Motilal Nehru National Institute of Technology, Allahabad
Department of Computer Science & Engineering
MCA First Semester
End Semester Examination 2017-18
Subject Code/Name: CA3104/Foundation of Logic

Duration: **3 hours** Max. Marks: **60**

Attempt all the questions.

(5*4=20)

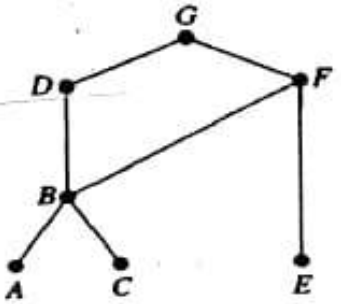
Question-1

a. i. Prove that the inverse of product of two elements of a group G is the product of the inverse taken in the reverse order, that is, $(ab)^{-1} = b^{-1}a^{-1}$
   ii. State and prove Lagrange's theorem.
b. Prove that the necessary and sufficient condition for a non-empty subset H of a group $(G,*)$ to be a subgroup is $a \in H, b \in H \rightarrow a * b^{-1} \in H$, where $b^{-1} is$ the inverse of b in G.
c. Prove that the fourth roots of unity $\{1,-1,i,-i\}$ forms a cyclic abelian multiplicative group. Find all the generators of this cyclic group.
d. If R is a Ring such that $a^2 = a, \forall a \in R$. Prove that
   i. $a + a = 0, \forall a \in R$, that is, each element of R is its own multiplicative inverse.
   ii. $a + b = 0 \Longrightarrow a = b$
   iii. R is a commutative ring.
e. Use Resolution principal to show that the hypotheses" It is not raining or Alice has her umbrella," "Alice does not have her umbrella or she does not get wet," and "it is raining or Alice does not get wet" imply the conclusion "Alice does not get wet."

(4*5=20)

Question-2

a. Define distributive and complemented lattice. Determine whether the following lattices are distributive or not:
   i. $(\{1,2,3,5,30\}, |)$
   ii. $(\{1,2,4,9,18\}, |)$
b. A development project at a computer company requires the completion of seven tasks. Some of these tasks can be started only after other tasks are finished. A partial ordering on tasks is setup by considering task X < task Y, i.e., task Y cannot be started until task X has been completed. The Hasse Diagram for the seven tasks with respect to the partial ordering is shown in the figure below. Find an order in which these tasks can be carried out to complete the project.



c. Determine whether these posets are lattices or not:
   i. $(\{1,3,6,9,12\}, |)$
   ii. $(\{1,2,3,4,5\}, |)$
   iii. $(P(S), \subseteq)$ where P(S) is the power set of S and S={a, b, c}.

d. i. Find the zero-one matrix of the transitive closure of the relation R where

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

   ii. What is the reflexive closure of the relation $R = \{(a,b)|a < b\}$ on the set of integers?
   iii. What is the symmetric closure of the relation $R = \{(a,b)|a > b\}$ on the set of positive integer?

Question-3

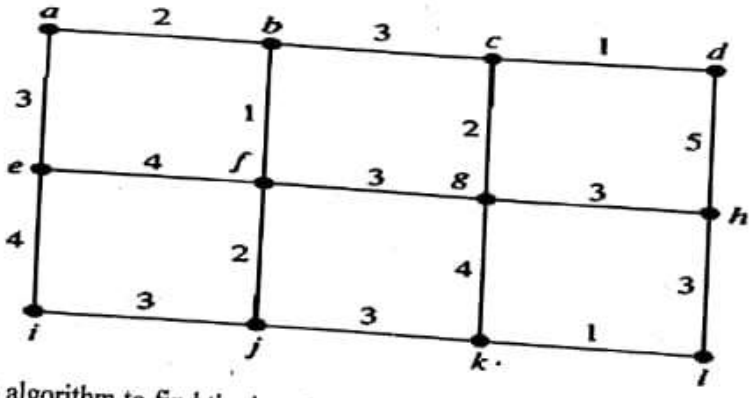
(5*4=20)

a. Find the post-order traversal of a binary tree whose pre-order and in-order traversal are given below:

   Pre-order: a b e j k n o p f c d g l m h i
   In-order: j e n k o p b f a c l g m d h i

b. Use Huffman encoding to encode the following symbols with the frequencies listed: A: 0.08, B: 0.10, C: 0.12, D: 0.15, E: 0.20, F: 0.35
What is the average number of bits used to encode a character?

c. Use Prim's Algorithm to design a minimum cost-communication network connecting all the computers represented by the following graph.



d. Use Dijkstra's algorithm to find the length of the shortest path between the vertices 'a' and 'z' in the weighted graph displayed by the following figure: