

**Motilal Nehru National Institute of Technology Allahabad**

**Department of Computer Science & Engineering**

**MCA First Semester**

**Mid Semester Examination 2017-18**

**Subject Code/Name: CA3104/Foundation of Logic**

**Duration: 90 Minutes**

**Max. Marks: 20**

**NOTE:**

- All questions are compulsory.
- Attempt the questions in sequential order.
- Answers should be justified & to the point.

- 1) a) Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer." (2)
- b) Use mathematical induction to prove that  $2^n < n!$  for every positive integer  $n$ , with  $n \geq 4$ . (2)
- c) Determine whether the given compound propositions is satisfiable using Truth Table. (1)  
 $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
- 2) a) Prove the following without using truth table: (2)
- i)  $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ . Is a tautology?
- ii)  $[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)] \leftrightarrow (p \vee q)$
- b) Define equivalence relation. Let  $N$  denote the set of all natural numbers and let  $R$  be the relation on  $N \times N$  defined by  
$$(a, b)R(c, d) \leftrightarrow ad(b + c) = bc(a + d)$$
  
Check Whether  $R$  is an equivalence relation on  $N \times N$ . (2)
- c) Let  $A$  and  $B$  be the subsets of a universal set  $U$ . Show that  $A \subseteq B$  if and only if  $\bar{B} \subseteq \bar{A}$ ? (1)
- 3) a) Let  $S(x)$  be the predicate of "x is a student"  $F(x)$  be the predicate of "x is a faculty member" and  $A(x, y)$  be the predicate "x has asked y a question" where the domain consist of all people associated with your school. Use Quantifier to express each of these statements. (3)
- i) Some student has asked every faculty member a question.
- ii) There is a faculty member who has asked every faculty member a question.
- iii) Some student has never been asked a question by a faculty member.

(1)

b) Translate the statement

$$\forall x \exists y ((F(x) \wedge P(x)) \rightarrow M(x, y))$$

into English where  $F(x)$  is “x is a female”,  $P(x)$  denotes “x is a parent” and  $M(x, y)$  denotes “x is the mother of y” and the domain for both x and y consists of all people.

c) How many reflexive and symmetric relations possible on a set of n elements. (1)

4) a) Check the validity of the following argument using inference rule:

Linda, a student in this class owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket. (2)

b) Use rules of inference to show that the hypothesis “If it does not rain or of it is not foggy then the sailing race will be held and the lifesaving demonstration will go on,” “if the sailing race is held, then the trophy will be awarded,” and “the trophy was not awarded.” imply the conclusion “it rained.” (2)

c) Use a direct proof to show that every odd integer is the difference of two squares. (1)