

Theory of concurrency

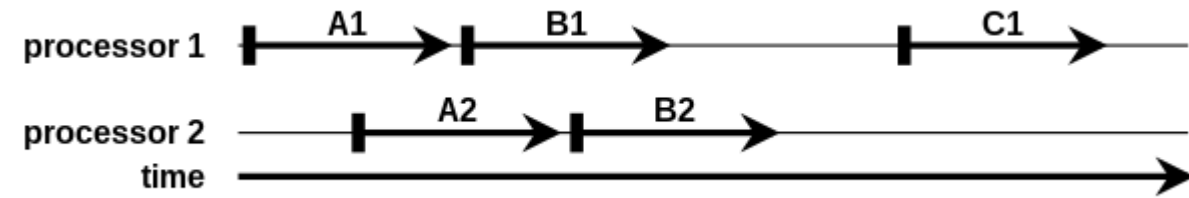
Lecture 14

Shared Resources

Shared Resources: **Shared storage**

- The behaviour of *systems* of concurrent processes can be implemented on
 - a *single* conventional stored program computer by *timesharing*:
 - a single processor executes each of the processes in *alternation*,
 - with process change on occurrence of *interrupt* from
 - an external *device* or from a regular *timer*.
- In this case, the concurrent processes may
 - *share locations* of common storage:
 - they are accessed and assigned by means of
 - the usual machine instructions within the code of the processes.

Shared Resources: **Shared storage**



Lamport: *sequential consistency* is met if

"the result of any execution is the same as

- if the operations of all the processors were executed in some sequential order,
- and the operations of each individual processor appear in this sequence in the order specified by its program."
- The *strong memory models*
 - processor architectures: AMD64, z/Architecture
 - programming languages: C/C ++ (default atomics), Java (volatile)
- The *weak memory models*
 - processor architectures: x86, Power, ARM, SPARC, DEC Alpha
 - programming languages: C/C ++, Java, OCaml.

Shared Resources: **Shared storage**

- A location of shared storage can be modelled a shared variable:

$$(count : VAR \parallel (count.left ! 0 \rightarrow (P \parallel Q)))$$

$$\begin{aligned} VAR &= left ? x \rightarrow VAR_x \\ VAR_x &= (left ? y \rightarrow VAR_y \mid right ! x \rightarrow VAR_x) \end{aligned}$$

- **Shared storage** vs. the **local storage** of sequential processes.
 - each variable is updated by at most one process
 - arbitrary interleaving of assignments from different processes.

Shared Resources: **Shared storage**

X1. (Interference) The shared variable *count* keeps a count of

- the total number of occurrences of some important event.
- On each occurrence of the event,
 - the relevant process *P* or *Q* tries to update the count by the pair of communications

$$count.right ? x; count.left ! (x + 1)$$

- These two communications may be interleaved by
 - a similar pair of communications from the other process:

$$count.right ? x \rightarrow count.right ? y \rightarrow count.left ! (y + 1) \rightarrow count.left ! (x + 1) \rightarrow \dots$$

- This kind of error is known as *interference*.
- The actual occurrence of the fault is *highly nondeterministic*
 - it is not reliably reproducible
 - it is very hard to diagnose the error by conventional testing techniques.

Shared Resources: **Shared storage**

- A *critical region* is a possible solution to this problem
 - to make sure that no change of process takes place during
 - a sequence of actions which must be protected from interleaving.
- On an implementation by a single processor,
 - the required exclusion can be achieved by
 - *inhibiting all interrupts* for the duration of the critical region.
- This solution has an undesirable effect in *delaying response to interrupts*; and
 - it fails completely as soon as *a second processor* is added to the computer.
- The binary exclusion *semaphore* by E. W. Dijkstra
 - is a better solution.



