The Fast Fourier Transform

Fourier Analysis and Synthesis (DFT, FFT)

Sine Waves

Power Spectrum (1D for temporal frequencies -- light and sound), (2D for spatial frequencies)

K-space

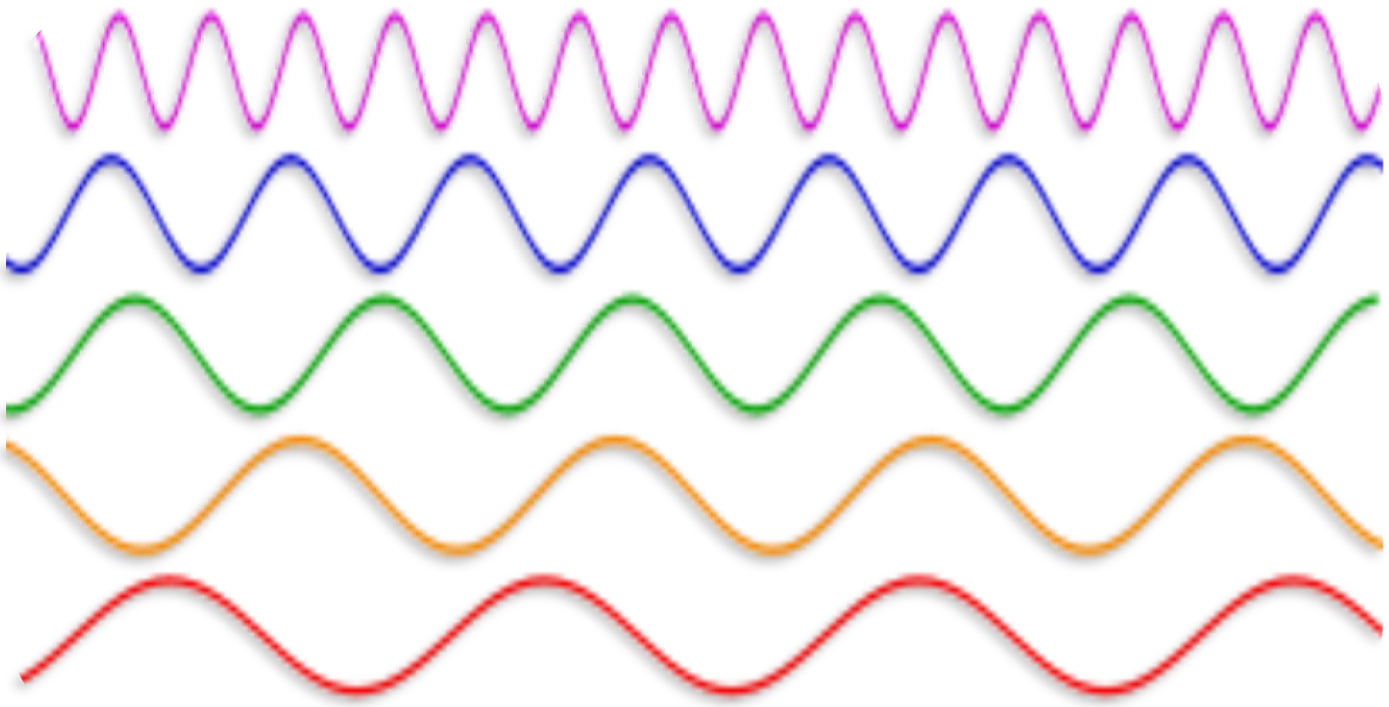
FFT is at the heart of digital signal processing,

so, let's look at how it works.

Remember sine waves?

They look like this:

High and Fast



Low and slow

A sine wave is a simple periodic waveform found in nature and used as the basis for Fourier analysis.

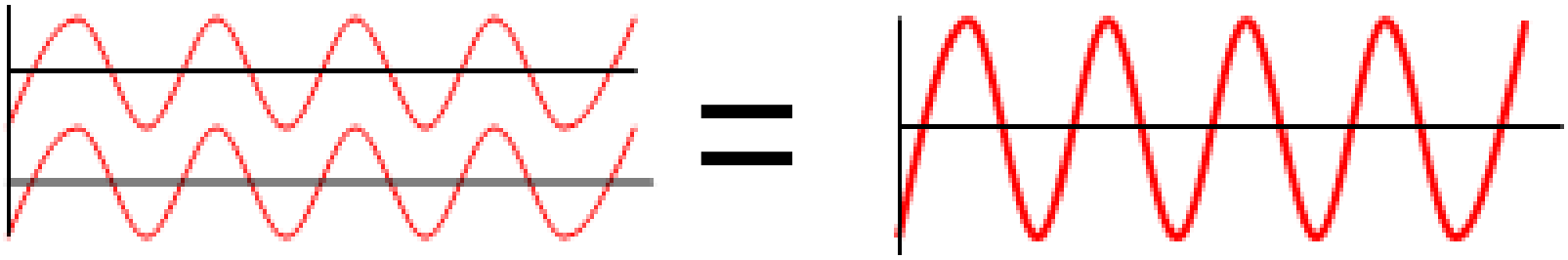
Sinusoidal waves can be used as simple building blocks

to 'make up' and describe nearly any periodic waveform.

