ISTA 116: Lab Assignment #5 (50 pts)

SOLUTION

Due Monday, October 31 by 11:59 P.M.

Problem 1:

(18 pts)

For each of the following procedures, identify the sample space of all possible, distinguishable outcomes, along with how many elements (individual outcomes) it contains (3 pts each).

a. Rolling two six-sided dice.

$\Omega = \{(1,1),(1,2),\ldots,(6,6)\}$

$|\Omega| = 6^2 = 36$

b. Picking a three-digit lottery number by drawing once from each of three buckets of ten ping-pong balls numbered 0 through 9.

$\Omega = \{(0,0,0),(0,0,1),\ldots,(0,1,0),(0,1,1),\ldots,(9,9,9)\}$

$|\Omega| = 10^3 = 1000$

c. Spinning a roulette wheel with one slot each for the numbers 1 through 36, as well as a slot for 0 and a slot for 00.

$\Omega = \{0, 00, 1, 2, 3, \ldots, 36\}$

$|\Omega| = 38$

d. Randomly selecting a letter from A to Z and then randomly selecting whether it is uppercase or lowercase.

$\Omega = \{A, B, \ldots, Z, a, b, \ldots, z\}$

$|\Omega| = 26 + 26 = 52$

e. Flipping a quarter, a nickel and a penny.

$\Omega = \{(H, H, H), (H, H, T), ..., (T, T, T)\}$

$|\Omega| = 2^3 = 8$

f. Choosing two pizza toppings from a list of four (pepperoni, mushrooms, peppers and black olives). Assume that order doesn't matter: for example, mushrooms and peppers is the same outcome as peppers and mushrooms.

Assuming that "double" orders are acceptable (e.g. "double pepperoni"), Ω consists of the 10 numbered pairs of toppings in the following table:

	pepperoni	mushrooms	peppers	olives
pepperoni	1	2	3	4
mushrooms		5	6	7
peppers			8	9
olives				10

Assuming that "double" orders are not acceptable, there are only 6 elements in Ω:

	pepperoni	mushrooms	peppers	olives
pepperoni		1	2	3
mushrooms			4	5
peppers				6
olives				

Problem 2:

(15 pts)

Consider rolling two balanced six-sided dice, where each individual pair of numbers [$(1, 1)$, $(1, 2)$, $\ldots$, $(2, 1)$,$(2, 2)$,$\ldots$,$(6, 5)$, $(6, 6)$] occurs with equal probability.

Compute the probability of each of the following events (3 pts each)

a. Rolling double ones (aka "snake eyes").

$E = \{(1, 1)\}$

$P(E) = \frac{1}{36}$

b. Rolling a total of three.

$E = \{(1, 2), (2, 1)\}$

$P(E) = \frac{2}{36}$

c. Rolling a total of seven.

$E = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$P(E) = \frac{6}{36}$

d. Rolling doubles of any kind.

$E = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$P(E) = \frac{6}{36}$

e. Rolling a total of four or less.

$E = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$

$P(E) = \frac{6}{36}$

Problem 3:

(17 pts)

Consider a standard deck of 52 playing cards. (See Figure 1 below if you're not familiar with playing cards)