$LP = acquire \rightarrow \mu X \cdot (left ? s \rightarrow h ! s \rightarrow X \mid release \rightarrow LP)$

$lp.acquire \rightarrow \dots lp.left ! \text{“A. JONES”} \rightarrow \dots lp.left ! nextline \rightarrow \dots lp.release \rightarrow$

Shared Resources: **Shared storage**

- A *semaphore* is a process which engages alternatively in actions named P and V

$$SEM = (P \rightarrow V \rightarrow SEM) \\ (mutex : SEM \parallel \dots)$$

Probeer (try) and *Verhoog* (increment)

- This is a shared resource
- Each process,
 - on entry into a critical region must send the signal $mutex.P$
 - on exit from the critical region must engage in the event $mutex.V$
- The critical region with the incremented count:
 $mutex.P \rightarrow count.right ? x \rightarrow count.left ! (x + 1) \rightarrow mutex.V \rightarrow \dots$
- Two processes cannot interfere with each other's updating of the count iff
 - all processes observe this discipline.
- If any process omits a P or a V , or gets them in the wrong order,
 - the effect is chaotic
 - risk a disastrous or a subtle error.

$$CT_0 = (up \rightarrow CT_1 \mid around \rightarrow CT_0)$$

$$CT_{n+1} = (up \rightarrow CT_{n+2} \mid down \rightarrow CT_n)$$

Shared Resources: **Shared storage**

- The design of the shared storage based on
 - knowledge of its intended pattern of usage
 - another way to prevent interference.
 - If a variable is only for counting, then
 - the increment operation is a single atomic operation *count.up*
 - the shared resource is CT_0

$$count : CT_0 \parallel (... P \parallel Q...)$$

- Shared resource specially designed for its purpose.
 - Pure (non-specialized) storage should not be shared in the design of a system using concurrency:
 - no accidental interference.
 - The design can be implemented efficiently on
 - networks of distributed processing elements, or
 - single-processor and multiprocessor computers with physically shared store.

$$LP = acquire \rightarrow \mu X \bullet (left ? s \rightarrow h ! s \rightarrow X \mid release \rightarrow LP)$$

$$lp.acquire \rightarrow \dots lp.left ! \text{“A. JONES”} \rightarrow \dots lp.left ! nextline \rightarrow \dots lp.release \rightarrow$$

Shared Resources: **Multiple resources**

- A single *serially reusable* resource
 - at any given time the resource is used by at most one of the sharing processes
 - alternating input and output, or
 - alternating acquire and release signals.
- Arrays of resources with identical behaviour
 - indices in the array ensure that
 - each element communicates safely with the process that has acquired it.

$$\parallel_{i < 12} P_i = (P_0 \parallel P_1 \parallel \dots \parallel P_{11})$$

$$\parallel_{i \geq 0} P_i = (P_0 \parallel P_1 \parallel \dots)$$

$$\parallel_{i < 4} P = (P \parallel P \parallel P \parallel P)$$

$$\square_{i \geq 0} (f(i) \rightarrow P_i) = (f(0) \rightarrow P_0 \mid f(1) \rightarrow P_1 \mid \dots)$$

- f is a one-one function
 - the choice between the alternatives is made solely by the environment.

Shared Resources: **Multiple resources**

X1. (Re-entrant subroutine)

- A serially reusable shared subroutine is used by only one calling process at a time.
- If the execution of the subroutine requires a considerable calculation,
 - there could be corresponding *delays* to the calling processes.
- Several instances of the subroutine may proceed concurrently on different processors
 - if several processors are available to perform the calculations.
- A subroutine capable of several concurrent instances is *re-entrant*,
 - it is defined as an array of concurrent processes:

doub : ($\parallel_{i < 27} (i : \text{DOUBLE})$) // ...

