Name: Krishanu Dey

Reg. no.: 2021CA055

Foundation of Logic
Assignment - 2

1) Sol. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

2)

a) Ans - This is the disjunction "Kwame will take a job in the industry, or go to graduate school". So the negation is "Kwame will not take a job in industry and not go to graduate school."

b) Ans - This is the conjunction "Yoshiko knows Java and calculus". So the negation is "Yoshiko does not knows Java, or Yoshiko does not knows Calculus.

c) Ans - This is the disjunction "Rita will move to Oregon, or Rita will move to Washington". So the negation is "Rita will not move to Washington and Rita will not move to Calculus"

3) a) $[\neg p \wedge (p \vee q)] \rightarrow q$

Sol.-

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Sol.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)]$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Sol-

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

d) $(\neg p \wedge (p \to q)) \to \neg q$

Sol-

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \wedge (p \to q)$	$(\neg p \wedge (p \to q)) \to \neg q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Hence, it is not a Tautology.

4) a) $[\neg p \wedge (p \vee q)] \to q$

Sol-
$= \neg[\neg p \wedge (p \vee q)] \vee q$ (Implication rule)
$= [\neg(\neg p) \vee \neg(p \vee q)] \vee q$ (De-Morgan's law)
$= [p \vee \neg(p \vee q)] \vee q$ (Double-Negation law)
$= [p \vee (\neg p \wedge \neg q)] \vee q$ (De-morgan's law)

$= [(p \vee \neg p) \wedge (p \vee \neg q)] \vee q$ (Distributive law)

$= [T \wedge (p \vee \neg q)] \vee q$ (Negation law)

$= [(p \vee \neg q) \wedge T] \vee q$ (Commutative law)

$= (p \vee \neg q) \vee q$ (Identity law)

$= (p \vee q) \vee (\neg q \vee q)$ (Distributive law)

$= (p \vee q) \vee T$ (Negation law)

$= T$ (Domination law)

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Sol- $= \neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$ (Implication law)

$= [(p \wedge \neg q) \vee (q \wedge \neg r)] \vee (\neg p \vee r)$ (De-Morgan law)

$= [(p \wedge \neg q) \vee (\neg p \vee r)] \vee (q \wedge \neg r)$ (Associative law)

$= [((\neg p \vee r) \vee p) \wedge ((\neg p \vee r) \vee \neg q)] \vee (q \wedge \neg r)$ (Distributive law)

$= [((\neg p \vee p) \vee r) \wedge ((\neg p \vee r) \vee \neg q)] \vee (q \wedge \neg r)$ (Associative law)

$= [(T \vee r) \wedge ((\neg p \vee r) \vee \neg q)] \vee (q \wedge \neg r)$ (Negation law)

$= [T \wedge ((\neg p \vee r) \vee \neg q)] \vee (q \wedge \neg r)$ (Domination law)

$= ((\neg p \vee r) \vee \neg q) \vee (q \wedge \neg r)$ (Identity law)

$= q \vee ((\neg p \vee r) \vee \neg q) \wedge \neg r \vee ((\neg p \vee r) \vee \neg q)$ (Distributive law)

$= ((q \vee \neg q) \vee (\neg p \vee r)) \wedge ((\neg r \vee r) \vee (\neg p \vee \neg q))$ (Associative, Commutative

$= (T \vee (\neg p \vee r)) \wedge (T \vee (\neg p \vee \neg q))$ (Negation law)

$= T \wedge T$ (Domination law)

$= T$ (Identity law)

c) $[p \wedge (p \rightarrow q)] \rightarrow q$

Sol. $\neg [p \wedge (\neg p \vee q)] \vee q$ (Implication law)

$= [\neg p \vee (p \wedge \neg q)] \vee q$ (De-morgan's law)

$= [(\neg p \vee p) \wedge (\neg p \vee \neg q)] \vee q$ (Distributive law)

$= [T \wedge (\neg p \vee \neg q)] \vee q$ (Negation law)

$= (\neg p \vee \neg q) \vee q$ (Associative, Identity law)

