Filters Meet Matrix Math

- Filter shape
- Kernel
- Rectangular Filter
- Box Filter
- Gaussian Filter
- Spatial filter
- Frequency filter

Digital filters are made up of numbers, arranged in a vector or matrix

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

0	0	0	1	1	1	1	0	0	0

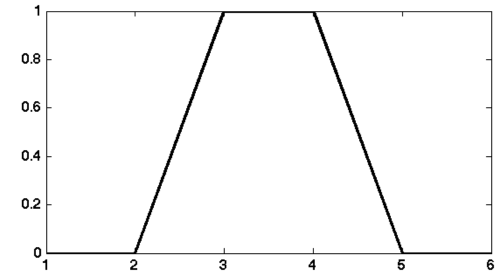
1	1	1	1	1	1	1	1	1
1	3	3.2	3.5	4	3.5	3.2	3	1
1	3.2	5	6.5	6	5.5	5	3.2	1
1	3.5	5.5	10	11	10	5.5	3.5	1
1	4	6	11	12	11	6	4	1
1	3.5	5.5	10	11	10	5.5	3.5	1
1	3.2	5	5.5	6	5.5	5	3.2	1
1	3	3.2	3.5	4	3.5	3.2	3	1
1	1	1	1	1	1	1	1	1

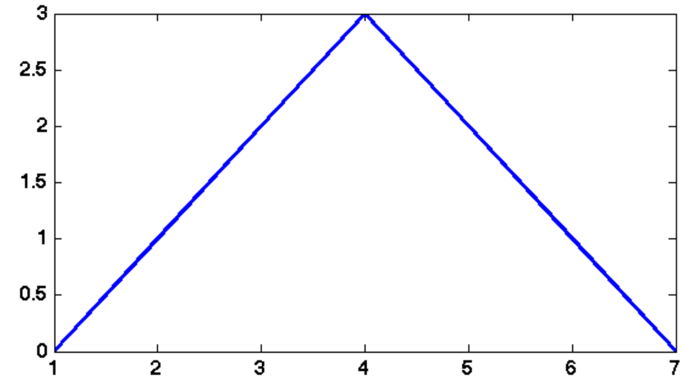
This matrix (or vector) is called a **kernel**

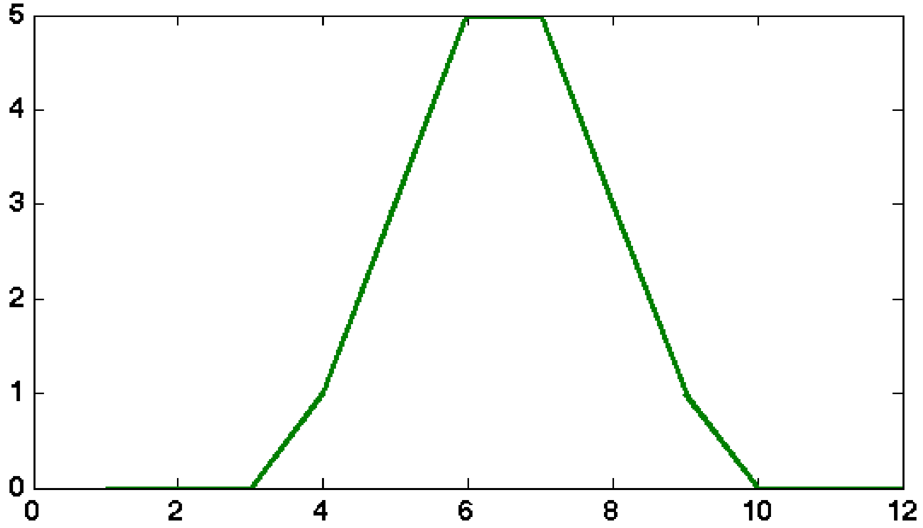
A **digital filter** is applied to a signal with matrix math operations (convolution or multiplication)

Vector Convolution

- Filter=[0 0 1 1 0 0]
- Wave=[0 1 2 3 2 1 0]
- conv(Filter, Wave)
- ans = [0 0 0 1 3 5 5 3 1 0 0