This building process is called

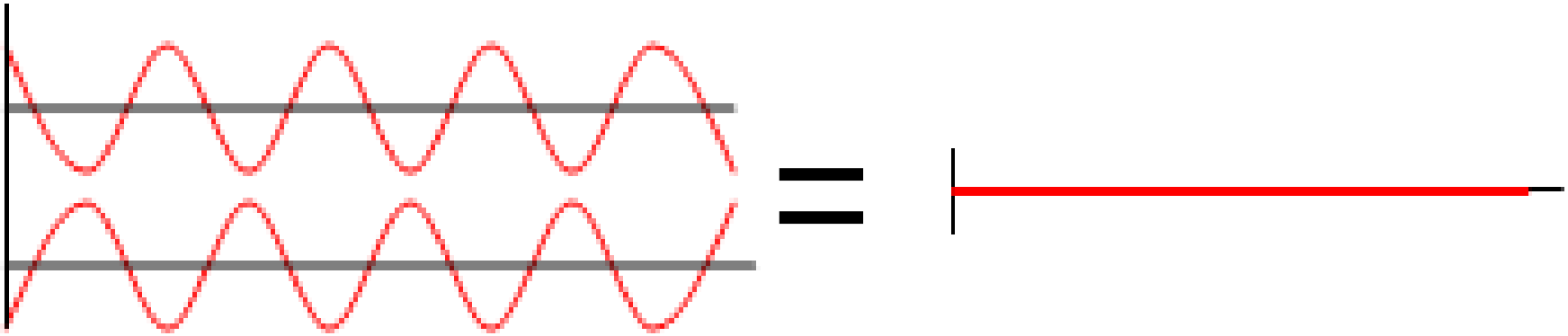
Fourier Synthesis

For example, we can add waves, and increase their amplitude:



Or, if we want noise cancelling headphones we can add waves that are 180 degrees out of phase,

to cancel everything out…

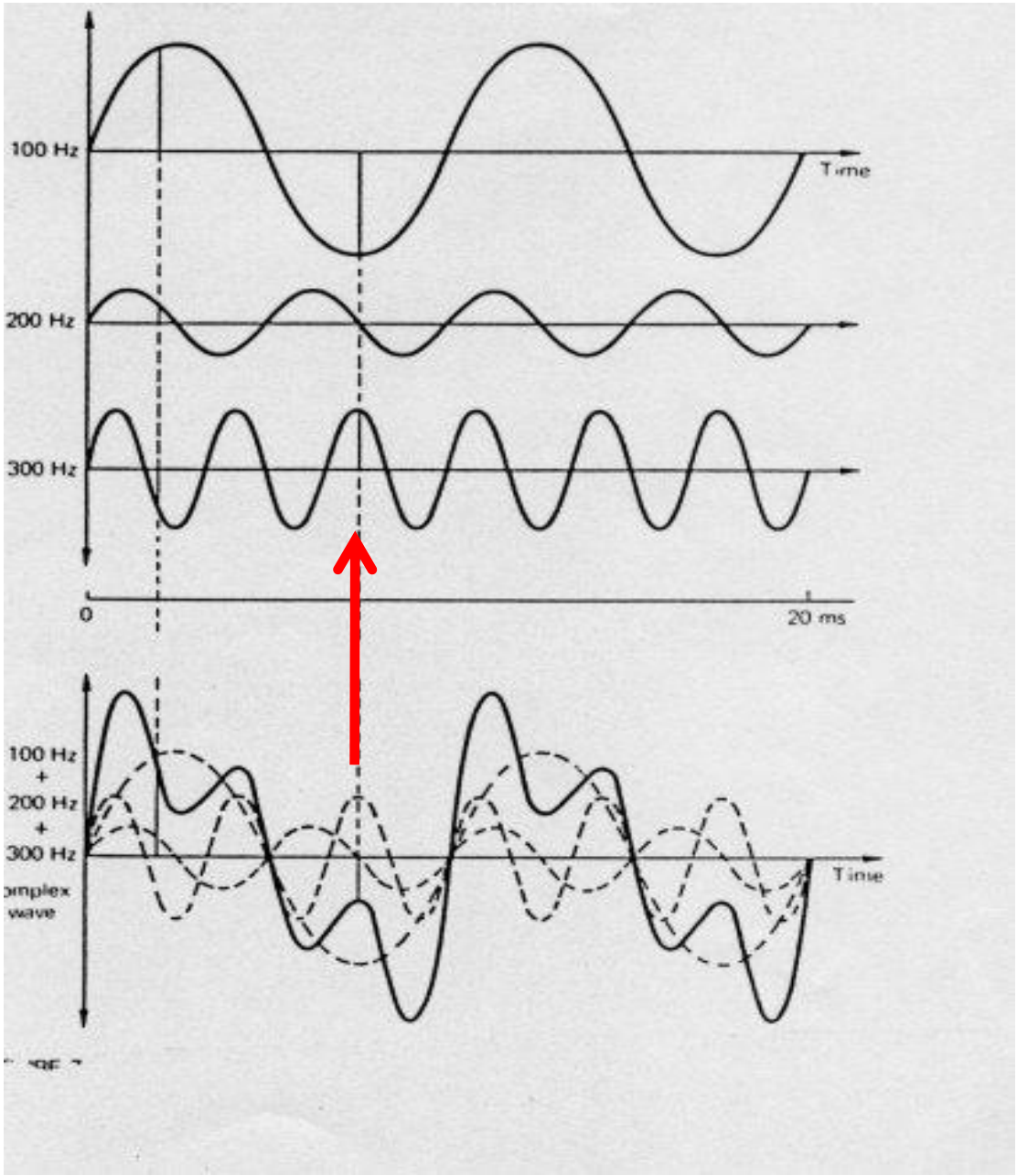


Any complex periodic wave can be deconvolved

into its component sine waves.

This decomposition is called

Fourier Analysis.


100 Hz
Time
200 Hz
300 Hz
0
20 ms
100 Hz
+
200 Hz
+
300 Hz
Time

The Fourier **transform** applies Fourier Analysis and its inverse, Fourier Synthesis.

The **Discrete Fourier Transform** (**DFT**) is discrete because it uses **digitized**, sampled data as input rather than continuous data.

The **Fast Fourier Transform** (**FFT**) is an efficient algorithm to compute the Discrete Fourier Transform (DFT).

