ISTA 116: Statistical Foundations for the Information Age

Multivariate Categorical Data

14 September 2011

Outline

Some Latin

- Recall: if a data set has one variable, we say that it is **univariate**
- Two variables: **bivariate**
- More than two: **multivariate**
- Verzani covers bivariate data in Ch. 3 and multivariate in Ch. 4. We'll cover them together, since the types of the variables makes a bigger difference in what you can do than the number.

Multiple Univariate vs. One Multivariate

- What's the difference between this...

Person	Sex
1	M
2	F
3	F
4	M
5	F
6	F
7	M
8	M

Person	Height (in.)
A	64
B	74
C	72
D	68
E	61
F	70
G	68
H	69

Multiple Univariate vs. One Multivariate

- and this?

Person	Sex	Height (in.)
1	M	74
2	F	64
3	F	61
4	M	68
5	F	70
6	F	69
7	M	72
8	M	68

Multiple Univariate vs. One Multivariate

- When we observe more than one characteristic from the *same person*, we can look at the *relationship* between the variables.
  - Are males taller than females on average?
  - Are males more variable than females?
- With two isolated variables, and no correspondence between the values, there's no way to examine relationships.

Three Cases

- The kinds of relationships we can identify depend on the types of variables we have
- Three Cases:
  - All categorical variables
  - A mix of categorical and numeric
  - All numeric

Contingency Tables

- Recall: with a univariate categorical data set, we summarized by *counting* the observations in each category
- With more than one variable, we do the same thing, but we keep track of *combinations*.
- With two variables, we can store the counts in a two-dimensional grid called a **contingency table**

A Simple Contingency Table

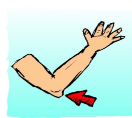
Sex	Computer
M	PC
F	Mac
F	PC
M	PC
F	PC
F	Mac
M	Mac
M	PC

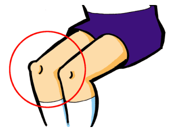
$\Longrightarrow$

		Computer	
		PC	**Mac**
Sex	**M**	3	1
	F	2	2

Joint Frequencies

- With one categorical variable, we had a **frequency table**: the entries represent how many times (how *frequently*) each category appears in the data
- With two or more variables, we can count how often two categories (say, male and PC) appear together (jointly).
- The counts of the combinations are called **joint frequencies**.





Marginal Frequencies

- We may be interested in how often one category appears, *regardless* of what it's combined with.
- To find the total number of females (say), sum across the row.
- We could write the sum in the "margin" of the contingency table.
- This sum is called the **marginal frequency** for females (say).

Marginal Frequencies

- We start with joint frequencies:

		Computer	
		PC	**Mac**
Sex	**M**	3	1
	F	2	2

Marginal Frequencies

- and compute the marginal frequencies for sex:

		Computer PC	Computer Mac	Marginal
Sex	**M**	3	1	4
	F	2	2	4

Marginal Frequencies

- We can also compute the marginal frequencies for Computer:

		Computer PC	Computer Mac	Marginal
Sex	**M**	3	1	4
	F	2	2	4
	Marginal	5	3	

- Notice that we can start with a set of marginal frequencies, and "marginalize" in the other direction to get the total number of observations ($= n$)

Marginal Frequencies

- Call our two variables X and Y, and let x and y represent particular values they can take on.
- If we use Freq$(\cdot)$ to represent the frequency of whatever we put inside the parentheses, then we have:

$$
\begin{aligned}
\text{Freq}(X = x) &= \sum_y \text{Freq}(X = x \text{ and } Y = y) \\
\text{Freq}(Y = y) &= \sum_x \text{Freq}(X = x \text{ and } Y = y) \\
\sum_x Freq(X = x) &= \sum_y Freq(Y = y) = n
\end{aligned}
$$

Marginal Frequencies

- For example:

$$
\text{Freq}(Sex = M) \\
= \sum_y \text{Freq}(Sex = M \text{ and } Computer = y)
$$

$$
\text{Freq}(Computer = PC) \\
= \sum_x \text{Freq}(Sex = x \text{ and } Computer = PC)
$$

Joint and Marginal Proportions

- With univariate categorical data, we computed **relative frequencies** for each category: the *proportion* of the time that category appeared.
- We can do the same thing for the frequencies in a contingency table, simply by dividing each frequency by the grand total.
- When applied to joint frequencies, we get **joint proportions**; when applied to marginal frequencies, we get **marginal proportions**

Joint and Marginal Proportions

- Starting with the frequencies...

		Computer		
		PC	**Mac**	**Marginal**
Sex	**M**	3	1	4
	F	2	2	4
	Marginal	5	3	$n = 8$

Joint and Marginal Proportions

- Divide each frequency by n (here $n = 8$):

		Computer		
		PC	**Mac**	**Marginal**
Sex	**M**	$3/n$	$1/n$	$4/n$
	F	$2/n$	$2/n$	$4/n$
	Marginal	$5/n$	$3/n$	n/n

Joint and Marginal Proportions

- The result is a table of joint and marginal proportions.