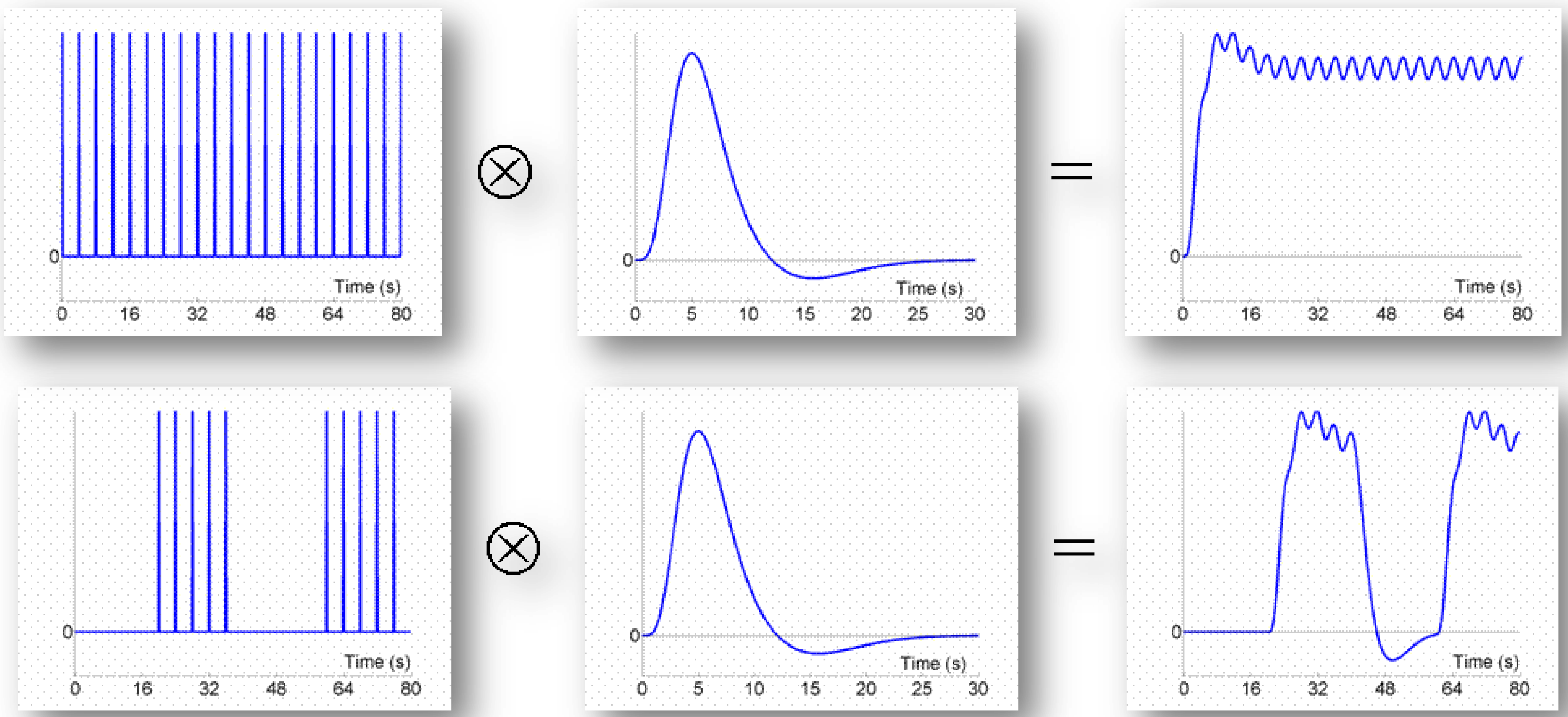






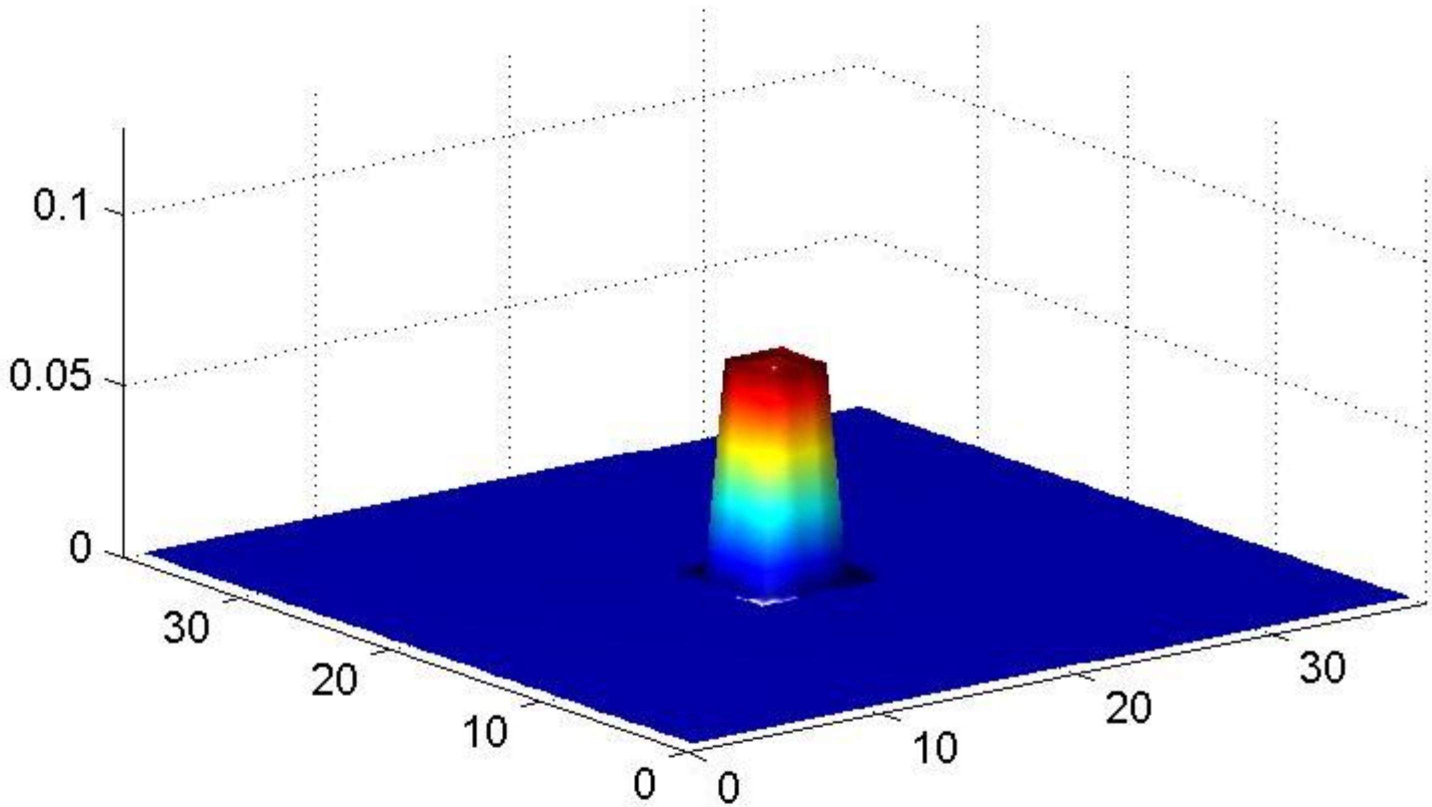
Convolution of Vectors in fMRI

Predicted Response | Filter | Result



If the kernel is a matrix of numbers, like this 2D box filter

0	**0**	**0**	**0**	**0**
