Accepted Manuscript

Title: A Multi-Objective Artificial Algae Algorithm

Authors: Ahmet Babalik, Ahmet Ozkis, S. Ali Uymaz, Mustafa Servet Kiran

PII: S1568-4946(18)30195-9
DOI: https://doi.org/10.1016/j.asoc.2018.04.009
Reference: ASOC 4813

To appear in: *Applied Soft Computing*

Received date: 22-12-2017
Revised date: 1-3-2018
Accepted date: 3-4-2018

Please cite this article as: Ahmet Babalik, Ahmet Ozkis, S.Ali Uymaz, Mustafa Servet Kiran, A Multi-Objective Artificial Algae Algorithm, Applied Soft Computing Journal https://doi.org/10.1016/j.asoc.2018.04.009

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

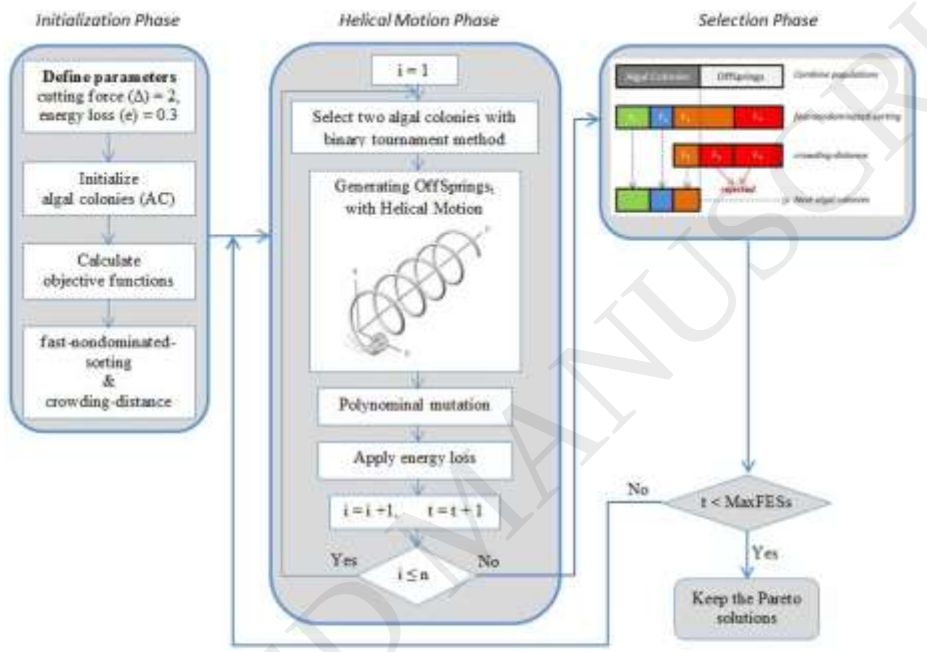
ACCEPTED MANUSCRIPT

A Multi-Objective Artificial Algae Algorithm

Ahmet Babalik, Ahmet Ozkis*, S.Ali Uymaz, Mustafa Servet Kiran

Selcuk University, Faculty of Engineering, Department of Computer Engineering, 42075 Konya, Turkey

GRAPHICAL ABSTRACT



Highlights

- Artificial Algae Algorithm (AAA) is modified for multi-objective optimization, MOAAA for short.
- The MOAAA has been tested on 36 different benchmark problems along with 4 other algorithms.
- Obtained results show that the MOAAA is highly successful and competitive algorithm.

Abstract

In this study, the authors focus on modification of the artificial algae algorithm (AAA), for multi-objective optimization. Basically, AAA is a population-based optimization algorithm inspired by the behavior of microalgae cells. In this work, a modified AAA with appropriate strategies is proposed for multi-objective Artificial Algae Algorithm (MOAAA) from the first

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

AAA that was initially presented to solve single-objective continuous optimization problems. To the best of our knowledge, the MOAAA is the first modification of the AAA for solving multi-objective problems. Performance of the proposed algorithm is examined on a benchmark set consisting of 36 different multi-objective optimization problems and compared with four different swarm intelligence or evolutionary algorithms that are well-known in literature., The MOAAA is highly successful in solving multi-objective problems, and has demonstrated an alternatively competitive algorithm according to experimental results and comparisons presented in this research topic.

Keywords: Artificial algae algorithm; multi-objective optimization; non-dominated sorting

1. Introduction

Many real-world optimization problems need making decisions that include two or more maximizing or minimizing objectives that simultaneously conflict with each-other. These problems are named as multi-objective optimization problems (MOOPs). In order to solve MOOPs, various classical methods such as linear programming, weighted sum method and goal programming method etc. have been proposed by researchers since 1950s [1]. These classical methods have some shortcomings for solving MOOPs [2]: i) getting stuck in the local optima issue ii) issues resulted from the structure of classical methods and iii) issue of high computational time. In order to overcome these issues, researchers have investigated on different fields. After the mid-1980s, researchers tend to study on metaheuristic-based approaches as they provided the following advantages: i) producing successful results independent of the nature of the problem ii) low calculation time [3]. The first Multi-Objective Evolutionary Algorithm (MOEA) called Vector Evaluated Genetic Algorithm (VEGA) was proposed by Schaffer in 1984 [4]. Goldberg in his study [5] criticized VEGA and proposed usage of non-dominated-sorting and selection methods for MOOPs. Goldberg's study encouraged researchers to modify single objective evolutionary algorithms that solve MOOPs. Some of the well-known multi-objective metaheuristic algorithms (MOMAs) are as follows: Non-dominated Sorting Genetic Algorithm II (NSGA-II) [6], Multi-Objective Cellular genetic algorithm (MOCell) [7], Indicator-based evolutionary algorithm (IBEA) [8], Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [9], Pareto Archived Evolution Strategy (PAES) [10], Strength–Pareto Evolutionary Algorithm 2 (SPEA2) [11], Generalized Differential Evolution 3 (GDE3) [12], Multi-Objective Particle Swarm Optimization (MOPSO) [13], Speed-constrained Multi-objective PSO (SMPSO) [14], Non-dominated Sorting Particle Swarm Optimizer [15] (NSPSO), Multi-Objective Artificial

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Bee Colony (MOABC) [16], Improved Bee Colony Algorithm for Multi-Objective Optimization [17], Bacterial Chemotaxis Multiobjective Optimization Algorithm (BCMOA) [1], Multi-Objective Vortex Search (MOVS) Algorithm [18], Multi-objective Bacteria Foraging Optimization Algorithm (MOBFOA) [19], Multi-objective Tabu Search Algorithm based on Decomposition (DMTS) [20], Adaptive Immune-Inspired Multi-objective Algorithm (AIMA) [21], Cellular Teaching-learning-based Optimization (CTLBO), Vector Evaluated TLBO (VECTLBO) [22] and Hybrid Multi-objective Firefly Algorithm (HMOFA) [23].

The proposed MOMAs are successfully used in solving real-world problems in many disciplines such as: big data optimization [23, 24], multi-objective scheduling [19, 25, 26], optimization of fuzzy models [27-29], optimization of cooling galleries [30-32], smart grid applications [33], optimization of Gene-pool Optimal Mixing [34], automatic melody generation [35, 36], engineering design problems [37, 38] etc.

