Synoptic meteorology: Inversion technique (11-week 1)

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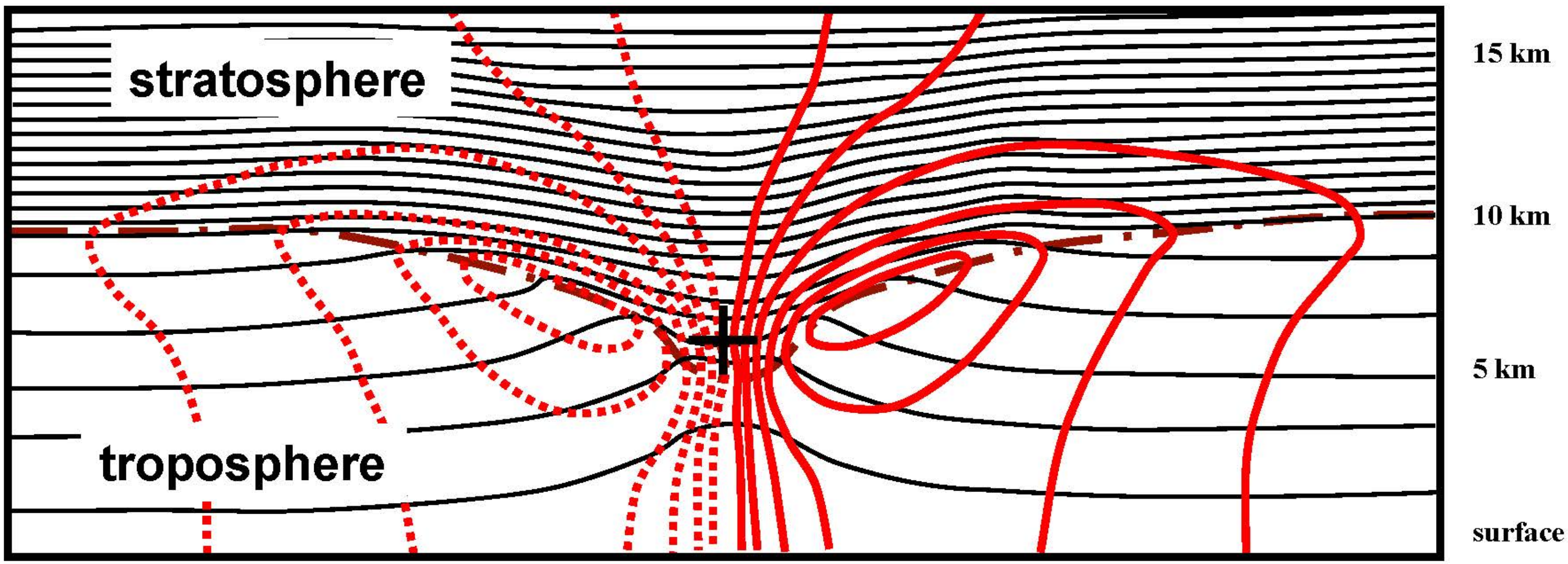
Geostrophic Vorticity Inversion

- Geostrophic relative vorticity
- $\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{1}{f_0}\nabla^2\Phi$
  - A point charge of PV (PV anomaly) is associated with a geopotential height field.
  - Flow related to PV anomaly extends throughout the domain.



Introduction to numerical methods for solving the Poisson's equation

The relationship between geostrophic vorticity ζ_g and geopotential ϕ (for a given level, evaluated at f_0) are of the form of Poisson's equation:

$$\nabla_H^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = f_0 \zeta_g(x, y) \tag{1}$$

Since this is a multivariable partial differential equation, we can't just solve this via integration.

Introduction to numerical methods for solving the Poisson's equation (Centered scheme)

Using centered difference, the second derivative of some scalar S with respect to some independent variable α can be written

$$\frac{\partial^2 S}{\partial \alpha^2} \approx \frac{S(\alpha - \Delta\alpha) - 2S(\alpha) + S(\alpha + \Delta\alpha)}{(\Delta\alpha)^2} \tag{2}$$

Using this, (1) can be rewritten

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta x)^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta y)^2} = f_0 \zeta_{i,j} \tag{3}$$

where i and j denote the x and y grid position of ϕ and ζ.

