Filtering

Filter

Noise

SNR

Processing Gain

Bandwidth

FWHM

High Pass Filter

- Drift

Low Pass Filter

- Smoothing

Bandpass Filter

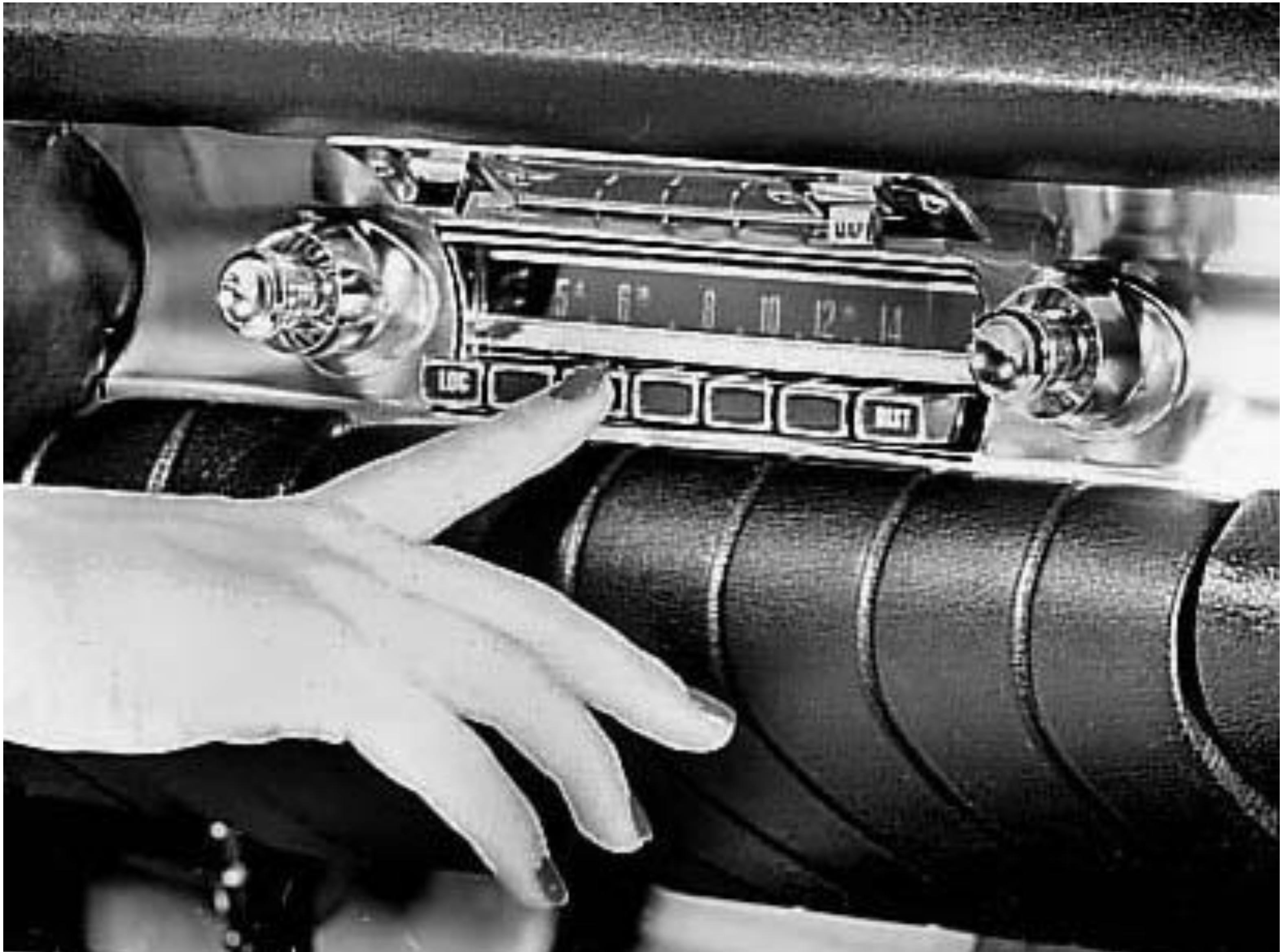
- Antialiasing

Why Filter?

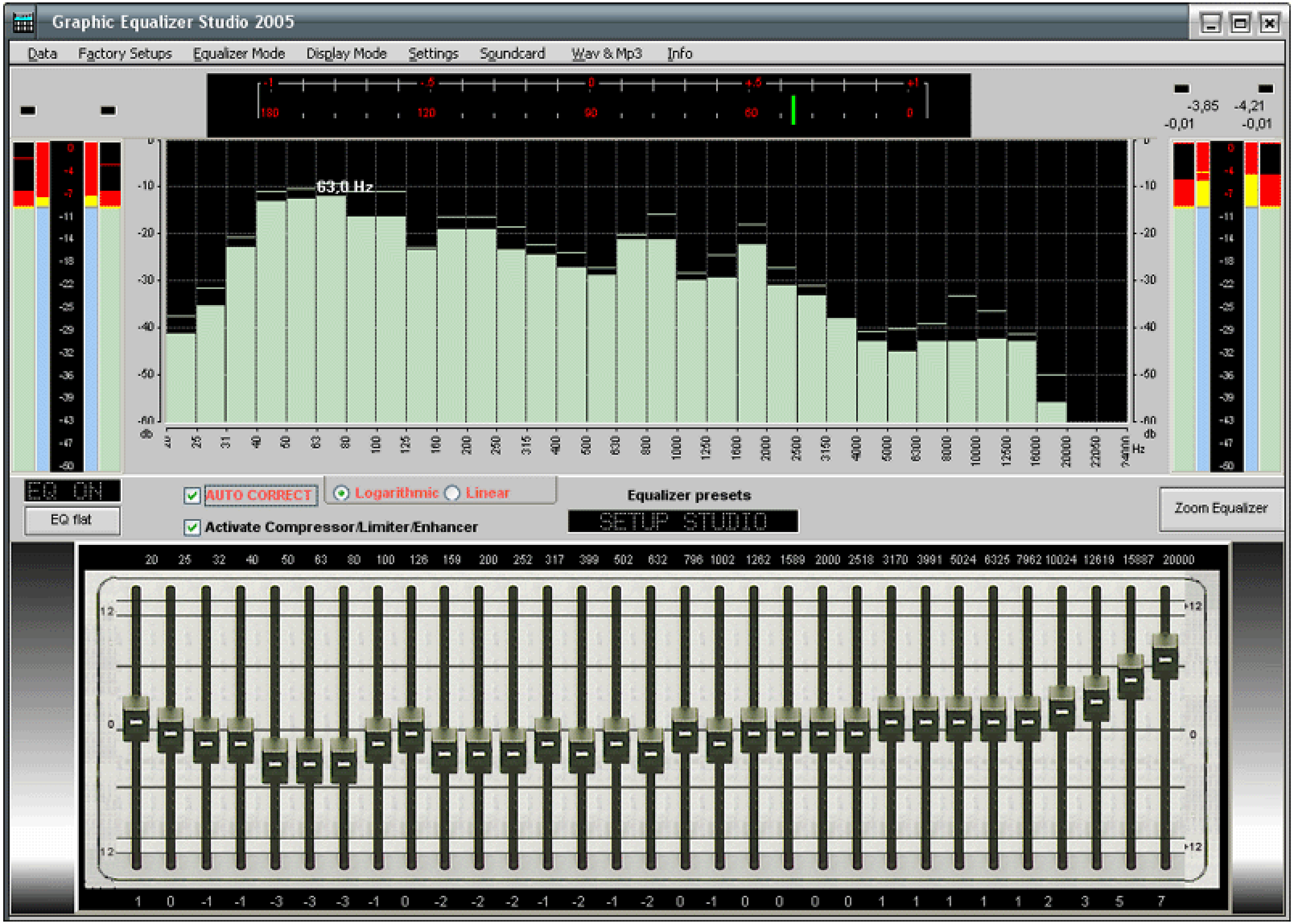
Filters remove or reduce frequencies you don't want in the signal and enhance frequencies you do want in the signal.

You use filters all the time:

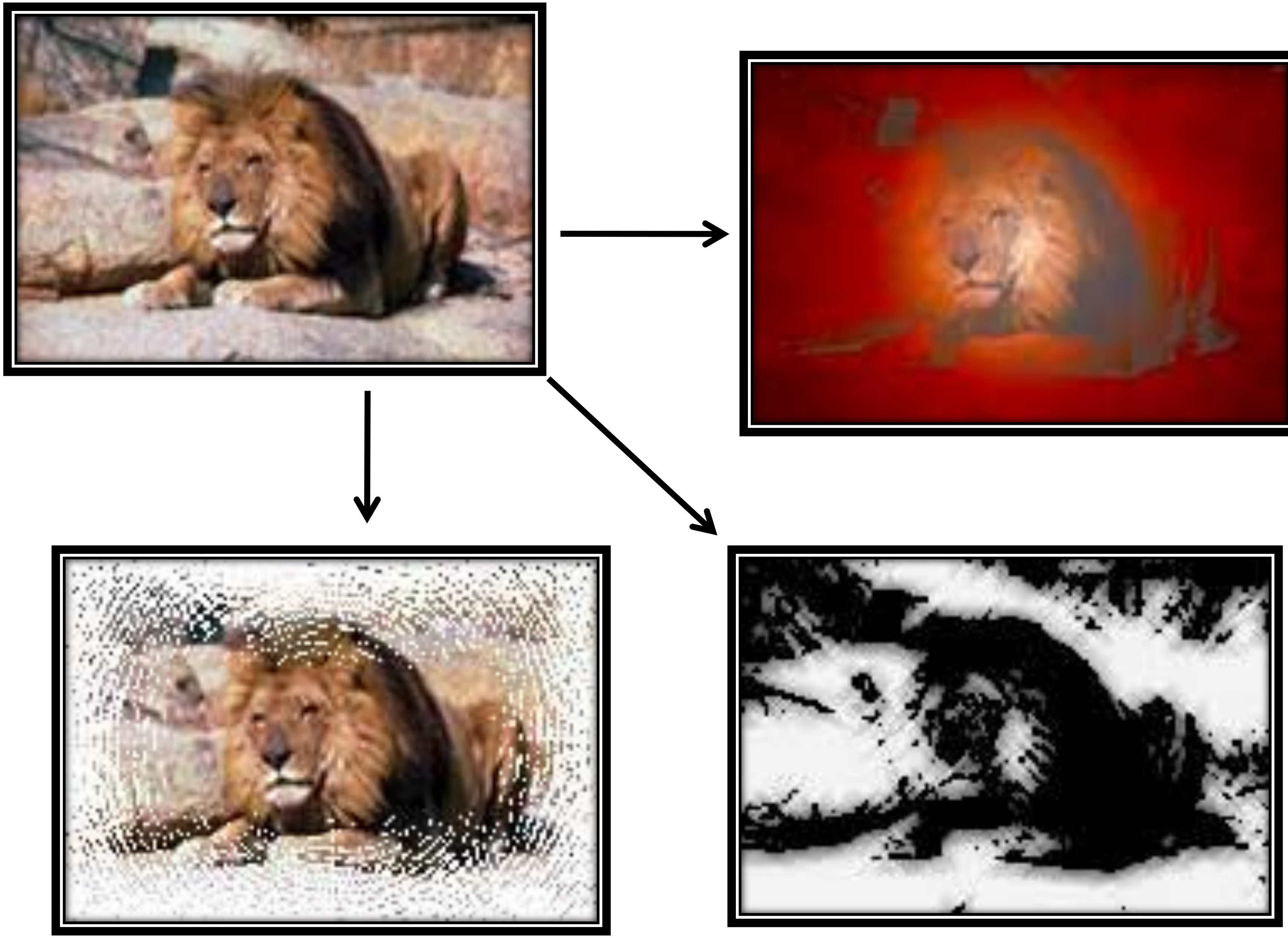
when you tune the radio to the frequency you want;


5 6 8 10 12 14
LOC
DIST

when you use a graphic equalizer to enhance the music;


Graphic Equalizer Studio 2005
Data
Factory Setups
Equalizer Mode
Display Mode
Settings
Soundcard
Wav & Mp3
Info
63,0 Hz
-3,85
-4,21
-0,01
-0,01
EQ ON
EQ flat
AUTO CORRECT
Logarithmic
Linear
Activate Compressor/Limiter/Enhancer
Equalizer presets
SETUP STUDIO
Zoom Equalizer
20 25 32 40 50 63 80 100 126 159 200 252 317 399 502 632 796 1002 1262 1589 2000 2518 3170 3991 5024 6325 7962 10024 12619 15887 20000
1 0 -1 -1 -3 -3 -3 -1 0 -2 -2 -2 -1 -2 -1 -2 0 -1 0 0 0 0 1 1 1 1 1 2 3 5 7

and when you use photoshop to change the way a picture looks.



One reason to filter is to remove noise.

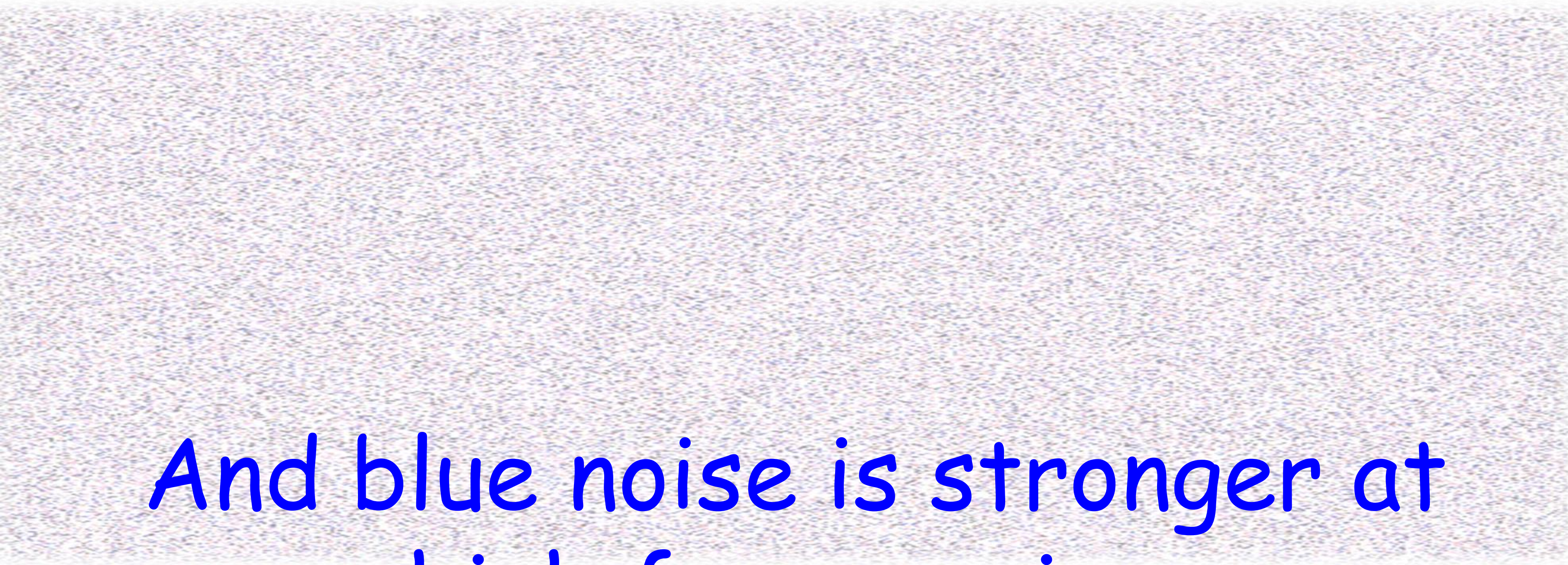
Noise is data that is not being used to transmit a signal, it is said to be random (stochastic).

You've heard noise on the radio…

And you've seen it on TV

White noise is equivalent across all frequencies, i.e., it has a flat power spectrum.

but brown noise is stronger
at low frequencies;



And blue noise is stronger at high frequencies.

And there's green noise, black noise, purple noise...etc.

Noise in MRI images is particularly bad, because the good signal is often tiny, (e.g., fMRI signal is ~1%)

We need a way to measure noise, so we can assess the quality of our signal, and the success of our fixes.

That measure is

Signal-to-Noise Ratio (SNR)

SNR compares the level of a desired signal (such as music) to the level of the background.

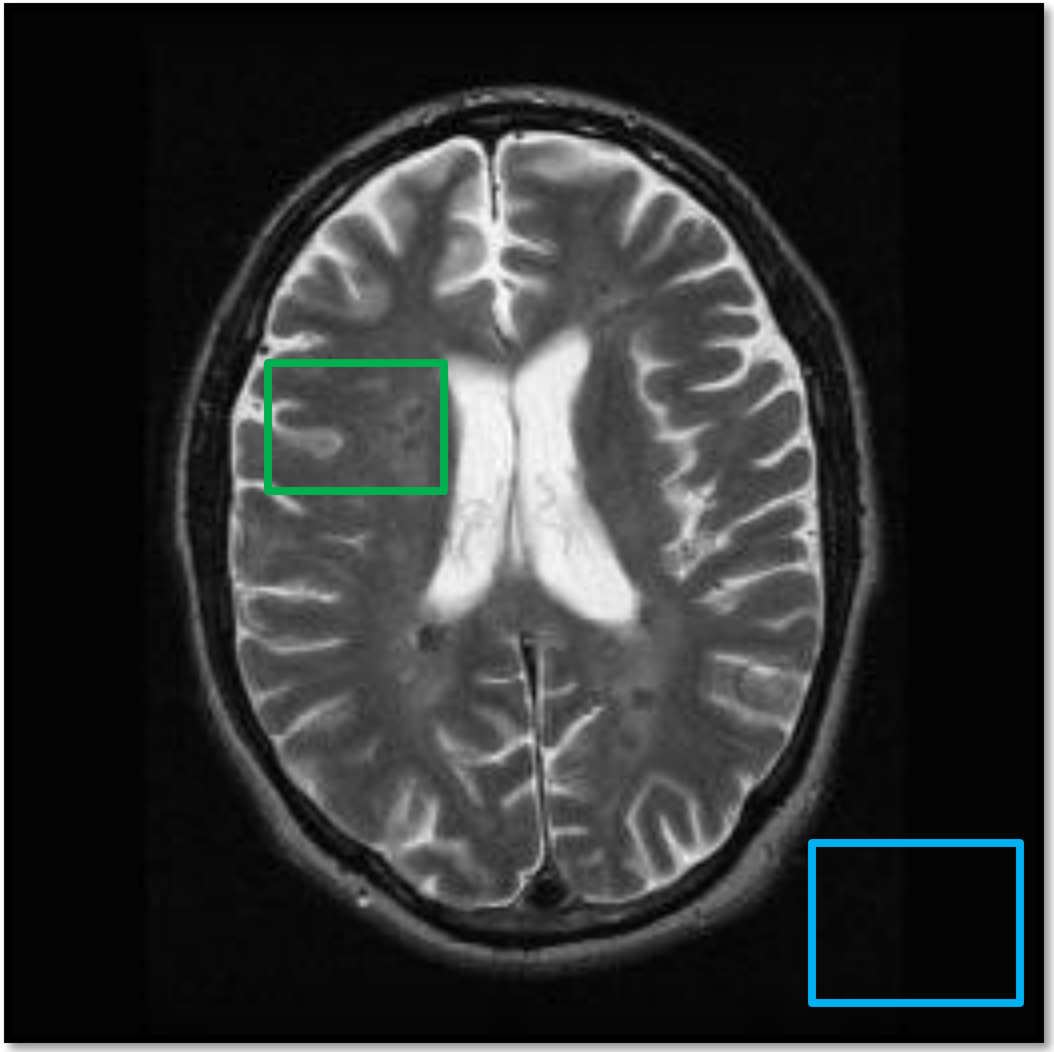
The higher the ratio, the less obtrusive the background noise is.