Basically, there are three types of approaches to solve multi-objective problems: (1) Pareto-based approaches, (2) decomposition based approaches, (3) indicator based approaches. In this study, a modification of the AAA [39] ─ a single-objective, bio-inspired algorithm proposed by Uymaz et al. in 2015 ─ by using Pareto-based approaches and a novel algorithm called multi-objective Artificial Algae Algorithm (MOAAA) is proposed.

The rest of the study is organized as follows: The main contribution of the study and the main motivation of the study are explained in section 1.1 and 1.2 respectively. In Section 2, multi-objective optimization problems (MOOPs) are explained. In Section 3, basic steps of the original AAA and the proposed MOAAA are presented. In Section 4, the experimental comparisons and results of the MOAAA and competitor algorithms are presented and analyzed. In Section 5, conclusions of the study and suggestion for further work are presented.

1.1 The Main Contribution of the Study

This paper presents a modified version of the AAA for solving multi-objective optimization problems. The performance of the proposed algorithm has been investigated on a large set of multi-objective optimization benchmarks based on indicators named hypervolume (HV), SPREAD, EPSILON and inverted generational distance (IGD). The main contribution of the study is that this work is the first modification of the AAA for MOOPs by using fast-nondominated-sorting, crowding-distance and polynominal mutation strategies therefore providing enough diversity and convergence. Consequently, a novel multi-objective optimization algorithm has been developed with better results compared to other algorithms on almost all cases based on the used metrics.

ACCEPTED MANUSCRIPT

1.2 The Motivation of the Study

A literature review analysis shows that the AAA is a successful and competitive algorithm on single-objective optimization problems (SOOPs). Solving of MOOPs is harder than solving of SOOPs due to number of objective functions and trade-off among these functions. Conducting comparisons between two solutions is not easy if the first one is better than the other one on the first function and at the same time the solution of the second one is better than the first one on the second function. Consequently, we need effective and successful optimizers for multi-objective optimization problems. This study handles modification of single objective AAA for multi-objective optimization.

2. Multi-objective Optimization Problems (MOOPs)

A multi-objective optimization problem must have at least two objectives and some restriction functions. It is aimed at finding mutually agreeable solutions that satisfy objectives and restrictions functions in a solution space. A MOOP is defined mathematically as follows [1]:

$$\begin{aligned} &Maximize/minimize: && F_m(X) \quad m = 1,2,\dots,M \\ &Subject\ to: && g_j(X) \geq 0,\ j = 1,2,\dots,J \\ & && h_k(X) = 0,\ k = 1,2,\dots,K \\ & && X_i^L \leq X_i \leq X_i^U,\ i = 1,2,\dots,n \end{aligned} \tag{1}$$

Where X is the decision variables vector, $F_m(X)$ states the m. objective function to be minimized or maximized. The values of X_i^L and X_i^U are the lower and the upper bounds for the decision variable X_i, J is the number of inequality constraints, K is the number of equality constraints, g_j is the *jth* inequality and h_k is the *kth* equality functions.

In MOOPs, there may be many alternative solutions that solve a problem, and a comparison of these solutions with each other is very important. One way to compare two solutions is by using the concept of Pareto dominance [40]. The mathematical model of the Pareto concept is given below [18, 41, 42]:

Let's assume that X and Z are candidate solutions for an optimization problem, specifically minimization, and Ω be the set of all the solutions that correspond to constraints of the problem. The Pareto dominance rules are given as follows:

Definition1. Pareto dominance: $X, Z \in \Omega$

$$\text{If} \begin{array}{ccc} \forall i & \in & \{1,2,\dots,m\} f_i(X) \leq f_i(Z) \\ \wedge \exists j & \in & \{1,2,\dots,m\} f_j(X) < f_j(Z) \end{array}, \quad X \succ Z \tag{2}$$

ACCEPTED MANUSCRIPT

If the rules given in Eq.2 are provided by X, then X is said to be a dominant solution. This means that X is not worse than Z for any objective but better than Z for at least one objective. This expression is given as $X \succ Z$.

Definition2. Pareto optimal:

If $\neg\exists \mathrm{Z} \in \Omega\colon \mathrm{Z} \succ \mathrm{X}$, X is a Pareto optimal solution.

Definition3. Pareto optimal set (PS):

$$\mathrm{PS} = \{\mathrm{X} \in \Omega \mid \neg\exists \mathrm{Z} \in \Omega\colon \mathrm{Z} \succ \mathrm{X}\} \tag{3}$$

Definition4. Pareto optimal Front (PF): The set of all the Pareto optimal objective vectors.

$$PF = \{F(X) \mid X \in PS\} \tag{4}$$

3. The Main Work

In Section 3, basic steps of the original AAA, and the proposed MOAAA algorithm are explained in detail.

3.1 Artificial Algae Algorithm (AAA)

The artificial algae algorithm [39] proposed by Uymaz et al. in 2015, was inspired by the characteristic and the living behavior of motile microalgae. The AAA consists of 3 basic processes: the helical movement process, evolutionary process and adaptation process. The helical movement is the motion of the algae in the fluid to get closer to light. The evolutionary process involves the reproduction of algae by mitosis while the adaptation process involves the algae adapting to their surroundings. In this algorithm, the algae are the main species and the whole population consists of algal colonies (an algal colony is a group of algae living together as shown in Eq.6) represented in Eq.5.

$$Population = \begin{bmatrix} x_1^1 & \cdots & x_1^D \\ \vdots & \ddots & \vdots \\ x_N^1 & \cdots & x_N^D \end{bmatrix} \tag{5}$$

$$x_i = [\, x_i^1 \,,\, x_i^2 \ldots\, x_i^D \,] \quad i = 1, 2, \ldots, N \tag{6}$$

Here, x_i^j represents *jth* cell of *ith* algal colony, *N* is the number of algal colony and *D* is the number of decision variables. The number of algae cells in each algal colony is therefore equal to the size of the problem. Each x_i represents a suitable solution in the solution

ACCEPTED MANUSCRIPT

space. It is assumed that all algae cells in the algae column move together to a suitable location in the solution space. An algae column is optimized when it reaches the ideal solution.

In the evolutionary process, when the artificial algae cell receives enough light, it grows into two artificial algae cells in a way comparable to real mitosis. On the contrary, the artificial algae cells that do not get enough light exposure survive for some time, but eventually die. In the adaptation process the algal colony, which cannot grow sufficiently in its environment, tries to adapt to a new environment to survive and eventually becomes the dominant species in the environment. They swim in the fluid by means of their flagella during the helical movement process because they need the light to survive (Fig. 1). Pseudo code of the AAA was given below in Fig. 2.