Introduction to numerical methods for solving the Poisson's equation

If we let $c = \frac{\Delta x}{\Delta y}$ after some algebra, (3) can be rewritten

$$c^{-1}\left(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}\right) + c\left(\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}\right) = (\Delta x \Delta y) f_0 \zeta_{i,j} \tag{4}$$

After bringing $\phi_{i,j}$ terms to the left hand side and simplifying,

$$\left[2(1+c^2)\right]\phi_{i,j} = \left(\phi_{i+1,j} + \phi_{i-1,j}\right) + c^2\left(\phi_{i,j+1} + \phi_{i,j-1}\right) - c(\Delta x \Delta y) f_0 \zeta_{i,j} \tag{5}$$

Introduction to numerical methods for solving the Poisson's equation

Finally, after solving for $\phi_{i,j}$, letting $\varepsilon = \left[2(1+c^2)\right]^{-1}$:

$$\phi_{i,j} = \varepsilon\left[\left(\phi_{i+1,j} + \phi_{i-1,j}\right) + c^2\left(\phi_{i,j+1} + \phi_{i,j-1}\right) - c(\Delta x \Delta y) f_0 \zeta_{i,j}\right] \tag{6}$$

Introduction to numerical methods for solving the Poisson's equation (Jacobi Iteration)

To numerically solve for $\phi_{i,j}$, we need the four points on a grid around $\phi_{i,j}$, but we don't have *any* points known for $\phi_{i,j}$. We can make some guess what the field ϕ may be, and use the fact that ϕ and ζ satisfy the Poisson equation to update our guess.

$$\phi_{i,j}^{n+1} = \varepsilon\left[\left(\phi_{i+1,j}^{n} + \phi_{i-1,j}^{n}\right) + c^2\left(\phi_{i,j+1}^{n} + \phi_{i,j-1}^{n}\right) - c(\Delta x \Delta y) f_0 \zeta_{i,j}\right] \tag{7}$$

This is the Jacobi iteration method, and we stop the iteration when ϕ^{n+1} and ϕ^{n} are no longer very different from one another.

Introduction to numerical methods for solving the Poisson's equation

(Successive Over-Relaxation; SOR)

It may be better to update $\phi_{i,j}^{n+1}$ using some combination of $\phi_{i,j}^{n}$ and the four points around $\phi_{i,j}^{n}$, rather than only using those four points. This method is called Successive Over-Relaxation (SOR). We modify (7) to include some weight, ω, to combine all five points

$$\phi_{i,j}^{n+1} = \phi_{i,j}^{n}(1-\omega) + \omega\varepsilon\Bigg[\left(\phi_{i+1,j}^{n} + \phi_{i-1,j}^{n}\right) + c^2\left(\phi_{i,j+1}^{n} + \phi_{i,j-1}^{n}\right) - c(\Delta x \Delta y) f_0 \zeta_{i,j}\Bigg] \tag{8}$$

Introduction to numerical methods for solving the Poisson's equation
(Successive Over-Relaxation; SOR)

$R^n_{i,j} = \phi^{n+1}_{i,j} - \phi^n_{i,j}$ be the residual

$$R^n_{i,j} = -\phi^n_{i,j}\omega + \omega\varepsilon\Bigg[\left(\phi^n_{i+1,j} + \phi^n_{i-1,j}\right) + c^2\left(\phi^n_{i,j+1} + \phi^n_{i,j-1}\right) - c(\Delta x \Delta y) f_0 \zeta_{i,j}\Bigg] \tag{9}$$

$$= \omega\varepsilon\Bigg[\left(\phi^n_{i+1,j} + \phi^n_{i-1,j}\right) + c^2\left(\phi^n_{i,j+1} + \phi^n_{i,j-1}\right) - \frac{1}{\varepsilon}\phi^n_{i,j} - c(\Delta x \Delta y) f_0 \zeta_{i,j}\Bigg] \tag{10}$$