		Computer		
		PC	**Mac**	**Marginal**
Sex	**M**	0.375	0.125	0.500
	F	0.250	0.250	0.500
	Marginal	0.625	0.375	1.000

- Notice that the joints still sum to the marginals, and all the joints, as well as any particular set of marginals, sum to 1.000.

Joint and Marginal Proportions

- If we define Prop$(\cdot)$ to be the proportion of whatever is inside, then using our previous notation:

$$\text{Prop}(\cdot) = \frac{1}{n}\text{Freq}(\cdot)$$

Joint and Marginal Proportions

- In particular:

$$\begin{aligned}
\text{Prop}(X = x, Y = y) &= \frac{1}{n}\text{Freq}(X = x, Y = y) \\
\text{Prop}(X = x) &= \frac{1}{n}\text{Freq}(X = x) \\
&= \frac{1}{n}\sum_y \text{Freq}(X = x, Y = y) \\
&= \sum_y \frac{1}{n}\text{Freq}(X = x, Y = y) \\
&= \sum_y \text{Prop}(X = x, Y = y)
\end{aligned}$$

Joint and Marginal Distributions

- If we take the set of all the joint proportions, we define a **joint distribution** over the variables.
- Similarly, each set of marginal proportions defines a **marginal distribution** for that variable.

Joint and Marginal Distributions

- Each color represents a different distribution.
  - Lavender: Joint distribution of Sex and Computer
  - Pink: Marginal distribution of Sex
  - Lime Green: Marginal distribution of Computer

		Computer		
		PC	**Mac**	**Marginal**
Sex	**M**	0.375	0.125	0.500
	F	0.250	0.250	0.500
	Marginal	0.625	0.375	1.000

- Notice that each distribution sums to 1.

Conditional Proportions

- Both joint and marginal proportions represent proportions of the *whole data set*.
- Sometimes, however, we want to ask about proportions within a *subset* of the data
  - Example: What proportion of males use a PC?
  - What proportion of Mac users are female?
- We can use these values to make interesting comparisons
  - Are women more likely than men to buy a Mac?
  - Are urban residents more likely to get lung cancer than rural or suburban residents?
  - Other examples?

Conditional Proportions

- Quantities like "what proportion *of males* use a PC?" are called **conditional proportions**: we "condition" on being male (i.e., restrict attention to males) before calculating the proportion.
- We write the above as $\text{Prop}_{Sex=M}(Computer = PC)$, which we can read as "the conditional proportion of PC users, *given* that Sex $=$ M" (later we will connect this to probability: "What's the probability someone will buy a PC, *given* that they are male?").

Conditioning on Sex

- Again, we start with the frequencies...

		Computer PC	Computer Mac	Marginal
Sex	**M**	3	1	4
	F	2	2	4
	Marginal	5	3	8

Conditioning on Sex

- But instead of dividing by n, we divide by the joints by the corresponding marginal for Sex.

		Computer PC	Computer Mac	Marginal
Sex	**M**	3/4	1/4	4/4
	F	2/4	2/4	4/4
	Marginal	5/8	3/8	8/8

- The lime green does not represent conditional proportions: it is the same marginal (unconditional) distribution from before.

Conditioning on Sex

- The resulting conditional proportions make up *two different* conditional distributions: one for each sex.

		Computer PC	Mac	Marginal
Sex	M	0.75	0.25	1.00
	F	0.50	0.50	1.00
	Marginal	0.625	0.375	1.00

- Notice that, as before, *each distribution* sums to one (but now each row is its own distribution: we consider males and females separately).

Conditioning on Computer

- Suppose instead we want to know "what proportion of Mac users are female?". Now we are conditioning on Computer: restrict attention to Mac users.
- Begin with frequencies as before, but group based on Computer:

		Computer PC	Mac	Marginal
Sex	M	3	1	4
	F	2	2	4
	Marginal	5	3	8

Conditioning on Computer

- Divide each frequency by the total *for that computer*:

		Computer PC	Mac	Marginal
Sex	M	3/5	1/3	4/8
	F	2/5	2/3	4/8
	Marginal	5/5	3/3	8/8

Conditioning on Computer

- Again we get two different conditional distributions: one for each computer.

		Computer PC	Mac	Marginal
Sex	M	0.60	0.33	0.50
	F	0.40	0.67	0.50
	Marginal	1.00	1.00	1.00

- Notice that the rows do *not* form distributions: the marginal distribution of sex is a *weighted average* of the conditional distributions (what are the weights?)

Conditional Proportions

- We calculate conditional proportions from frequencies in the obvious way: restrict attention to ("condition on") a particular category, and divide the frequency by the total in that category.
- In our notation:

$$\begin{aligned}\text{Prop}_{X=x}(Y=y) &= \frac{1}{n_{X=x}}\text{Freq}_{X=x}(Y=y) \\ &= \frac{\text{Freq}(X=x,\, Y=y)}{\text{Freq}(X=x)}\end{aligned}$$

Debt Ceiling Vote

Figure: Display of House votes by party on the recent debt-ceiling deal

Exercise

- Compute the distribution of votes, both unconditionally and conditioned on party.

		Vote			
		Yea	**Nay**	**No Vote**	**Marginal**
Party	**GOP**	171	59	10	
	Dem	68	77	48	
	Marginal				