High SNR=Good

Low SNR=Bad

There are a variety of approaches to measuring SNR in MR images, but here is one of the simplest:

$$SNR = \frac{\text{mean (center)}}{\text{SD (outside)}}$$



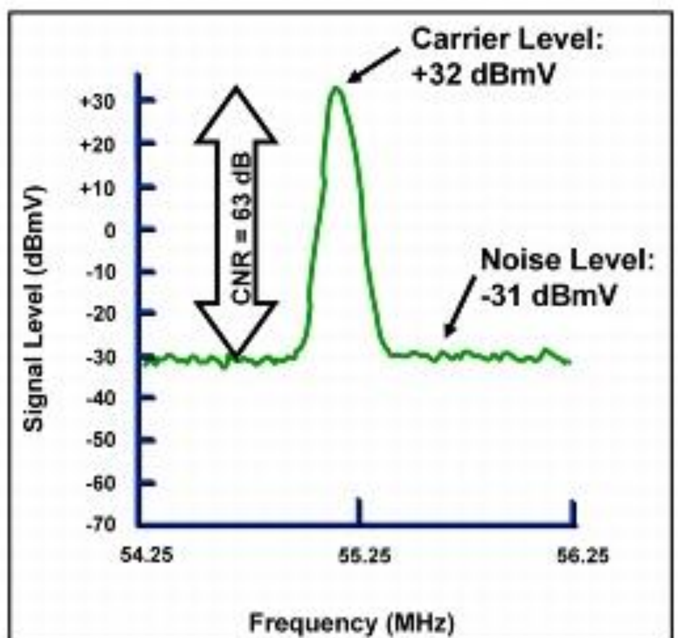
SNR is useful for understanding the quality of other kinds of signals, as well.

Identify part of the wave that does **not** contain the signal.

Compare it to part of the wave that **does** contain the signal.

How different are they?

More different->Higher (Better) SNR



Improved SNR is called **processing gain.**

Summary

So far, we've looked at common filters (radios, graphic equalizers, photoshop effects),

Types of noise (white, blue, brown etc),

Signal to Noise Ratio: it's bad to be low, but its good to be high.

Higher SNR→processing gain.

Another useful concept is

Bandwidth

Bandwidth is the **range** of frequencies in a signal.

Let’s re-examine Nyquist’s Theorum with the notion of bandwidth in mind.

Nyquist's theorum says we have to sample at twice the bandwidth of the signal.

…And that is usually manifested as twice the highest frequency.

For example, if the highest frequency is 200 Hz, then we have to sample at **400** times per second.

1 hz ⟷ 200 hz

But, sampling at twice the **bandwidth** may, or may not, be equivalent to twice the **highest** frequency.

For example,

If the signal ranges from 100-200 Hz, then we only need to sample 200 times per second, to prevent aliasing.

100 hz ⟷ 200 hz

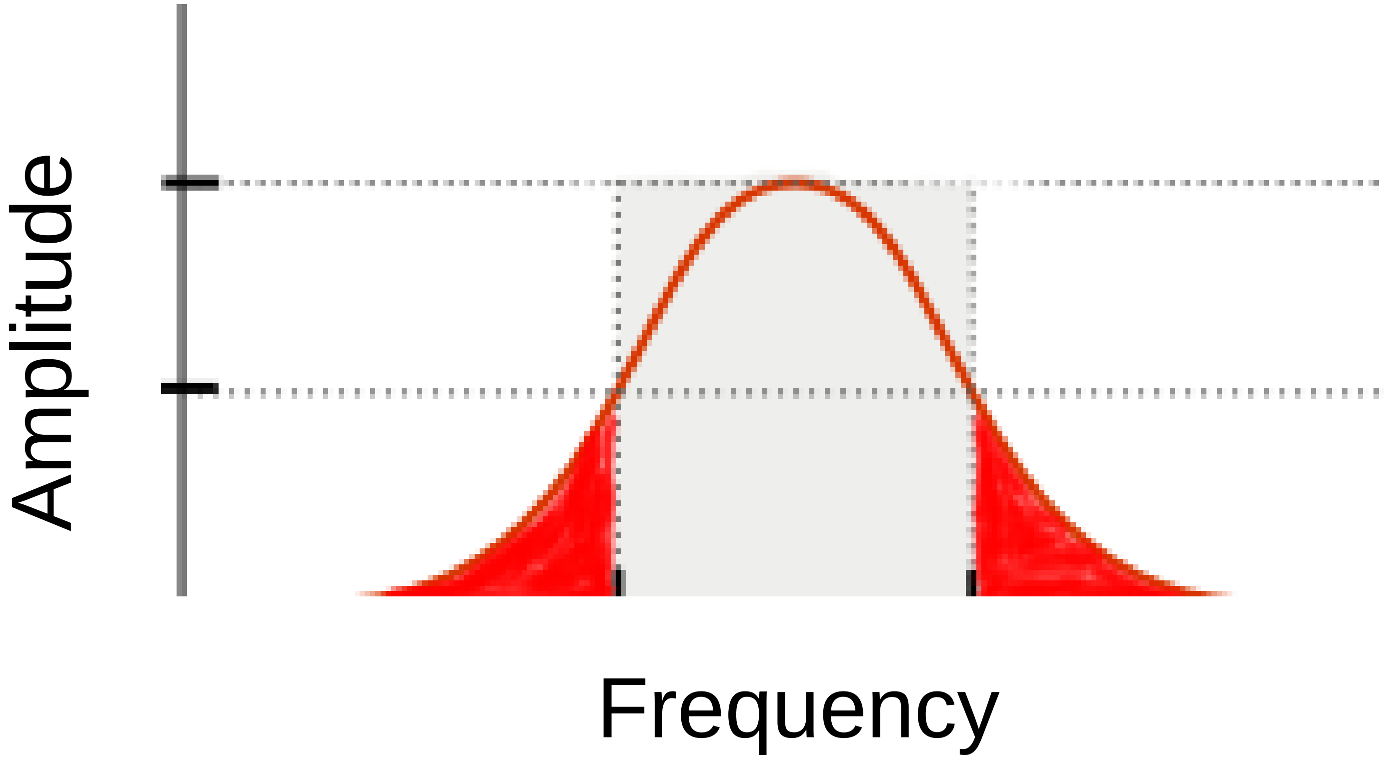
Because even though the highest frequency is still 200 Hz, the range (bandwidth) is only 100 Hz!!

100 hz ⟷ 200 hz

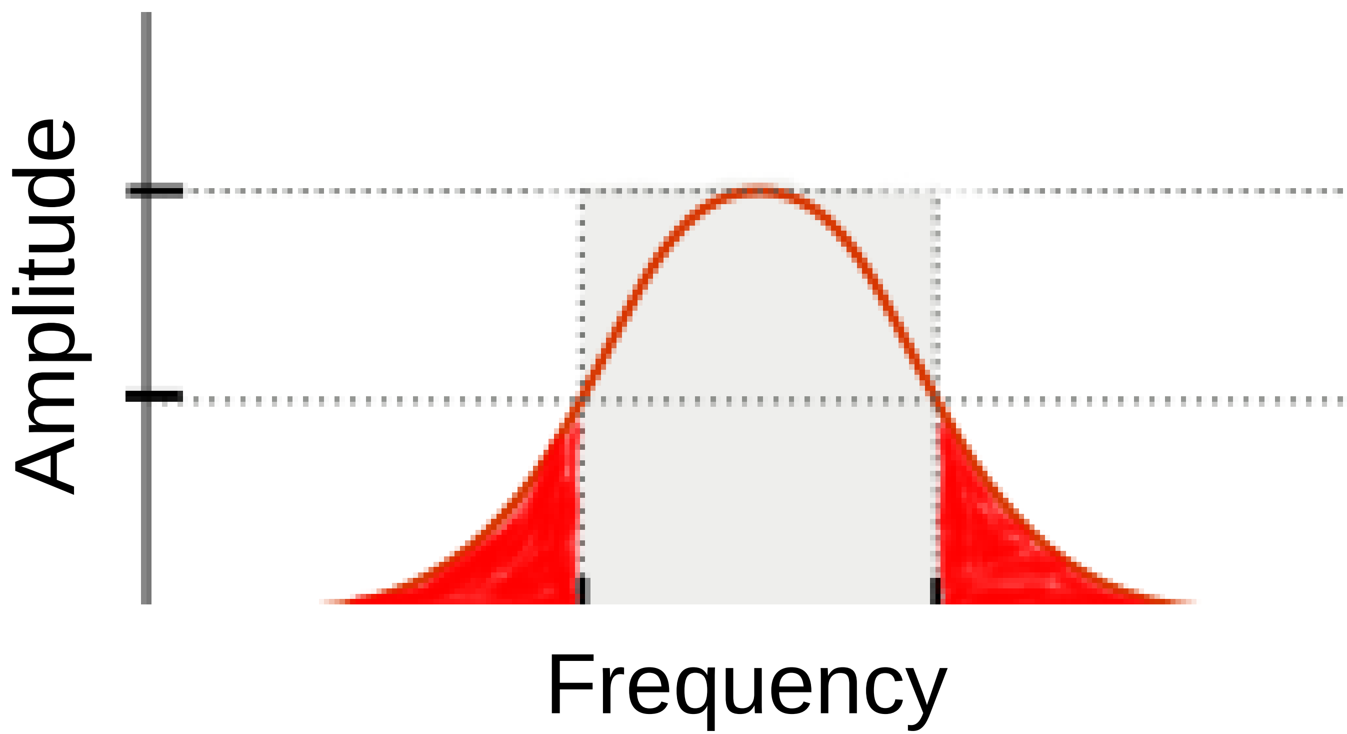
But, don't get all happy yet

There's a problem.

