NTC Thermistor Sensor

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1 Introduction

The NTC thermistors display non-linear resistance characteristics with temperature. The resistance of an NTC will decrease at the temperature increases. This behaviour is related to it's constant value B. This phenomenon allows for use of an NTC thermistor as a temperature sensor. In the discussed experiments a Vishay NTCLE100E3 thermistor was used, with $B = 3977°K$.

2 Theory

2.1 R-T characteristics

In the following experiments measurements are compared against the linear sensor model:

$$O(I) = KI + a \tag{1}$$

where K is the sensor's sensitivity, I is the input, a is the y-intercept, and $O(I)$ is the sensor's output at a given input. The sensitivity is calculated by the following equation:

$$K = \frac{O(I_{max}) - O(I_{min})}{I_{max} - I_{min}} \tag{2}$$

where I_{max} and I_{min} are maximum and minimum input in the range of operation. Finally, the y-intercept can be found:

$$a = O(I_{min}) - KI_{min} \tag{3}$$

The non-linearity of a measurement is defined by:

$$N(I) = |O_{actual}(I) - K \cdot I - a| \tag{4}$$

where O_{actual} is the measured output of a sensor given input I. In the case of a NTC thermistor the input is temperature and the output is resistance. However, the relation between temperature is said to be non-linear, for the NTCLE100E3 it can be described with equation for expected intermediate temperatures, taken from the NTC's datasheet:

$$R_T(T) = R_{ref} \cdot e^{A + \frac{B}{T} + \frac{C}{T^2} + \frac{D}{T^3}} \tag{5}$$

where A, B, C, and D are constant values which are dependent on the thermistor; R_{ref} is the resistance at a reference temperature—for the thermistor used in the experiment (Brown, Black, and Orange bands) it is 10000Ω, and the constant values:

R_{ref}	10000Ω
A	-14.6337
B[1]	4791.842
C	-115334
D	$-3.730535 \cdot 10^6$

Since, the temperature T in the equation 5 is in $°K$ for measurements in $°C$ conversion is required:

$$T_K = T_C + 273.15° \tag{6}$$

When considering the equation 5 the thermistor is expected to have significant non-linearity. This error can be reduced, the Siemens Handout describes how the output can be linearised in a range by use of a parallel resistor. The equation

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[1]Note that this value is different to the β "B" value of a thermistor, constants from this table should only be used in combination with eqation 5

for the value of such resistor is given by:

$$R_p = R_{T_{ctr}} \cdot \frac{B - T_{ctr}}{B + 2 \cdot T_{ctr}} \tag{7}$$

where $R_{T_{ctr}}$ and T_{ctr} are thermistor's resistance and temperature at the center of the temperature range, and B is the B (β) value of the thermistor.
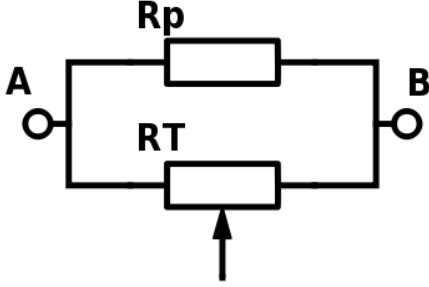

Figure 1: Parallel circuit

The expected output $R_{AB}(T)$ for a linearised thermistor setup shown in Fig.2.1 can be derived from a basic equation for resistor:

$$R_{AB}(T) = \frac{R_p \cdot R_T(T)}{R_p + R_T(T)} \tag{8}$$

2.2 Time Constant

A time constant, τ is defined by the time it takes for an element to respond to the change in the surrounding temperature. For example, in this lab experiment, the time constant will define how long it will take for the thermistor to respond to the change from ambient room temperature, to the temperature of iced water. The method of determining the time constant is as follows.