0	**1**	**1**	**1**	**0**
0	**1**	**1**	**1**	**0**
0	**1**	**1**	**1**	**0**
0	**0**	**0**	**0**	**0**



2D convolution

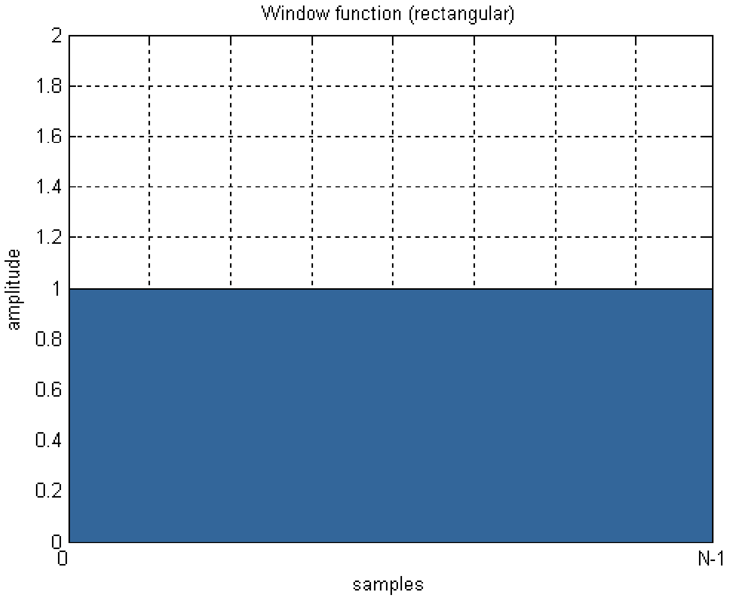
We can convolve it with an image by sliding the kernel over the image.

