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Foundation of Logic
Assignment - 3

1) a) P(orange) $\rightarrow$ T, since 'orange' contains the letter 'a'.
b) P(lemon) $\rightarrow$ F, since 'lemon' doesn't contain the letter 'a'.

2) Consider the statement: "if P(x) then $x = 1$" which is equivalent to "if $x > 1$ then $x = 1$"

a) If $x = 0$, then statement P(0) = "$0 > 1$" is False and thus the value of $x = 0$ after the statement "if P(x), then $x = 1$"

b) If $x = 3$ then statement P(3) = "$3 > 1$" is True and thus the value of $x = 1$ after the statement "if P(x), then $x = 1$"

3) N(x): "x has visited MNNIT," where the domain consists of the student in your college

a) $\exists x\, N(x) \rightarrow$ Some student in my college has visited MNNIT.
b) $\forall x\, N(x) \rightarrow$ Every student in my college have visited MNNIT
c) $\neg \exists x\, N(x) \rightarrow$ No student in my college has visited MNNIT.
d) $\exists x\, \neg N(x) \rightarrow$ Some student in my college hasn't visited MNNIT.
e) $\neg \forall x\, N(x) \rightarrow$ Not all student in my college has visited MNNIT.
f) $\forall x\, \neg N(x) \rightarrow$ All student in my college haven't visited MNNIT.

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4. $R(x)$ is "x is a Rabbit"
$H(x)$ is "x hops"
Domain consists of all animals.

a) $\forall x(R(x) \rightarrow H(x))$
Ans- Among the set of all animals, if an animal is a rabbit, then it hops.

b) $\exists x(R(x) \rightarrow H(x))$
Ans- There exist atleast one animal and if it is a rabbit, then it hops.

c) $\exists x(R(x) \land H(x))$
Ans- There exist atleast one animal which is a rabbit and it hops.

5) a) $\forall x((x=1) \rightarrow P(x))$
Ans- $P(1)$

b) $\exists x((x \geq 0) \land P(x))$
Ans- $P(1) \lor P(3) \lor P(5)$

c) $\exists x(\neg P(x)) \land \forall x((x<0) \rightarrow P(x))$
Ans- $(\neg P(-5) \lor \neg P(-3) \lor \neg P(-1) \lor \neg P(1) \lor \neg P(3) \lor \neg P(5)) \land (P(-5) \land P(-3) \land P(-1)$

6) a) Everyone speak Hindi.

Ans- The following domain makes the statement True, because all Indians speak Hindi.

"True" domain = Indian People.

The following domain makes it false, because there are many people in the world that cannot speak Hindi.

"False" domain = All people in the world

b) Every two people have the same first name.

Ans - "True" domain = Ajay Devgan, Ajay Singh, Ajay Kumar

"False" domain = All people in the world.

c) Someone knows more than two other people.

Ans - "True" Domain = All people in the world

"False" Domain = All people that live alone on an unhabited island.

7) Domain of x is all people.

Let $P(x)$ be "x is perfect" & $F(x)$ be "x is your friend".

a) No one is perfect.

Ans. $\forall x \neg P(x)$

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b) Not everyone is perfect.
Ans- $\exists x \neg P(x)$

c) All your friends are perfect.
Ans- $\forall x (F(x) \rightarrow P(x))$

d) At least one of your friends is perfect.
Ans- $\exists x (F(x) \wedge P(x))$

e) Everyone is your friend and is perfect
Ans- $\forall x (F(x) \wedge P(x))$

f) Not everybody is your friend or someone is not perfect.
Ans- $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$

8) a) Some drivers do not obey the speed limit
Ans- statement: $\exists x \neg S(x)$
Negation : $\neg \exists x \neg\neg S(x) \equiv \forall x S(x)$
Hence, All drivers obey the speed limit.

b) All swedish movies are serious.
Ans- statement: $\forall x M(x)$
Negation: $\neg \forall x \neg M(x) \equiv \exists x \neg M(x)$
Hence, Some swedish movies is not serious.

c) No one can Keep a secret.
Ans- statement : $\neg \forall x K(x)$
Negation : $\neg\neg \forall x \neg K(x) \equiv \forall x \neg K(x)$
Hence Everyone cannot Keep a secret.

d) There is someone in the class who does not have a good attitude

Ans- statement : $\exists x \neg H(x)$

Negation : $\neg \exists x \neg\neg H(x) \equiv \forall x\, H(x)$

Hence, Everyone in the class have a good attitude

9) a) $\forall x (x^2 \neq x)$

Ans. For $x = 1$, we have $x^2 = 1 = x$

and for $x = 0$, we have $x^2 = 0 = x$.