Fourier analysis converts a time domain signal,

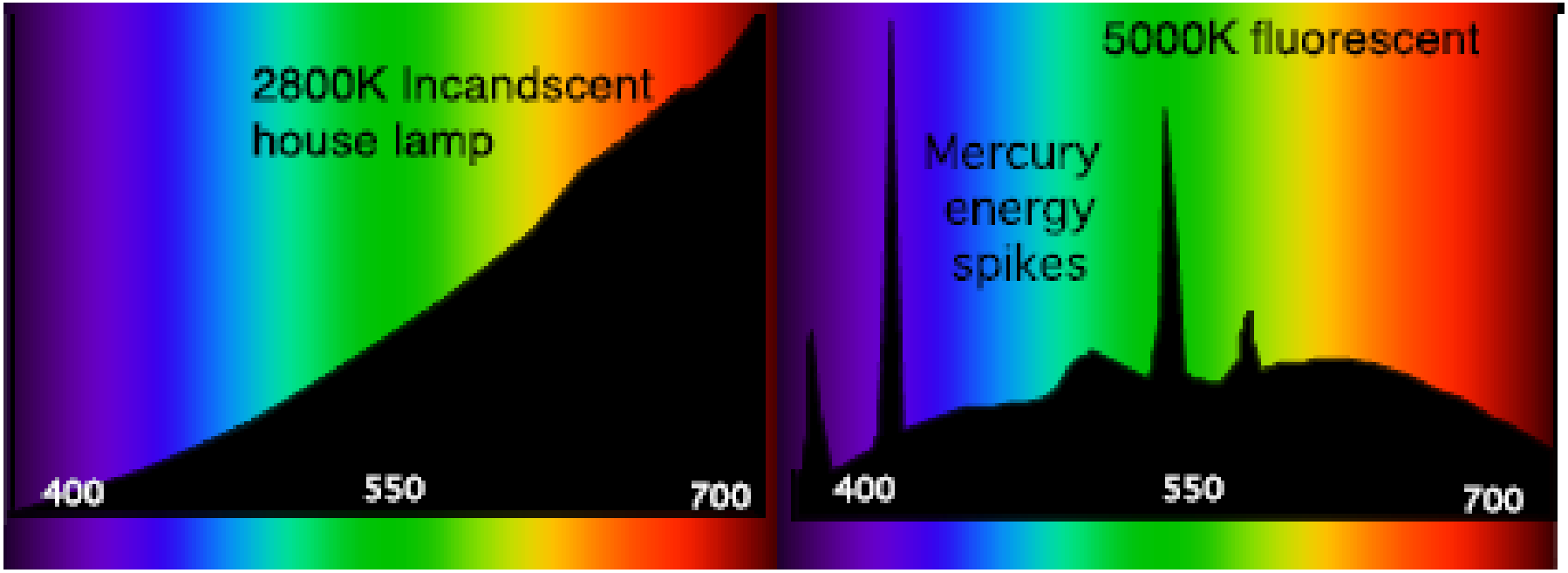
or a spatial domain signal,

into the frequency domain.

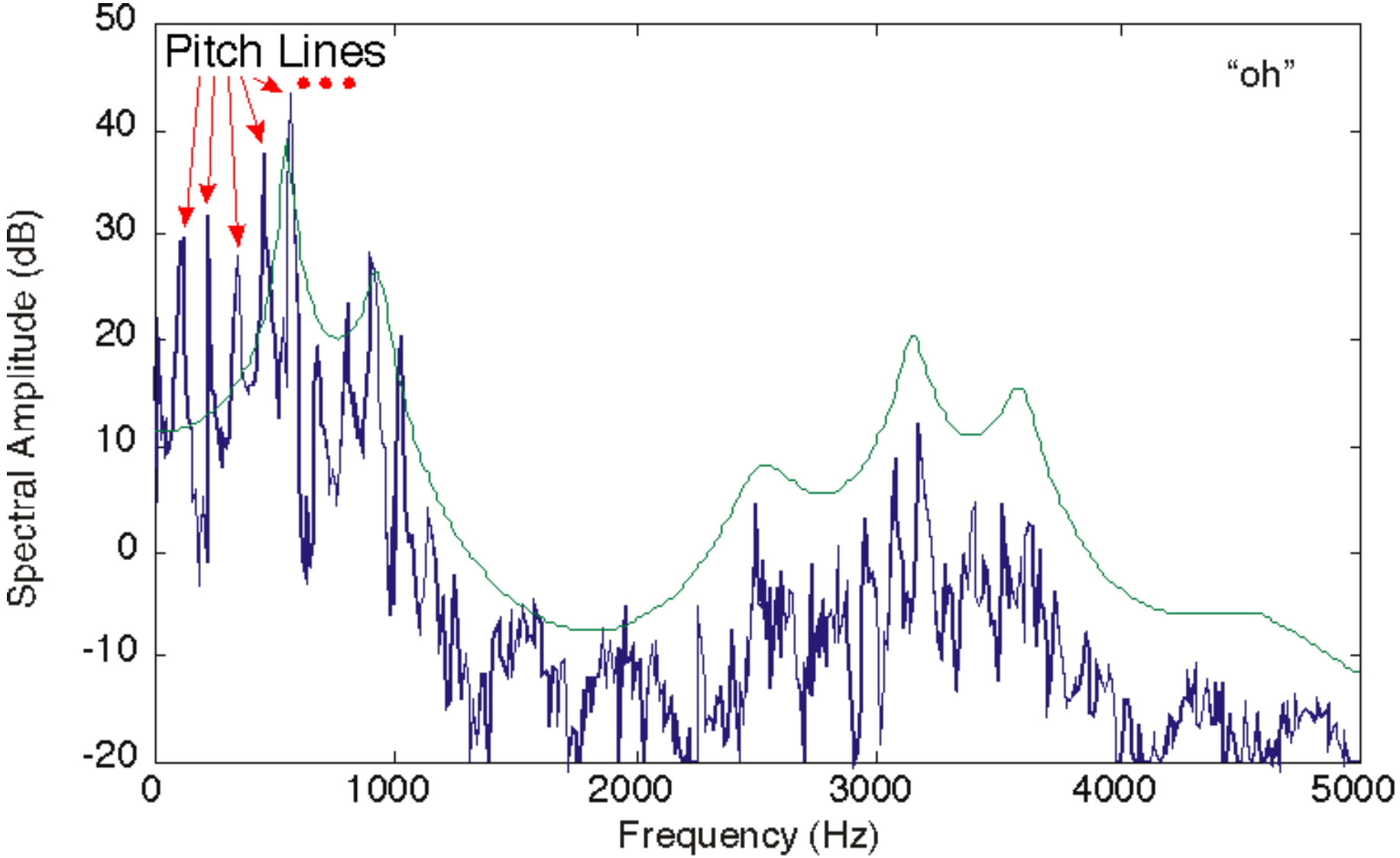
This frequency domain signal can be displayed as a Power Spectrum.