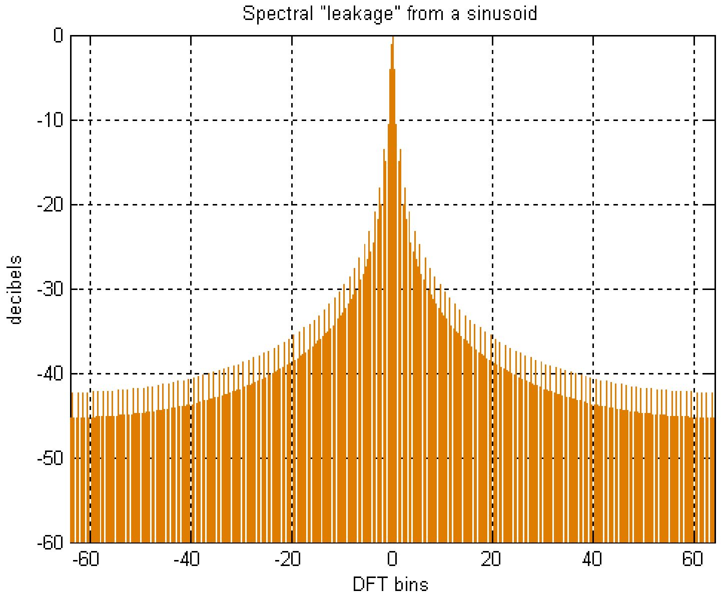
Generally, start at the top left corner.

Move the kernel through all the positions where the kernel fits entirely within the boundaries of the image.

Not all filters are created equal.

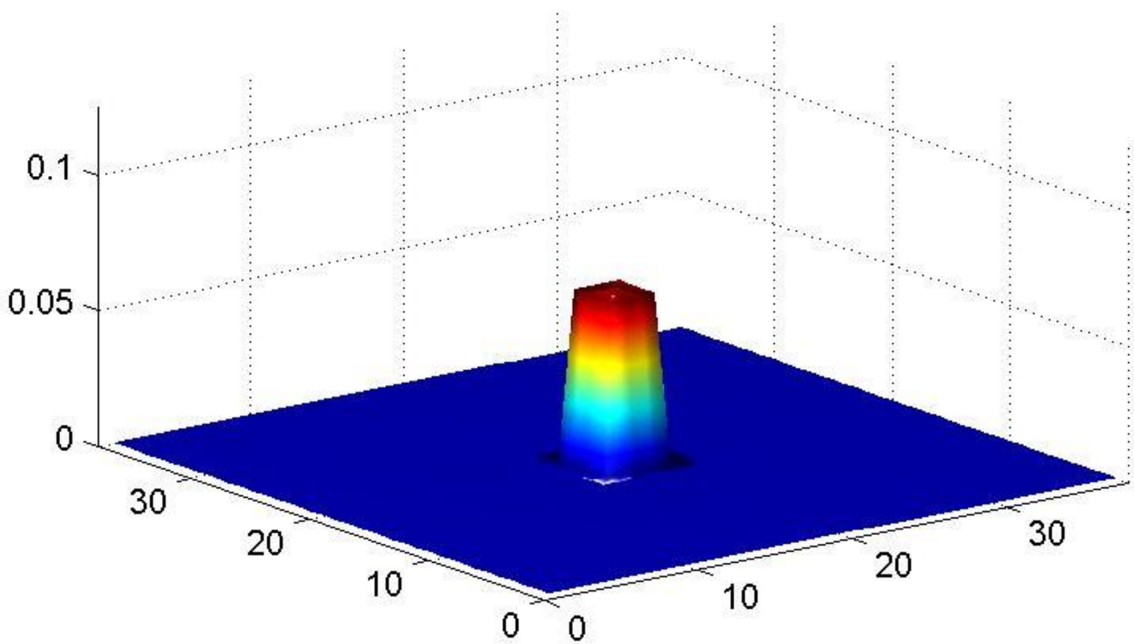
For example, the rectangular filter produces pops and clicks at the beginning and end of synthetic speech output*.





