Name: _____ Roll No.: _____

TRIBHUVAN UNIVERSITY
KHWOPA COLLEGE OF ENGINEERING
Dept. of Computer Engineering
2075 Asar

Exam	**CT – Chapter 4 & 5**		
Level	BE	**Full Marks**	25
Program	BEL, BEX, BCT, B. Agri.	**Pass Marks**	15
Year/Part	II / II	**Time**	1 Hrs.

Subject: - Numerical Method *(SH553)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ Attempt **All** questions.
- ✓ The figures in the margin indicate **Full Marks**.
- ✓ Assume suitable data if necessary.

1. By the method of least squares, find the curve $y = ax + bx^2$ that best fit the following: [8]

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

[$y = 1.52x + 0.49x^2$]

2. Fit a parabola $y = ax^2 + bx + c$, by the method of least square to the following data: [8]

x	1	2	3	4	5
y	5	12	26	60	97

[$y = 5.7143x^2 – 11.0258x + 10.4001$]

3. Fit a parabola to the following data: [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

[Hint: - Change the origin: $u = (x-x_0)/h$; $v = (y-y_0)/h$; $x0 = 5$; $y0 = 7$; $h = 1$ & the equation of a parabola is $v = a + bu + cu^2$]

[$v = 3 + 0.85u – 0.27u^2$; Changing the origin: $y = -1 + 3.55x – 0.27x^2$]

4. Fit a second degree parabola to the following data: [8]

x	1	2	3	4	5
y	1090	1220	1390	1625	1915

[Hint: - Change the origin: $u = (x-3)$; $v = (y-1450)/5$; & the parabolic curve is $v = a + bu + cu^2$]

[$v = -11 + 41.1u + 5.5u^2$; Changing the origin: $y = 1024 + 40.5x + 27.5x^2$]

5. Determine the constants a & b by the method of least square method such that $y = ae^{bx}$ fits the following data: [8]

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

[$y = 1.4999e^{0.5x}$]

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6. Determine curve of the form $y = ax^b$ which is the best fit to the following data according to least squares principle: [8]

x	1.0	1.5	2.0	2.5	3.0	3.5
y	0.01	0.405	0.693	0.916	1.098	1.252

[$y = 0.0306e^{3.5004}$]

7. Fit a curve $y = ab^x$ to the following data: [8]

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

[$y = 100.0322(1.1998)^x$]

8. The pressure and volumn of a gas are related by the equation $PV^r = K$, r and K being constants. Fit this equation to the following set of observations: [8]

P (kg/cm^2)	0.5	1.0	1.5	2.0	2.5	3.0
V (liters)	1.62	1.00	0.75	0.62	0.52	0.46

[$PV^{1.276} = 1.039$]

9. The pressure of the gas corresponding to various volumes V is measured, given by the following data: [8]

V (cm^3)	50	60	70	90	100
P (kg/cm^2)	64.7	51.3	40.5	25.9	78

Fit the data to the equation $PV^r = C$ [$PV^{7.84068} = 1.71199 \times 10^{16}$]

10. Fit a curve $y = ax + b/x$ to the data given below: [8]

x	1	2	3	4	5	6	7	8
y	5.43	6.28	8.23	10.32	12.63	14.84	17.27	19.51

[$y = 2.396x + 3.0289/x$]

11. A person runs the same race track for 5 consecutive days and is times as follows: [8]

Days (x)	1	2	3	4	5
Time (y)	15.3	15.1	15	14.5	14

Make a least square fit to the above data using the function $y = a + \frac{b}{x} + \frac{c}{x^2}$

[$y = 12.6751 + 8.2676/x - 5.7071/x^2$]

12. Use the method of least squares to fit the curve $y = \frac{c0}{x} + c1\sqrt{x}$ to the following table of values: [8]

x	0.1	0.2	0.4	0.5	1	2
y	21	11	7	6	5	6

[$y = 1.9733/x + 3.2818\sqrt{x}$]

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13. Find the value of sin 30º 15' 30" from the following table: [8]

Angle x^o	30º	31º	32º	33º	34º
sin x^o = y	0.5000	0.5150	0.5299	0.5446	0.5592

[Hint: x_0 = 30º; 15' 30" = 0.2583º] [f(30º 15' 30") = 0.5039]

14. The following table gives the population of a town during the last six censuses. Estimate the increase in the population during the period from 1946 to 1948: [8]

Year	1911	1921	1931	1941	1951	1961
Population (in thousands)	12	15	20	27	39	52

[2.53 thousands (Approx)]

15. Using Newton's interpolation formula, find the area of a circle of diameter 82 & 98 from the given table of diameter and area of circle: [8]

Diameter	80	85	90	95	100
Area	5026	5674	6362	7088	7854

[5279.856,]

16. The hourly declination of the moon on a day is given below. Find the declination at $3^h35^m15^s$: [8]

Hour	0	1	2	3	4
Decl,	8º29'53.7"	8º18'19.4"	8º6'43.5"	7º55'6.1"	7º43'27.2"

[Hint: $x_n = 4^h$; $p = (x-x_n)/h = (3^h35^m15^s-4^h)/1^h = -(0^h24^m45^s)/1^h = -1485^s/3600^s = -0.4125$]

[$y(3^h35^m15^s) = 7º48'16"$]

17. Following are the populations of a district: [8]

Year	1881	1891	1901	1911	1921	1931
Population (in thousands)	363	391	421	?	467	501