- A typical call of this subroutine:

(doub.3.left ! 30 \rightarrow doub.3.right ? y \rightarrow SKIP)

Shared Resources: **Multiple resources**

- The use of the index 3 ensures that
 - the result of the call is obtained from the same instance of *doub* to
 - which the arguments were sent,
 - even though some other concurrent process may
 - at the same time call another instance of the array:
 $doub.3.left.30, \dots doub.2.left.20,$
 $\dots doub.3.right.60, \dots doub.2.right.40, \dots$
- When a process calls a *re-entrant* subroutine,
 - it really does not matter *which* element of the array responds to the call.
- A calling process should leave the selection arbitrary
 - rather than specifying a particular index 2 or 3

$$\square_{i \geq 0} (doub.i.left ! 30 \rightarrow doub.i.right ? y \rightarrow SKIP)$$

- The same index is used for sending the arguments and for receiving the result.

Shared Resources: **Multiple resources**

- An arbitrary limit of 27 simultaneous activations of the subroutine.
- For single processor which divides its attention among a larger number of processes,
 - we introduce an infinite array of concurrent processes:

$$doub : (\parallel_{i \geq 0} i : D)$$

$$doub : (\parallel_{i < 27} (i : DOUBLE)) // \dots$$

- D serves only a single call and then stop.
- A *procedure* is a subroutine with no bound on its re-entrancy.
- In a procedure, the effect of each call

$$\Box_{i \geq 0} (doub.i.left ! x \rightarrow doub.i.right ? y \rightarrow SKIP)$$

- is identical to the call of a subordinate process D
 - declared immediately adjacent to the call

$$(doub : D // (doub.left ! x \rightarrow doub.right ? y \rightarrow SKIP))$$

Shared Resources: **Multiple resources**

- A *local* procedure call suggests execution of the procedure on
 - the same processor as the calling process;
- A *remote* call is the call of a shared procedure
 - it suggests execution on a separate possibly distant processor.
- The effect of remote and local calls is intended to be the same
 - the reasons for using the remote call can be political or economic
 - to keep the code of the procedure secret,
 - to run it on a machine with special expensive facilities.
 - a high-volume backing store: a disk or bubble memory.

LOC: $(doub : D \parallel (doub.left ! x \rightarrow doub.right ? y \rightarrow SKIP))$

REM: $\square_{i \geq 0} (doub.i.left ! x \rightarrow doub.i.right ? y \rightarrow SKIP)$

$$COPY = \mu X \bullet (left ? x \rightarrow right ! x \rightarrow X)$$

$$VAR = left ? x \rightarrow VAR_x$$

$$VAR_x = (left ? y \rightarrow VAR_y \mid right ! x \rightarrow VAR_x)$$

Shared Resources: **Multiple resources**

X2. (Shared backing storage)

A storage medium is split into B sectors which can be read and written independently.

- Each sector stores one block of information,
 - which it inputs on the left and outputs on the right.
- The storage medium is implemented in a technology with *destructive read-out*,
 - each block written can be *read only once*.
 - Each sector behaves like *COPY* rather than *VAR*.
- The whole backing store is an array of such sectors: $BSTORE = \parallel_{i < B} i : COPY$
- This store is intended for use as a subordinate process $(back : BSTORE \parallel \dots)$
- Within its main process, the store may be used by the communications

$$back.i.left ! bl \rightarrow \dots back.i.right ? y \rightarrow \dots$$

Shared Resources: **Multiple resources**

X2. (Shared backing storage)

- The backing store may also be shared by concurrent processes.
- The action simultaneously *acquires* an arbitrary free sector with number i , and
 - *writes* the value of bl into it: $\Box_{i < B}(back.i.left ! bl \rightarrow \dots)$
- The single action both *reads* the content of sector i into x and
 - *release* this sector for use on another occasion: $back.i.right ? x$
- Successful sharing of this backing store requires the utmost discipline:
 - A process may input from a sector only if
 - the same process has most recently output to that very sector.
 - Each output must eventually be followed by such an input.
- Failure to observe such disciplines will lead to deadlock, or even worse confusion.