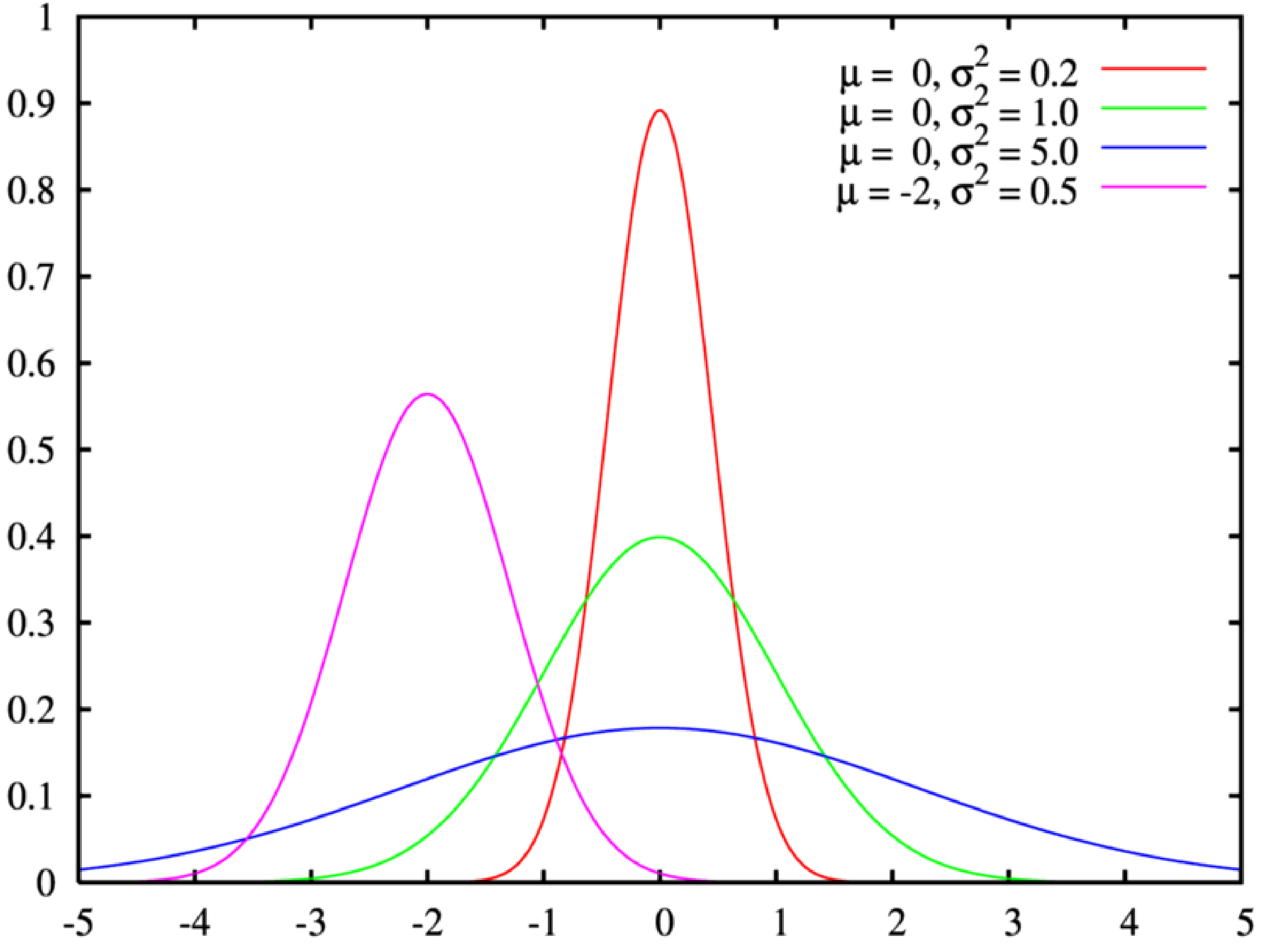
*(Personal Communication, Susan LeGendre)

"The box filter produces strong ringing effects in frequency space and is therefore not considered a high quality smoothing filter."