Find the population of the year 1991 & 1930. [445.2 thousands,]

18. Compute $\frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx$ when x = 0.6538 using Stirling's formula and Bessel's formula, and compare the answers given that: [8]

x	0.62	0.63	0.64	0.65	0.66	0.67	0.68
y	0.6194	0.6270	0.6348	0.6420	0.6493	0.6566	0.6637

[y = 0.6448,]

19. From the following table, find the value of $\log_{10}337.5$ by Stirling's formula and Bessel's formula, and compare the answers: [8]

x	310	320	330	340	350	360
$\log_{10}x$	2.4913617	2.505100	2.5185139	2.5314789	2.5440680	2.5563025

[$\log_{10}337.5$ = 2.52827374, 2.52827374]

20. Using Stirling's formula and Bessel's formula, estimate $(46.24)^{1/3}$ given: [8]

x	41	45	49	53
$y=x^{1/3}$	3.4482	3.5569	3.6593	3.7563

[$(46.24)^{1/3}$ = 3.589276,]

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21. Determine f(x) as a polynomial in x and find by divided difference formula f(7), f(1), f(-5), f(-2): [8]

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

[$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$, , , ,]

22. Given $\log_{10}654 = 2.8156$, $\log_{10}658 = 2.8182$, $\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$, find the polynomial equation; $\log_{10}665$; $\log_{10}650$ and $\log_{10}656$. [8]

[, , , 2.8168]

23. Given $\log_{10}654 = 2.8156$, $\log_{10}658 = 2.8182$, $\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$, find $\log_{10}656$ using Newton's Divided Difference & Lagrange's formula. [8]

[2.8168, 2.8170]

24. Given $\log_{10}654 = 2.8156$, $\log_{10}658 = 2.8182$, $\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$, find the polynomial equation by both methods. [8]

[ ]

25. Find the age corresponding to the annuity value 13.6 from the given table: [8]

Age	30	35	40	45	50
Annuity Value	15.9	14.9	14.1	13.3	12.5

[$43.141851 \approx 43$]

26. Obtain cubic spline for every sub-interval, given in the tabular form: [8]

x	0	1	2	3
f(x)	1	2	33	244

With the end conditions $M_0 = 0 = M_3$.

$$\left[f(x) = \begin{cases} -4x^3 + 5x + 1 \; for \; 0 \le x \le 1 \\ 50x^3 - 162x^2 + 167x - 53 \; for \; 1 \le x \le 2 \\ -46x^3 + 414x^2 - 985x + 715 \; for \; 2 \le x \le 3 \end{cases} \right]$$

27. Use Stirling's formula to find f'(0.6) and f"(0.6) from the following table: [8]

x	0.4	0.5	0.6	0.7	0.8
f(x)	1.5836	1.7974	2.0442	2.3275	2.6510

[2.6445, 3.6484]

28. Use Bessel's formula to find f'(0.04) from the following table: [8]

x	0.01	0.02	0.03	0.04	0.05	0.06
f(x)	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

[0.25625]

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[0.25625]

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30. From the following table, find the value of x for which f(x) is maximum. Also find the value of f(x) from the table of values given below: [8]

x	60	75	90	105	120
f(x)	28.2	38.2	43.2	40.9	37.7

[f(x) is maximum at x = 92.1135 and the maximum value is 43.27]

31. Find the value of x for which f(x) is minimum and find the minimum value from the table below: [8]

x	0.60	0.65	0.70	0.75
f(x)	0.6221	0.6155	0.6138	0.6170

[f(x) is minimum at x = 0.6923 and the minimum value is 0.6137426]

32. Evaluate I = $\int_0^6 \frac{1}{1+x}\,dx$ using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule where intervals of integrations is sub-divided into six equal parts. Also, check by direct integration. [8]

[2.021429, 1.958730, 1.966071, 1.945910]

33. Find $\int_0^1 \frac{1}{1+x^2}\,dx$ using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[0.78424, , ,]

33. Evaluate the value of the integral $\int_4^{5.2} log_e x dx$ using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[1.827648, , ,]

33. Find the value of $\int_0^{0.6} e^x dx$, taking n=6, correct to five significant figures using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[, 0.82212, ,]

33. Calculate an approximate value of the integral $\int_0^{\pi/2} sinx\,dx$, using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[0.99795, , ,]

34. Evaluate the value of the integral $\int_{0.2}^{1.4} (sinx - lnx + e^x)\,dx$, using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[4.05617, 4.05106, 4.051879, 4.05095]

35. A river is 80 meter wide. The depth 'y' of the river at a distance 'x' from one bank is given in the following table: [8]

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

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Find approximately the area of cross-section of the river using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule.

[, 710 sq. meter,]

33. Evaluate $\int_0^{\pi/2} e^{sinx} dx$ correct to four decimal places, using (i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule. Also, check by direct integration. [8]

[, , 3.10166,]

34. Evaluate $\int_0^2 \frac{1}{x^2+4}\, dx$, using Romberg's method. Hence, obtain an approx. value of π. [8]

[I = 0.3927 = π/8; π = 3.1416]

35. Evaluate $\int_0^1 \frac{1}{x^2+1}\, dx$, using Romberg's method correct to 4 decimal places. Hence, deduce an approximate value of π. [8]

[I = 0.7854 = π/4; π = 3.1416]

36. Find $\int_0^{\pi/2} sinx\, dx$, by two & three point Gauss quadrature formula. [8]

[0.9985, 1.000008116]

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