Shared Resources: **Multiple resources**

X3. (Two line printers)

- Two identical line printers are available to serve the demands of a collection of processes.
- They both need the kind of protection from interleaving that was provided by LP .
- An array of two instances of LP :

$$LP2 = (0 : LP \parallel 1 : LP)$$

- This array may itself be given a name for use as a shared resource

$$(lp : LP2 \parallel \dots)$$

- Each instance of LP is now prefixed twice
 - once by a name and once by an index
 - communications with the using process have three or four components:

$$lp.0.acquire, lp.1.left.\text{“A.JONES”}, \dots$$

Shared Resources: **Multiple resources**

X3. (Two line printers)

- When a process needs to acquire one of an array of identical resources
 - it cannot matter which element of the array is selected on a given occasion.
 - as in the case of a re-entrant procedure.
 - any element which is ready to respond to the acquire signal will be acceptable:

$$\Box_{i \geq 0} (lp.i.acquire \rightarrow \dots lp.i.left ! x \rightarrow \dots lp.i.release \rightarrow SKIP)$$

- The initial *lp.i.acquire*
 - acquires whichever of the two *LP* processes is ready for this event.
 - If neither is ready, the acquiring process will wait.
 - If both are ready, the choice between them is nondeterministic.
- After the initial acquisition,
 - the bound variable *i* takes as its value the index of the selected resource, and
 - all subsequent communications will be correctly directed to that same resource.

$(doub : D \parallel (doub.left ! x \rightarrow doub.right ? y \rightarrow SKIP))$

Shared Resources: **Multiple resources**

X3. (Two line printers)

- The shared resource is intended to behave exactly like
 - a locally declared subordinate process, communicating only with its using process:

$(myfile :: lp \parallel \dots myfile.left ! x \dots)$

- instead of $\Box_{i \geq 0} (lp.i.acquire \rightarrow \dots lp.i.left ! x \dots ; lp.i.release \rightarrow SKIP)$
- The local name *myfile* is stand for the indexed name *lp.i*, and
 - the technicalities of acquisition and release have been conveniently suppressed.
- The *remote subordination* “::” is distinguished from the “:” in that
 - it takes on its right, not a complete process, but
 - the name of a remotely positioned array of processes.

$(lp : LP2 \parallel \dots)$

$LP2 = (0 : LP \parallel 1 : LP)$

$LP = acquire \rightarrow \mu X \bullet (left ? x \rightarrow h ! s \rightarrow X \mid release \rightarrow LP)$

$\Box_{i \geq 0} (lp.i.acquire \rightarrow \dots lp.i.left ! x \dots ; lp.i.release \rightarrow SKIP)$

Shared Resources: **Multiple resources**

X4. (Two output files)

- A using process requires simultaneous use of two line printers
 - to output two files, $f1$ and $f2$

$(f1 :: lp \parallel (f2 :: lp \parallel \dots f1.left ! s1 \rightarrow f2.left ! s2 \rightarrow \dots))$

- The using process interleaves output of lines to the two different files;
 - each line is printed on the appropriate printer.
- **Deadlock will be the result if**
 - **declare *three* printers simultaneously**
 - declaring two printers simultaneously in each of two concurrent processes
 - Ann and Mary $(myfile :: lp \parallel (A \parallel M))$

$(f1 :: lp \parallel (f2 :: lp \parallel (f3 :: lp \parallel \dots$

$(lp : LP2 \parallel \dots) \quad LP2 = (0 : LP \parallel 1 : LP)$

$LP = acquire \rightarrow \mu X \bullet (left ? x \rightarrow h ! s \rightarrow X \mid release \rightarrow LP)$

Shared Resources: **Multiple resources**

X5. (Scratch file) A scratch file is used for output of a sequence of blocks.

- When the output is complete,
 - the entire sequence of blocks is read back from the beginning.
- When all the blocks have been read,
 - the scratch file will then give only *empty* signals; no further reading/writing.
- A scratch file behaves like
 - a file output to compact disc, which must be set to the start before being read.
- The *empty* signal serves as an end-of-file marker.