3.2 Multi-objective Artificial Algae Algorithm (MOAAA)

NSGA-II [6] is a well-known multi-objective algorithm in the evolutionary optimization field used to solve the multi-objective optimization problems in different research field [43, 44]. In NSGA-II, two basic strategies ─ fast-non-dominated-sorting and crowding-distance calculation ─ are used to transform the single-objective Genetic Algorithm. These strategies are still successfully used to obtain results in many recent works such as [3, 45]. With the motivation we have taken from these studies, we proposed MOAAA by applying fast-non-dominated-sorting and crowding-distance calculation to the AAA algorithm. Another important aspect of this work is that, as far as we know, this is the first study to convert the AAA to a multi-objective algorithm.

The algal colony population consisting of N agents is considered as the main population. From the main population, N offspring agents are created by using parent agents obtained by a tournament selection method. The tournament selection is made according to the quality ranking (QR) value and calculation of the QR value is given in section 3.2.2.

The agents of main and offspring populations are combined (2N) and the best N agents are selected by using fast-non-dominated-sorting and crowding distance calculation strategies. These agents form the main population of the next generation and these operations continue until the last generation. When last generation is reached, the first front which contains the non-dominated solutions composes the Pareto front (PF). In order to improve the MOAAA, modifications are made to the AAA algorithm and expressed using the pseudo code given in Fig.5.

ACCEPTED MANUSCRIPT

3.2.1. The NSGA-II-based Strategies

The core of NSGA-II-based approaches are fast-nondominated-sorting and crowding distance calculation strategies that are briefly given below. The detailed information on these strategies can be found in [6].

(i) Fast-nondominated-sorting

Each solution is compared with the other members of the population using Pareto domination rules. The nondominated solutions form the first nondominated front. Afterwards, nondominated solutions of the remaining population is selected to determine the second front. Similarly, all the remaining solutions are positioned at the front in compliance with their nondomination degree.

(ii) Crowding distance Calculation

This strategy refers to the density of the objective space for each solution in the population. The crowding-distance value is calculated with the aid of the nearest neighborhood of the solution and extreme solutions in its rank. The extreme solutions of each objective function are set to an infinite distance value since these extreme solutions need to be protected within the population. A large distance value denotes that this region needs to be searched in detail while a small distance value denotes that the region has been sufficiently searched.

3.2.2. Quality Ranking (QR) Value

Greatness value of an algal colony indicates the quality of a solution. In the original AAA, greatness was evaluated by using the objective function value and previous greatness value of the solution. In MOOPs, there are at least two objective functions hence there is no scalar objective value to calculate greatness of the solution. To overcome this problem, we proposed a quality ranking (QR) value for each algal colony by using the fast-nondominated-sorting and crowding-distance approaches at each front. In the first front, solutions are ranked inversely according to their crowding-distance values and these QR values increase at each successive front. At the start, QR = 1 for extreme solutions whose crowding-distance values are infinitive in the first front and the remaining solutions take incremental QR values. In the second front, extreme solutions take 1 more QR values from the maximum QR value in the first front. In this way, each solution takes a QR value where small QR values indicate more quality solutions as shown in Fig. 3.

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

3.2.3 Polynomial Mutation

In this study, the evolutionary process (EP) and adaptation process (AP) strategies have not been used in multi-objective version because these two strategies decrease diversity and convergence on the Pareto front. Instead, polynomial mutation (PM) [46] is used to provide enough diversity and convergence in the population. The pseudo code of the PM is given below.

```
Polynominal mutation
p: is an algal colony,  p(i): value of the ith algae cell,
D: the number of algae cell,
LB(i) & UB(i): lower and upper bound of the ith algae cell respectively
prob  = 1/D,  eta_m = 20, Distribution Index (DI) = 20

while  i = 1 to D
     if rand <= prob
            y = p(i)
            yl = LB(i)
            yu = UB(i)
            delta1 = (y - yl) /(yu - yl)
            delta2 = (yu - y) /(yu - yl)
            rnd = random(0,1)
            mut_pow = 1/ (eta_m + 1)

            if(rnd <= 0.5)
                   xy = 1 – delta1
                   val = 2*rnd + (1 – 2*rnd)*(xy^(DI+1))
                   deltaq = (val^mut_pow )- 1
            else
                   xy = 1 – delta2
                   val = 2*(1 - rnd) + 2*(rnd-0.5)*(xy^(DI+1))
                   deltaq = 1 – (val^mut_pow)
            end if

            y = y + deltaq * (yu - yl)
            y = max( min(y, UB(i)), LB(i))
            p(i) = y
     end if
end while
```

In Fig.4, the effect of PM is visually demonstrated by solving ZDT4 and DTLZ1 problems with two versions of the MOAAA. The Pareto fronts given in Fig. 4 show that using PM instead of EP and AP improves convergence and diversity performances of the MOAAA.

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

4. Experiments

In this section, the experimental materials are introduced, some properties on them given and the parameters of the algorithms defined before the experimental results and comparisons are presented.

4.1 Benchmark Problems

In this paper, the performance of the algorithms has been tested by a wide set consisting of 36 multi-objective benchmark problems. These are (1) ZDT [41], (2) WFG [47], (3) DTLZ [48], (4) LZ09 [49] problem families; OKA1 – OKA2 [50], Viennet2 – Viennet3 [51], Schaffer [4], Fonseca [52], Kursawe [53] and Poloni problems. In this set, 27 problems have 2 objective functions while the remaining 9 problems (DTLZs, LZ09_F6, Viennet2 and Viennet3) have 3 objective functions. In order to investigate the performance of the algorithms from different perspectives, the benchmark suite is selected from the problems with different characteristics. Further information about the characteristics of the most of the used problems can be found [54-56].

[please insert Table 1 about here]

In Table 1, DV and OBJ stand for "decision variables" and "objective functions" respectively.

4.2 The Performance Metrics

*P** is an array of objective function vectors distributed uniformly throughout the true Pareto front (PF_t) on the objective space and *A* is an array of objective function vectors that forecasts the PF_t. The success of the solutions that belong to A is calculated by using the performance metrics (indicators) explained below.

4.2.1 Hypervolume (HV) Metric

HV [57, 58] metric is used to calculate the volume of the objective space and is covered by solutions belonging to set *A*. Mathematically, for each $i \in A$ solution, a v_i hypercube is formed with *W as* the reference point where the *i* solution is the diagonal of the hypercube. This reference point is gained by forming a vector from the worst values of the objective functions. The union of all the hypercubes is obtained and their HV values are calculated as shown in Eq. (7) [18].

$$HV = volume\left(\bigcup_{i=1}^{|A|} v_i\right). \tag{7}$$

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

HV gives us knowledge of the convergence and diversity successes of the A array where large HV value indicates that the algorithm is performing better.

4.2.2 SPREAD Metric

SPREAD signifies the size of spread of the estimated A array and is shown as Δ. In [57], two different *SPREAD* indicators are given:

(i) The first one can be used for problems that have two-objectives,

(ii) The second one (Generalized *SPREAD*) is suitable for all multi-objective problems regardless of the number of objective functions.