A Power Spectrum is a graph of frequency vs amplitude.

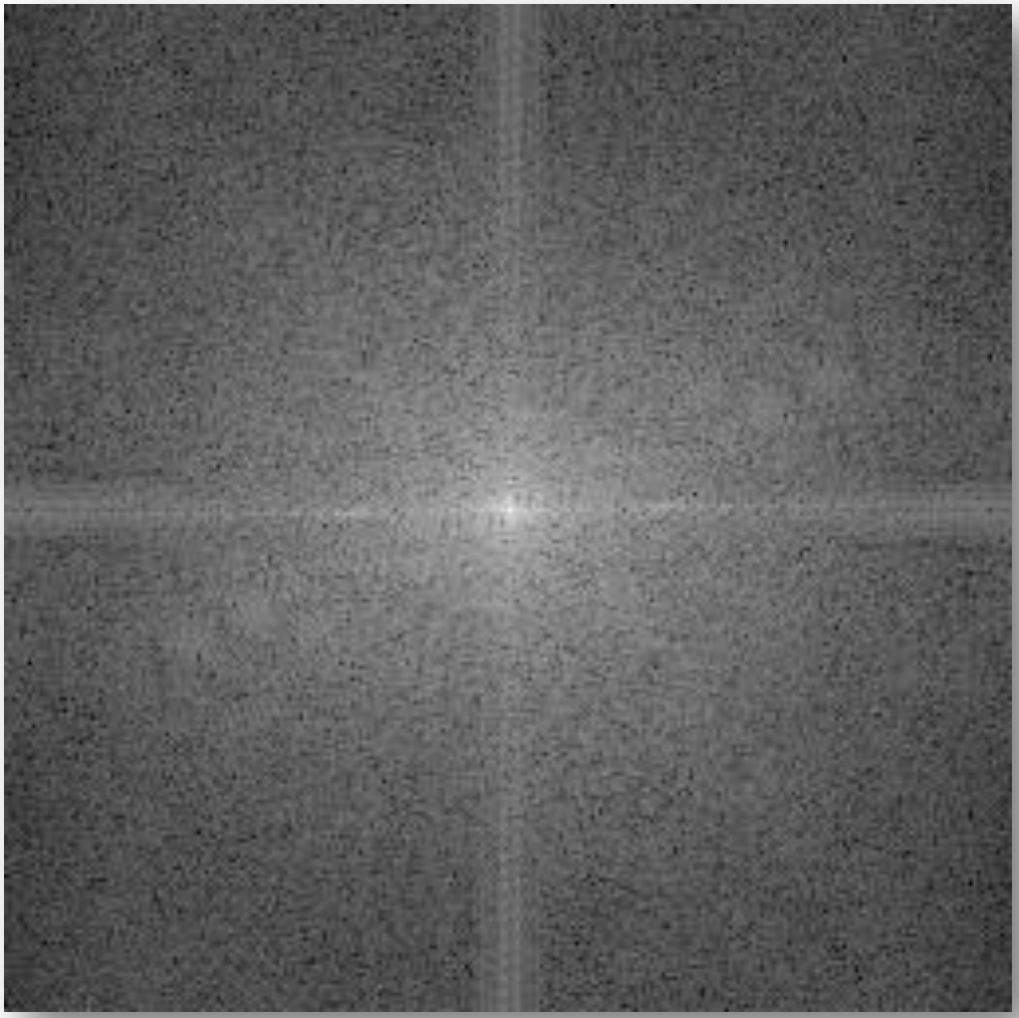
Whether it is applied to light…



or sound…

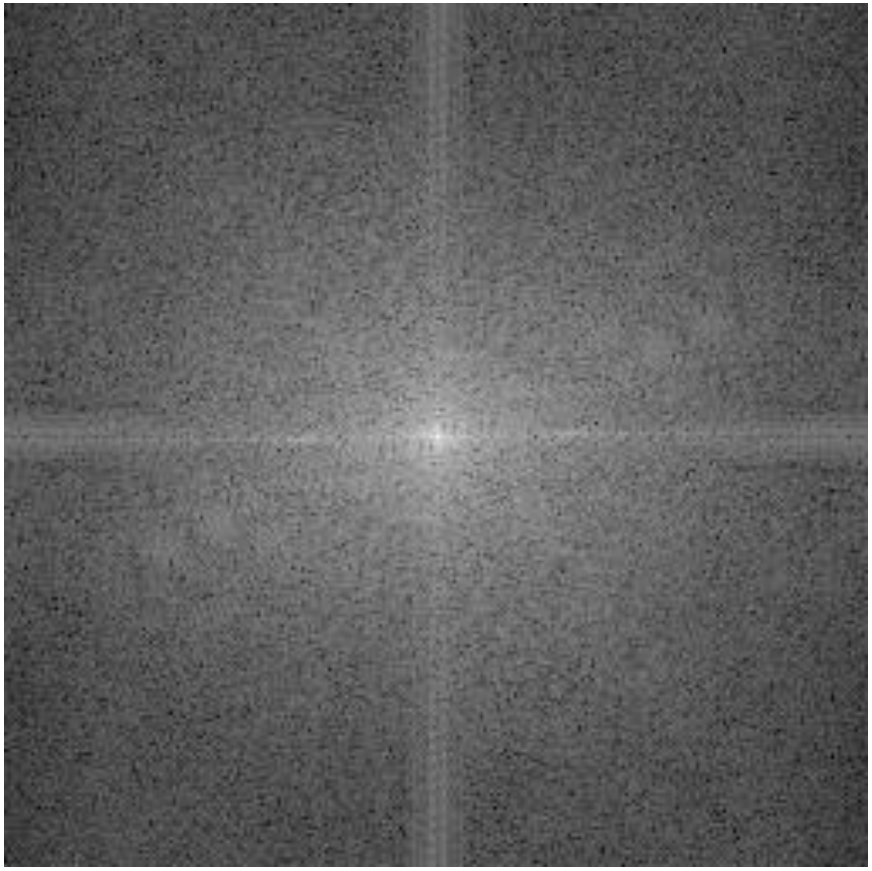

Pitch Lines
"oh"
Spectral Amplitude (dB)
50
40
30
20
10
0
-10
-20
0
1000
2000
3000
4000
5000