Here's a power spectrum of a typical natural signal:



And the red stuff is signal you have to sample, even though it is low amplitude.



Because if you don't sample it, it'll fold up on you, and cause icky aliasing.

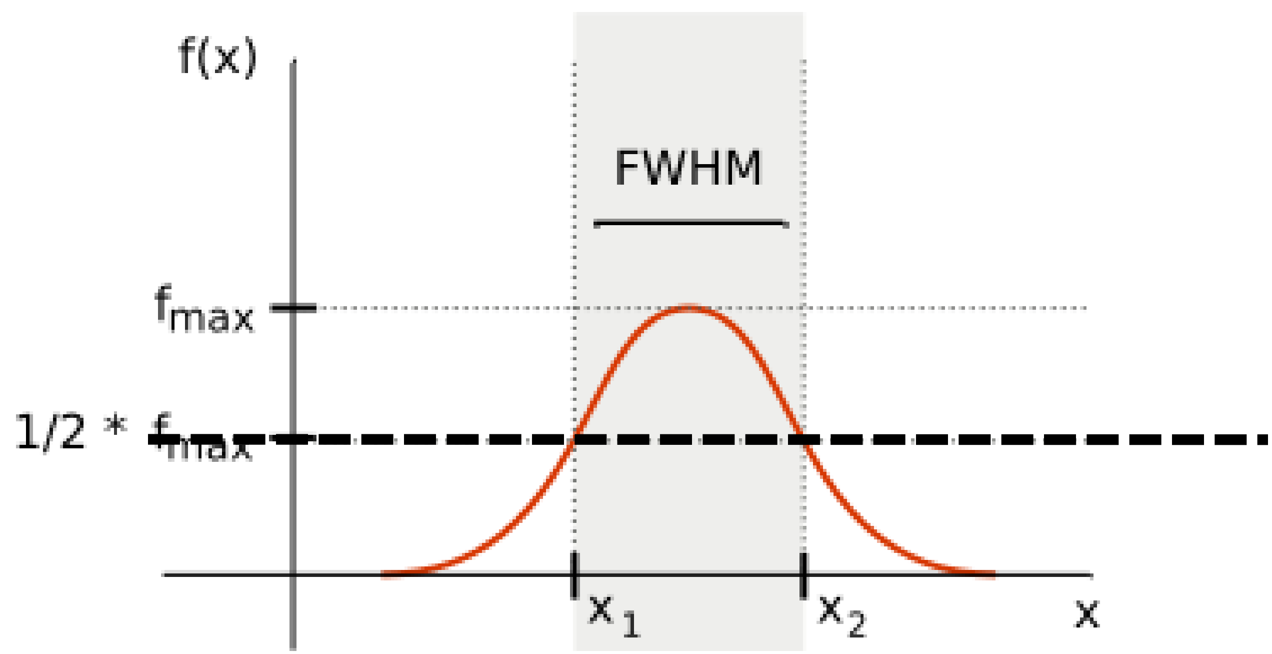
If you have to count these low amplitude tails of a typical natural signal, then the bandwidth may be VERY wide, and so we get no savings on sampling rate.

Fortunately,
we have a standard cutoff criterion (Full Width at Half Maximum) that we can apply to limit bandwidth.

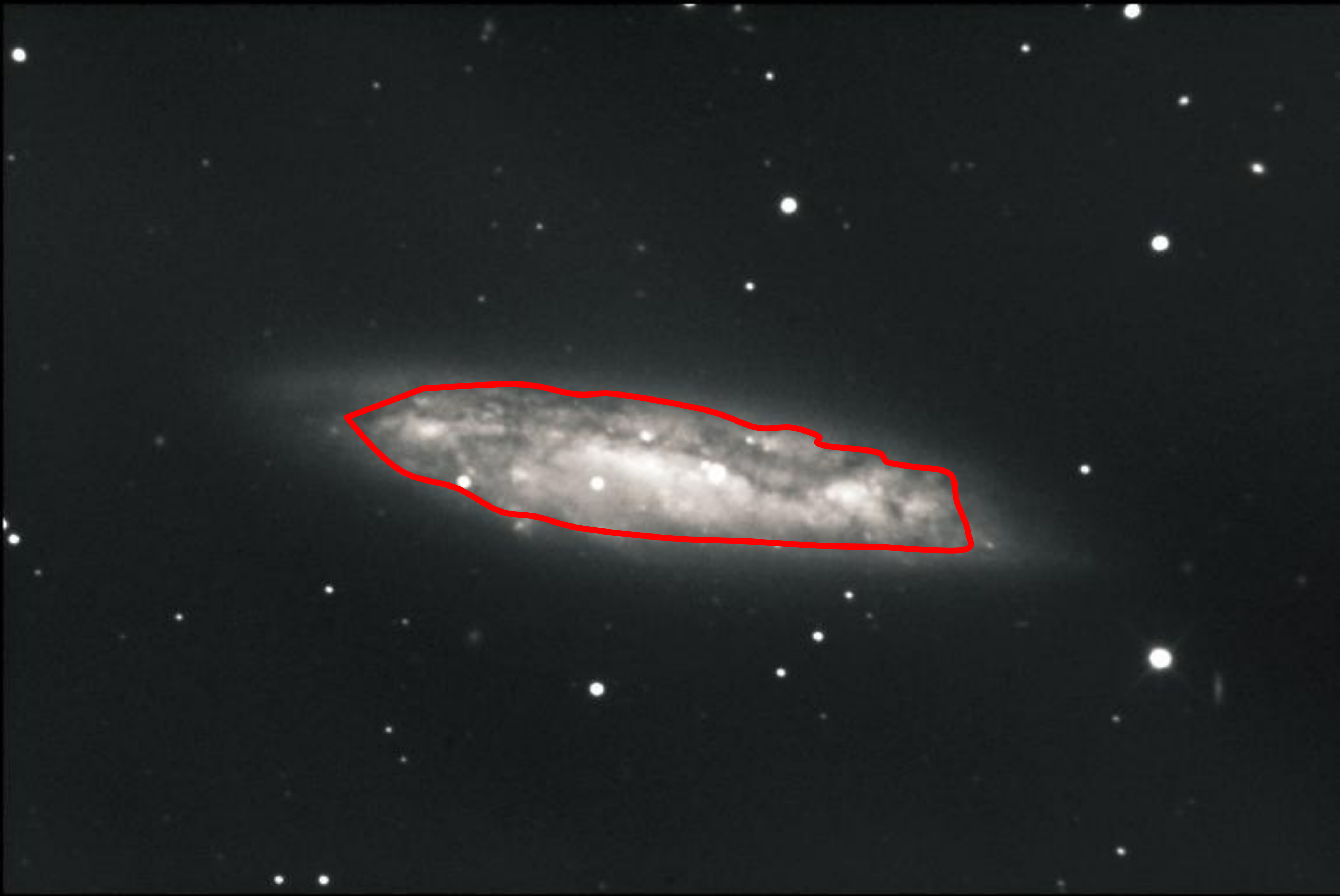
By limiting bandwidth, we can reduce sampling rate without aliasing.

FWHM criterion says that the full width of the signal is that point where the signal reaches $\frac{1}{2}$ its maximum intensity.

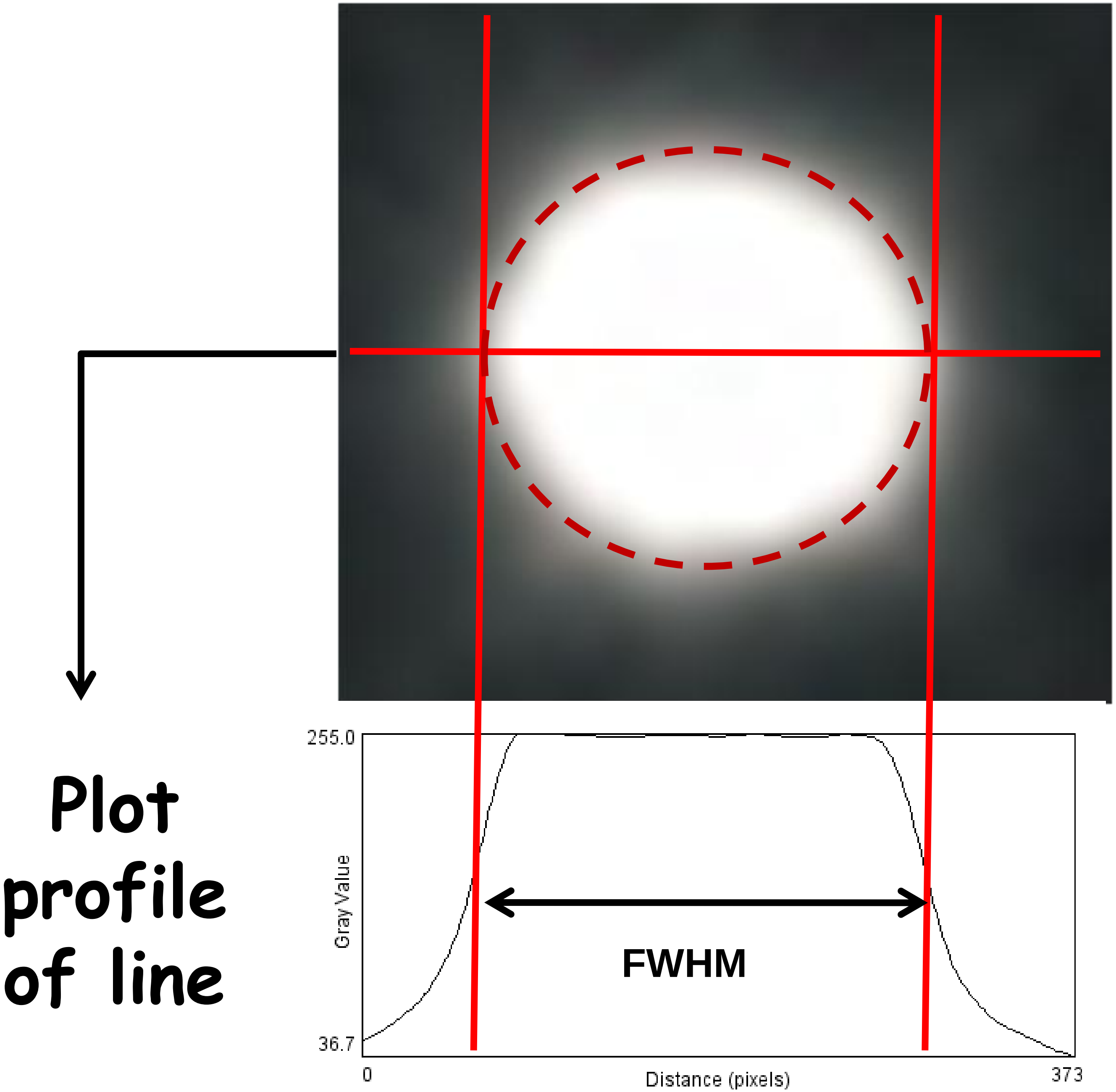
So instead of having the infinitely long tails on the signal, we can cleanly and consistently identify the extent of the signal.



