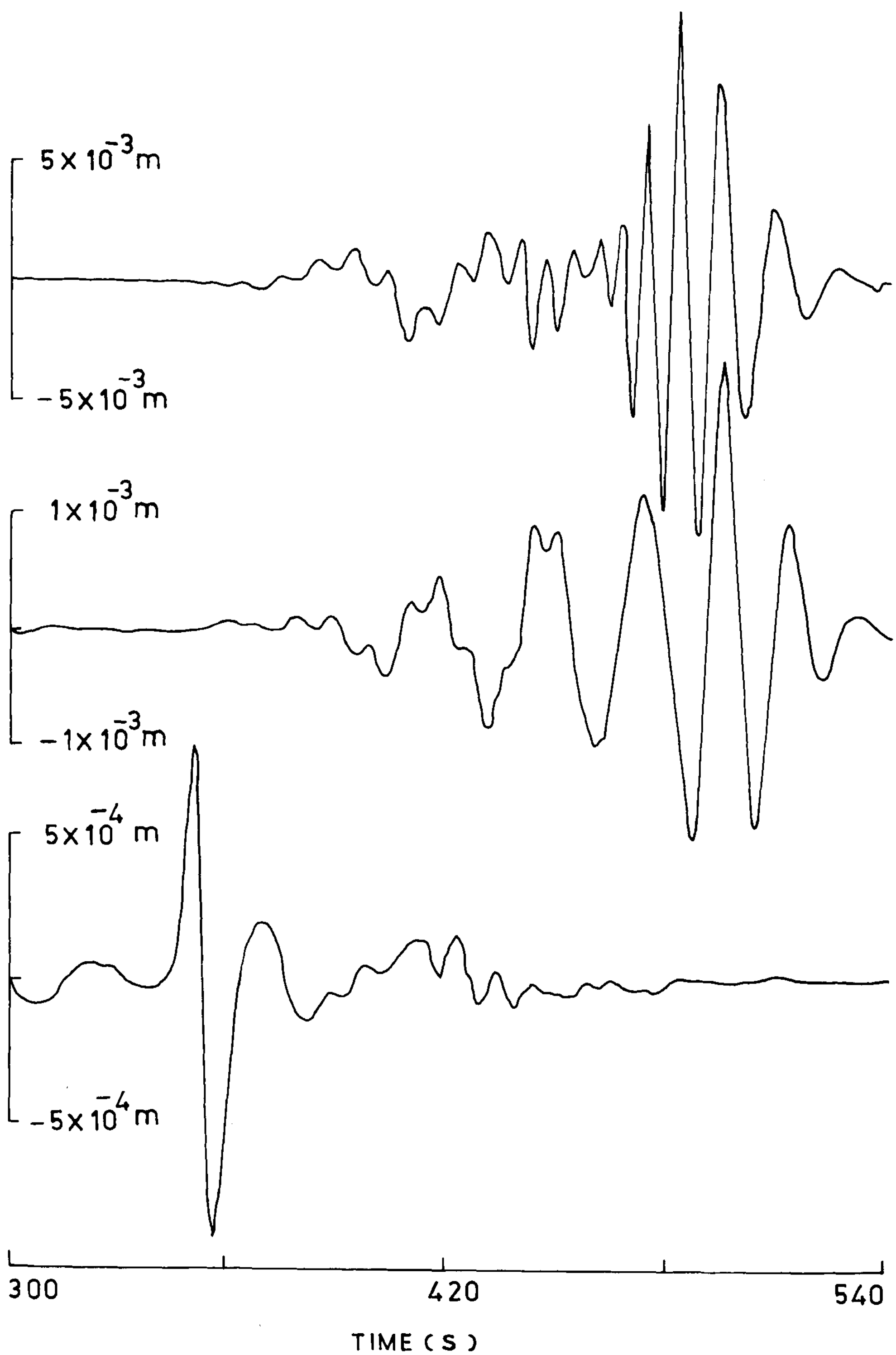

Figure 5. Vertical component of displacement (ZSS) at epicentral distance 1.50×10^6 m for vertical strike-slip source with moment 10^{22} dyn m for focal depths 1.50×10^4 (top), 5.50×10^4 (middle) and 2.75×10^5 m (bottom) as recorded by a long period seismograph (15–100 s) with peak magnification 1500.

In model GBA, six figure accuracy of $\bar{u}_r$, $\bar{u}_z$ is achieved with $\bar{Q} = 12$. The layer reduction with $\bar{Q} = 12$ is shown in Fig. 4. The vertical component of displacement (ZSS) for a vertical strike–slip source is shown in Fig. 5. Here, ZSS has the same meaning as in Herrmann & Wang (1985).

6 CONCLUSIONS

(i) When $\mathscr{Re}\, Q_l$ is large, the layer l behaves as the terminating half-space to obtain surface motion. This occurs when $\beta_l > c$ and the thickness of the lth layer is large compared to the wavelength; this is further aided when β_l is large compared to c.

(ii) If the source is below the lth layer, it has no contribution to the surface motion for c–ω pair at which the lth layer becomes the terminating half-space.

(iii) Without layer reduction, the above mentioned lth layer during computation creates an overflow which can be avoided by the present structure minimization. However, we also need to apply the usual normalization (Schwab & Knopoff 1970; Kind & Odom 1983).

(iv) Once the level of accuracy is given, the present algorithm gives the depth below which the terminating half-space should be taken to get the solution in the full structure. Thus we are economizing computation time by layer reduction without any loss of precision.

(v) A time saving is also achieved as the highest frequency limit decreases when the focal depth increases.

(vi) The method is most economical in low phase velocity and high frequency ranges.

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