$SCRATCH = WRITE_{\triangleleft}$

$WRITE_s = (left ? x \rightarrow WRITE_{s \wedge \triangleleft x} \mid rewind \rightarrow READ_s)$

$READ_{\triangleleft x \wedge s} = (right ! x \rightarrow READ_s) \quad READ_{\triangleleft} = (empty \rightarrow READ_{\triangleleft})$

- This may conveniently be used as a simple unshared subordinate process

$(myfile : SCRATCH \parallel \dots myfile.left ! v \dots myfile.rewind \dots$
 $(myfile.right ? x \rightarrow \dots \mid myfile.empty \rightarrow \dots) \dots)$

Shared Resources: **Multiple resources**

X6. (Scratch files on backing store)

- The scratch file X5 can be implemented by
 - holding the stored sequence of blocks in the *main* store of a computer.
- If the blocks are large and the sequence is long,
 - it would be better to store the blocks on a *backing* store.
- A backing store X2 with destructive read-out will suffice
 - each block in a scratch file is read and written only once.
- An ordinary scratch file (held in main store) is used to
 - hold the sequence of indices of the sectors of backing store on which
 - the corresponding actual blocks of information are held;
 - this ensures that the correct blocks are read back, and in the correct sequence.

X2. $BSTORE = \parallel_{i < B} i : COPY$

X5. $SCRATCH = WRITE_{\triangleleft}$
 $WRITE_s = (left ? x \rightarrow WRITE_{s \wedge \langle x \rangle} \mid rewind \rightarrow READ_s)$
 $READ_{\langle x \rangle \wedge s} = (right ! x \rightarrow READ_s) \quad READ_{\triangleleft} = (empty \rightarrow READ_{\triangleleft})$

Shared Resources: **Multiple resources**

$BSCRATCH = (pagetable : SCRATCH //$
 $\mu X \cdot (left ? x \rightarrow (\sqcap_{i < B} back.i.left ! x \rightarrow pagetable.left ! i \rightarrow X)$
 $\mid rewind \rightarrow pagetable.rewind \rightarrow$
 $\mu Y \cdot (pagetable.right ? i \rightarrow back.i.right ? x \rightarrow right ! x \rightarrow Y$
 $\mid pagetable.empty \rightarrow empty \rightarrow Y)))$

- $BSCRATCH$ uses the name *back* to address a backing store X2 as a subordinate process:

$SCRATCHB = (back : BSTORE // BSCRATCH)$

- $SCRATCHB$ can be used as a simple unshared subordinate process as in X5:

$(myfile : SCRATCHB // \dots myfile.left ! v \dots)$

- The effect is exactly the same as use of $SCRATCH$, except that
 - the maximum length of the scratch file is limited to B blocks.

X2. $BSTORE = \parallel_{i < B} i : COPY \quad COPY = \mu X \cdot (left ? x \rightarrow right ! x \rightarrow X)$

Shared Resources: **Multiple resources**

X7 (Serially reused scratch files)

- Share the scratch file on backing store among a number of interleaved users:
 - they acquire, use, and release it one at a time (a shared line printer X3).
- Adapt *BSCRATCH* to accept *acquire* and *release* signals.
- If a user releases his scratch file before reading to the end,
 - the unread blocks on backing store may never be reclaimed.
- A loop reads back these blocks and discards them
$$SCAN = \mu X \cdot (pagetable.right ? i \rightarrow back.i.right ? x \rightarrow X \mid pagetable.empty \rightarrow SKIP)$$
- A shared scratch file acquires its user, and then behaves as *BSCRATCH*:
$$SHBSCR = acquire \rightarrow (BSCRATCH \triangle (release \rightarrow SCAN))$$
 - The release signal causes an interrupt to the *SCAN* process
- The serially reusable scratchfile is provided by $back : BSTORE \parallel *SHBSCR$
 - the simple loop $*SHBSCR$ which uses *BSTORE* as a subordinate process

Shared Resources: **Multiple resources**

X8. (Multiplexed scratch files)

- A backing store is usually sufficiently large for simultaneously existing many scratch files
 - each occupying a disjoint subset of the available sectors.
- The backing store can be shared among an *unbounded* array of scratch files.
- Each scratch file acquires a sector on outputting to it, and
 - releases it automatically on inputting that block again.