First, the difference between the voltage at ambient room temperature, V_A and the final voltage, V_O in iced water must be found:

$$V_D = V_O - V_A$$

Then, based on the equation

$$f(t) = 1 - exp(-t) = 0.63, \ t = 1$$

from Page 59 of J.P. Bentley's Principles and Measurement Systems, this shows that when $\frac{t}{\tau}$

is equal to 1, i.e. one time constant has passed, that 63% of the total change that will occur due to the new temperature will have happened after τ has passed. So, multiplying V_D by 0.63 will determine the voltage that will change after one time constant has occurred, V_τ:

$$V_\tau = V_D \times 0.63$$

From knowing this value, it is then only necessary to observe the time it takes for the voltage across the thermistor to reach this value from the moment it is submerged into the iced water in order to be able to determine the time constant. Ideally, the system will act in such a way that with each passing time constant, the voltage across the thermistor will increase in relation to the exponential equation listed above, and by 5τ, 99% of the change will have occurred.

3 Experiments

For this lab assignment, two experiments were conducted. The first was to determine the following characteristics of the non-linear response of the NTC: the resistance-temperature R-T and temperature-resistance T-R, the maximum non-linearity $\hat{N}$ as % of $f.s.d$ (full scale deflection), and the response of the system linearised by a parallel resistor. The second experiment was done to find the time constant τ of the measurement system. Raw data from all measurements are presented in the Appendix.

Both of these experiments followed the same basic steps, where the thermistor was placed in an environment with a temperature significantly different from the ambient room temperature — in a cup filled with either boiling or iced water. The temperature of the environment was measured with a Fluke Thermocouple.

3.1 R-T and T-R Characteristics

To measure the R-T and T-R characteristics, the resistance of the thermistor was measured with an AMPROBE AM-510-EUR multimeter, and recorded over a temperature range of 90–45°C. This was achieved by measuring the temperature of boiling water in a cup, which was cooled

from 100° to 45°. As the temperature and the resistance were constantly changing, it was decided to record the setup in video format, in order to collect the necessary data points accurately when some significant temperature change has occurred, for example, every drop of 5 degrees. Having collected this data, the non-linearity of the system can be determined.

Next, a parallel resistor was placed in the system, and the same procedure as previous was followed. In order to establish the necessary value of the parallel resistor, $R_{T_{ctr}}$ was recorded at $T_{ctr} = 72.5°C$ (the midpoint of the temperature range in which the resistance was measured) and calculated using equation 7, which can also be found in the Siemens handout. Ideally, placing a parallel resistor in the system would help to linearise the change in resistance of the thermistor as a result of the change in temperature of the environment.

As the R-T and T-R characteristics of the thermistor rely on its resistance, it was decided to measure the resistance from the multimeter directly, rather than measure the voltage across as is done in the next experiment.

3.2 Time Constant

In order to determine the time constant, the thermistor was again subjected to change in environment temperature, however this time the it was placed in iced water. The iced water throughout this experiment remained at a temperature between 2° and 3°, which was as close to the triple point of water as the conditions would allow. This time, a 2kΩ resistor was placed in series with the thermistor, and, with the same multimeter, the voltage across the thermistor was measured. This process was recorded again by video, and this was done in order to accurately record the near-exact time it took for the voltage across the thermistor to go from the value at ambient room temperature, to the voltage at the first time constant. It was decided to measure the voltage across the thermistor in this part of the lab assignment, rather than the resistance itself, because the multimeter used in this experiment had a better accuracy for voltage (0.8%

Rdg) than it does for resistance (1.0% Rdg). As the time constant simply relies on the change in the system over time, and since the voltage across the thermistor is anyway proportional to its resistance, it was decided that the higher accuracy of the voltage reading would lead to a better understanding of the time constant of the thermistor.

This was done three times in order to collect an average on the operational time constant, i.e. the time constant of the thermistor during prolonged use.

