

Simplify and express the following expressions with positive indices.

1. $\frac{(x^{-3}y^3)^{-3}}{x^5y^{-3}}$

Solution:

$$\begin{aligned}\frac{(x^{-3}y^3)^{-3}}{x^5y^{-3}} &= \frac{x^9y^{-9}}{x^5y^{-3}} \\ &= x^{9-5}y^{-9-(-3)} \\ &= x^4y^{-6} \\ &= \frac{x^4}{y^6}\end{aligned}$$

2. $\frac{(xy^{-3})^{-1}}{x^2y^{-1}}$

Solution:

$$\begin{aligned}\frac{(xy^{-3})^{-1}}{x^2y^{-1}} &= \frac{x^{-1}y^3}{x^2y^{-1}} \\ &= x^{-1-2}y^{3-(-1)} \\ &= x^{-3}y^4 \\ &= \frac{y^4}{x^3}\end{aligned}$$

3. $\frac{(x^3y^2)^4}{x^{-5}y^4}$

Solution:

$$\begin{aligned}\frac{(x^3y^2)^4}{x^{-5}y^4} &= \frac{x^{12}y^8}{x^{-5}y^4} \\ &= x^{12-(-5)}y^{8-4} \\ &= x^{17}y^4\end{aligned}$$

4. $\frac{(x^{-3}y^{-5})^{-4}}{x^{-3}y}$

Solution:

$$\begin{aligned}\frac{(x^{-3}y^{-5})^{-4}}{x^{-3}y} &= \frac{x^{12}y^{20}}{x^{-3}y} \\ &= x^{12-(-3)}y^{20-1} \\ &= x^{15}y^{19}\end{aligned}$$

5. $\frac{(x^2y^2)^{-2}}{x^4y^{-3}}$

Solution:

$$\begin{aligned}\frac{(x^2y^2)^{-2}}{x^4y^{-3}} &= \frac{x^{-4}y^{-4}}{x^4y^{-3}} \\ &= x^{-4-4}y^{-4-(-3)} \\ &= x^{-8}y^{-1} \\ &= \frac{1}{x^8y}\end{aligned}$$

6. $\frac{xy}{(xy^{-3})^3}$

Solution:

$$\begin{aligned}\frac{xy}{(xy^{-3})^3} &= \frac{xy}{x^3y^{-9}} \\ &= x^{1-3}y^{1-(-9)} \\ &= x^{-2}y^{10} \\ &= \frac{y^{10}}{x^2}\end{aligned}$$

7. $\frac{x^2y^3}{(x^4y^{-2})^{-3}}$

Solution:

$$\begin{aligned}\frac{x^2y^3}{(x^4y^{-2})^{-3}} &= \frac{x^2y^3}{x^{-12}y^6} \\ &= x^{2-(-12)}y^{3-6} \\ &= x^{14}y^{-3} \\ &= \frac{x^{14}}{y^3}\end{aligned}$$

8. $\frac{x^{-5}y}{(x^4y^4)^{-5}}$

Solution:

$$\begin{aligned}\frac{x^{-5}y}{(x^4y^4)^{-5}} &= \frac{x^{-5}y}{x^{-20}y^{-20}} \\ &= x^{-5-(-20)}y^{1-(-20)} \\ &= x^{15}y^{21}\end{aligned}$$

9. $\frac{x^4y^{-5}}{(xy^{-1})^{-2}}$

Solution:

$$\begin{aligned}\frac{x^4 y^{-5}}{(x y^{-1})^{-2}} &= \frac{x^4 y^{-5}}{x^{-2} y^2} \\ &= x^{4-(-2)} y^{-5-2} \\ &= x^6 y^{-7} \\ &= \frac{x^6}{y^7}\end{aligned}$$

10. $\frac{x^{-2} y^2}{(x^2 y^{-5})^{-4}}$

Solution:

$$\begin{aligned}\frac{x^{-2} y^2}{(x^2 y^{-5})^{-4}} &= \frac{x^{-2} y^2}{x^{-8} y^{20}} \\ &= x^{-2-(-8)} y^{2-20} \\ &= x^6 y^{-18} \\ &= \frac{x^6}{y^{18}}\end{aligned}$$

11. $\frac{x^{-3}}{y^{-2}} \left(\frac{y^{-2}}{x} \right)^{-2}$

Solution:

$$\begin{aligned}\frac{x^{-3}}{y^{-2}} \left(\frac{y^{-2}}{x} \right)^{-2} &= \frac{x^{-3}}{y^{-2}} \cdot \frac{y^4}{x^{-2}} \\ &= x^{-3-(-2)} y^{4-(-2)} \\ &= x^{-1} y^6 \\ &= \frac{y^6}{x}\end{aligned}$$

12. $\frac{x^5}{y^{-3}} \left(\frac{y^{-3}}{x} \right)^{-3}$

Solution:

$$\begin{aligned}\frac{x^5}{y^{-3}} \left(\frac{y^{-3}}{x} \right)^{-3} &= \frac{x^5}{y^{-3}} \cdot \frac{y^9}{x^{-3}} \\ &= x^{5-(-3)} y^{9-(-3)} \\ &= x^8 y^{12}\end{aligned}$$

13. $\frac{x^2}{y^5} \left(\frac{y^3}{x^{-3}} \right)^3$

Solution:

$$\begin{aligned}\frac{x^2}{y^5} \left(\frac{y^3}{x^{-3}} \right)^3 &= \frac{x^2}{y^5} \cdot \frac{y^9}{x^{-9}} \\ &= x^{2-(-9)} y^{9-5} \\ &= x^{11} y^4\end{aligned}$$

14. $\frac{x^4}{y^{-5}} \left(\frac{y}{x^4} \right)^{-4}$

Solution:

$$\begin{aligned}\frac{x^4}{y^{-5}} \left(\frac{y}{x^4} \right)^{-4} &= \frac{x^4}{y^{-5}} \cdot \frac{y^{-4}}{x^{-16}} \\ &= x^{4-(-16)} y^{-4-(-5)} \\ &= x^{20} y\end{aligned}$$

15. $\frac{x^3}{y^{-3}} \left(\frac{y^{-1}}{x^{-1}} \right)^{-3}$

Solution:

$$\begin{aligned}\frac{x^3}{y^{-3}} \left(\frac{y^{-1}}{x^{-1}} \right)^{-3} &= \frac{x^3}{y^{-3}} \cdot \frac{y^3}{x^3} \\ &= x^{3-3} y^{3-(-3)} \\ &= x^0 y^6 \\ &= y^6\end{aligned}$$