Frequency (Hz)

or spatial frequencies.



A spatial domain image

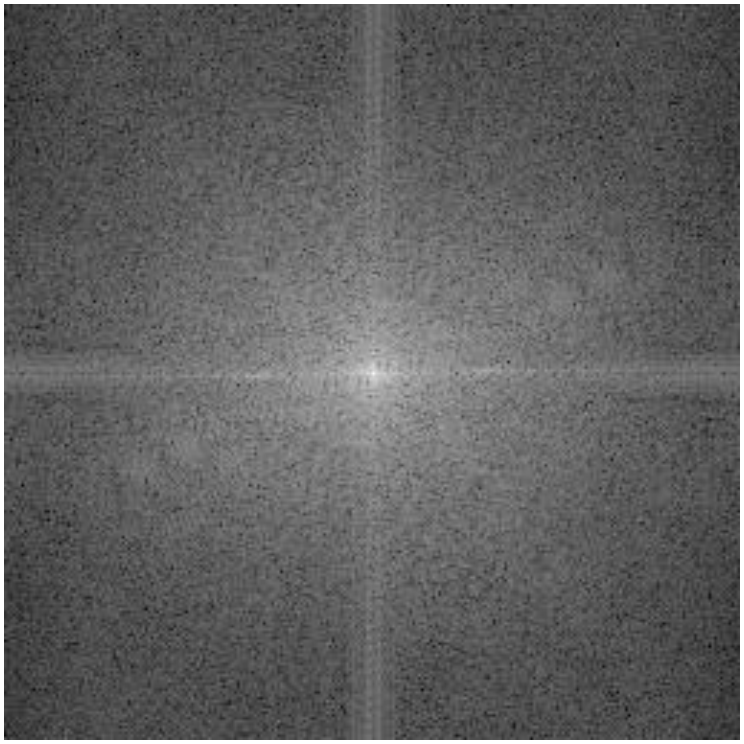
can be represented by a 2D spectral analysis (power spectrum) in the frequency domain.



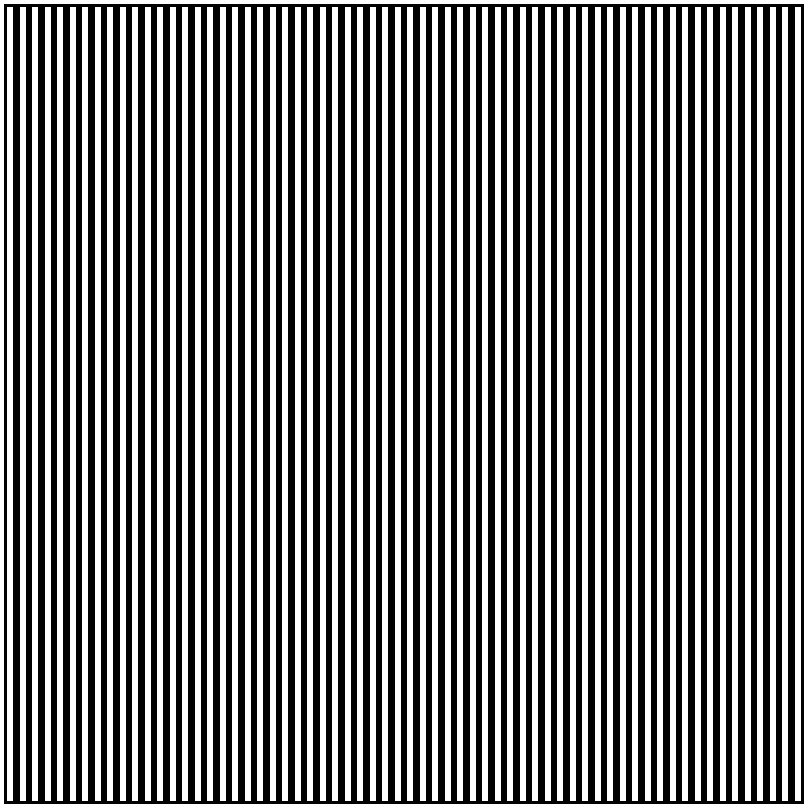
This Fourier image has two dominant directions:

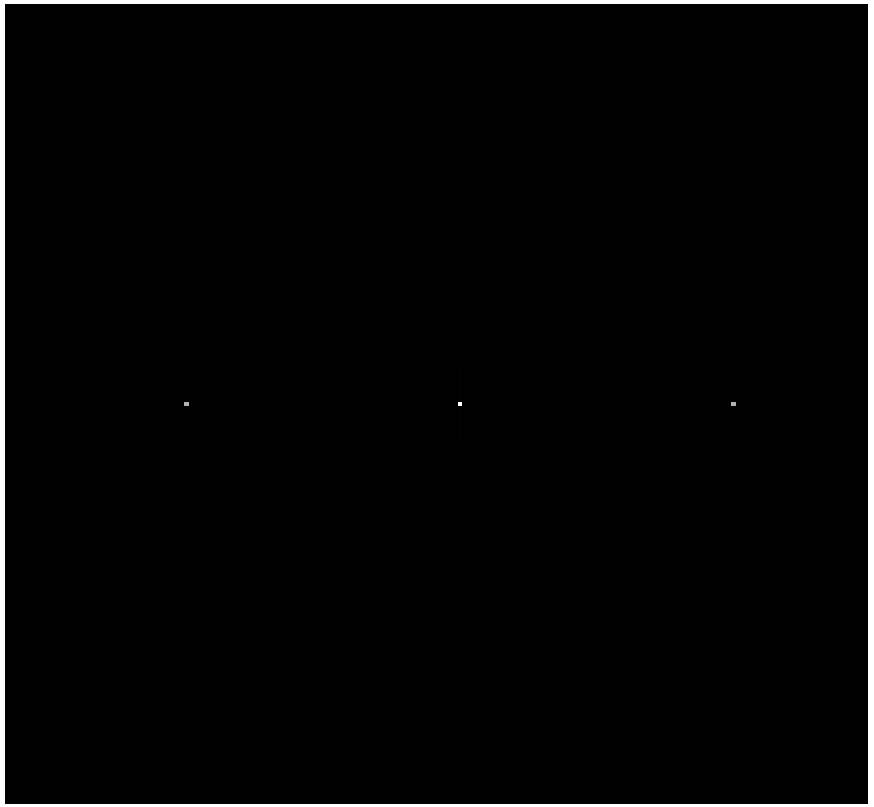
- one passing vertically and one horizontally through the center.

These originate from the regular patterns in the background of the original image.