4 Results

Here, the data from each part the experiments are analysed and discussed. Important graphs, equations, and tables are shown directly in these subsections. Complete calculated data will be put into tables, graphs that can be seen in the Appendix.
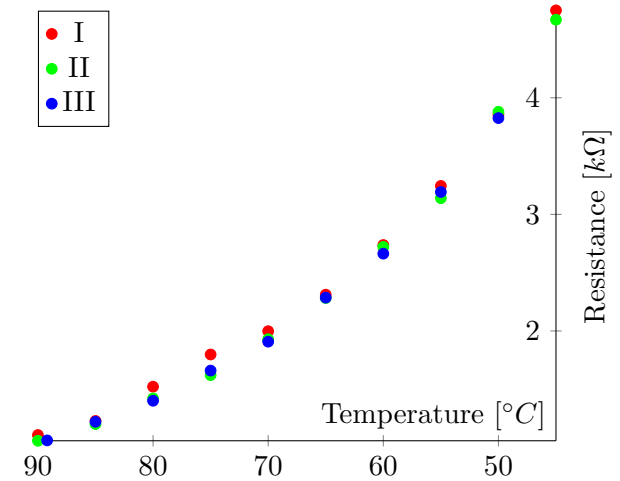
4.1 R-T Characteristics

4.1.1 Non-Linear

Fig. 4.1.1 Raw non-linear R-T

The equation for the ideal straight line for input range 45–90° was found using the equations 5

and [6]:

$$K = \frac{R_T(90) - R_T(45)}{90 - 45}$$
$$= \frac{0.915443 - 4.37181}{90 - 45} \tag{9}$$
$$= -0.07681 \frac{k\Omega}{^\circ C}$$

$$a = R_T(45) - K \cdot 45$$
$$= 4.37181 + 0.07681 \cdot 45 \tag{10}$$
$$= 7.828 k\Omega$$

The sensitivity and y-intercept of measured data were found by taking the mean of sensitivities and y-intercept of all runs. Shown in the table ??.

Run	K	a
I	-0.07628	8.183
II	-0.07697	8.134
III	-0.06792	7.222
Mean	-0.07373	7.846

Next, the maximum non-linearity $\hat{N}$ was found by use of equation 4:

$$\hat{N} = |N(60) = O_{actual}(60) - K(60) - a|$$
$$= |2.663 + 0.07681 \cdot 60 - 7.828| \tag{11}$$
$$= 0.5564 k\Omega$$

So $\hat{N}$ as a percentage of $f.s.d$ is

$$\hat{N} \ as \ \% \ of \ f.s.d = \frac{0.5564}{4.75 - 1.058} \cdot 100\% \tag{12}$$
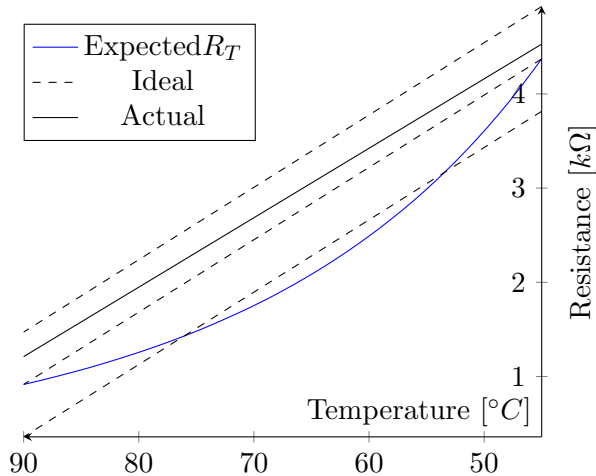$$= 15.1\%$$

Fig.4.1.1 Non-Linear R-T

From the Fig.4.1.1 it can be seen that the thermistors output is non-linear, and the curve shape is similar to the expected response. However, a static offset can be observed, which will be discussed the Error Discussion 5.
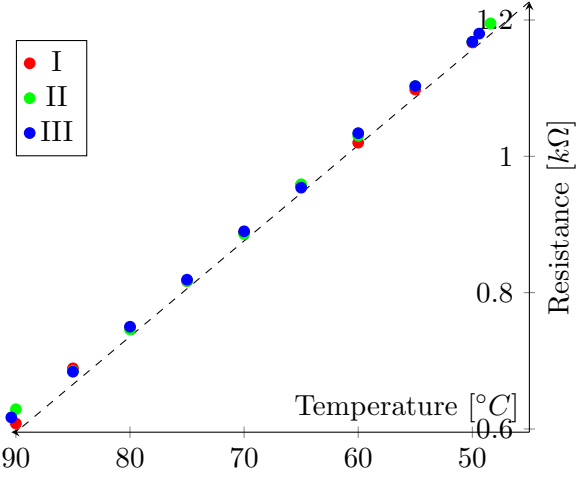
4.1.2 Linear

