Assignment#7

Solⁿ. of Ordinary Differential Equations

Initial Value Problems

- 1. Using **Euler's method**, find y(0.2) from the equation: y' = x + y, y(0) = 0, take h = 0.1
- 2. **Derive Euler's formula** for solving initial value problem.
- 3. Solve $y' = y/(x^2 + y^2)$, y(0) = 1 using **RK-2 method** in the range 0, 0.5, 1.
- 4. Solve $y' = \sin x + \cos y$ subject to initial condition y(0) = 2 in the range 0(0.5)2 using the RK second order method.
- 5. Using the RK-2, obtain a solution of the equation $y' = xy + y^2$ with the initial condition y(0) = 1 for the range $0 \le x \le 0.6$ with increments of 0.2.
- 6. Solve $y' = 4e^{0.8x} 0.5y$; subject to initial condition y(0) = 2, for y(0.5) and y(0.1) using Runge-Kutta 2^{nd} order method.
- 7. Solve $\frac{dy}{dx} = \frac{y^2 x^2}{y^2 + x^2}$ using RK-4 method, for y(0.4). Given: y(0) = 1, h = 0.2
- 8. Write an **algorithm**, **pseudo-code** and a **program** in any high level language (C/C++/FORTRAN) to solve a first-order initial value problem using classical RK-4 method.
- 9. Solve the differential equation, $dy/dx = (1 + x^2)y$ within $x \le 0(0.2)0.4$ and y(0) = 1 using RK 4th order method.
- 10. Solve $y' = xy + y^2$, y(0) = 1 for y(0.1) & y(0.2) using RK method of fourth order.
- 11. Solve the following simultaneous differential equations using RK 2nd order method at x = 0.1 & 0.2; $\frac{dy}{dx} = xz + 1$; $\frac{dz}{dx} = -xy$; with initial conditions y(0) = 0, z(0) = 1.
- 12. Solve by RK-2 method for x = 0(0.1)0.2

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$
; y(0) = 1, y'(0) = 0

- 13. Using the RK-2 method, obtain a solution of the equation y'' = y + xy' with the initial condition y(0) = 1, y'(0) = 0 to find y(0.2) and y'(0.2) [Take h = 0.1]
- 14. Solve the ordinary differential equation, $y'' = x(y')^2 y^2$ for x = 0.6 with the initial condition y(0) = y'(0) = 0 by using RK-2 method. [Take h = 0.3]
- 15. Solve the following differential equation within 0≤x≤0.4 using RK-2 method,

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x$$
; y(0) = y'(0) = 1, h = 0.2.

16. Solve:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 4y = 3x; y(0) = 0, y'(0) = 1, h = 0.5 \text{ within } 0 \le x \le 1.0 \text{ using RK } 4^{th} \text{ order method.}$$

17. Solve the following initial value problem for y(1.2) using the RK-4th order method:

$$y'' - 3y' + y = sinx$$
; $y(1) = 1.2$, $y'(1) = 0.5$

Boundary Value Problems

- 1. Using the finite difference method, find y(0.25), y(0.5) and y(0.75) satisfying the differential equation xy'' + y = 0, subject to the boundary conditions y(0) = 1, y'(1) = 2.
- 2. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

$$\frac{d^2y}{dx^2} = x + y$$
; $y(0) = y(1) = 0$

- 3. Solve the BVP: $y'' + 3y' = y' + x^2$, y(0) = 2, y(2) = 5 at x = 0.5, 1, 1.5 using finite difference method.
- 4. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

$$y'' = e^x + 2y' - y$$
; $y(0) = 1.5$, $y(2) = 2.5$

5. Using Finite difference method to solve the BVP:

$$y'' = 4y' - 4y + e^{2x}$$
; $y(0) = 0$, $y(1) = 2$ for three internal points in $(0, 1)$

- 6. Write an algorithm to solve two-point BVP using shooting method.
- 7. Solve the following BVP using shooting method:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x$$
 with y(1) = 1 and y(2) = 5; Take h = 0.25

8. Using shooting method, solve the BVP:

$$\frac{d^2y}{dx^2} = 6y^2$$
 with y(0) = 1 and y(0.5) = 0.44;

[Taking
$$m0 = -1.8$$
, $m1 = -1.9$, we get $m2 = -2 & y(0.5) = 0.4441$]