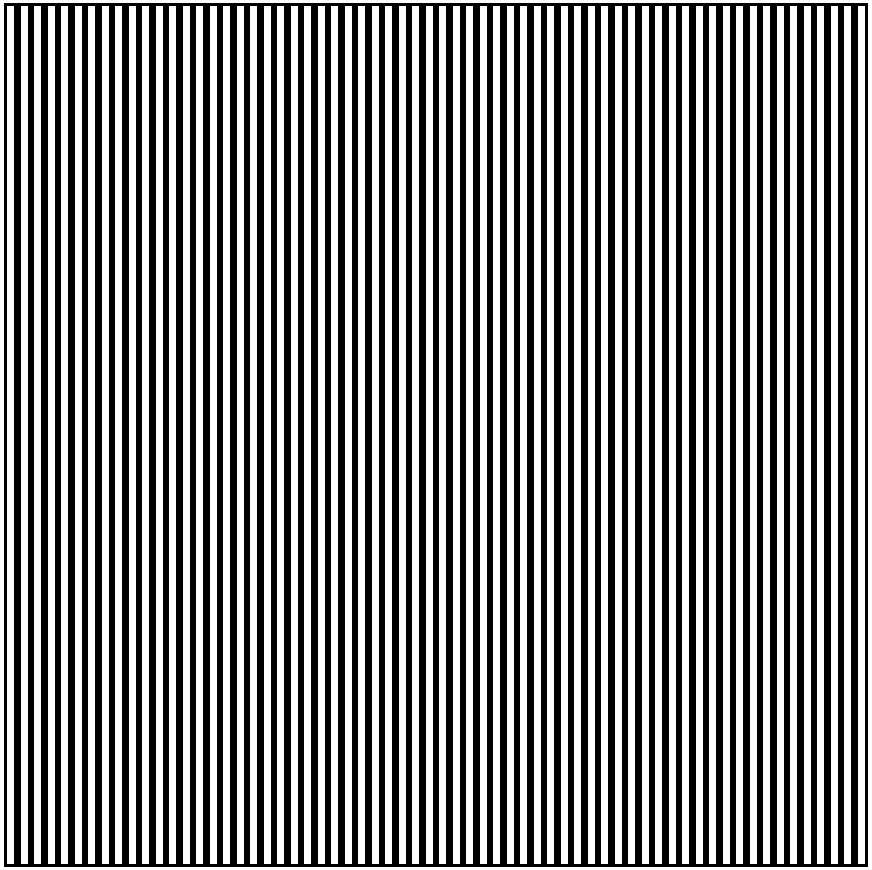


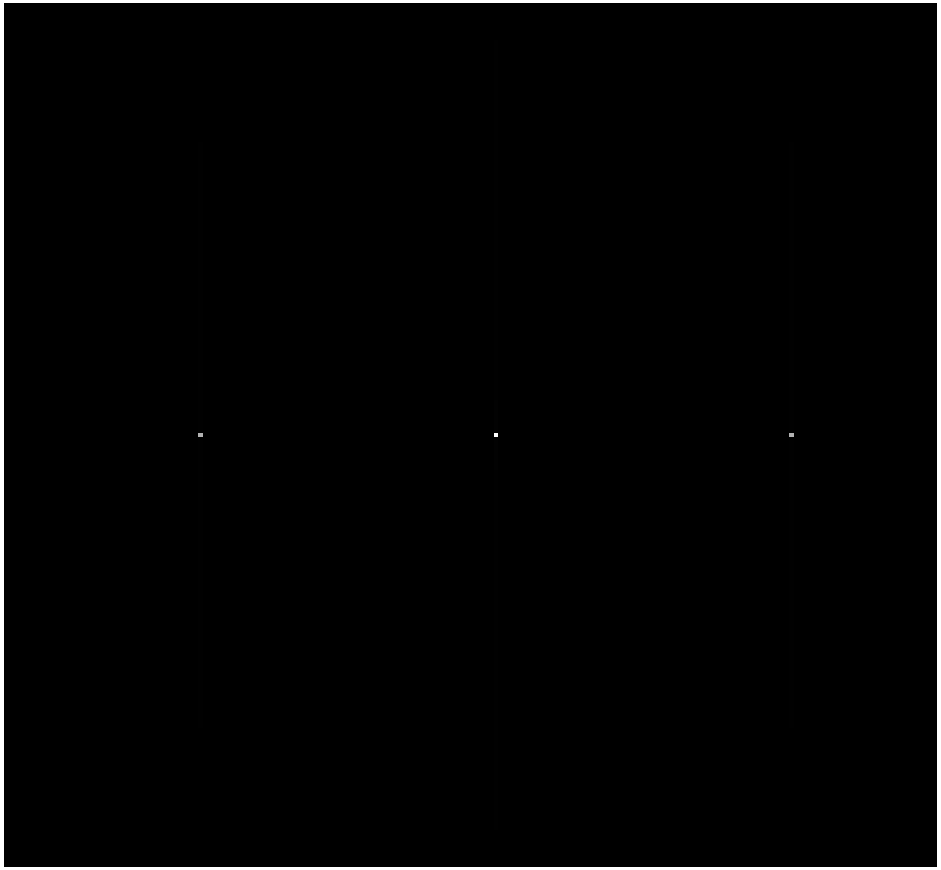
An image with a simple repeating pattern has a simple representation in the frequency domain.



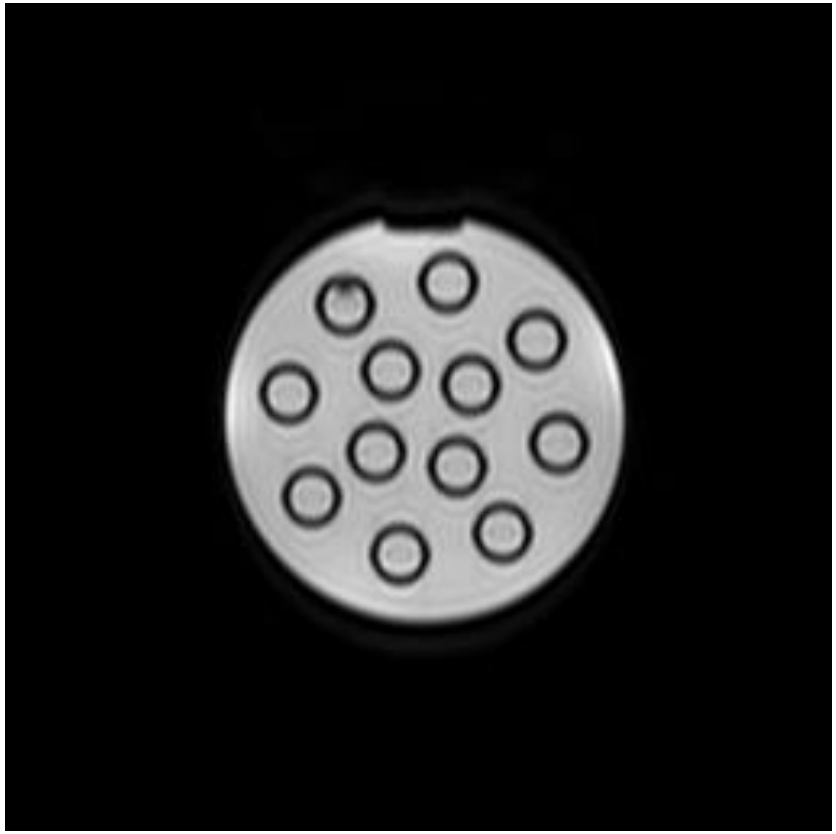


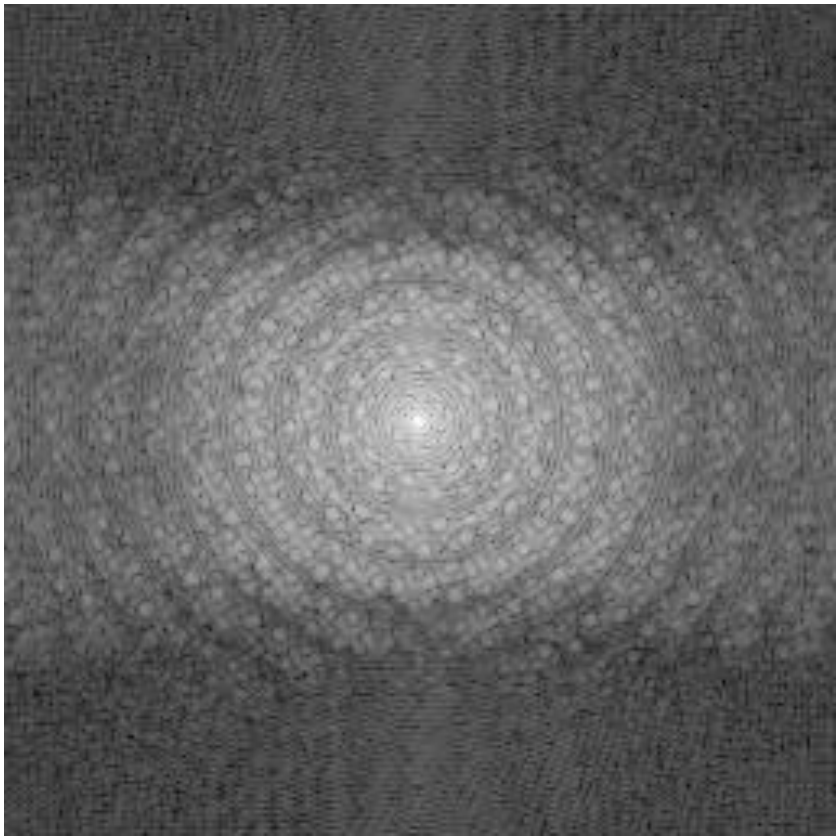
The dots lie on the horizontal, because all the changes occur along the horizontal.