In image processing, the FWHM criterion is used to identify edges consistently, (even when those edges are fuzzy).



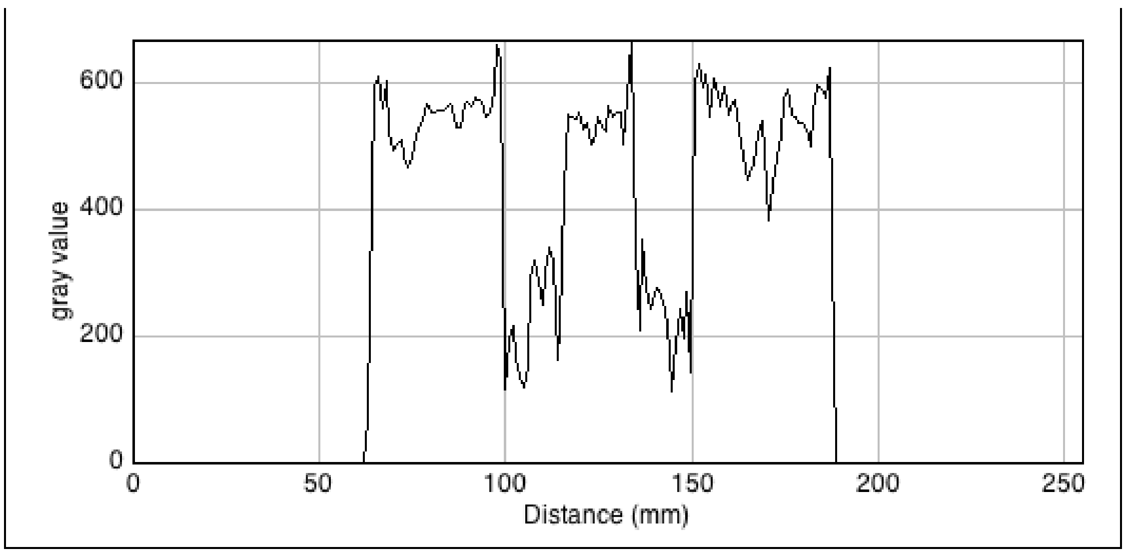
This is useful if we want to measure an object in a picture, and we need to decide where the edges are in some consistent way.

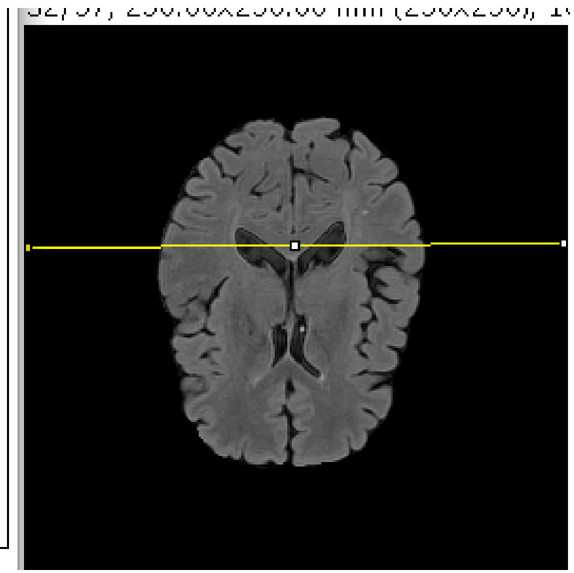


Plot profile of line
255.0
36.7
Gray Value
FWHM
0
Distance (pixels)
373

Edge Detection

- If edges are well defined, it is really easy to find them.
- Lots of image processing tools take advantage of this property to identify objects: Brain extraction is an obvious example (though it does other stuff too).

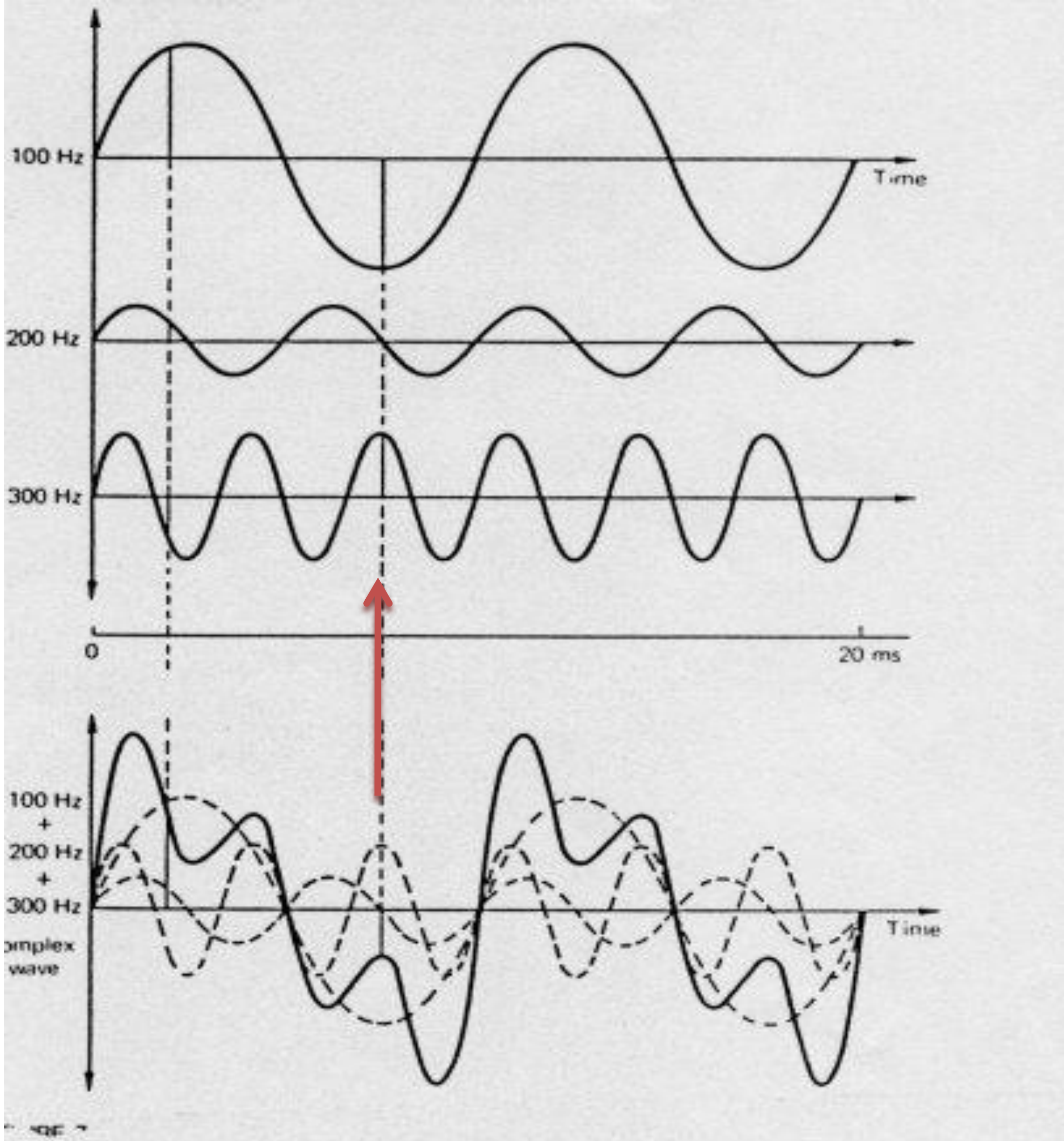




Alright, how is FWHM related to filters?

