Department of Computer Science and Engineering
Motilal Nehru National Institute of Technology, Allahabad
End-semester Examination 2018-19
Foundation of Logic CA-31104
M.C.A. I Semester

All Questions are compulsory. Assume any missing data and mention it at the top of the answer.

Time: 3 Hours

M.M: 60

Q1 Aladdin, Abu and Jasmine find themselves trapped in a dark and cold dungeon (how they arrived there, is another story). After a quick search they find three doors, first one is red, second one is blue, and the third one is green. Behind one of the doors there is a path to freedom. Behind the other two doors, however, is an evil fire-breathing dragon. Opening a door to the dragon means death. **10 marks**

On each door there is an inscription:

Red Door: *Freedom is behind This door.*

Blue Door: *Freedom is not behind This door.*

Green Door: *Freedom is not behind the blue door.*

Given the fact that at least one of the three statements on the three doors is true and at least one of them is false, which door would lead the them to safety?

Q2 Use rules of logical equivalence to show the equivalence/non-equivalence of following statements (Please do not use truth table): **2*5 marks**

a) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

b) $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

c) $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

d) $\neg(p \oplus q)$ and $(p \leftrightarrow q)$

e) $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$

Q3 **5+5 marks**

a) Give an inductive definition for the relation R on set of non-negative integers N. In each case, use your definition to show $X \in R$.

$R = \{<a, b> | a = 2b\}$; $X = <8, 4>$

b) Use mathematical induction to show that $\neg(p_1 \vee p_2 \vee \ldots \vee p_n)$ is equivalent to $\neg p_1 \wedge \neg p_2 \wedge \ldots \wedge \neg p_n$ whenever $p_1, p_2, \ldots, p_n$ are propositions, where $n \geq 2$.

Q4 Answer these questions for the poset $(\{3, 5, 9, 15, 24, 45\}, |)$; where $a|b$ means a divides b. **1*10 marks**

a) Draw the Hasse diagram of this poset.

b) Find the maximal elements.

c) Find the minimal elements.

d) Is there a greatest element?

e) Is there a least element?

f) Find all upper bounds of {3, 5}.

g) Find the least upper bound of {3, 5}, if it exists.

h) Find all lower bounds of {15, 45}.

i) Find the greatest lower bound of {15, 45}, if it exists.

j) Is the given poset a lattice? Explain.

[P.T.O]

Q5 a) Show that if $a^2 = e$ for all a in a group $G = (A, *)$, then G is commutative. 2.5+2.5 marks

b) Show that the same is true in any *monoid.*

Q6 Let $(A, *)$ be a semi group. Furthermore, for every a and b in A 1+2+2 marks
If $a \neq b$, then $a * b \neq b * a$
i.e., if $a * b = b * a$, then $b = a$

a) Show that for every a in A
$a * a = a$
b) Show that for every a, b in A
$a * b * a = a$
c) Show that for every a, b, c in A
$a * b * c = a * c$

Q7 a) Give the recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if 5+5 marks
i) $a_n = 6n$
ii) $a_n = 10^n$
iii) $a_n = 4n - 2$
iv) $a_n = n(n + 1)$
v) $a_n = n^2$

b) The Game of Logic, has these two assumptions:
1. "Logic is difficult or not many students like logic."
2. "If mathematics is easy, then logic is not difficult."
By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:
i) That mathematics is not easy, if many students like logic.
ii) That not many students like logic, if mathematics is not easy.
iii) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

*********End of Paper*********