Here's another one



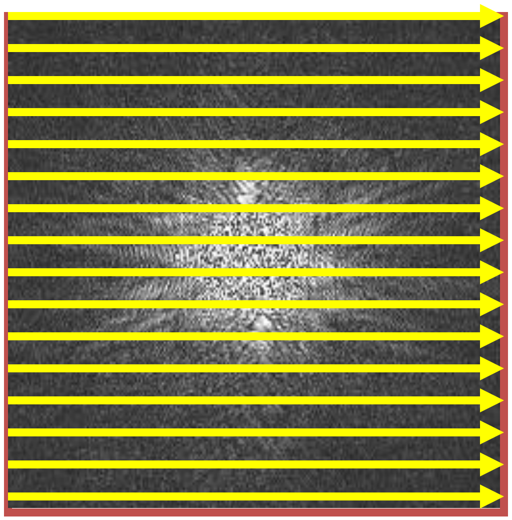


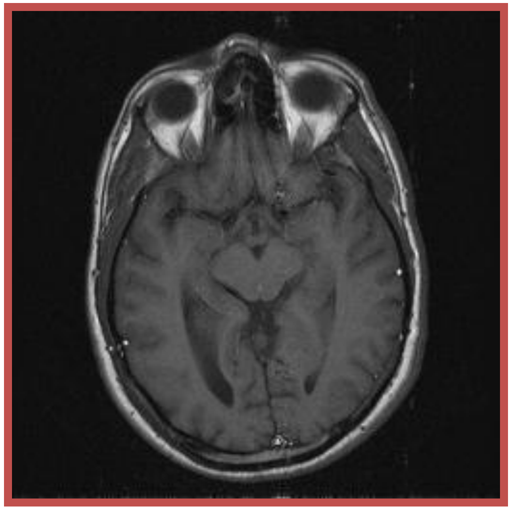
In the MR scanner, images are initially stored in the frequency domain.

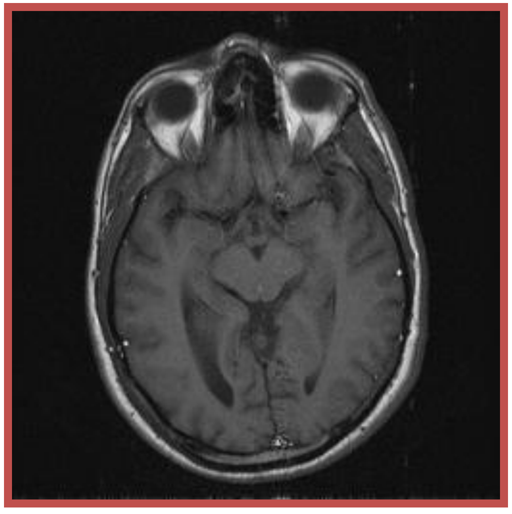
And this temporary image space is called k-space.

K-space

Image space







(This is Ted Trouard's very cool k-space animation)

As we pass through the center, we suddenly get most of the image

In fact, data is high frequency edge data

And that's this detail edge frequency edges

Let's Summarize

Fourier described a way to analyze any complex periodic wave into its component sine waves.

And this, combined with sufficient computing power, revolutionized signal processing.

Fourier analysis can be applied to a digitized signal (DFT) using a clever algorithm that makes it fast (FFT).

The analysis algorithm can be run in reverse to generate the complex wave from the components (Fourier Synthesis).

Once Fourier Analysis has been applied, we can examine new representations of the signal, like power spectra, in the frequency domain.

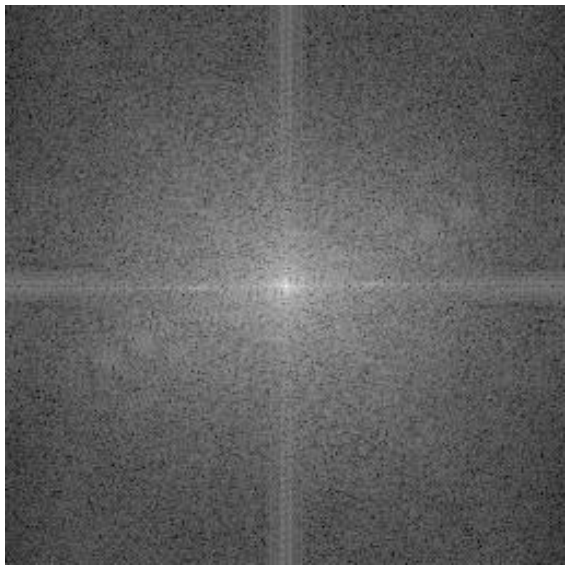
These representations allow us to manipulate the signal in new ways.

In imaging, Fourier analysis is applied to spatial frequencies

and despite looking very odd,

these representations in "k-space", turn out to be in MR scanners,

and in all kinds of image processing.



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