Fig.4.1.2 Raw linear R-T

Using the equation ?? R_p was calculated for linear range 45–90$^\circ C$. B value is given in $^\circ K$ in the datasheet, but temperature range and measurements are in $^\circ C$, therefore for the calculations it must be converted to Celsius:

$$B_C = B_K + 273.15$$
$$= 3977 - 273.15 \tag{13}$$
$$= 3703.85 ^\circ C$$

$$T_{ctr} = \frac{90 - 45}{2} + 45$$
$$= 72.5 ^\circ C \tag{14}$$

$$R_p = 1807 \cdot \frac{3703.85 - 72.5}{3703.85 + 2 \cdot 72.5} \tag{15}$$
$$= 1.705 k\Omega$$

this resistance is then used to find R_{AB} at T_{min} and T_{max}

$$R_{AB}(45) = 1.227 k\Omega$$
$$R_{AB}(90) = 0.5956 \tag{16}$$

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which allows finding the sensitivity and the y-intercept for the ideal straight line:

$$K = \frac{R_{AB}(T_{max}) - R_{AB}(T_{min})}{T_{max} - T_{min}}$$
$$= \frac{0.9154 - 4.372}{90 - 45} \qquad (17)$$
$$= -0.01403$$

and y-intercept

$$a = R_{AB}(I_{min}) - K \cdot I_{min}$$
$$= 1.227 + 0.01403 \cdot 45 \qquad (18)$$
$$= 1.858$$

The sensitivity and y-intercept of measured data were found by taking the mean of sensitivities and y-intercept of all runs. Shown in the following table:

Run	K	a
I	-0.01382	1.858
II	-0.01380	1.863
III	-0.01384	1.864
Mean	-0.01382	1.862

Non-linearity:

$$\hat{N} = |N(90) = R_{actual}(90) - K \cdot 90 - a|$$
$$= |0.629 + 0.01403 \cdot 90 - 1.858| \qquad (19)$$
$$= 3.37\Omega$$

and $\hat{N}$ as a percentage of $f.s.d$:

$$\hat{N} \; as \; \% \; of \; f.s.d = \frac{3.37}{1.195 - 0.608} \cdot 100\% \qquad (20)$$
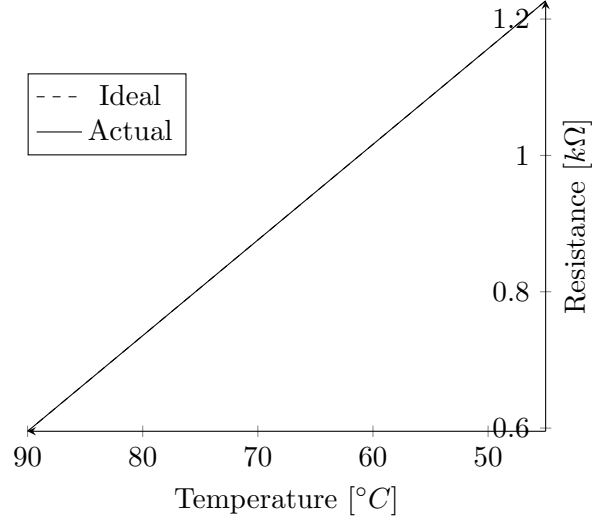$$= 0.574\%$$

Fig.4.1.2 Linear R-T

Using the parallel resistor greatly decreased the $\hat{N}$ of the response.

