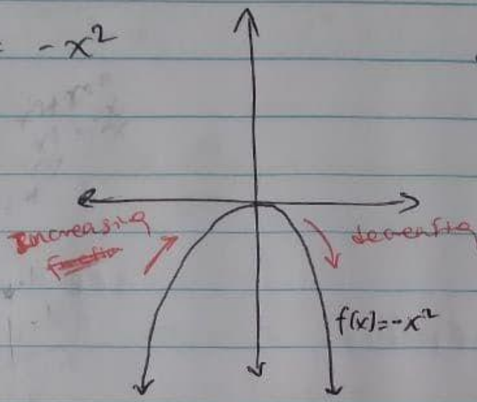
Problem 1: which one increasing? eventually non-decreasing?

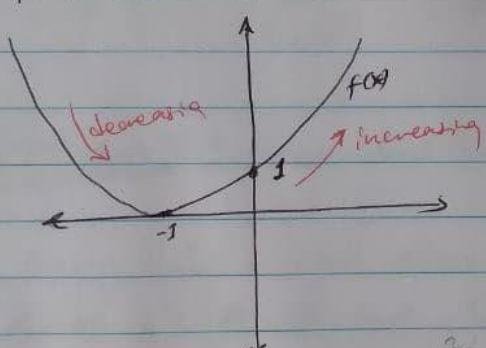
(1) $f(x) = -x^2$



∴ the function is neither increasing or nor non-decreasing.

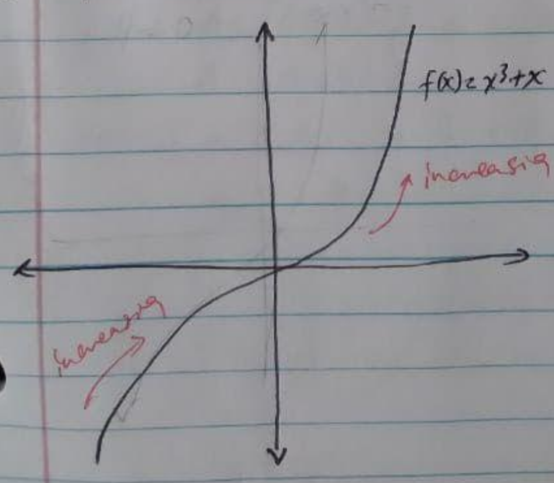
It's eventually decreasing

(2) $f(x) = x^2 + 2x + 1$



∴ this function is increasing for $[-1, \infty)$ which will make our function to be eventually non-decreasing or eventually increasing

(3) $f(x) = x^3 + x$



∴ the following graph is increasing for $(-\infty, \infty)$

Increasing

Problem 2:

1) $\lim_{n\to\infty} \frac{2n^2+3n}{n^3-4} = \frac{\frac{2n^2}{n^2}+\frac{3n}{n^2}}{\frac{n^3}{n^2}+\frac{4}{n^2}} = \frac{1}{n}$

$\therefore \lim_{n\to\infty} \frac{1}{n} = 0$

2) $\lim_{n\to\infty} \frac{n^2}{2^n}$ by using L' Hopital rule we can reduce to

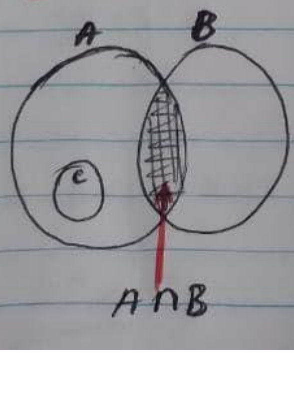
$\lim_{n\to\infty} \frac{n^2}{2^n} = \frac{\frac{d}{dn}(n^2)}{\frac{d}{dn}(2^n)} = \lim_{n\to\infty} \frac{2^n}{2^n \cdot \ln 2} = \lim_{n\to\infty} \frac{2^n}{(\ln 2)(2^n)} = 0$

Problem 3: from what is given in our problem we can find C.