Example for PV inversion

- Case study for Hurricane Katrina
  - Data: ECMWF Reanalysis interim (ERA-interim) 0.75 x 0.75 degree
  - Variable: Geopotential height field at 500 hPa at 12 UTC on 28 Aug 2005
- Options
  - Vorticity partitioned within 18N to 34N and -97E to -79E
  - Vorticity threshold set to value of vorticity at 25N and -60E
  - Reference latitude for f_0 at 30N.
  - Initial condition (IC)
    - Without Katrina: Original geopotential height
    - Only Katrina: geopotential height = 0
  - Boundary condition (BC)
    - Without Katrina: Original geopotential height along the boundary (Dirichlet BC)
    - Only Katrina: geopotential height = 0 (Dirichlet BC)
  - Options
    - Tolerance = 0.001 (0.1%) change from
    - Max number of iteration = 1000000
    - Weights for SOR method = 0.8

Original geopotential and vorticity

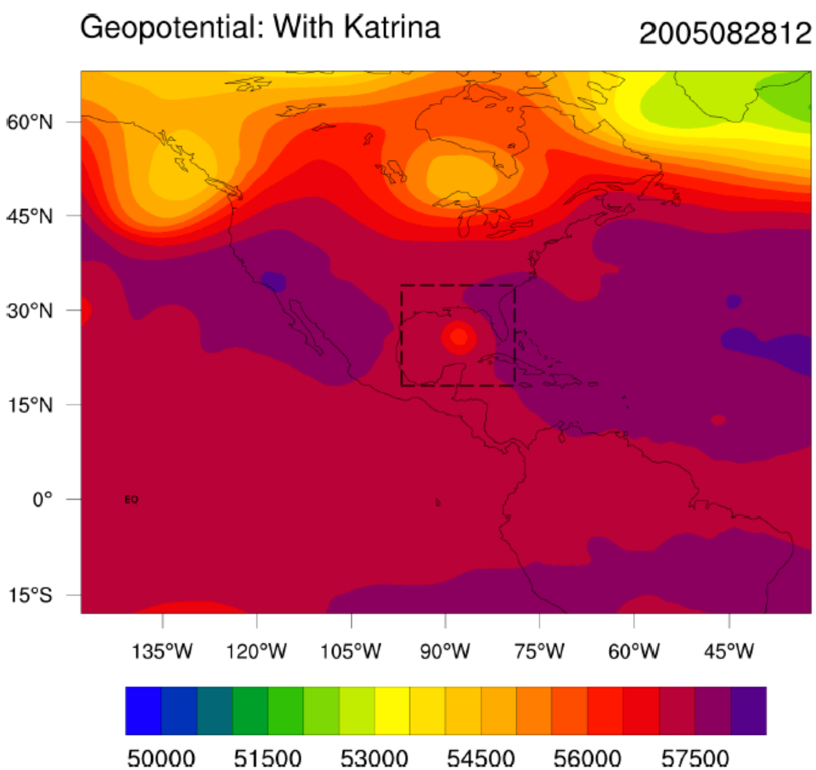


Figure 1: Geopotential with Katrina

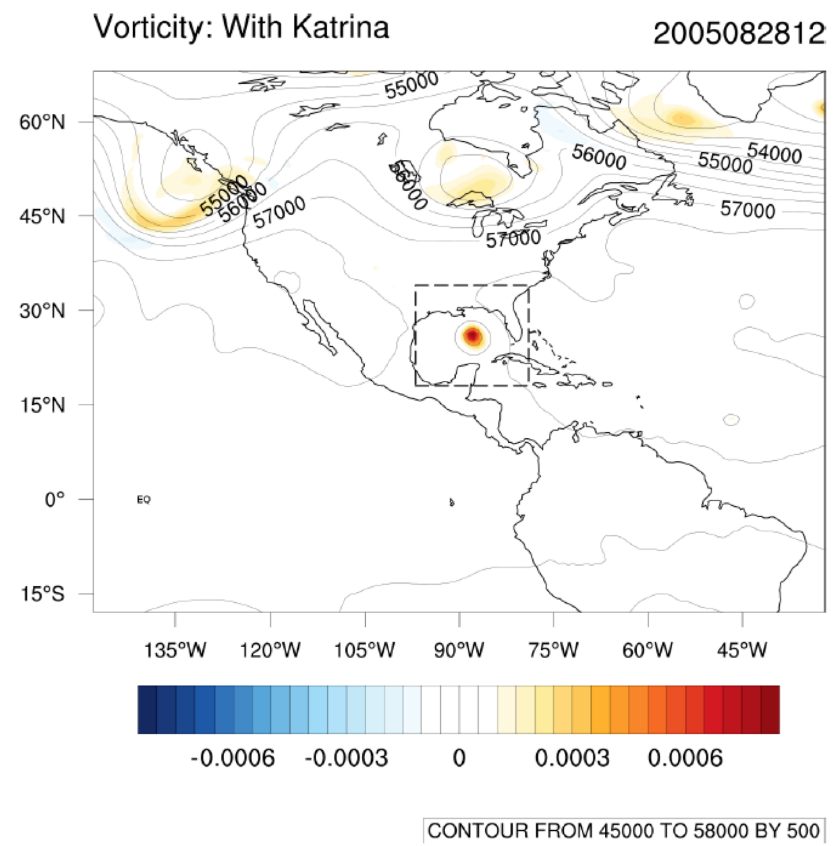