Hence, $x = 0, 1$ is a counterexample.

b) $\forall x\ (x^2 \neq 2)$

Ans - For $x = \sqrt{2}$, whe have $(\pm\sqrt{2})^2 = 2$

Hence, $x = \pm\sqrt{2}$ is a counter example.

c) $\forall x (|x| > 0)$

Ans- For $x = 0$, we have $|x| = |0| = 0$ which is neither greater than nor less than equal to 0.

Hence, $x = 0$, is a counterexample.

10) To proof: $\forall x (P(x) \longleftrightarrow Q(x)) \equiv \forall x\, P(x) \longleftrightarrow \forall x\, Q(x)$

Ans- Let take an example that $x = 0, 1$

Let at $P(0) =$ False, $P(1) =$ True

and $Q(0) =$ True, $Q(1) =$ False

Now checking truth value at $\forall x (P(x) \longleftrightarrow Q(x))$

For $x = 0$: $P(0) =$ False, $Q(0) =$ True

$\therefore$ $P(0) \longleftrightarrow Q(0) \equiv$ False $\longleftrightarrow$ True $\equiv$ False

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Now checking truth value at $\forall x\, P(x) \longleftrightarrow \forall x\, Q(x)$

$\Rightarrow$ $\forall x\, P(x) = \text{False}$, as $P(0) = \text{False}$.

$P(1) = \text{True}$, $\forall x\, P(x) = \text{False}$

$\Rightarrow$ $\forall x\, Q(x) = \text{False}$. as $Q(0) = \text{True}$

$Q(1) = \text{False}$, $\forall x\, Q(x) = \text{False}$

$\therefore$ $\forall x\, P(x) \longleftrightarrow \forall x\, Q(x) \equiv \text{False} \longleftrightarrow \text{False} \equiv \text{True}$

Since False $\neq$ True

Hence, these are not logically equivalent to each other.

ii) To proof: a) $(\forall x\, P(x)) \vee A \equiv \forall x\, (P(x)$

Sol. As x is a free variable in A. So

Case 1: A is false

i) If $\forall x\, P(x)$ is also False, then

LHS: False $\vee$ False = False

ii) If $\forall x\, P(x)$ is true, then

LHS: True $\vee$ False = True

iii) If $P(x)$ is True, then

RHS: True $\vee$ False = True

iv) If $P(x)$ is False, then

RHS: False $\vee$ False = False

As when A is False, then for $P(x)$ and $\forall x\, P(x)$ at False or True both condition are equivalent as i, iv & ii, iii

Case 2: A is True $\Rightarrow$ All conditions are always True i.e. at every condition propositions are always logically equivalent.

b) $(\exists x P(x)) \vee A \equiv \exists x (P(x) \vee A)$

Ans- Case 1: A is False

Then for $\exists x P(x)$ and $P(x)$ is False, LHS & RHS both False

And for $\exists x P(x)$ and $P(x)$ is True, LHS & RHS both True.

Case 2: A is True $\Rightarrow$ All conditions are always True.

Hence, both are logically equivalent propositions.

12) a) $\exists! x (x > 1)$

Ans- As x belongs to all integers, so x has many values greater than 1, there is no any unique value.

Hence, the statement is False.

b) $\exists! x (x + 3 = 2x)$

Ans. In this scenario, we are getting a unique value of x i.e. 3, for which $P(x)$ is True.

Hence, the statement is True.

c) $\exists! x (x = x + 1)$

Ans- As in all Integers, there is no any value exist for which $x = x + 1$ condition is satisfied.

Hence, the statement is False.