* Since $C \subseteq A$, all elements in C are in A
* Since $C \cap B = \phi$, and $C \cup B = A \cup B$ $\therefore$ C is part of A But does not intersect with B.

$C = A - (A \cap B)$
$= A \; (A \cap B)$ } A without $(A \cap B)$

or $C = A \cap B'$, $B' = B$ Compliment



Problem 4: In how many dlt ways can two of the Five applicants be ranked first and second?

$\therefore \; 5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5\times4\times\cancel{3!}}{2!\times\cancel{3!}} = \underline{\underline{10}}$

$\therefore$ we have $\underline{\underline{10}}$ dlt ways.

Problem 5: Use Induction to show that for all $n > 4$, $2^n < n!$.

⊛ Base Case:- $\underline{\underline{n=5}}$ Since $n > 4$

$LHS \longleftrightarrow RHS$

$2^5 < 5!$

$2\times2\times2\times2\times2 < 5\times4\times3\times2\times1$

$\underline{\underline{32 < 120}}$ which is $\underline{\underline{\text{True}}}$

⊛ Inductive Step

assume $2^n < n!$ for $n > 4$ is **True**

$\therefore$ we want to show $2^{n+1} < (n+1)!$

$LHS \Rightarrow 2^{n+1}$ $\qquad$ $RHS \Rightarrow (n+1)!$

$= 2^n \times 2^1$ $\qquad$ $= (n+1)\times(n!)$

$= 2^n \times 2$ $\qquad$ $= (n+1)\times n!$

$\Rightarrow$ This mean $= (n+1)(2^n)$ From inductive hypothesis

$\therefore LHS < RHS \Rightarrow \cancel{2^n}\times 2 < (n+1)\cancel{(2^n)}$

$2 < n+1$

$\therefore \underline{\underline{2^n < n!}}$ For $n \geq 4$ holds $\underline{\underline{\text{true}}}$

```java
/**
 * @author KidusMT
 * Date: 5/23/2021
 */
public class Problem6 {
    public static void main(String[] args) {
        System.out.println(log2(5));
    }

    public static double log2(double x){
        return Math.log(x)/Math.log(2);
    }
}
```

Run: Problem6 (1)

```
"C:\Program Files\Java\jdk-15.0.2\bin\java.exe" "-javaagent:C:\Program Files\Je
2.321928094887362

Process finished with exit code 0
```

Problem 7 : Compute the following derivative:

~~recall~~

$$\frac{d}{dx}\frac{3\log x}{2^x}$$

from division rule $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{g(x)^2}$

$$= \frac{\frac{d}{dx}(3\log x)\, 2^x - \frac{d}{dx}(2^x)\,(3\log x)}{2^{2x}}$$

$$= \frac{\frac{3}{x\ln 2}\cdot 2^x - (2^x \ln 2)(3\log x)}{2^{2x}}$$

$$= \frac{3\left(2^x\cdot(\log'(x)) - \log(x)\cdot(2^x)'\right)}{(2^x)^2}$$

$$= \frac{3\left(\frac{1}{x\ln 2} - \log x \ln 2\right)}{2^x}$$

Problem 8

(A) Assum: $|x| = n \quad n \geq 1$

$x = \{x_1 \dots x_n\}$

$x^- = \{x_1 \dots x_{n-1}\}$

$|x| = 2^n$ and $|x^-| = 2^{n-1}$

$P(x^-) \subseteq P(x)$ because x^- Contains all elements of x except the n^{th}.