Figure 2: Vorticity with Katrina

Geopotential and vorticity without Katrina

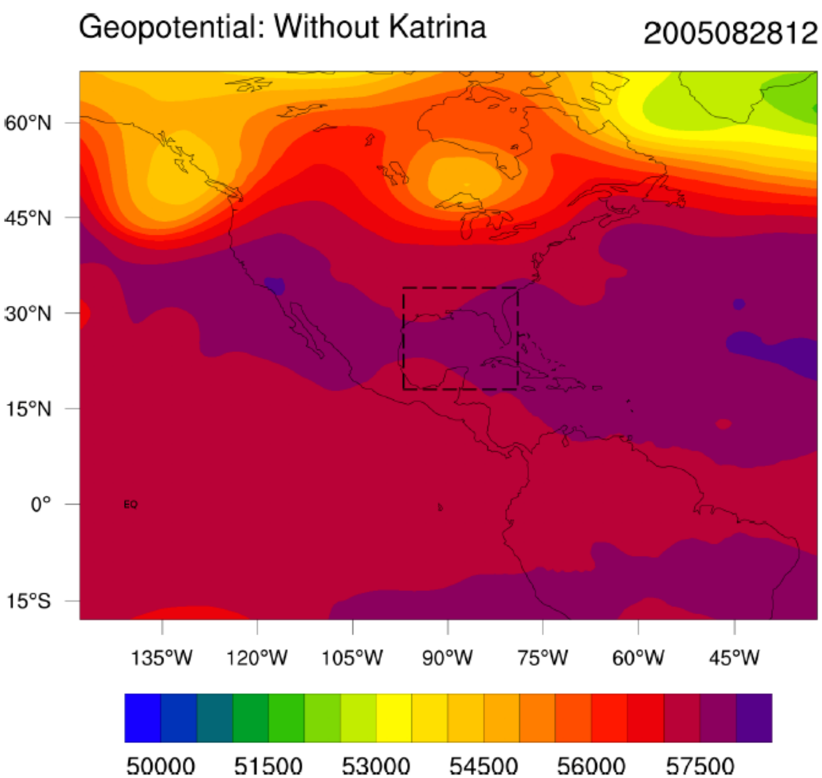


Figure 3: Geopotential without Katrina

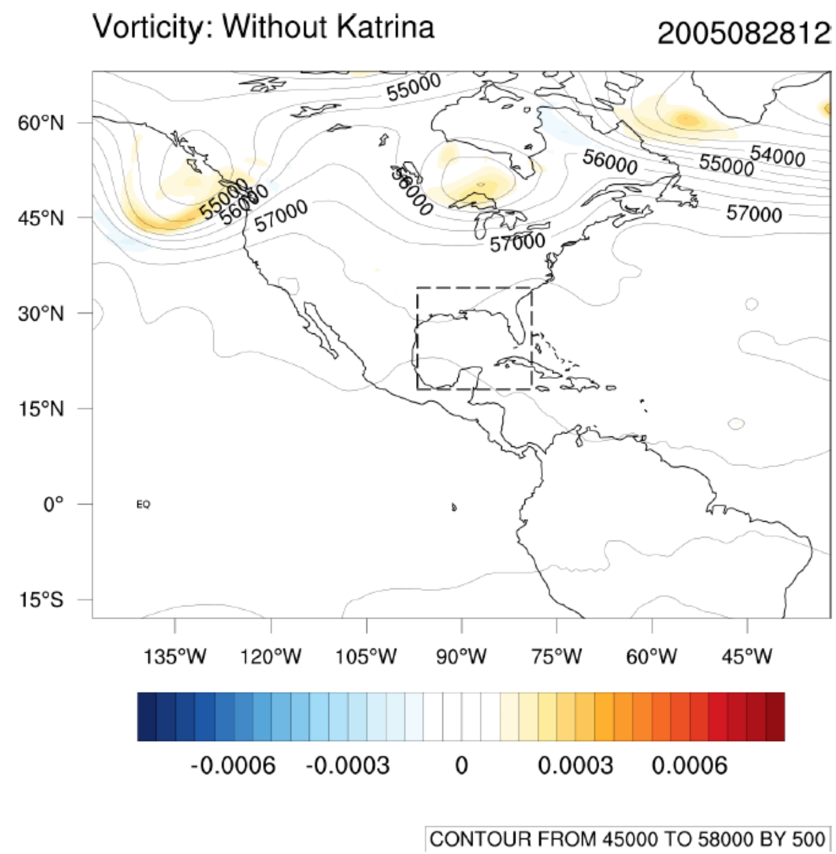


Figure 4: Vorticity without Katrina

Geopotential and vorticity only Katrina

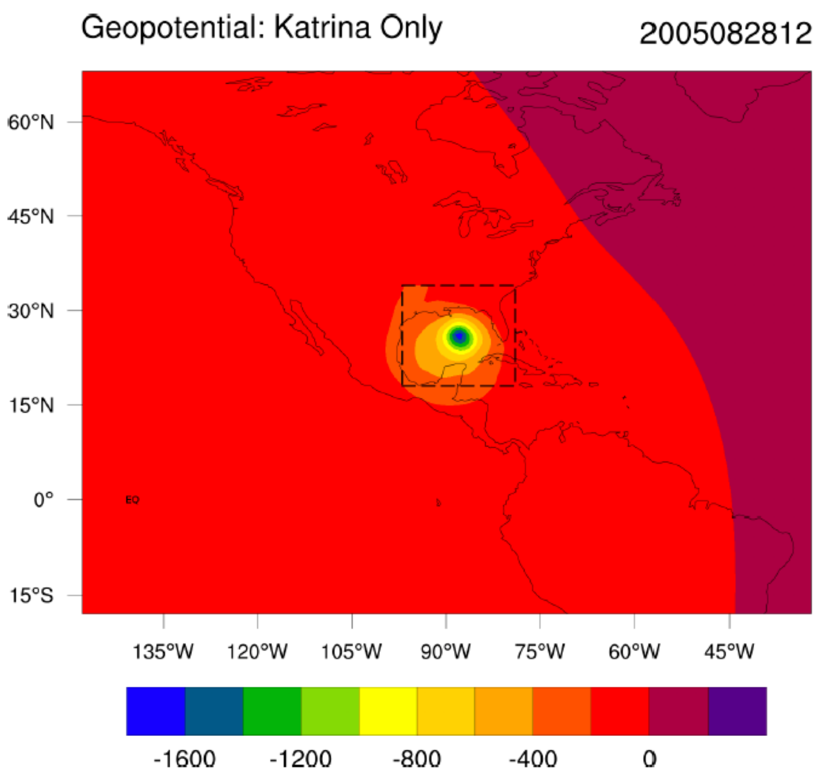


Figure 5: Geopotential with only Katrina

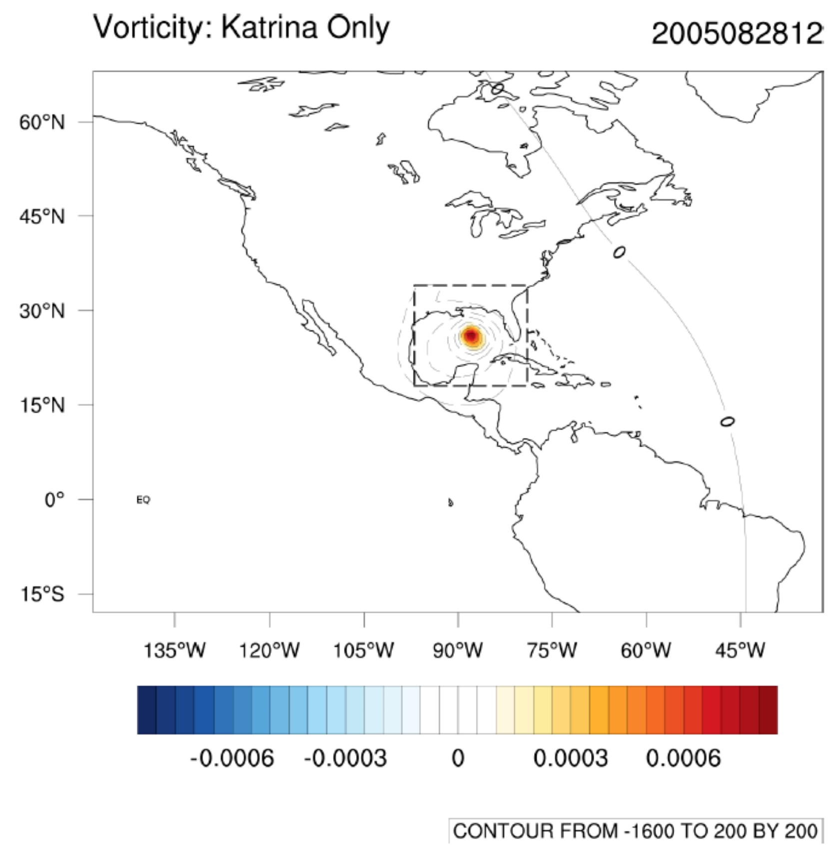


Figure 6: Vorticity with only Katrina

Instruction for the final-exam team project

- Goal of this final-exam team project
  - Gain experience with boundary conditions, iterative equation solution techniques, map scale factors, and appreciation for the concept of PV inversion
- Assignment
  - Obtain a gridded dataset (Reanalysis or operational analyses) of 500-hPa geopotential height for any Typhoon cases in East Asia for a time when the storm was located in the ocean.
  - Develop NCL, Grads, Matlab, Fortran, or other codes that accomplish the following:

    1) Read the 500-hPa geopotential height field for 1 time during the event

    2) Compute and plot the geostrophic relative vorticity on a pressure surface, superimposed with geopotential height.

    3) Partition the vorticity into various anomalies so as to isolate the vorticity associated with hurricane Katrina (or other Typhoon).

    4) Invert the geostrophic vorticity, recovering the full geopotential height field.

    5) Piecewise invert Katrina's vorticity as well as all non-Katrina vorticity in order to obtain the height field with the storm removed.

    6) Produce a plot of the height field without Katrina (or other Typhoon).

Instruction for the final-exam team project

- Assignment
  - Each step of the way, examine output and check the results.
  - Answer those questions:

    1) The initial geostrophic vorticity. Does it look the way you expected ? Discuss what you see, and explain any unusual features.

    2) The partitioned vorticity field. Has your code partitioned the vorticity field in a meaningful and defensible way ? What assumptions and choices were made, and why ?

    3) How does the code set lateral boundary conditions? What other options are available for this?

    4) Was the separation of Katrina successful ?

    5) Suppose that one wanted to use geostrophic vorticity inversion to remove Katrina (or Typhoon) from model initial condition, and then run an experiment without the storm. How could one go about doing this ?

    6) Discuss any difficulties for accomplishing this. Discuss any other ideas to apply this technique for other studies.

Instruction for final-exam team project

- 3 Teams
  - Team 1 (김정희, 박인규, 하은조, 김중진)
  - Team 2 (김범석, 유승우, 신예원)
  - Team 3 (윤현석, 김익호, 백관구)
- Final exam project
  - Consider my program as a baseline
  - Each group should build your own program for PV inversion and plots
  - Each group should submit the reports and programs
  - Group presentation on 15 June 2020