Mathematical formulation of the Generalized *SPREAD* metric is given in Eq.8.

$$\Delta = \frac{\sum_{i=1}^{m} d(e_i, A) + \sum_{X\in A} |d(X,A) - \overline{d}|}{\sum_{i=1}^{m} d(e_i, A) + |A| \cdot \overline{d}}, \tag{8}$$

Here, $e_1, \ldots e_m$ express the m extreme points in the array P^*, $d(e_i, A)$ expresses the minimum Euclidean distance between array A and e_i while m expresses number of the objective function. The other formula used in Eq.8 are given in Eq.9.

$$d(X, A) = \min_{Y\in A, Y\neq X} \parallel X - Y \parallel^2,$$
$$\overline{d} = \frac{1}{|A|} \sum_{X\in A} d(X, A). \tag{9}$$

4.2.3 EPSILON Metric

Let A be the estimated solution array for a problem, *EPSILON* is the measure of the minimum distance necessary for converting every solution in *A* so as to be able to dominate the PF_t of the problem [57].

$$E(A, P^*) = \inf_{\epsilon\in\mathbb{R}^+}\{\forall\vec{p} \in P^*, \exists\vec{a} \in A: \vec{a} \prec_\epsilon \vec{p}\}, \tag{10}$$

Here, $\vec{a} = (a_1, \ldots, a_m)$, $\vec{p} = (p_1, \ldots, p_m)$, m expresses the number of the objective functions and ∈ expresses a small positive number.

$\vec{a} \prec_\epsilon \vec{p}$ if and only if $\forall 1 \leq i \leq m: a_i < \epsilon + p_i$.

4.2.4 Inverted Generational Distance (IGD) Metric

IGD [59, 60] is utilized to measure the average from P^* to *A*.

ACCEPTED MANUSCRIPT

$$IGD(A, P^*) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|}, \tag{11}$$

Here, $d(v,A)$ is the minimum Euclidean distance between the set A and v. If $|P^*|$ has reached a number that well represents PF, diversity and convergence performance of array A can be measured by using $IGD(A, P^*)$. Decreasing of the metric value indicates that the array A has got closer to the PF_t.

4.3 Experimental Setting

MOAAA has been compared with different multi-objective algorithms known as NSGA-II, MOCell, IBEA and MOEA/D on the performance of *HV*, *SPREAD*, *EPSILON* and *IGD* metrics. In the experiments, the compared algorithms were run from a software package developed for multi-objective optimization known as jMetal 4.5 [57, 61]. The authors have coded the proposed MOAAA compatible with the jMetal 4.5. In jMetal 4.5, default values of population size and the maximum function evaluation number (maxFES) are set to 100 and 25,000 respectively. In accordance with this tool, population size of all the algorithms is set to 100, the maxFES to 25,000 and each algorithm has been run with the default values defined in the *jmetal.experiments.settings* package of the jMetal 4.5 [62].

In single-objective AAA, *cutting force* (Δ) and *energy loss* (e) parameters are set as 2 and 0.3 respectively. In this study, we have tested both parameters for multi-objective version of the AAA by considering the values of performance metrics. The tested versions of MOAAA for both parameters are given in Table 2 and Table 3 under the name "Exp". First, the *cutting force* parameter was tested for values from 1 to 5, keeping the parameter *energy loss* constant at 0.3. The performance metrics have been calculated using the Pareto optimal solutions obtained for each value of *cutting force* in the MOAAA. Results of the Friedman test [63] – gives the average rankings of success achieved by the algorithms – are given instead of all results, due to page limitation. In Table 2, it is seen that MOAAA has the best average rankings value when the *cutting force* parameter is set to 2.

Secondly, *energy loss* parameter was tested for values from 0.1 to 1, keeping the parameter *cutting force* constant at 2. Table 3 gives the Friedman test results obtained for each *energy loss* parameter. It is seen that MOAAA has the best average rankings value in three out of four performance metrics when the *energy loss* parameter is set to 0.3. As a result, the *cutting force* and *energy loss* parameters in the MOAAA are set to 2 and 0.3 respectively.

4.4 Experimental Results

ACCEPTED MANUSCRIPT

All the algorithms have been run 50 times with random seeds for the each problem and the obtained *HV*, *SPREAD*, *EPSILON* and *IGD* results are given in Tables 4, 5, 6 and 7. For clarity, the best and the 2nd-best metric values were shown in dark grey and light grey respectively. For a more quality Pareto front, *HV* metric must have a bigger value, while the other metrics must have a smaller one.

[please insert Table 4 about here]

In Table 4, when the *HV* metric values are examined, it is shown that the MOAAA and MOCell have become the most competitive algorithms, as they have the best or the 2nd-best value in 20 (8+12) and in 16 (12+4) problems out of 36, respectively. NSGA-II in 14 problems (6+8), MOEA/D in 14 (7+7) and IBEA in 8 (3+5) obtained the best or the 2nd-best values. When we examine the results according to the number of objectives, MOAAA has been observed to be the first or second most successful algorithm in 12 of the 27 two objective problems and 8 of the 9 three objective problems. In conclusion, if the number of objective function is increased, the algorithm shows better performance than the performance of compared algorithms.

[please insert Table 5 about here]

It is clearly visible that MOCell is by far the most competitive algorithm when the *SPREAD* metric values are examined in Table 5 by obtaining 27 (24+3) of the best or the 2nd-best values. The proposed MOAAA has the best or the 2nd-best values in 21 (6+15) problems, NSGA-II in 9 (0+9), MOEA/D in 8 (3+5) and IBEA in 7 (3+4). Eventually NSGA-II, IBEA and MOEA/D have displayed a worse performance than MOCell and MOAAA using the SPREAD metric. MOAAA has been the first or second most successful algorithm in 15 of the 27 two objective problems and 6 of the 9 three objective problems when we examine the results according to the number of objectives.

[please insert Table 6 about here]

The *EPSILON* metric indicates that the MOAAA is the most competitive algorithm with the best or the 2nd-best value in 22 (10+12) problems as observed from the Table 6.

NSGA-II is the 2nd-best algorithm with the best or the 2nd-best values in 16 (8+8) problems. MOCell has the best or the 2nd-best values in 15 (10+5) problems, while MOEA/D in 12 (5+7) problems has the best or the 2nd-best values. IBEA has the best or the 2nd-best values in 7 (3+4) problems, hence performance of IBEA has become weaker than the other algorithms. MOAAA has been the first or second most successful algorithm in 14 of the 27

ACCEPTED MANUSCRIPT

two objective problems and 8 of the 9 three objective problems when we examine the results according to the number of objectives.

[please insert Table 7 about here]

In Table 7, when we examine the *IGD* metric values, it is shown that the MOAAA has become the most competitive algorithm with the best or the 2nd-best values in 21 (10+11) problems. NSGA-II and MOCell was closest to the MOAAA with the best and 2nd-best values in 17 problems. While MOEA/D has the best or the 2nd-best values in 13 (6+7) problems, IBEA gained the best or the 2nd-best values in 4 (0+4) problems. MOAAA has been the first or second most successful algorithm in 14 of the 27 two objective problems and 7 of the 9 three objective problems when we examine the results according to the number of objectives.