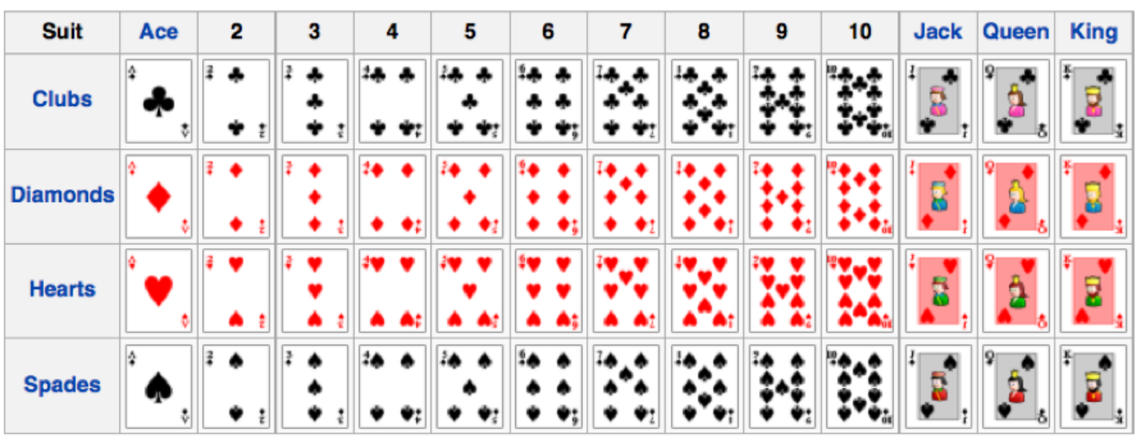

Figure 1: A standard deck of 52 playing cards

Let *Ace*, *Two*, *Three*, ..., *Queen*, *King* represent the events corresponding to drawing an Ace, Two, etc. Similarly let *Club*, *Diamond*, *Heart* and *Spade* represent the events corresponding to drawing a club, diamond, etc.

For each of the following events, represent it using unions, intersections and complements of the basic events above. For example, if $E =$ "Drawing a black Queen", then

$$E = Queen \cap (Spade \cup Club)$$

or

$$E = (Queen \cap Spade) \cup (Queen \cap Club)$$

Then, use the probability axioms and corollaries to write its probability in terms of probabilities of basic events and probabilities of intersections of basic events.

For the example above:

$$P(E) = P(Queen \cap Spade) + P(Queen \cap Club)$$

where we used the disjoint additivity rule (Axiom 2), because we can't draw a card that is both the Queen of spades and the Queen of clubs.

Check your equations by computing the probabilities of each term directly by counting, assuming each individual card has an equal probability of being selected.

Example:

$$\begin{aligned} P(E) = P(\text{"Black Queen"}) &= {}^2/_{52} \\ P(Queen \cap Spade) &= {}^1/_{52} \\ P(Queen \cap Club) &= {}^1/_{52} \\ P(Queen \cap Spade) + P(Queen \cap Club) &= {}^1/_{52} + {}^1/_{52} = {}^2/_{52} = P(\text{"Black Queen"}) \end{aligned}$$

a. (4 pts) $E =$ Drawing a face card (Jack, Queen or King)

$E = Jack \cup Queen \cup King$
$P(E) = P(Jack) + P(Queen) + P(King)$
$P(Jack) = \frac{4}{52}$
$P(Queen) = \frac{4}{52}$
$P(King) = \frac{4}{52}$
$P(Jack) + P(Queen) + P(King) = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52} = \frac{3}{13} = \mathbf{0.2308}$

b. (6 pts) $E =$ Drawing a spade or an Ace.

$E = Spade \cup Ace$
$P(E) = P(Spade \cup Ace)$
$P(Spade \cup Ace) = P(Spade) + P(Ace) - P(Spade \cap Ace)$
$P(Spade) = \frac{13}{52}$

$P(Ace) = \frac{4}{52}$
$P(Spade \cap Ace) = \frac{1}{52}$
$P(Spade) + P(Ace) - P(Spade \cap Ace) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} = \mathbf{0.3077}$

c. (7 pts) $E =$ Drawing a card that is neither red nor a Jack

Then the event of interest is "not red and at the same time not a Jack":

$$E = red^C \cap Jack^C$$

But, the problem asked us to use the basic events only and we have no *red* event. So:

$$\mathbf{E = (Heart \cup Diamond)^C \cap Jack^C}$$

This reminds us of one of DeMorgan's laws:
$(Heart \cup Diamond)^C \cap Jack^C = ((Heart \cup Diamond) \cup Jack)^C$
That part inside the parentheses looks easy to calculate.
It's the complement of what I really want, but that's ok.
$E^C = (Heart \cup Diamond) \cup Jack$

Now we calculate the probability.
Hearts and Diamonds are disjoint so:
$P(Heart \cup Diamond) = P(Heart) + P(Diamond) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52}$

Red cards and Jacks are not disjoint, so we need to remember to subtract the intersection:

$$\begin{aligned}
P(E^C) &= P((Heart \cup Diamond) \cup Jack) \\
&= P(Heart \cup Diamond) + P(Jack) - P((Heart \cup Diamond) \cap Jack) \\
&= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} \\
&= \frac{28}{52} \\
&= \frac{7}{13} \\
&= 0.5385
\end{aligned}$$

But remember, all this time we've been calculating the complement of what we really want, so:

$$\mathbf{P(E) = 1 - P(E^C) = 1 - \frac{7}{13} = \frac{6}{13} = 0.4615}$$

NOTE: the steps described here are not the only way to solve the problem.