$= (\neg q \vee q) \vee \neg p$ (Associative law)

$= T \vee \neg p$ (Negation law)

$= T$ (Domination law)

d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Sol- $= \neg [(p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)] \vee r$ (Implication rule)

$= \neg (p \vee q) \vee \neg (\neg p \vee r) \vee \neg (\neg q \vee r) \vee r$ (De-morgan's law)

$= \neg (p \vee q) \vee (p \wedge \neg r) \vee (q \wedge \neg r) \vee r$ (De-morgan's law)

$= \neg (p \vee q) \vee [(p \vee q) \wedge \neg r] \vee r$ (Distributive law)

$= \neg (p \vee q) \vee [((p \vee q) \vee r) \wedge (\neg r \vee r)]$ (Distributive law)

$= \neg (p \vee q) \vee [((p \vee q) \vee r) \wedge T]$ (Negation law)

$= \neg (p \vee q) \vee ((p \vee q) \vee r)$ (Identity law)

$= [\neg (p \vee q) \vee (p \vee q)] \vee r$ (Associative law)

$= T \vee r$ (Negation law)

$= T$ (Domination law)

Teacher's Signature ....

5) Sol. $p \vee \neg q \equiv T \vee F \equiv T$

$\neg p \vee q \equiv F \vee T \equiv T$

$q \vee r \equiv T \vee T \equiv T$

$q \vee \neg r \equiv T \vee F \equiv T$

$\neg q \vee \neg r \equiv F \vee F \equiv F$

Hence, we get four truth values.

6) i) $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

Sol. $(p \to r) \vee (q \to r) \equiv (\neg p \vee r) \vee (\neg q \vee r)$ Implication law

$\equiv (\neg p \vee \neg q) \vee r$ Distributive law

$\equiv \neg (p \wedge q) \vee r$ De-morgan's law

$\equiv (p \wedge q) \to r$ Implication law

ii) $\neg p \to (q \to r)$ and $q \to (p \vee r)$

Soln $\neg p \to (q \to r) \equiv p \vee (\neg q \vee r)$ (Implication rule)

$\equiv \neg q \vee (p \vee r)$ (Associative law)

$\equiv q \to (p \vee r)$ (Implication rule)

iii) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$

Soln

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

7) Sol- p : The user enters a valid password
q : Access is granted
r : The user has paid the subscription fee.

a) $r \wedge \neg p$
b) $(r \wedge p) \rightarrow q$
c) $\neg r \rightarrow \neg q$
d) $(\neg p \wedge r) \rightarrow q$

8) Sol. Let the following statements be represented symbolically as:
p : The system software is being upgraded
q : Users can access the file system
r : Users can save new files.

Now, system verifications can be written as:
a) $p \rightarrow \neg q$
b) $q \rightarrow r$
c) $\neg r \rightarrow \neg p$

So, If we take p (False), q (False) & r (True), then system is consistent.

specification	Condition	Truth Value
$p \rightarrow \neg q$	$F \rightarrow T$	T
$q \rightarrow r$	$F \rightarrow T$	T
$\neg r \rightarrow \neg p$	$F \rightarrow T$	T

Hence, the specifications are consistent.

Teacher's Signature........................

9) Sol. Let the following statements be represented symbolically as:

p: The file system is locked.

q: New messages will be queued.

r: The system is functioning normally ~~and conversely~~

s: The new messages will be sent to the message buffer.

The specifications can be written as:

a) $\neg p \rightarrow q$

b) $\neg p \leftrightarrow r$

c) $\neg q \rightarrow s$

d) $\neg p \rightarrow s$

e) $\neg s$

So, If we take p (True), q (True), r (False) & s (False) then the system is consistent.

Specification	Condition	Truth Value
$\neg p \rightarrow q$	$F \rightarrow T$	T
$\neg p \leftrightarrow r$	$F \leftrightarrow F$	T
$\neg q \rightarrow s$	$F \rightarrow F$	T
$\neg p \rightarrow s$	$F \rightarrow F$	T
$\neg s$	T	T

Hence, the specifications are consistent.