Filters **cut off** frequencies you don't want in the signal

If we use the FFT to deconvolve the signal into its component sine waves…

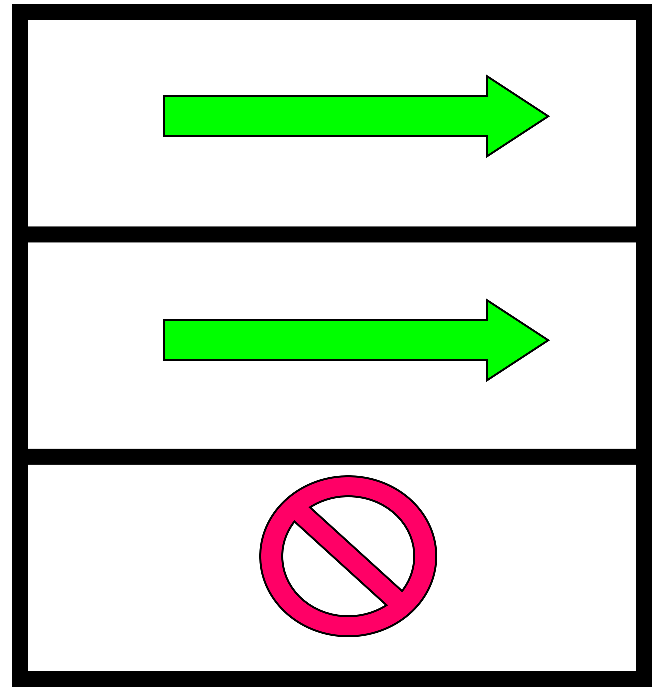

100 Hz
Time
200 Hz
300 Hz
0
20 ms
100 Hz
+
200 Hz
+
300 Hz
omplex
wave
Time

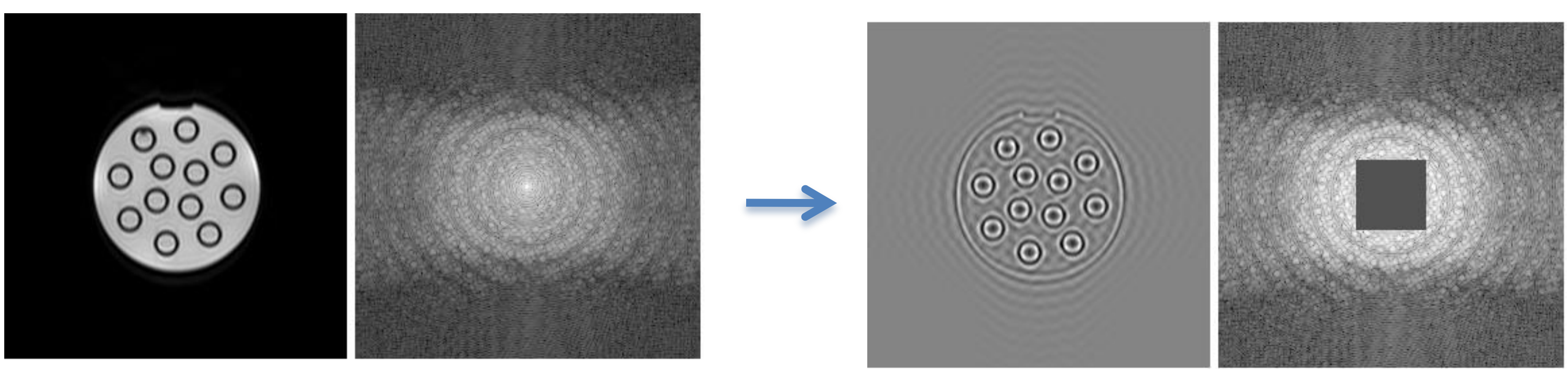
then it is much easier to remove some frequencies, but leave other frequencies intact.

Let's illustrate with 3 simple filters

High Pass Filter

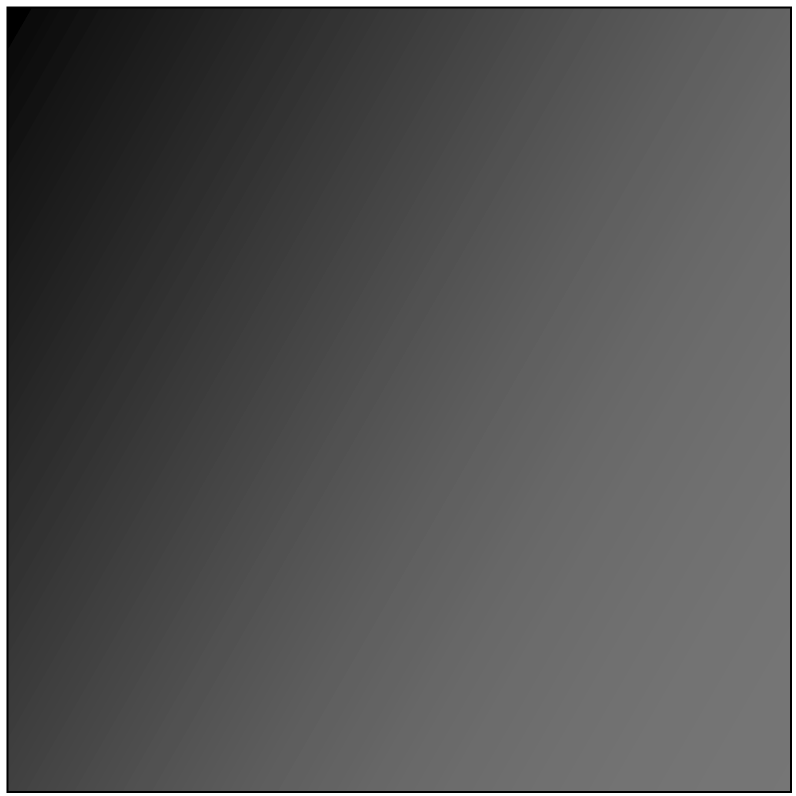
Removes low frequency information, **allows high frequencies to pass**





Why high pass filter?

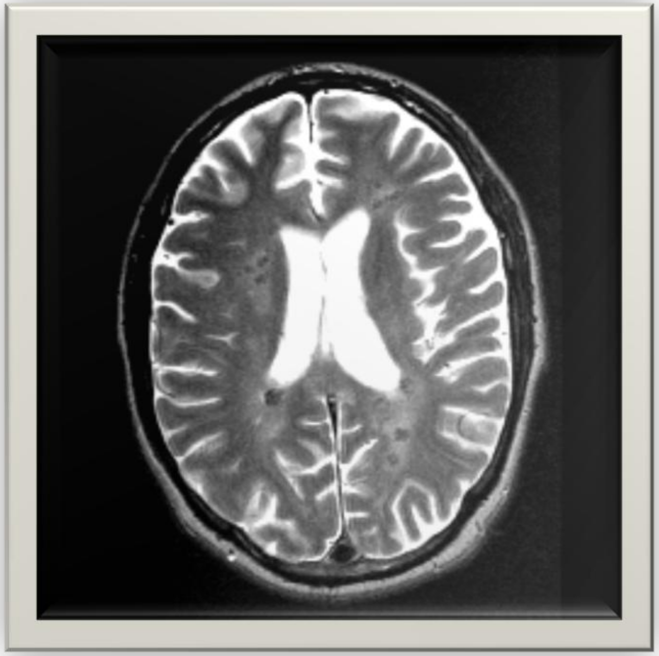
Remember Low Spatial Frequencies?



Frequently, in the MR scanner, low frequency variation alters the intensity on one side of the image, as compared to the other.

We call this drift.

Since intensity changes are supposed to be **anatomically** meaningful, we hate drift.



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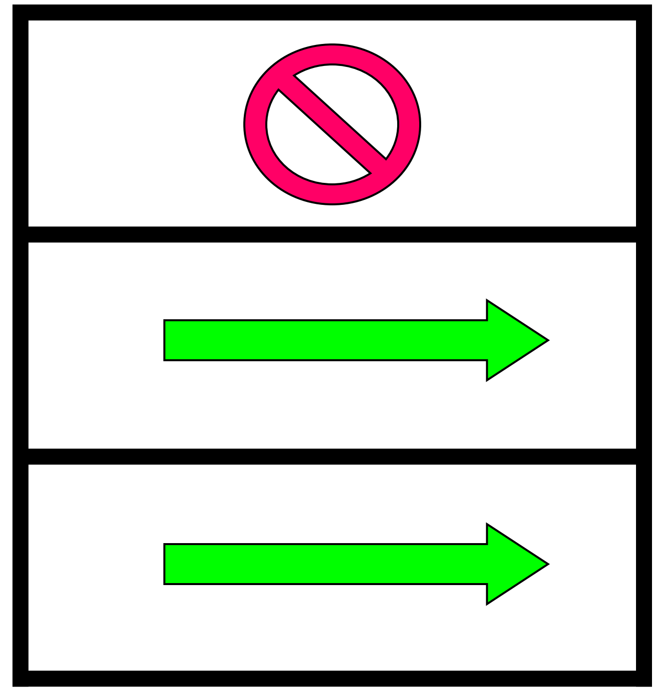
So we want to remove it with high pass filtering.