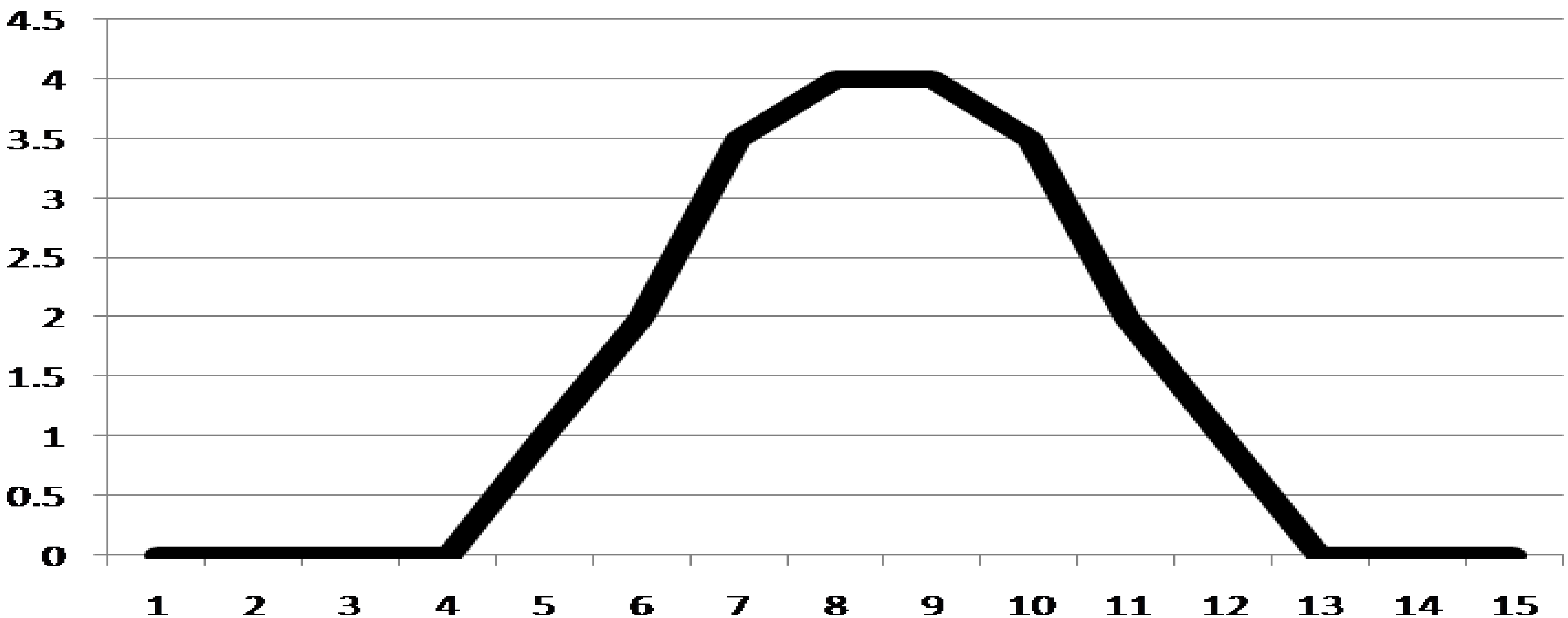


(Burger and Burge, 2008, p 97)

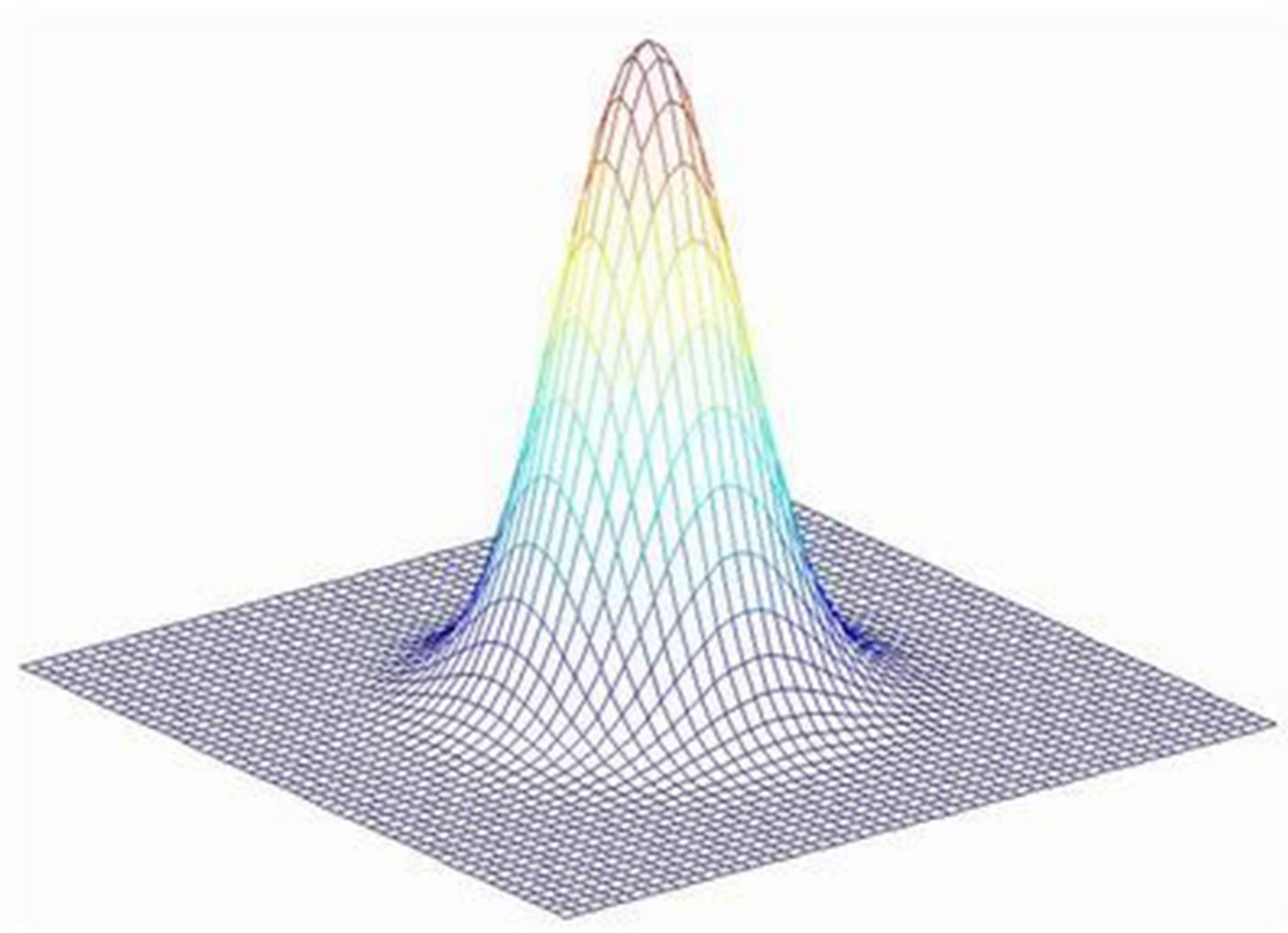
- Let's consider another filter shape.
- It looks like a bell curve (in 1D)



