All Questions are compulsory. Assume any missing data and mention it at the top of the answer.

M.M: 20   **Time: 90 minutes**

Q1   Suppose that on an island there are three types of people, knights, knaves, and normals (also   **5 marks** known as spies). Knights always tell the truth, knaves always lie, and normals sometimes lie and sometimes tell the truth. Detectives questioned three inhabitants of the island—Amy, Brenda, and Claire—as part of the investigation of a crime. The detectives knew that one of the three committed the crime, but not which one. They also knew that the criminal was a knight, and that the other two were not. Additionally, the detectives recorded these statements: Amy: "I am innocent." Brenda: "What Amy says is true." Claire: "Brenda is not a normal." After analyzing their information, the detectives positively identified the guilty party. Who was it?

Q2   Let S(x) be the predicate "x is a student," F(x) the predicate "x is a faculty member," and   **5 marks** A(x, y) the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.
a)   Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
b)   Some student has not asked any faculty member a question.
c)   Some student has never been asked a question by a faculty member.
d)   There is a faculty member who has asked every other faculty member a question.
e)   There is a faculty member who has never been asked a question by a student.

Q3   a)   Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$?   **2.5+2.5**
b)   Suppose that A, B, and C are sets such that $A \oplus C = B \oplus C$. Must it be the case that A = **marks** B?

Q4   Use mathematical induction to prove the following generalization of one of De Morgan's   **5 marks** laws:

$$\overline{\bigcap_{j=1}^{n} A_j} = \bigcup_{j=1}^{n} \overline{A_j}$$

Whenever $A_1, A_2, \ldots A_n$ are subsets of a universal set U and $n \geq 2$.