[please insert Table 8 about here]

Table 8 gives the average rankings calculated by applying the Friedman test [63] to the results of all algorithms. From the Friedman test, it is clearly seen that the proposed MOAAA is the most competitive algorithm. The MOAAA obtains the best ranking values with the HV, EPSILON and IGD metrics whereas MOCell is the most successful algorithm in accordance with the SPREAD metric. NSGA-II is the second most successful algorithm by gaining the 2nd-best ranking values with the HV, EPSILON and IGD metrics.

Wilcoxon's rank sum test [64], statistical test for non-parametric independent samples, is applied at the 5% significance level to decide if the results gained with the MOAAA is different from the final results of the other algorithms in a statistically significant way. Obtained p-values through the rank sum test over metrics of all the competitors are given in Tables 9 and 10. If the p-values are less than 0.05 (5% significance level) and marked with "+". This situation is robust evidence that the results of the MOAAA are statistically significant and have not gained incidentally.

[please insert Table 9 about here]

MOAAA has produced statistically different results from NSGA-II, MOCell, IBEA and MOEA/D in 26, 30, 32 and 31 problems, respectively, out of 36, when gaining the p-values for HV metric in Table 9. Results of the MOAAA are different from results of the NSGA-II, MOCell, IBEA and MOEA/D in 30, 29, 32 and 35 problems, respectively, out of 36 when the p-values of the SPREAD metric are examined.

[please insert Table 10 about here]

ACCEPTED MANUSCRIPT

Results of the MOAAA are statistically different from the results of the NSGA-II, MOCell, IBEA and MOEA/D in 23, 31, 28 and 33 problems, respectively, out of 36 when gaining the p-values for EPSILON metric in Table 10. The MOAAA has produced statistically different results from NSGA-II, MOCell, IBEA and MOEA/D in 28, 34, 33 and 32 problems, respectively, out of 36 when the p-values of the IGD metric are examined. The results in Tables 9 and 10 mean that the MOAAA is clearly different from the competitor algorithms.

To observe convergence and diversity performances of the algorithms, the true Pareto fronts (PF_t) of some problems have been drawn in Figs. 6, 7 and 8. PF_t has been indicated using a light blue color. Box plot drawings that were formed after 50 runs for each metric calculated from the obtained results of the problems have also been shown in Figs. 9 and 10. We can see that the MOAAA has produced better and robust results than the compared algorithms in many cases when we analyze Pareto fronts and the box plot drawings.

[please insert Fig. 6. about here]

[please insert Fig. 7. about here]

[please insert Fig. 8. about here]

[please insert Fig. 9. about here]

[please insert Fig. 10. about here]

4.5 Experimental Environment and Computation Time

All experiments in this study were conducted using a computer with Windows 7 64 bit operating system, Intel Core i7-3930K 3.20 GHz processor and 16 GB Ram. The average computational times of the algorithms are given in Table 11. It is observed that the MOAAA is the 2nd best algorithm in terms of computation time together with NSGA-II.

[please insert Table 11 about here]

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

5. Conclusion and Future Works

In this study, Artificial Algae Algorithm (AAA) which is a single-objective, bio-inspired algorithm modified and multi-objective Artificial Algae Algorithm (MOAAA) is proposed. To the best of our knowledge, MOAAA is the first algorithm that modifies AAA to solve multi-objective problems. For MOAAA, the basic AAA has been integrated with fast-nondominated-sorting and crowding-distance calculation approaches in order to compare the obtained solutions and solve multi-objective optimization problems. Additionally, the polynomial mutation strategy has been applied to each offspring algal colony after helical movement process. The evolutionary process and adaptation process strategies have not been used in multi-objective version because these strategies decrease convergence and diversity and prevent the collection of solutions on the true Pareto front. The performance of the proposed MOAAA was investigated on 36 different benchmark problems, and compared with the performance of NSGA-II, MOCell, IBEA, MOEA/D algorithms in terms of *HV*, *SPREAD*, *EPSİLON* and *IGD* performance metrics. The experimental results and statistical analyses on these results show that MOAAA achieves more outstanding performance than the compared algorithms on many benchmark problems, and therefore, it can be said that MOAAA is an alternative and competitive algorithm in the multi-objective optimization field. Even though the MOAAA has been quite successful, the algorithm could still be improved for the diversity (*SPREAD*) metric. Of the 36 test problems used in this study, 27 have 2 objective functions and 9 have 3 objective functions. MOAAA is the first or second best algorithm with 50.9% (55/108) for 2-objective problems and 80.5% (29/36) for 3-objective problems when the obtained performance metric values are considered. According to the problems used in this study, the proposed MOAAA presents more successful results in solving 3-objective problems. Additionally, the MOAAA has achieved a general success in solving the problems of different characteristics. The proposed MOAAA uses the fast-nondominated-sorting and the crowding-distance strategies similar to NSGA-II algorithm. It will be more explanatory to evaluate performance metrics instead of problem characteristics if the performance of the NSGA-II and the MOAAA are compared separately. According to the Friedman test results, MOAAA obtained better results than the NSGA-II algorithm for all the performance metrics. This indicates that MOAAA has achieved better performance in terms of convergence and distribution for the problems used in this study.

In future works, the performance of the proposed MOAAA will be investigated on different types of multi-objective optimization problems including constrained, dynamic and discrete. Additionally, hybridization alternatives for MOAAA will be studied to improve the

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

performance on *SPREAD* metrics. Finally, it is said that the MOAAA can be applied to real-world problems such as antenna design, aerodynamic design, interactive aircraft design, optimal power flow, synthesis gas, industrial neural network design and others as a future direction for the researchers and practitioners.

Acknowledgment

The authors of this study would like to thank the Scientific Research Projects Coordinatorship at Selcuk University and The Scientific and Technological Research Research Council of Turkey for their institutional support. The second author also wishes to thank the Selcuk University Academic Staff Training Program Coordination Unit (OYP Program, 2014-OYP-059) for their institutional support.

References

[1] M. Alejandra Guzmán, A. Delgado, J. De Carvalho, A novel multiobjective optimization algorithm based on bacterial chemotaxis, Engineering Applications of Artificial Intelligence, 23 (2010) 292-301.
[2] C.-L. Yu, Y. Lu, J. Chu, Multi-objective optimization with combination of particle swarm and extremal optimization for constrained engineering design, WSEAS Trans Syst Control, 4 (2012) 129-138.
[3] I.-D. Psychas, M. Marinaki, Y. Marinakis, A. Migdalas, Non-dominated sorting differential evolution algorithm for the minimization of route based fuel consumption multiobjective vehicle routing problems, Energy Systems, (2016).
[4] J.D. Schaffer, Multiple Objective Optimization with Vector Evaluated Genetic Algorithms, in: Proceedings of the 1st International Conference on Genetic Algorithms, L. Erlbaum Associates Inc., 1985, pp. 93-100.
[5] D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley Longman Publishing Co., Inc., 1989.
[6] K. Deb, A. Pratap, S. Agarwal, T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II, Ieee Transactions on Evolutionary Computation, 6 (2002) 182-197.
[7] A.J. Nebro, J.J. Durillo, F. Luna, B. Dorronsoro, E. Alba, MOCell: A cellular genetic algorithm for multiobjective optimization, International Journal of Intelligent Systems, 24 (2009) 726-746.
[8] E. Zitzler, S. Kunzli, Indicator-based selection in multiobjective search, Lect Notes Comput Sc, 3242 (2004) 832-842.
[9] Q.F. Zhang, H. Li, MOEA/D: A multiobjective evolutionary algorithm based on decomposition, Ieee T Evolut Comput, 11 (2007) 712-731.
[10] J. Knowles, D. Corne, The Pareto archived evolution strategy: a new baseline algorithm for Pareto multiobjective optimisation, in: Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on, 1999, pp. 1-105 Vol. 101.
[11] E. Zitzler, M. Laumanns, L. Thiele, SPEA2: Improving the Strength Pareto Evolutionary Algorithm, in, ETH Zurich, 2001, pp. 21.
[12] S. Kukkonen, J. Lampinen, Performance assessment of generalized differential evolution 3 with a given set of constrained multi-objective test problems, in: Proceedings of the Eleventh conference on Congress on Evolutionary Computation, IEEE Press, Trondheim, Norway, 2009, pp. 1943-1950.
[13] C.A.C. Coello, G.T. Pulido, M.S. Lechuga, Handling multiple objectives with particle swarm optimization, Ieee T Evolut Comput, 8 (2004) 256-279.

