- 8. Calculus with curves
  - 8.1. Parametrized curves
    - 8.1.1 Simple regular parametrizations
    - 8.1.2 Curves and piecewise curves
    - 8.1.3 Reparametrizations and orientation
  - 8.2. Arc length
    - 8.2.1 Definition and invariance
    - 8.2.2 Arc length parametrization
    - 8.2.3 Derivation of arc length
    - 8.2.4 Line integrals of scalar functions
    - 8.2.5 Elements and infinitesimals
  - 8.3. Line integrals
    - 8.3.1 Oriented curves
    - 8.3.2 Line integrals of vector fields
      - $$\int_C F\cdot T\,ds = \int_a^b F(\gamma(t))\cdot\gamma'(t)\,dt.$$
    - 8.3.3 Derivation of work done along a curve
  - 8.4. Fundamental theorem of line integrals
    - 8.4.1 Statement and proof
    - 8.4.2 Conservative vector fields and potentials
    - 8.4.3 Irrotational vector fields
      - Lemma 8.4.11 Let F be a vector field in $\mathbb{R}^n$ that is C^1 on an open set U. If F is conservative on U, then F is irrotational on U.
  - 8.5. Conservative vector fields
    - 8.5.1 Physical viewpoints
    - 8.5.2 Path independence
      - Theorem 8.5.2 Let F be a vector field in $\mathbb{R}^n$ that is continuous on an open path-connected set $U \subseteq \mathbb{R}^n$. The following are equivalent:
        (a) There exists C^1 real-valued function f such that $F = \nabla f$ on U.
        (b) $\int_{C_1} F\cdot d\gamma = \int_{C_2} F\cdot d\gamma$ for oriented piecewise curves C_1, C_2 in U with the same endpoints.
        (c) $\int_C F\cdot d\gamma = 0$ for any closed piecewise curve C in U.
    - 8.5.3 Irrotational vector fields and sets without holes
      - Lemma 8.5.7 (Poincaré's lemma) Let F be a vector field in $\mathbb{R}^n$ that is C^1 on the open set $U \subseteq \mathbb{R}^n$. If U is convex and F is irrotational on U, then F is conservative on U.

        Conservative —> irrotational
        Convex + irrotational —> conservative
      - Theorem 8.5.10 (Jordan curve theorem) Let C be a simple closed curve in $\mathbb{R}^2$. Then C divides $\mathbb{R}^2$ into two regions: an open bounded region Ω (the inside) and an unbounded region $\mathbb{R}^2 \setminus \Omega$ (the outside). Moreover, the inside Ω is Jordan measurable and $\partial\Omega = C$.
      - Definition 8.5.11 A set $D \subseteq \mathbb{R}^2$ is a **simply connected domain** if D is open, path-connected, and, for every simple closed curve lying in D, its inside is a subset of D.
        - *A set $D \subseteq \mathbb{R}^3$ is a **simply connected domain** if D is open, path-connected, and any simple closed curve can shrink continuously to a point while staying entirely inside D.*
      - Theorem 8.5.14 Let F be a vector field in $\mathbb{R}^2$ (or $\mathbb{R}^3$) that is C^1 on the open path-connected set D. If D is simply connected and F is irrotational on D, then F is conservative on D.
  - 8.6. Circulation and flux in 2D
    - 8.6.1 Circulation and curl in 2D
      - Definition 8.6.1 (Circulation) Let F be a vector field in $\mathbb{R}^2$ defined on an oriented curve C in $\mathbb{R}^2$. Assume C is simple and closed. The **circulation of F along C** is given by
        $$\int_C F\cdot T\,ds.$$
      - Remark 8.6.6 Notice that F is irrotational if and only if $\operatorname{curl}(F) = 0$. You can therefore also refer to such vector fields as **curl-free**.
        - turns counter-clockwise, $(\operatorname{curl} F)(p) > 0$.
        - turns clockwise, $(\operatorname{curl} F)(p) < 0$.
        - does not turn at all, $(\operatorname{curl} F)(p) = 0$.
    - 8.6.2 Unit normal and positive orientation
      - $T_1(t)\cdot n_1(t) = 0$ for all $t \in [0, 2\pi]$.
      - $\|n(t)\| = 1$ for all $t \in [0, 2\pi]$.
      - $\{n_1(t), T_1(t)\}$ is a positively oriented ordered basis of $\mathbb{R}^2$.
    - 8.6.3 Flux and divergence in 2D
      - $$\operatorname{div}(G) = \partial_1 G_1 + \cdots + \partial_n G_n.$$
      - A point $p \in \mathbb{R}^2$ is a **source** if $\operatorname{div}(F)(p) > 0$ and a **sink** if $\operatorname{div}(F)(p) < 0$.
      - A vector field F is **sourceless** if $\operatorname{div}(F) = 0$.
      - Imagine the vector field represents a flowing body of water. For any point p in this body, $(\operatorname{div} F)(p) > 0$ if p is a source. In other words, there seems to be more water emerging from p than draining into it. Similarly, $(\operatorname{div} F)(p) < 0$ if p is a sink. In other words, there seems to be more water draining into p than emerging from it. Otherwise, $(\operatorname{div} F)(p) = 0$.
  - 8.7. Green's theorem and curl
    - 8.7.1 Regular regions and orienting the boundary
      - *A regular region is a compact set whose boundary always touches its interior.*
    - 8.7.2 Statement and proof
      - *The total infinitesimal circulation over R is the circulation along its boundary ∂R.*
    - 8.7.3 Examples with Green's theorem
      - Example 8.7.9 (Three conservative methods)
        - calculate a line integral directly from the definition.
        - fundamental theorem of line integrals (Theorem 8.4.1)
        - apply your newly minted Green's Theorem
      - Example 8.7.10 (Moving the curve)
  - 8.8. Green's theorem and divergence
    - 8.8.1 Regular regions and orienting the boundary
    - 8.8.2 Statement and proof
      - The total infinitesimal flux over R is the flux across its boundary ∂R.
    - 8.8.3 Examples with Green's theorem