4.2 Time Constant

In order to find the time constant, the voltage across the thermistor was noted at ambient room temperature, V_A, and after it has seemingly stabilised in the ice water, V_O. Here, these data are shown for the three runs in this part of the lab assignment:

Run	V_A	V_O
I	2.095	2.204
II	2.096	2.205
III	2.098	2.201

The calculation for the time constant of Run I will be demonstrated here, following that which is mentioned in Section 2.2. First, the difference between V_A and V_O is taken to give V_D:

$$V_D = 2.205 - 2.095$$

$$V_D = 0.11$$

Then, considering that the first time constant, τ happens at the time at which the change in temperature has reached 63% of V_D,

$$V_\tau = 0.108 \times 0.63$$

$$V_\tau = 0.068$$

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making V_t to be

$$V_t = 2.095 + 0.068$$

$$V_t = 2.164$$

the time constant can then be found by determining the change in time for the voltage across the thermistor to reach V_t from the moment the thermistor had been submerged in the iced water. From the video, this time needs to be estimated, as the value on the multimeter jumps over the desired voltage reading. From estimation (by taking the average time between the voltage read above and below the desired value), the time constant for the first run is 1.382s.

Here, a table is shown with the time constants of the three runs:

Run	τ, s
I	1.382
II	1.91
III	1.498

and here, a table with the expected time, t_E, to reach V_O compared to the actual time, t_A:

Run	t_E, s	t_A, s
I	6.91	6.025
II	9.55	61.809
III	7.49	6.358

An average of the three results have been taken, and put into a graph in Fig. 4.2.1 in order to analyse the nature of response of the thermistor to that shown on Page 59 of J.P. Bentley's Principles and Measurement Systems, modified in Fig. 4.2.2. It can be seen that in the average time response function of the thermistor, it appears to approach the average V_O, however when compared to the expected time response function, it does not follow this trend.
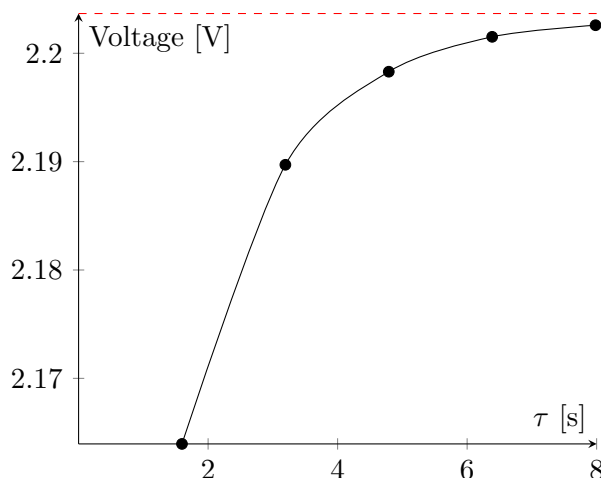

Figure 4.2.1: Time Response Function of Thermistor Compared to Average V_O
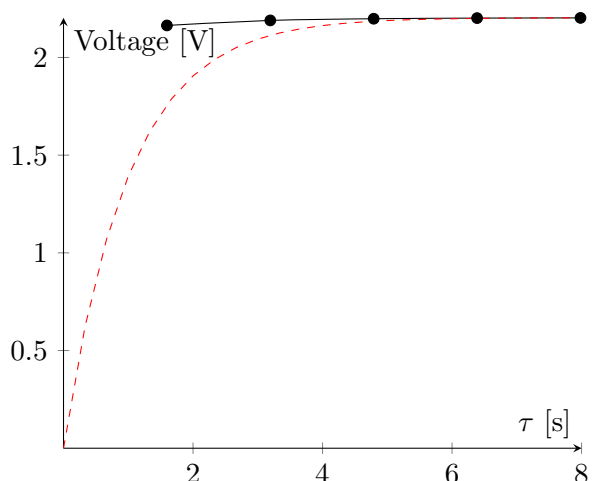

Figure 4.2.2: Time Response Function of Thermistor Compared to Expected Result

5 Error Discussion

There was no significant error in from the R-T characteristics experiment, so this section will focus on the significant error from the time constant experiment.