ACCEPTED MANUSCRIPT

[14] A.J. Nebro, J.J. Durillo, J. Garcia-Nieto, C.A.C. Coello, F. Luna, E. Alba, SMPSO: A new PSO-based metaheuristic for multi-objective optimization, in: Computational intelligence in miulti-criteria decision-making, 2009. mcdm '09. ieee symposium on, 2009, pp. 66-73.
[15] X. Li, A non-dominated sorting particle swarm optimizer for multiobjective optimization, in: Proceedings of the 2003 international conference on Genetic and evolutionary computation: PartI, Springer-Verlag, Chicago, IL, USA, 2003, pp. 37-48.
[16] R. Hedayatzadeh, B. Hasanizadeh, R. Akbari, K. Ziarati, A multi-objective Artificial Bee Colony for optimizing multi-objective problems, in: 2010 3rd International Conference on Advanced Computer Theory and Engineering(ICACTE), 2010, pp. V5-277-V275-281.
[17] T. SAĞ, M. ÇUNKAŞ, A new ABC-based multiobjective optimization algorithm with an improvement approach (IBMO: improved bee colony algorithm for multiobjective optimization), Turkish Journal of Electrical Engineering & Computer Sciences, 24 (2016).
[18] A. Özkış, A. Babalık, A novel metaheuristic for multi-objective optimization problems: The multi-objective vortex search algorithm, Information Sciences, 402 (2017) 124-148.
[19] M. Kaur, S. Kadam, A novel multi-objective bacteria foraging optimization algorithm (MOBFOA) for multi-objective scheduling, Applied Soft Computing.
[20] Y. Zhou, J. Wang, Z. Wu, K. Wu, A multi-objective tabu search algorithm based on decomposition for multi-objective unconstrained binary quadratic programming problem, Knowledge-Based Systems, 141 (2018) 18-30.
[21] Q. Lin, Y. Ma, J. Chen, Q. Zhu, C.A.C. Coello, K.-C. Wong, F. Chen, An adaptive immune-inspired multi-objective algorithm with multiple differential evolution strategies, Information Sciences, 430-431 (2018) 46-64.
[22] A. Birashk, J. Kazemi Kordestani, M.R. Meybodi, Cellular teaching-learning-based optimization approach for dynamic multi-objective problems, Knowledge-Based Systems, 141 (2018) 148-177.
[23] H. Wang, W. Wang, L. Cui, H. Sun, J. Zhao, Y. Wang, Y. Xue, A hybrid multi-objective firefly algorithm for big data optimization, Applied Soft Computing, (2017).
[24] Y. Zhang, J. Liu, M. Zhou, Z. Jiang, A multi-objective memetic algorithm based on decomposition for big optimization problems, Memetic Computing, 8 (2016) 45-61.
[25] R. Jena, Task scheduling in cloud environment: A multi-objective ABC framework, Journal of Information and Optimization Sciences, 38 (2017) 1-19.
[26] G.-s. Yao, Y.-s. Ding, K.-r. Hao, Multi-objective workflow scheduling in cloud system based on cooperative multi-swarm optimization algorithm, Journal of Central South University, 24 (2017) 1050-1062.
[27] O. Bahri, E.-G. Talbi, N. Ben Amor, A generic fuzzy approach for multi-objective optimization under uncertainty, Swarm and Evolutionary Computation.
[28] R.-E. Precup, M.-C. Sabau, E.M. Petriu, Nature-inspired optimal tuning of input membership functions of Takagi-Sugeno-Kang fuzzy models for anti-lock braking systems, Applied Soft Computing, 27 (2015) 575-589.
[29] S. Vrkalovic, T.-A. Teban, I.-D. Borlea, Stable Takagi-Sugeno fuzzy control designed by optimization, Int. J. Artif. Intell, 15 (2017) 17-29.
[30] X. Deng, J. Lei, J. Wen, Z. Wen, L. Shen, Multi-objective optimization of cooling galleries inside pistons of a diesel engine, Applied Thermal Engineering, 132 (2018) 441-449.
[31] B.D. Raja, R. Jhala, V. Patel, Multi-objective optimization of a rotary regenerator using tutorial training and self-learning inspired teaching-learning based optimization algorithm (TS-TLBO), Applied Thermal Engineering, 93 (2016) 456-467.
[32] H. Safikhani, S. Eiamsa-ard, Pareto based multi-objective optimization of turbulent heat transfer flow in helically corrugated tubes, Applied Thermal Engineering, 95 (2016) 275-280.
[33] K. Salameh, R. Chbeir, H. Camblong, Multi-objective cooperative scheduling: An application on smart grids, Applied Computing and Informatics, (2017).
[34] N.H. Luong, H. La Poutré, P.A.N. Bosman, Multi-objective Gene-pool Optimal Mixing Evolutionary Algorithm with the interleaved multi-start scheme, Swarm and Evolutionary Computation.

