

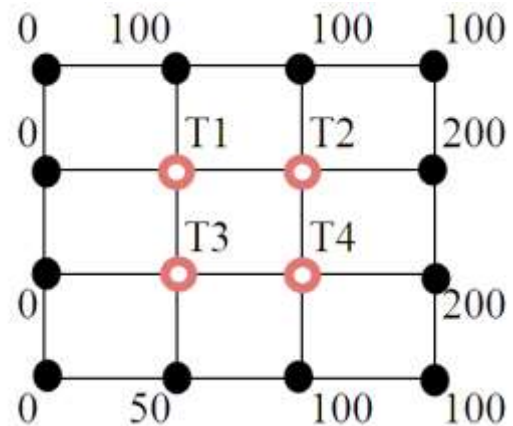
Assignment#9

Numerical Solⁿ. of Partial Differential Equations

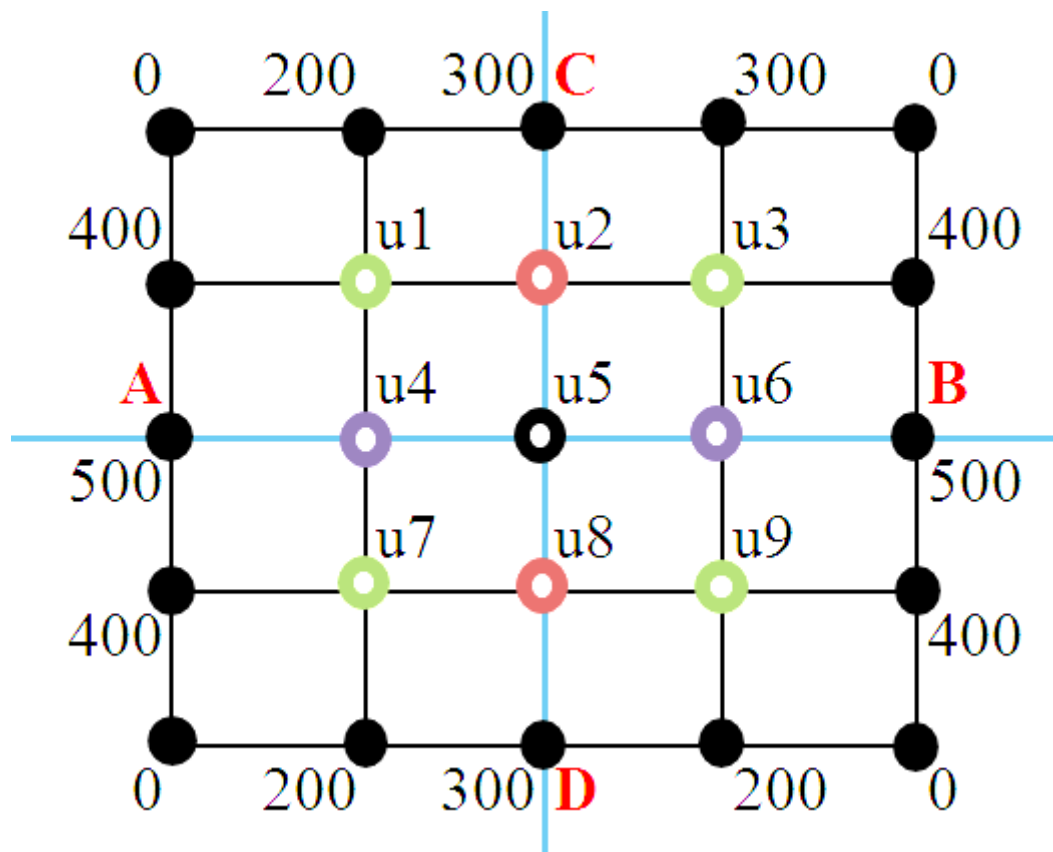
1. The steady state two dimensions heat-flow in a metallic plate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Given the boundary conditions as shown in the figure below, find the temperatures T1, T2, T3 & T4. Solve the equations using Gauss-Seidel method.

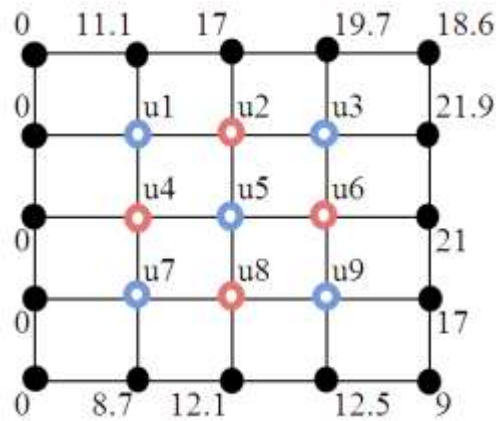


2. Torsion on a rectangular bar subject to twisting is governed by $\nabla^2 T = -4$. Given conditions: $T = 0$ on boundary, find T over a cross section of a bar of size **9cm x 9cm**, use the small grid size of **3cm x 3cm**.
3. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ given that



4. Solve for the steady-state temperature in rectangular plate **8cm x 10cm**, if one 10cm side is held at 50C, and the other 10cm side held at 30C and the other sides are held at 10C. Assume square grids of **2cm x 2cm**. [Hint: 4x5 data grid]

5. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ given that



6. Solve the equation $\nabla^2 f = F(x, y)$ with $F(x, y) = xy$ and $f = 0$ on boundary. The domain is a square with corners at $(0, 0)$ & $(3, 3)$. Use $h=1$
7. Given $\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial t} = 0$; $f(0, t) = f(5, t) = 0$, $f(x, 0) = x^2(25-x^2)$; find the values of f for $x = ih$ ($i = 0, 1, \dots, 5$) and $t = jk$ ($j = 0, 1, \dots, 6$) with $h = 1$ and $k = \frac{1}{2}$, using the explicit method.
8. Estimate the values at grid points of the following equations using recurrence formula [$h = 1$]:
- $f_{xx} - 0.5f_t = 0$
Given: $f(0, t) = 0$; $f(5, t) = 0$; $f(x, 0) = x(5-x)$;
 - $9f_{xx} = f_t$
Given: $f(0, t) = -5$; $f(5, t) = 5$;
$$f(x, 0) = \begin{cases} -5 & \text{for } 0 \leq x \leq 2.5 \\ 5 & \text{for } 2.5 \leq x \leq 5 \end{cases}$$
9. Solve by relaxation method, the equation $\nabla^2 u = 0$ in the square region with square meshes starting with the initial values $u_1 = u_2 = u_3 = u_4 = 1$.

[Ans: $u_1 = 1$, $u_2 = 1.3$, $u_3 = 0.7$, $u_4 = 1$]

