

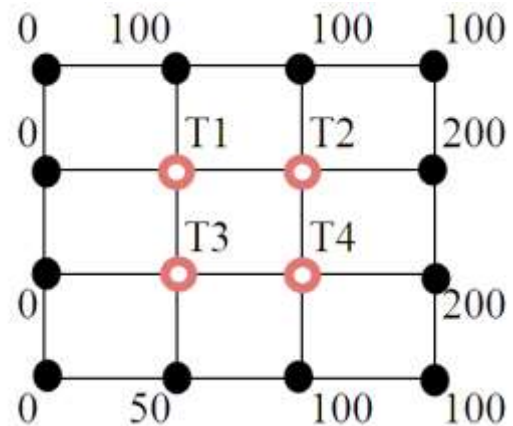
Assignment#9

Numerical Solⁿ. of Partial Differential Equations

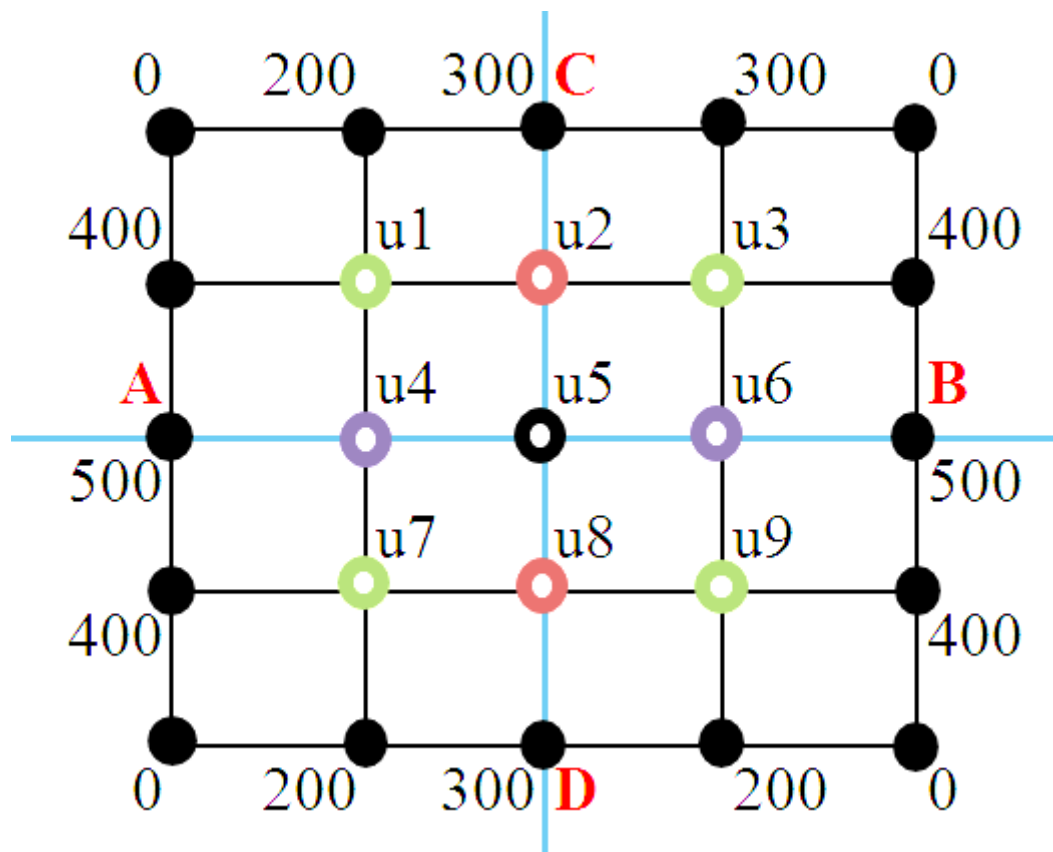
1. The steady state two dimensions heat-flow in a metallic plate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Given the boundary conditions as shown in the figure below, find the temperatures T1, T2, T3 & T4. Solve the equations using Gauss-Seidel method.