ACCEPTED MANUSCRIPT

[35] J. Jeong, Y. Kim, C.W. Ahn, A multi-objective evolutionary approach to automatic melody generation, Expert Systems with Applications, 90 (2017) 50-61.
[36] M. Scirea, J. Togelius, P. Eklund, S. Risi, Metacompose: A compositional evolutionary music composer, in: International Conference on Evolutionary and Biologically Inspired Music and Art, Springer, 2016, pp. 202-217.
[37] S. Mirjalili, A.H. Gandomi, S.Z. Mirjalili, S. Saremi, H. Faris, S.M. Mirjalili, Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems, Advances in Engineering Software, 114 (2017) 163-191.
[38] A. Sadollah, H. Eskandar, J.H. Kim, Water cycle algorithm for solving constrained multi-objective optimization problems, Applied Soft Computing, 27 (2015) 279-298.
[39] S.A. Uymaz, G. Tezel, E. Yel, Artificial algae algorithm (AAA) for nonlinear global optimization, Applied Soft Computing, 31 (2015) 153-171.
[40] V. Pareto, Cours d'economie politique, (1964) 424.
[41] E. Zitzler, K. Deb, L. Thiele, Comparison of Multiobjective Evolutionary Algorithms: Empirical Results, Evol. Comput., 8 (2000) 173-195.
[42] C. Dai, Y. Wang, M. Ye, A new multi-objective particle swarm optimization algorithm based on decomposition, Information Sciences, 325 (2015) 541-557.
[43] M.H. Ahmadi, M.A. Ahmadi, Multi objective optimization of performance of three-heat-source irreversible refrigerators based algorithm NSGAII, Renewable and Sustainable Energy Reviews, 60 (2016) 784-794.
[44] S.A. Sadatsakkak, M.H. Ahmadi, M.A. Ahmadi, Thermodynamic and thermo-economic analysis and optimization of an irreversible regenerative closed Brayton cycle, Energy Conversion and Management, 94 (2015) 124-129.
[45] A. Kishor, P.K. Singh, J. Prakash, NSABC: Non-dominated sorting based multi-objective artificial bee colony algorithm and its application in data clustering, Neurocomputing, 216 (2016) 514-533.
[46] K. Deb, R.B. Agrawal, Simulated binary crossover for continuous search space, Complex Systems, 9 (1994) 1-15.
[47] S. Huband, P. Hingston, L. Barone, L. While, A review of multiobjective test problems and a scalable test problem toolkit, Ieee T Evolut Comput, 10 (2006) 477-506.
[48] K. Deb, L. Thiele, M. Laumanns, E. Zitzler, Scalable Test Problems for Evolutionary Multiobjective Optimization, in: A. Abraham, L. Jain, R. Goldberg (Eds.) Evolutionary Multiobjective Optimization: Theoretical Advances and Applications, Springer London, London, 2005, pp. 105-145.
[49] H. Li, Q. Zhang, Multiobjective Optimization Problems With Complicated Pareto Sets, MOEA/D and NSGA-II, Ieee T Evolut Comput, 13 (2009) 284-302.
[50] T. Okabe, Y. Jin, M. Olhofer, B. Sendhoff, On Test Functions for Evolutionary Multi-objective Optimization, in: X. Yao, E.K. Burke, J.A. Lozano, J. Smith, J.J. Merelo-Guervós, J.A. Bullinaria, J.E. Rowe, P. Tiňo, A. Kabán, H.-P. Schwefel (Eds.) Parallel Problem Solving from Nature - PPSN VIII: 8th International Conference, Birmingham, UK, September 18-22, 2004. Proceedings, Springer Berlin Heidelberg, Berlin, Heidelberg, 2004, pp. 792-802.
[51] R. Viennet, C. Fonteix, I. Marc, Multicriteria optimization using a genetic algorithm for determining a Pareto set, Int J Syst Sci, 27 (1996) 255-260.
[52] C.M. Fonseca, P.J. Fleming, Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation, IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 28 (1998) 26-37.
[53] F. Kursawe, A Variant of Evolution Strategies for Vector Optimization, in: Proceedings of the 1st Workshop on Parallel Problem Solving from Nature, Springer-Verlag, 1991, pp. 193-197.
[54] A. Al-Dujaili, S. Suresh, BMOBench: Black-Box Multi-Objective Optimization Benchmarking Platform, arXiv preprint arXiv:1605.07009, (2016).
[55] C.A.C. Coello, G.B. Lamont, D.A. Van Veldhuizen, Evolutionary algorithms for solving multi-objective problems, Springer, 2007.
[56] C.C. Coello, C. Dhaenens, L. Jourdan, Advances in multi-objective nature inspired computing, Springer, 2009.

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

[57] J.J. Durillo, A.J. Nebro, jMetal: A Java framework for multi-objective optimization, Adv Eng Softw, 42 (2011) 760-771.
[58] Y. Xiang, Y.R. Zhou, H.L. Liu, An elitism based multi-objective artificial bee colony algorithm, Eur J Oper Res, 245 (2015) 168-193.
[59] M.R. Sierra, C.A.C. Coello, Improving PSO-Based multi-objective optimization using crowding, mutation and epsilon-dominance, Evolutionary Multi-Criterion Optimization, 3410 (2005) 505-519.
[60] Q. Zhang, A. Zhou, S. Zhao, P.N. Suganthan, W. Liu, S. Tiwari, Multiobjective optimization test instances for the CEC 2009 special session and competition, in, 2009.
[61] J.J. Durillo, A.J. Nebro, E. Alba, The jMetal Framework for Multi-Objective Optimization: Design and Architecture, Ieee C Evol Computat, (2010).
[62] jMetal4.5. , in, 2016.
[63] M. Friedman, The use of ranks to avoid the assumption of normality implicit in the analysis of variance, Journal of the american statistical association, 32 (1937) 675-701.
[64] S. Biswas, S. Das, P.N. Suganthan, C.A.C. Coello, Evolutionary multiobjective optimization in dynamic environments: A set of novel benchmark functions, in: Evolutionary Computation (CEC), 2014 IEEE Congress on, IEEE, 2014, pp. 3192-3199.

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

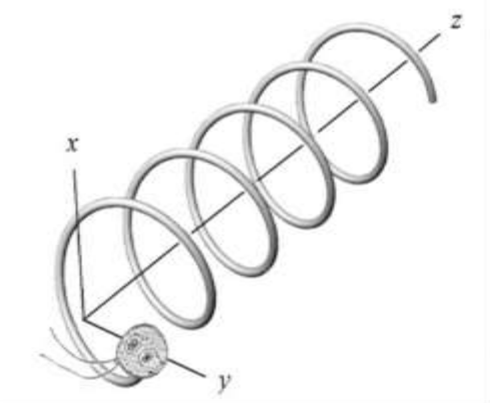


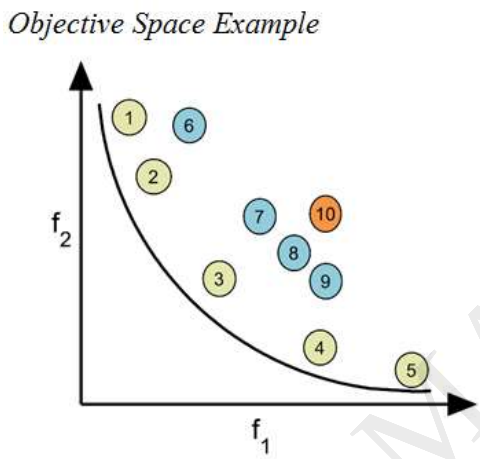
Fig. 1. Helical movement of an algal colony [39]