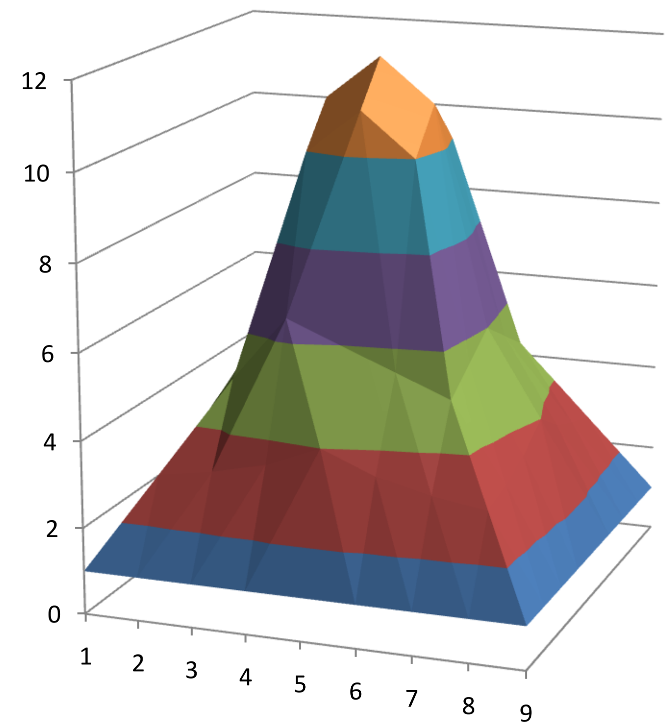
0	0	1	2	3.5	4	4	3.5	2	1	0	0



Or like a cone in (2D)



1	1	1	1	1	1	1	1	1
1	3	3.2	3.5	4	3.5	3.2	3	1
1	3.2	5	6.5	6	5.5	5	3.2	1
1	3.5	5.5	10	11	10	5.5	3.5	1
1	4	6	11	12	11	6	4	1
1	3.5	5.5	10	11	10	5.5	3.5	1
1	3.2	5	5.5	6	5.5	5	3.2	1
1	3	3.2	3.5	4	3.5	3.2	3	1
1	1	1	1	1	1	1	1	1



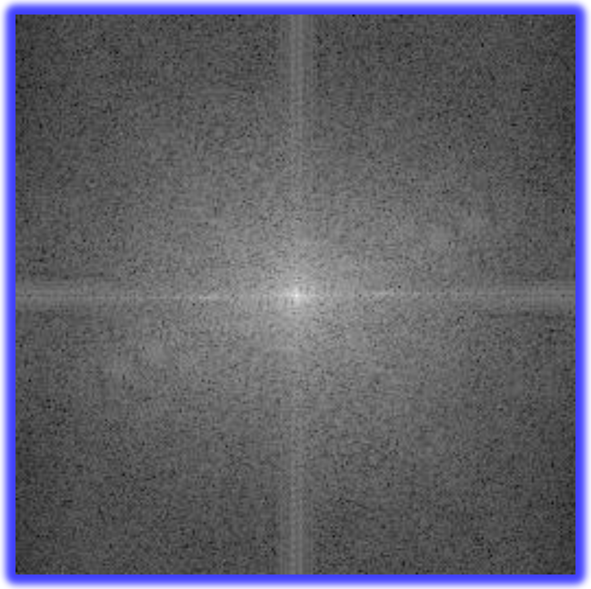
These are Gaussian filters.

2D Gaussian filters for image smoothing are generally beloved.

And it is not uncommon to find Gaussian filters in other applications,

including speech processing

- Image filtering can be occur in the frequency domain or the spatial domain.