- The backing store is shared by the technique of multiple labelling,
 - using as labels the same indices which are used for the array of sharing processes

FILESYS = N : (back : BSTORE) // ($\parallel_{i \geq 0} i : SHBSCR$), where $N = \{i \mid i \geq 0\}$

- This filing system is a subordinate process, shared by among any number of users:

filesys : FILESYS //... (USER1 \parallel USER2 \parallel ...)

- In each user, a fresh scratch file is acquired, used, & released by remote subordination

myfile : filesys // (... myfile.left ! v... myfle.rewid... myfile.right ? x...)

- which has the same effect as the simple subordination of a private scratch file X5:

(myfile : SCRATCH // ... myfile.left ! v... myfile.rewind... myfile.right ? x...)

Shared Resources: **Multiple resources**

- Inside a user processor a scratch file is created by remote subordination

myfile :: filesys // (... myfile.left ! v... myfile.rewind... myfile.right ? x...)

- By definition of remote subordination this is equivalent to

$$(\Box_{i \geq 0} \text{filesys}.i.\text{acquire} \rightarrow \text{filesys}.i.\text{left} ! v \dots \text{filesys}.i.\text{rewind} \dots \\ \text{filesys}.i.\text{right} ? x \dots \text{filesys}.i.\text{release} \rightarrow \text{SKIP})$$

- All communications between *filesys* and its users begin with *filesys.i...* where
 - *i* is the index of the *particular instance* of *SHBSCR* which is
 - acquired by a particular user on a particular occasion.
 - Each occasion of its use is surrounded by a matching pair of signals
 - *filesys.i.acquire* and *filesys.i.release*.

Shared Resources: **Multiple resources**

- Each virtual scratchfile begins by acquiring its user, and
 - then continues according to the pattern of X5 and X6

(acquire → ... left ? x ... rewind ... right ! v ... release ...)

- All other communications of the virtual scratch file are
 - with the subordinate *BSTORE* process, and
 - concealed from the user.
- Each instance of the virtual scratch file is indexed by a different index *i*,
 - and then named by the name *filesys*.
- The externally visible behaviour of each instance is
 - (filesys.i.acquire → filesys.i.left ? x ... filesys.i.rewind ...*
filesys.i.right ! v ... filesys.i.release)
- The matching pairs of *acquire* and *release* signals ensure that
 - no user can interfere with a scratch file that has been acquired by another user.

Shared Resources: **Multiple resources**

- Communications within *FILESYS* between the virtual scratch files and the backing store.
 - are concealed from the user,
 - do not have the name *filesys* attached to them.
- The relevant events are:
 - $i.back.j.left.v$ — communication of block v from
 - the i^{th} element of the array of scratch files to the j^{th} sector of backing store
 - $i.back.j.right.v$ — a communication in the reverse direction.
- Each sector of the backing store behaves like *COPY*.
- After indexing with a sector number j and naming by *back*, the j^{th} sector behaves like

$$\mu X \bullet (back.j.left ? x \rightarrow back.j.right ! x \rightarrow X)$$

- After multiple labelling by natural numbers it behaves like

$$\mu X \bullet (\Box_{i \geq 0} i.back.j.left ? x \rightarrow (\Box_{k \geq 0} k.back.j.right ! x \rightarrow X))$$

- communicate on any occasion with any element of the array of virtual scratch files.
- Each scratch file reads only from sectors the most recently written by it.

