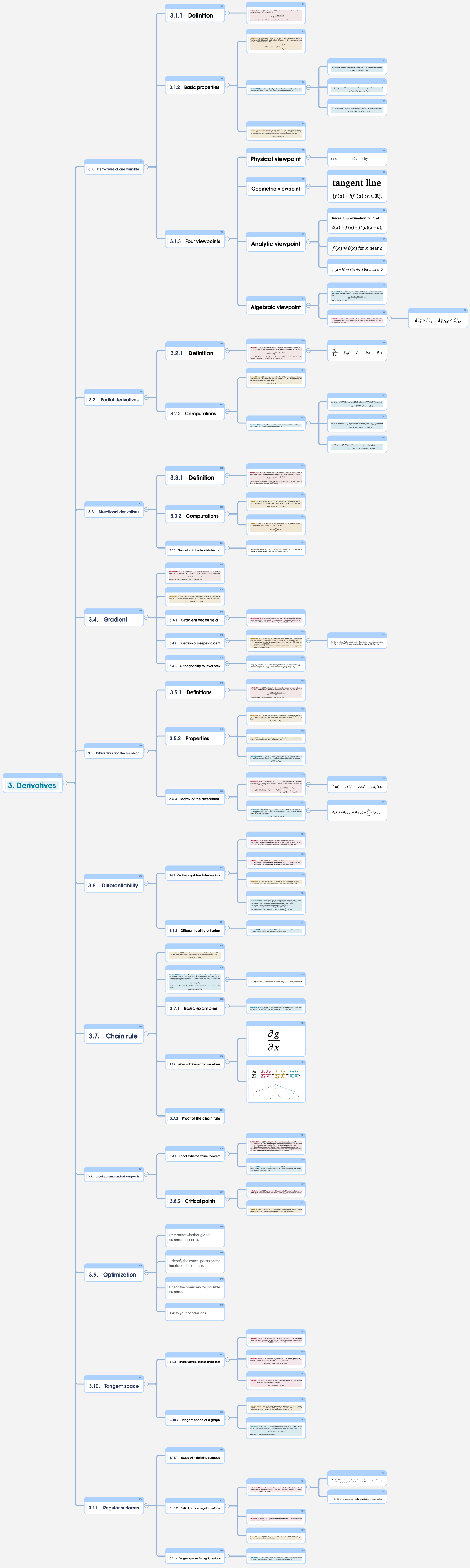

3. Derivatives
3.1. Derivatives of one variable
3.1.1 Definition
3.1.2 Basic properties
3.1.3 Four viewpoints
Physical viewpoint
(instantaneous) velocity
Geometric viewpoint
tangent line
$\{f(a)+hf'(a) : h \in \mathbb{R}\}$.
Analytic viewpoint
linear approximation of $f$ at $a$
$\ell(x) = f(a) + f'(a)(x-a)$,
$f(x) \approx \ell(x)$ for $x$ near $a$.
$f(a+h) \approx \ell(a+h)$ for $h$ near 0.
Algebraic viewpoint
$d(g \circ f)_a = dg_{f(a)} \circ df_a$.
3.2. Partial derivatives
3.2.1 Definition
$\frac{\partial f}{\partial x_j}$ $D_{e_j}f$ $f_{x_j}$ $D_jf$ $\partial_{x_j}f$
3.2.2 Computations
3.3. Directional derivatives
3.3.1 Definition
3.3.2 Computations
3.3.3 Geometry of directional derivatives
3.4. Gradient
3.4.1 Gradient vector field
3.4.2 Direction of steepest ascent
3.4.3 Orthogonality to level sets
3.5. Differentials and the Jacobian
3.5.1 Definitions
3.5.2 Properties
3.5.3 Matrix of the differential
3.6. Differentiability
3.6.1 Continuously differentiable functions
3.6.2 Differentiability criterion
3.7. Chain rule
3.7.1 Basic examples
3.7.2 Leibniz notation and chain rule trees
$\frac{\partial g}{\partial x}$
3.7.3 Proof of the chain rule
3.8. Local extrema and critical points
3.8.1 Local extreme value theorem
3.8.2 Critical points
3.9. Optimization
Determine whether global extrema must exist.
. Identify the critical points on the interior of the domain.
Check the boundary for possible extrema.
Justify your conclusions.
3.10. Tangent space
3.10.1 Tangent vectors, spaces, and planes
3.10.2 Tangent space of a graph
3.11. Regular surfaces
3.11.1 Issues with defining surfaces
3.11.2 Definition of a regular surface
3.11.3 Tangent space of a regular surface