Low Pass Filter

Removes high frequency information, **allowing low frequencies to pass**



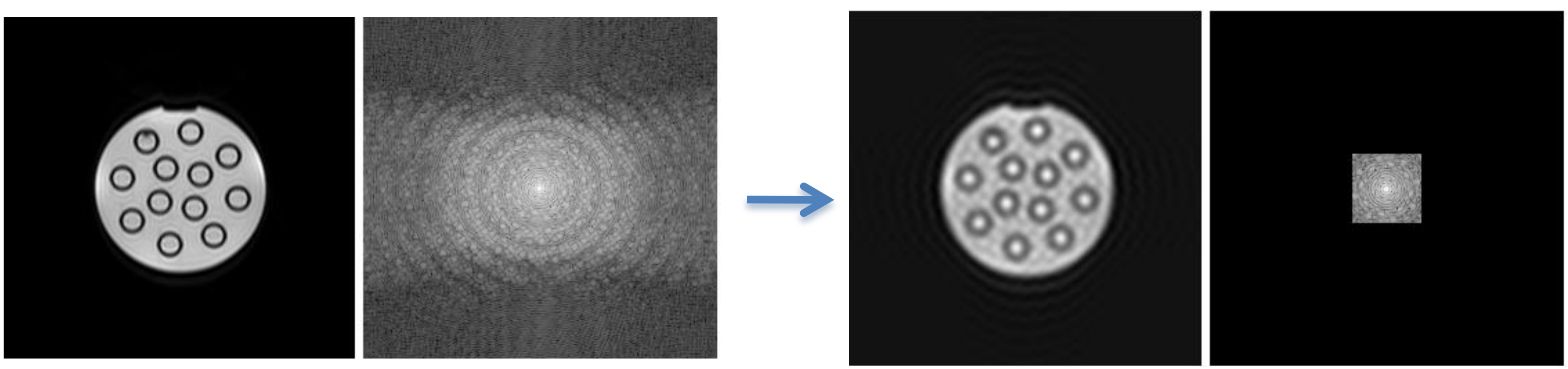
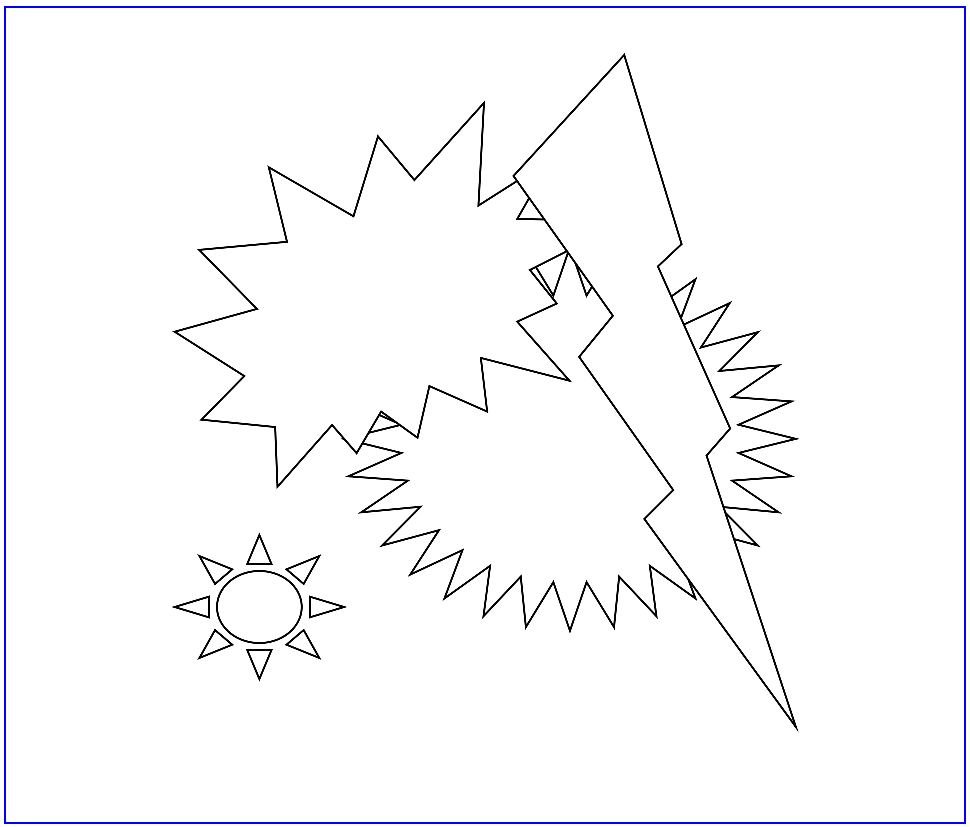


Image smoothing is an example of low pass filtering applied to spatial frequencies…

Remember High Spatial Frequencies?



Well, we use a low pass filter to remove these high spatial frequencies (i.e., smooth sharp edges).

Why smooth?

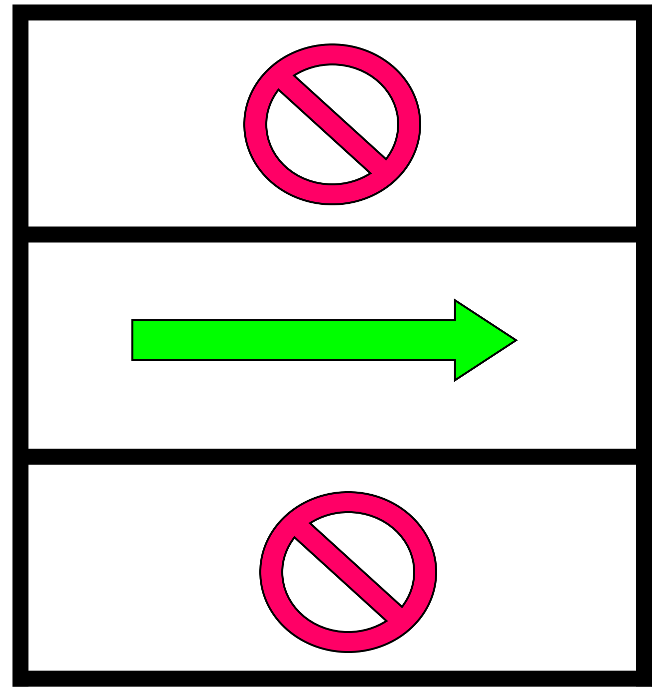
Smoothing reduces high frequency noise.

By removing the noise, we increase signal to noise ratio.

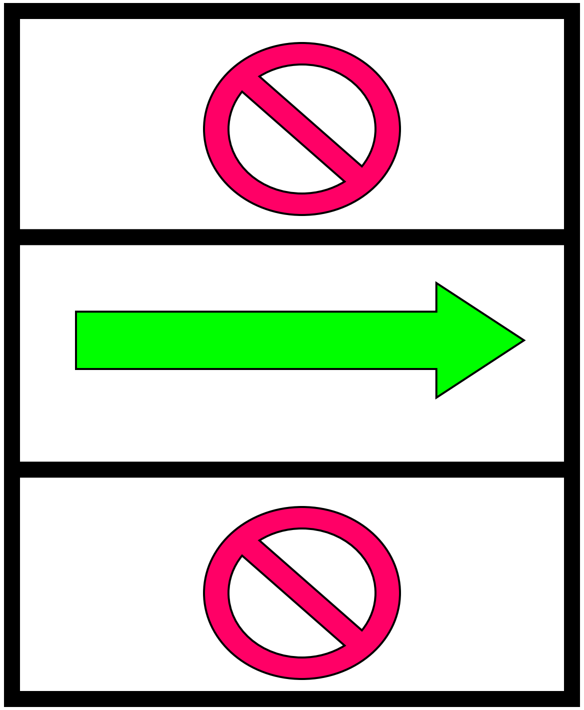
Higher SNR is better (it results in a processing gain).

Band Pass Filter

Removes high and low frequencies, leaving only a middle band.