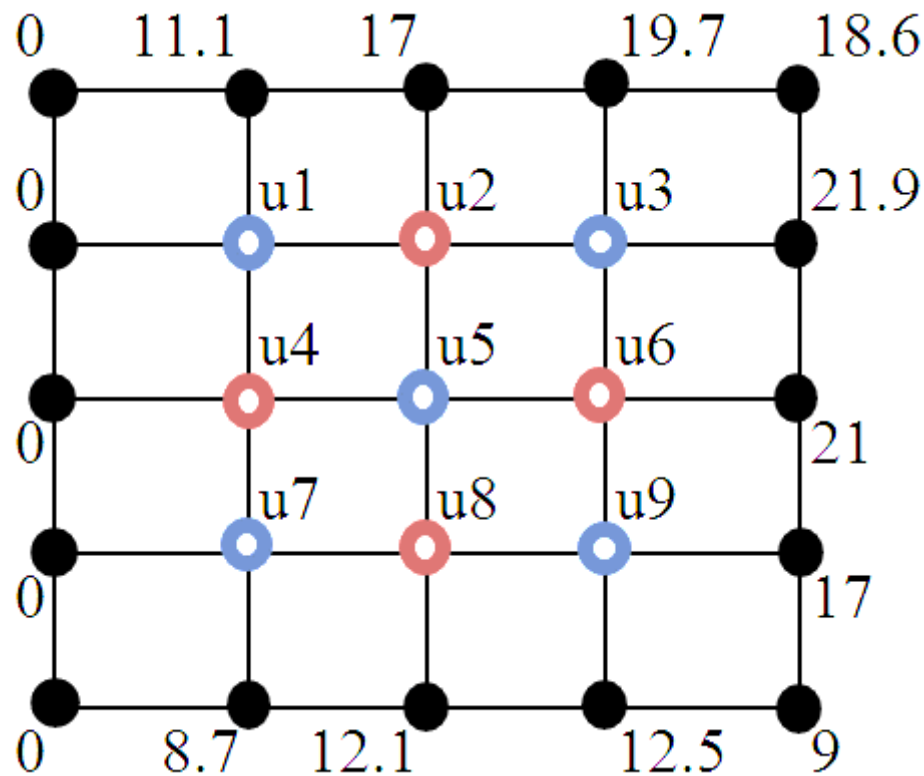


2. Torsion on a rectangular bar subject to twisting is governed by $\nabla^2 T = -4$. Given conditions: $T = 0$ on boundary, find T over a cross section of a bar of size **9cm x 9cm**, use the small grid size of **3cm x 3cm**.
3. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ given that



4. Solve for the steady-state temperature in rectangular plate **8cm x 10cm**, if one 10cm side is held at 50C, and the other 10cm side held at 30C and the other sides are held at 10C. Assume square grids of **2cm x 2cm**. [Hint: 4x5 data grid]

5. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ given that



6. Solve the equation $\nabla^2 f = F(x, y)$ with $F(x, y) = xy$ and $f = 0$ on boundary. The domain is a square with corners at $(0, 0)$ & $(3, 3)$. Use $h=1$
7. Estimate the values at grid points of the following equations, assume $h = 1$:
- $f_{xx} - 0.5f_t = 0$
Given: $f(0, t) = 0$; $f(5, t) = 0$; $f(x, 0) = x(5-x)$;
 - $9f_{xx} = f_t$
Given: $f(0, t) = -5$; $f(5, t) = 5$;
 $f(x, 0) = \begin{cases} -5 & \text{for } 0 \leq x \leq 2.5 \\ 5 & \text{for } 2.5 \leq x \leq 5 \end{cases}$
8. Solve the following hyperbolic equations:
- $f_{tt} = uf_{xx}$
Given: $f(0, t) = 0$; $f(5, t) = 0$; $f(x, 0) = 100x^2(5-x)$; $f_t(x, 0) = 0$
 - $f_{tt} = uf_{xx}$
Given: $f(0, t) = 0$; $f(1, t) = 0$; $f(x, 0) = f(x) = \sin(\pi x) + \sin(2\pi x)$; $f_t(x, 0) = 0$