5.1 Time Constant

The calculated function including the time constant for this part of the lab assignment appears to tend towards V_O, however, this may simply be due to the nature of how the function was calculated. It can be seen that the function does

not follow that of the expected outcome, and this may be attributed to the fact that there was no accurate way of gathering the data from the time. As the video was not started the exact moment the thermistor was submerged into the iced water, a guess needed to be made towards when exactly the time measurement should stop. Additionally, as the voltage values assumed to be seen at the time of the first time constant V_τ were never read by the multimeter, there was more guessing that needed to be done in order to determine the time at which this voltage was reached. Additional error can come from the delay that the multimeter has when displaying the voltage, as it is not capable of displaying the voltage accurately in relation to the time that it is reached. Furthermore, there is some fluctuation in the voltage reading near what is assumed to be V_O, and this can mainly be attributed to the temperature of the iced water not being constant, i.e. some parts of the iced water may be colder or warmer than others, and the thermistor moving around in the different temperatures.

6 Conclusion

This lab assignment had two main expected outcomes. Firstly, that adding a parallel resistor would decrease the non-linearity of the R-T characteristic of the thermistor. This expected outcome was met with minimal errors. However, with the second expected outcome, the error was significant enough to render analysing the data rather difficult. In future renditions of this lab assignment, it should be considered to stir the iced water, in order to try and keep the overall temperature inside the cup more consistent, and thereby limiting the fluctuations in voltage.

References

[1] J.P. Bentley, *Principles of Measurement Systems, 4th Edition*, (Pearson Education Limited, England, 2005).

[2] *AM-510 Digital Multimeter*, available at `https://content.amprobe.com/`
DataSheets/AM-510%20Digital
%20Multimeter.pdf.

[3] *NTC Thermistors, Radial Leaded, Standard Precision*, available at `https://e-learn.sdu.dk/bbcswebdav/pid-4633321-dt-content-rid-7067738_2/courses/MC-DIM-U1-1-E16/ntcle100.pdf`.

[4] *Siemens Note, NTC*, available at `https://e-learn.sdu.dk/bbcswebdav/pid-4623642-dt-content-rid-7043312_2/courses/MC-DIM-U1-1-E16/Siemens_note_NTC.pdf`.

A Tables

Table 1: Non-Linear R-T Data I

$T,^\circ C$	$R, k\Omega$
90	1.107
85	1.228
80	1.522
75	1.798
70	1.998
65	2.311
60	2.737
55	3.244
50	3.858
45	4.75

Table 2: Non-Linear R-T Data II

$T,^\circ C$	$R, k\Omega$
90	1.058
85	1.201
80	1.421
75	1.621
70	1.929
65	2.282
60	2.721
55	3.139
50	3.88
45	4.67

Table 3: Non-Linear R-T Data III

$T, ^\circ C$	$R, k\Omega$
89.2	1.061
85	1.221
80	1.401
75	1.66
70	1.908
65	2.286
60	2.663
55	3.192
50	3.826

Table 4: Linear R-T Data I

$T, ^\circ C$	$R, k\Omega$
90	0.608
85	0.689
80	0.749
75	0.817
70	0.888
65	0.958
60	1.02
55	1.098
50	1.167

Table 5: Linear R-T Data II

$T, ^\circ C$	$R, k\Omega$
90	0.629
85	0.685
80	0.746
75	0.817
70	0.886
65	0.959
60	1.03
55	1.103
50	1.168
48.4	1.195

Table 6: Linear R-T Data III

$T, ^\circ C$	$R, k\Omega$
90.4	0.617
85	0.684
80	0.75
75	0.819
70	0.89
65	0.954
60	1.034
55	1.103
50	1.168
49.4	1.18

Table 7: Time Constant: Run I

τ, s	Expected Voltage, V
1.382	2.164
2.764	2.191
4.146	2.196
5.528	2.203
6.91	2.204

Table 8: Time Constant: Run II

τ, s	Expected Voltage, V
1.91	2.164
3.82	2.191
5.73	2.196
7.64	2.203
9.55	2.204

Table 9: Time Constant: Run III

τ, s	Expected Voltage, V
1.498	2.164
2.996	2.190
4.494	2.198
5.992	2.202
7.49	2.203