Shared Resources: **Multiple resources**

- The natural numbers i and j
 - permit any scratch file to communicate
 - with any sector on disc,
 - safely with the user that has acquired it.
 - a mathematical description of a kind of crossbar used in
 - a telephone exchange to allow any subscriber to communicate with another one.
- If the number of sectors in the backing store is infinite,
 - *FILESYS* behaves like an array of simple scratch files

$$\parallel_{i \geq 0} i : (acquire \rightarrow (SCRATCH \triangle (release \rightarrow STOP)))$$

- With a backing store of finite size,
 - there is a danger of deadlock if
 - the backing store gets full at a time when all users are still writing to their files.
- In practice, this risk is usually reduced by
 - delaying acquisition of new files when the backing store is nearly full.

Shared Resources: **Multiple resources**

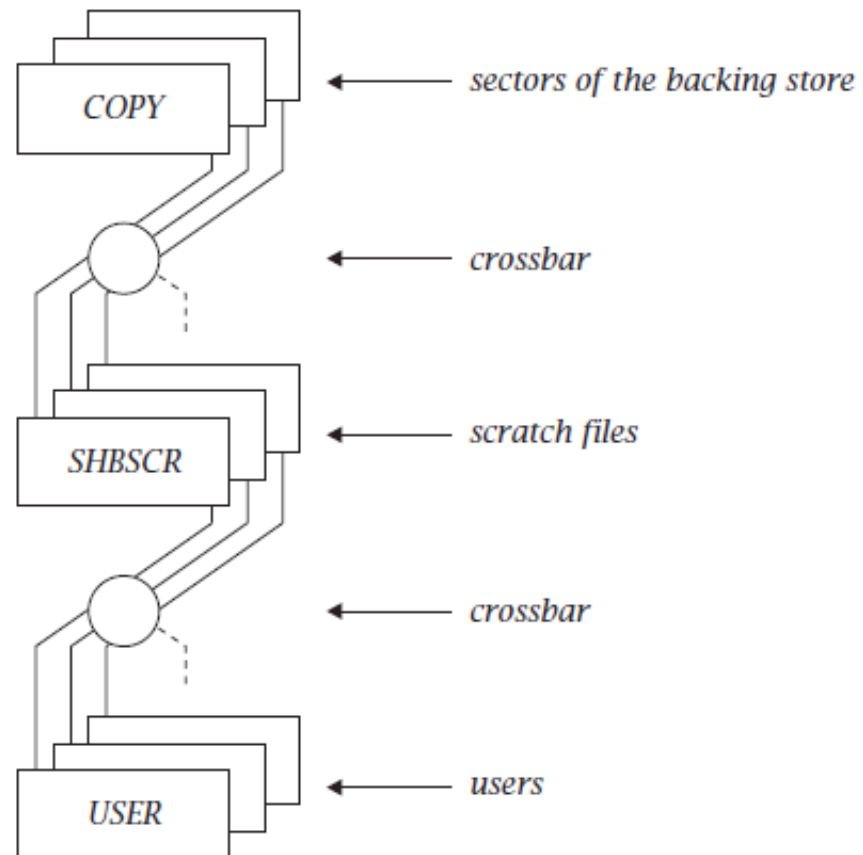


Figure 6.2

Shared Resources: **Multiple resources**

- The problem of sharing a *limited number* of resources among
 - an *unknown number* of users.
 - the structure of the filing system X8 and its mode of use.
- The users do not communicate directly with the resources;
 - there is an intermediary *virtual resource* (the *SHBSCR*) which
 - they declare and use as though it were a private subordinate process.
- The function of the virtual resource is twofold.
 1. It provides a nice clean interface to the user;
 - *SHBSCR* glues together into a single contiguous scratch file
 - a set of sectors scattered on backing store.
 2. It guarantees a proper, disciplined access to the actual resources;
 - the process *SHBSCR* ensures that each user reads only from sectors allocated to it,
 - and cannot forget to release sectors on finishing with a scratch file.
- Point (1) ensures that the discipline of Point (2) is painless.