- i.e., **frequency filters** & **spatial filters**

Convolution of image and filter in the spatial domain=multiplication of image and filter in the frequency domain.

In practice, it is computationally less expensive to work in the spatial domain.

But, if no straightforward kernel can be created…

then the frequency domain may be more efficient to work in.

Some examples of convolution filters

0	1	0
1	1	1
0	1	0

Smoothing averages the current pixel and at least 4 neighbors.

0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0

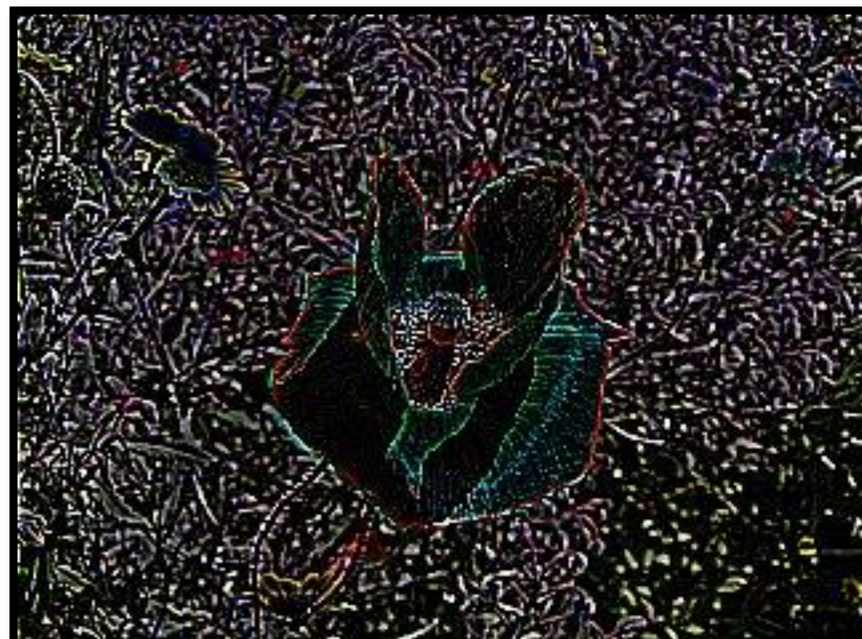
More smoothing is achieved by using a bigger kernel (averaging more pixels)

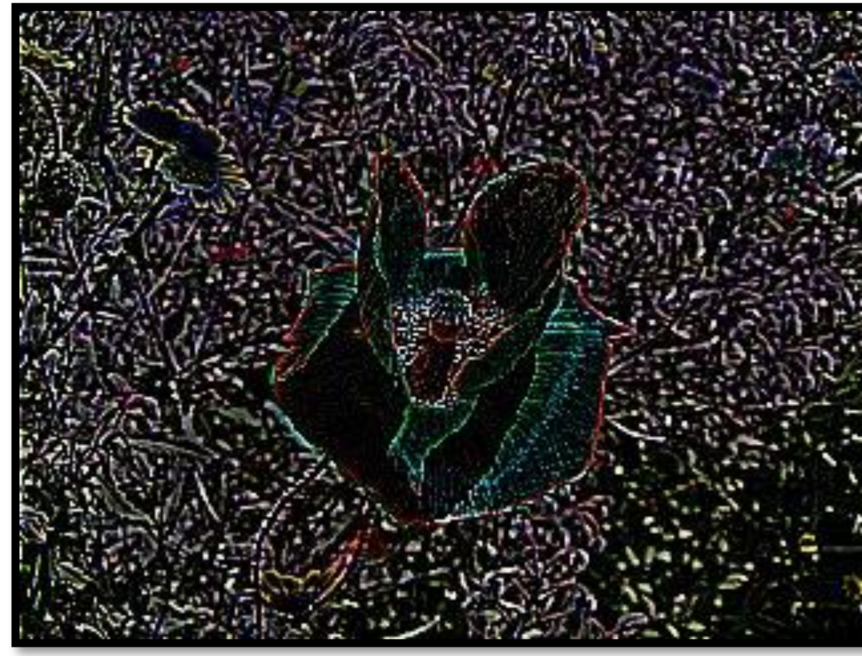
1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

Motion blur is achieved by smoothing in only one direction (averaging pixels in one direction)

0	-1	0
-1	8	-1
0	-1	0

Edge detection enhances differences between a pixel and its neighbours





original + edge detection = sharpening

Summary

Filtering is an application of matrix math.

Image filtering can be accomplished through convolution of a kernel and an image in the spatial domain;

or multiplication of a kernel and Fourier transformed image (power spectrum of image).

The Gaussian filter, which looks like a bell curve, is an extremely common and useful filter

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