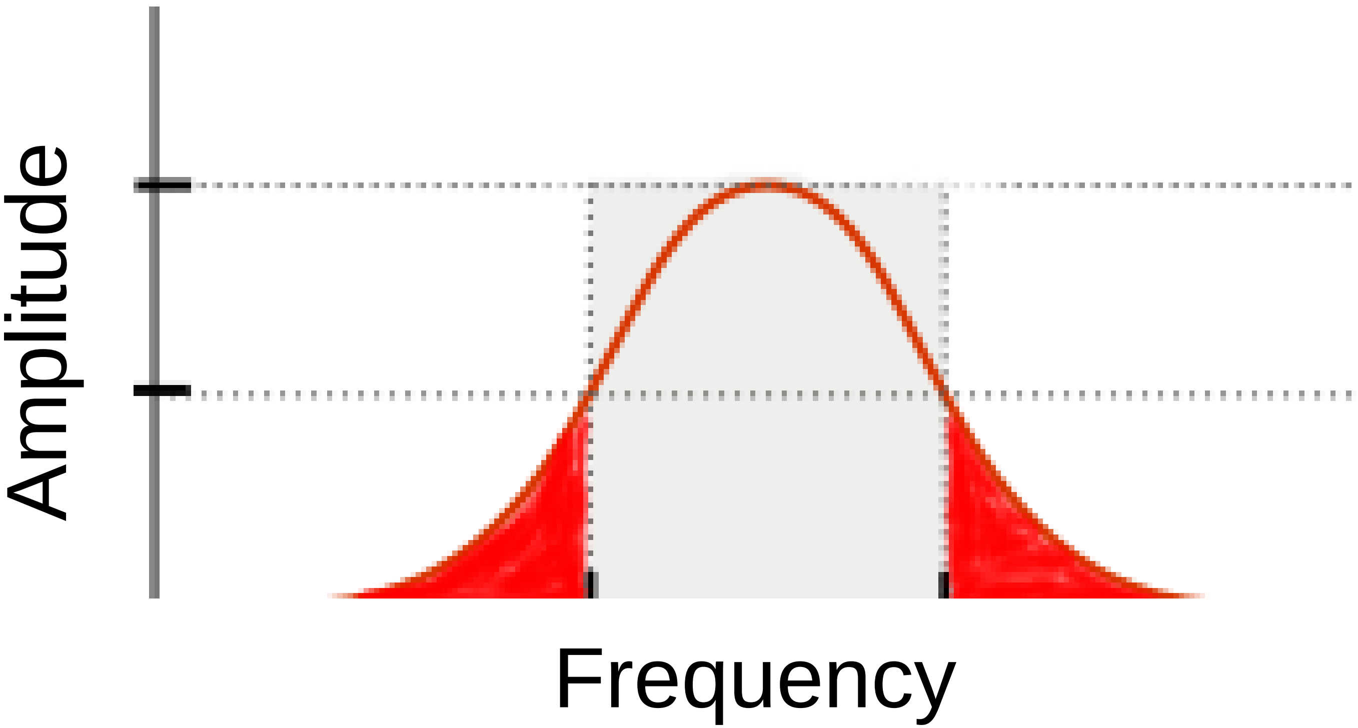
lowpass + highpass = bandpass



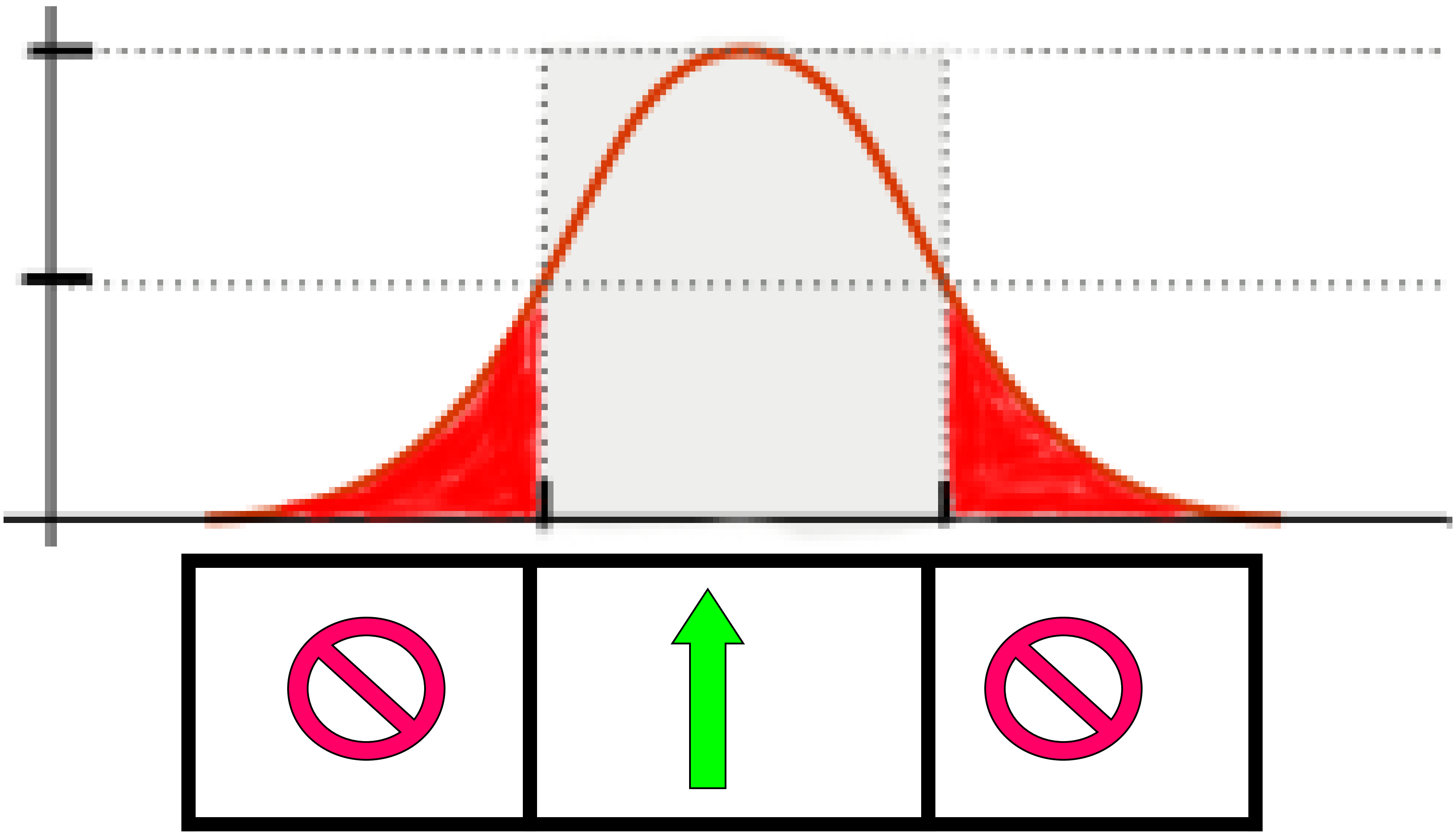
Now, we know that one reason to filter is to limit a signal's bandwidth so that we can reduce sampling frequency.

And we know that a smart way to limit bandwidth is to use FWHM to set those limits.

Remember this picture?



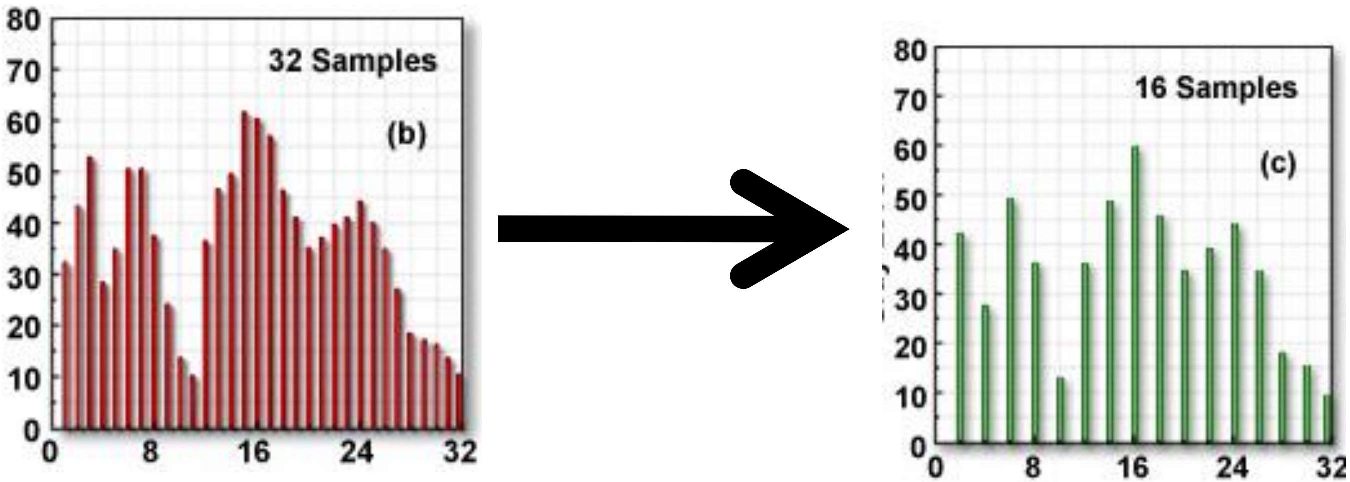
We can use the FWHM to set the bandpass filter!



So now the MR scanner can sample at a lower rate,

get the picture faster

and still prevent aliasing!

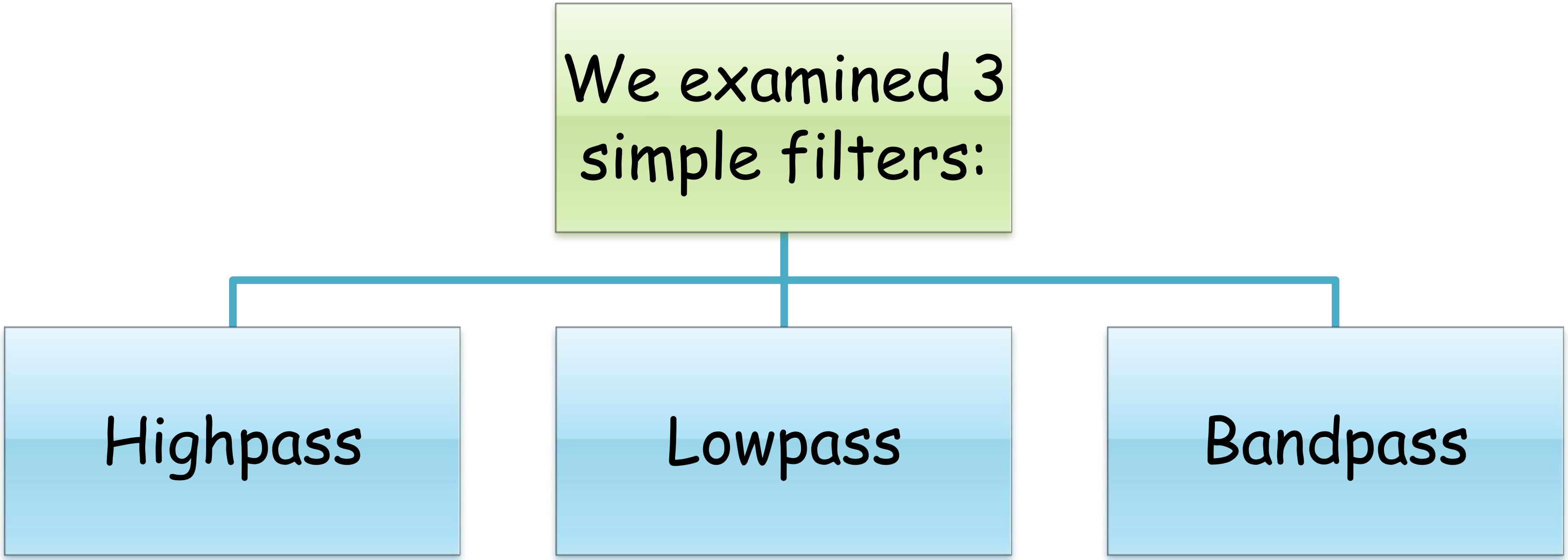


Let's Summarize

Filters remove or reduce frequencies we don't want; and enhance frequencies we do want.

We described the FWHM (Full Width Half Max) criterion as a tool for finding edges in images.

We also used FWHM as a criterion for applying a bandpass filter, so that we could limit sampling rate in a sensible way.


We examined 3 simple filters:
Highpass
Lowpass
Bandpass

And we pointed out that the FFT helps provide filters with more precise control.

Our highpass filter example cleaned up drift in an MRI image

Our lowpass filter smoothed an MRI image to enhance signal to noise ratio.

A bandpass filter, (lowpass + highpass),

can clean up low amplitude frequencies
at the tails of the signal distribution.

so sampling rate can be reduced
without aliasing

resulting in better faster MRI images

Filter

Noise

SNR

Processing Gain

Bandwidth

FWHM

High Pass Filter

- Drift

Low Pass Filter

- Smoothing

Bandpass Filter

- Antialiasing