

# Assignment#7

## Sol<sup>n</sup>. of Ordinary Differential Equations

### Initial Value Problems

1. Using **Euler's method**, find  $y(0.2)$  from the equation:  $y' = x + y$ ,  $y(0) = 0$ , take  $h = 0.1$
2. **Derive Euler's formula** for solving initial value problem.
3. Solve  $y' = y/(x^2 + y^2)$ ,  $y(0) = 1$  using **RK-2 method** in the range 0, 0.5, 1.
4. Solve  $y' = \sin x + \cos y$  subject to initial condition  $y(0) = 2$  in the range 0(0.5)2 using the RK second order method.
5. Using the RK-2, obtain a solution of the equation  $y' = xy + y^2$  with the initial condition  $y(0) = 1$  for the range  $0 \leq x \leq 0.6$  with increments of 0.2.
6. Solve  $y' = 4e^{0.8x} - 0.5y$ ; subject to initial condition  $y(0) = 2$ , for  $y(0.5)$  and  $y(0.1)$  using Runge-Kutta 2<sup>nd</sup> order method.
7. Solve  $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$  using RK-4 method, for  $y(0.4)$ . Given:  $y(0) = 1$ ,  $h = 0.2$
8. Write an **algorithm**, **pseudo-code** and a **program** in any high level language (C/C++/FORTRAN) to solve a first-order initial value problem using classical RK-4 method.
9. Solve the differential equation,  $dy/dx = (1 + x^2)y$  within  $x \leq 0(0.2)0.4$  and  $y(0) = 1$  using RK 4<sup>th</sup> order method.
10. Solve  $y' = xy + y^2$ ,  $y(0) = 1$  for  $y(0.1)$  &  $y(0.2)$  using RK method of fourth order.
11. Solve the following simultaneous differential equations using RK 2<sup>nd</sup> order method at  $x = 0.1$  &  $0.2$ ;  $\frac{dy}{dx} = xz + 1$ ;  $\frac{dz}{dx} = -xy$ ; with initial conditions  $y(0) = 0$ ,  $z(0) = 1$ .
12. Solve by RK-2 method for  $x = 0(0.1)0.2$

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0; y(0) = 1, y'(0) = 0$$

13. Using the RK-2 method, obtain a solution of the equation  $y'' = y + xy'$  with the initial condition  $y(0) = 1$ ,  $y'(0) = 0$  to find  $y(0.2)$  and  $y'(0.2)$  [Take  $h = 0.1$ ]
14. Solve the ordinary differential equation,  $y'' = x(y')^2 - y^2$  for  $x = 0.6$  with the initial condition  $y(0) = y'(0) = 0$  by using RK-2 method. [Take  $h = 0.3$ ]
15. Solve the following differential equation within  $0 \leq x \leq 0.4$  using RK-2 method,

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - 3y = 6x; y(0) = y'(0) = 1, h = 0.2.$$

16. Solve:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 4y = 3x; y(0) = 0, y'(0) = 1, h = 0.5 \text{ within } 0 \leq x \leq 1.0 \text{ using RK 4}^{\text{th}} \text{ order method.}$$

17. Solve the following initial value problem for  $y(1.2)$  using the RK-4<sup>th</sup> order method:

$$y'' - 3y' + y = \sin x; y(1) = 1.2, y'(1) = 0.5$$

## Boundary Value Problems

1. Using the finite difference method, find  $y(0.25)$ ,  $y(0.5)$  and  $y(0.75)$  satisfying the differential equation  $xy'' + y = 0$ , subject to the boundary conditions  $y(0) = 1$ ,  $y'(1) = 2$ .
2. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

$$\frac{d^2y}{dx^2} = x + y; y(0) = y(1) = 0$$

3. Solve the BVP:  $y'' + 3y' = y' + x^2$ ,  $y(0) = 2$ ,  $y(2) = 5$  at  $x = 0.5, 1, 1.5$  using finite difference method.
4. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

$$y'' = e^x + 2y' - y; y(0) = 1.5, y(2) = 2.5$$

5. Using Finite difference method to solve the BVP:

$$y'' = 4y' - 4y + e^{2x}; y(0) = 0, y(1) = 2 \text{ for three internal points in } (0, 1)$$

6. Write an algorithm to solve two-point BVP using shooting method.
7. Solve the following BVP using shooting method:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \text{ with } y(1) = 1 \text{ and } y(2) = 5; \text{ Take } h = 0.25$$