For each U in $P(x^-)$ $U \cup \{x_n\}$ and ~~Placing~~ Substituting by Y

$|Y| = 2^{n-1}$

Since Y contains x_n

because Y's properties are

$Y = \{ \{x_n\}, \{x_1, x_n\}, \{x_2, x_n\}, \dots \}$

$\therefore P(x^-) \cup Y \subseteq P(x)$ and

$P(x^- \cup Y) = |P(x)| \rightarrow P(x^-) \cup Y = P(x)$

(B) Among the dff ways to optain Y from $P(x^-)$ the easiest way would be ~~[illegible]~~ to union the set $\{x_n\}$ with all elements (Subsets) of $P(x^-)$. Therefore, we can create a set containing all including x_n as an element Because in power set we can have for example like the one we used in our class

(Ex) $\{1,2,3\} \Rightarrow \{ \{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{3\}, \{2,3\}, \{1,2,3\} \}$

Problem 9 :-

(A) Suppose d/m and d/n show $d/m \bmod n$

$$m = n \cdot \left\lfloor \frac{m}{n} \right\rfloor + m \bmod n$$

$$m \bmod = m - n \cdot \left\lfloor \frac{m}{n} \right\rfloor$$

since $d/n \Rightarrow d/n : \left\lfloor \frac{m}{n} \right\rfloor$ and $d/m \Rightarrow \left(\frac{d}{m - n \cdot \left\lfloor \frac{m}{n} \right\rfloor} \right)$

$$\Rightarrow d/m \bmod n$$

(B) By hypothesis d/n, and $d/m \bmod n$ so di kiden each sum and of right hand side of (x)

It follows that d/m

or simply $\frac{m}{d} = \frac{n}{d} \left\lfloor \frac{m}{n} \right\rfloor + \frac{m \bmod n}{d}$ $\therefore$ $\frac{d}{m}$

(C) $\gcd(m, n) = \gcd(n, m \bmod n)$

Because A and B show that an integer d is a Common divisor of m and n if and only if Comm divisor of n, (m mod n).

Therefore the greatest Common divisor of m and n is equal to the gcd of n, (m mod n).

Problem 10: prove

(A) $4n^3+n$ is $\Theta(n^3)$

From defn: $f(n)$ is $\Theta(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0$

$$\lim_{n\to x} \frac{4n^3+n}{n^3} \times \frac{1/n^3}{1/n^3}$$

$$\lim_{n\to x} \frac{4n^3/n^3 + n/n^3}{1} = \lim_{n\to\infty} \frac{4}{1} = 4$$ since $\neq 0$ then True

(B) $\log n$ is $o(n)$

$$\lim_{n\to\infty} \frac{(\log n)'}{n'}$$ → L'Hopital rule

$$\lim \frac{1/n \log e}{1} = 0$$ ∴ $\log(n)$ is $o(n)$ → is True

(C) 2^n is $\omega(n^2)$ by

$$= \lim_{n\to\infty} \frac{(n^2)'}{(2^n)'} =$$

$$= \lim_{n\to\infty} \frac{2n}{2^n \ln 2} = 0$$ Therefore, 2^n is Lower Bound by n^2

(D) 2^n is $o(3^n)$

$$\lim_{n\to\infty} \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n = 0$$ ∴ 2^n is $\Theta(3^n)$ from defn

$f(n)$ is $\Theta(g(n))$ iff $\lim \frac{f(n)}{g(n)} = 0$

(E) 2^n is $\theta(2^{n-1})$

$$\lim_{n\to\infty} \frac{2^n}{2^{n-1}} = \frac{2^n}{\frac{2^n}{2}}$$

$$\lim 2 \cdot \frac{\cancel{2^n}}{\cancel{2^n}} = 2$$ so 2^n is $\underline{\underline{\theta(2^{n-1})}}$ holds

(F) $\log n$ is $\theta(\log_3 n)$

$$\lim_{n\to\infty} \frac{(\log n)'}{(\log_3 n)'}$$ L'Hopital

$$\lim_{n\to\infty} \frac{1/n\ln 2}{1/n\ln 3} = \frac{1}{\cancel{n}\ln 2} \cdot \frac{\cancel{n}\ln 3}{1} = \lim_{n\to\infty} = \frac{\ln 3}{\ln 2} = \frac{\ln 3}{\ln 2}$$

so $\log(n)$ is $\underline{\underline{\theta(\log_3 n)}}$