Objective function f*(x)*, $x = (x_1, x_2, \ldots, x_d)$
  Define parameters (cutting force (Δ) = 2, energy loss $(e) = 0.3$, and adaptation parameter (A_p) = 0.5
  Start the population consisting of *n* times algal colony with random solutions
  Define the size of each algae colonies with 1 and the hunger value with 0
  Define the aim function value and the size (G) of each aim function
While (t < $MAXFES_s$)   *t: time,* $MAXFES_s$ *: Maximum fitness evaluation number*
  Calculate the energy (E) and friction surface (τ) of the algae colonies
  For i=1: *n*
    iStarve = 1
    While ($E(x_i) > 0$)
      Select the algal colony called j according to levels of the aim function with the tournament method
      Select three random sizes (algae cell) (k, l, and m) for the helical motion
$$x_{im}^{t+1} = x_{im}^{t} + \left(x_{jm}^{t} - x_{im}^{t}\right)(\Delta - \tau_i)p$$
$$x_{ik}^{t+1} = x_{ik}^{t} + \left(x_{jk}^{t} - x_{ik}^{t}\right)(\Delta - \tau_i)\cos\alpha$$
$$x_{il}^{t+1} = x_{il}^{t} + \left(x_{jl}^{t} - x_{il}^{t}\right)(\Delta - \tau_i)\sin\beta$$
      α, β ∈ [0,2π]; p ∈ [-1,1];
      Calculate the new solution
      $E(x_i) = E(x_i) - (\frac{e}{2})$ motion-based energy loss
      if the new solution is better, update the algal colony called i and *iStarve* = 0
      else $E(x_i) = E(x_i) - (\frac{e}{2})$ metabolism-related energy loss **end if**
    end While
    if *iStarve*=1, $A(x_i)$ adds 1 to the hunger level **end if**   *A holds the starvation value of each algal colony*
  end For
  Calculate the size (G) of the algae colonies
  r = rand(1,d) $smallest_r^t = biggest_r^t$ *, Copy the* r_{th} *cell of the biggest colony to the smallest algal colony*
  Select the best algal colony
  if rand<A_p
$$starving^{t+1} = starving^{t} + (biggest^{t} - starving^{t}) \times rand$$
  end if
  Keep the best solution
end While

Fig. 2. The pseudo code of the Artificial Algae Algorithm (AAA)

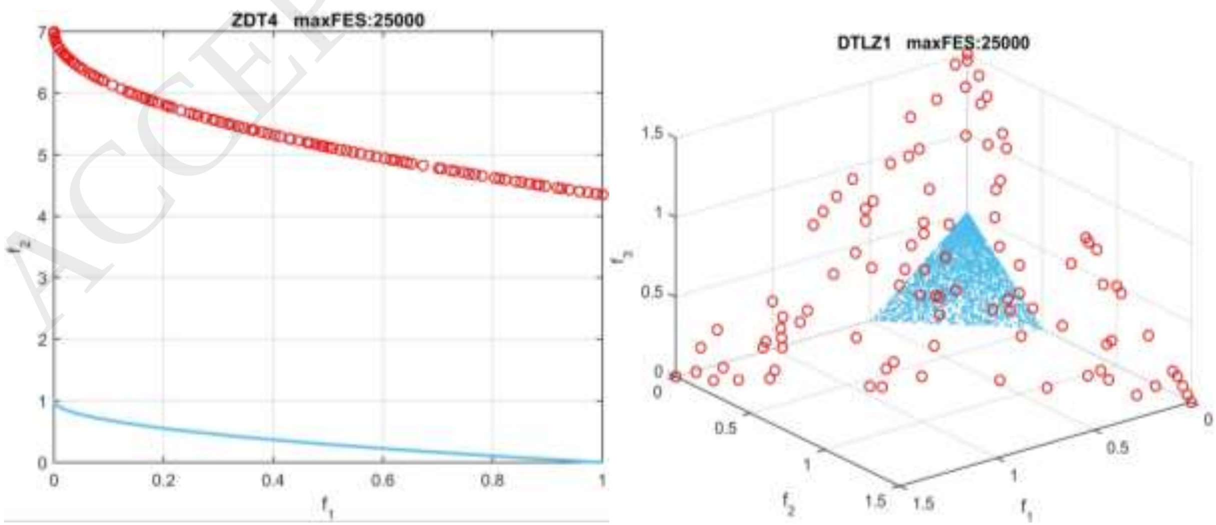
ACCEPTED MANUSCRIPT

```
QR calculation (POP)
NDR = Fast-nondominated-sorting(POP)      sort solutions by front
CRD =  Crowding-distance(POP)             sort solutions by crow-dis.
q = 0                                      quality rank of solutions
   while i = 1 to max(NDR)                 i hold the current front no
        q = q+1
        For each p ∈ NDRi
            if (CRDp is infinitive)        extreme solutions in the same front take the same QR value
               QRp = q
           else
               q = q+1
               QRp  = q
           end if
        end for each p
   end while i
```



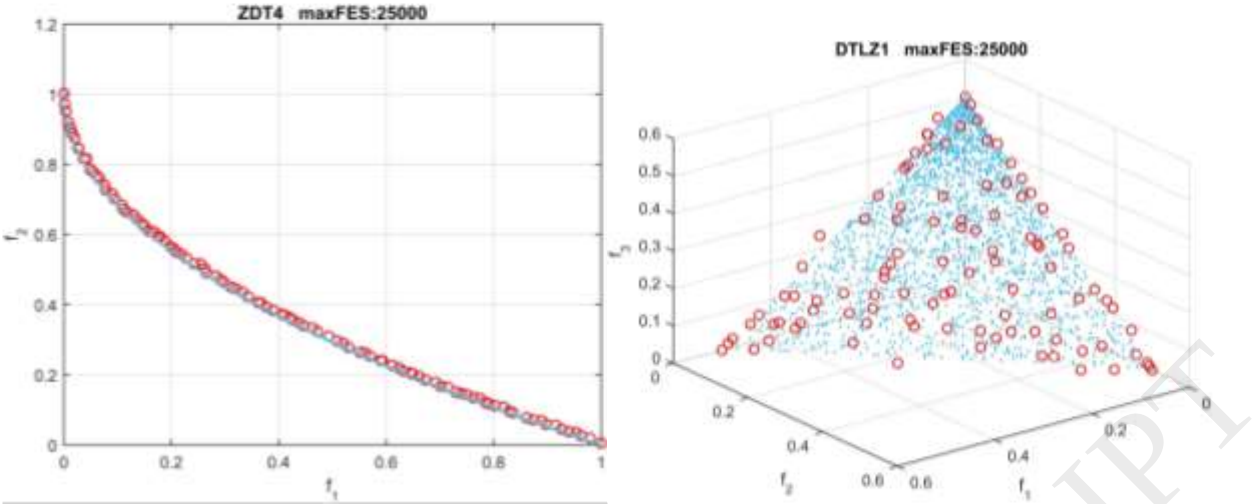
Solution No	NDR	CRD	QR
1	1	inf	1
5	1	inf	1
3	1	0.9	2
4	1	0.7	3
2	1	0.65	4
6	2	inf	5
9	2	inf	5
7	2	0.61	6
8	2	0.12	7
10	3	inf	8

Fig. 3. QR value calculation example



a) Pareto fronts of ZDT4 and DTLZ1 problems obtained with MOAAA using EP + AP

ACCEPTED MANUSCRIPT



b) Pareto fronts of ZDT4 and DTLZ1 problems obtained with MOAAA using PM

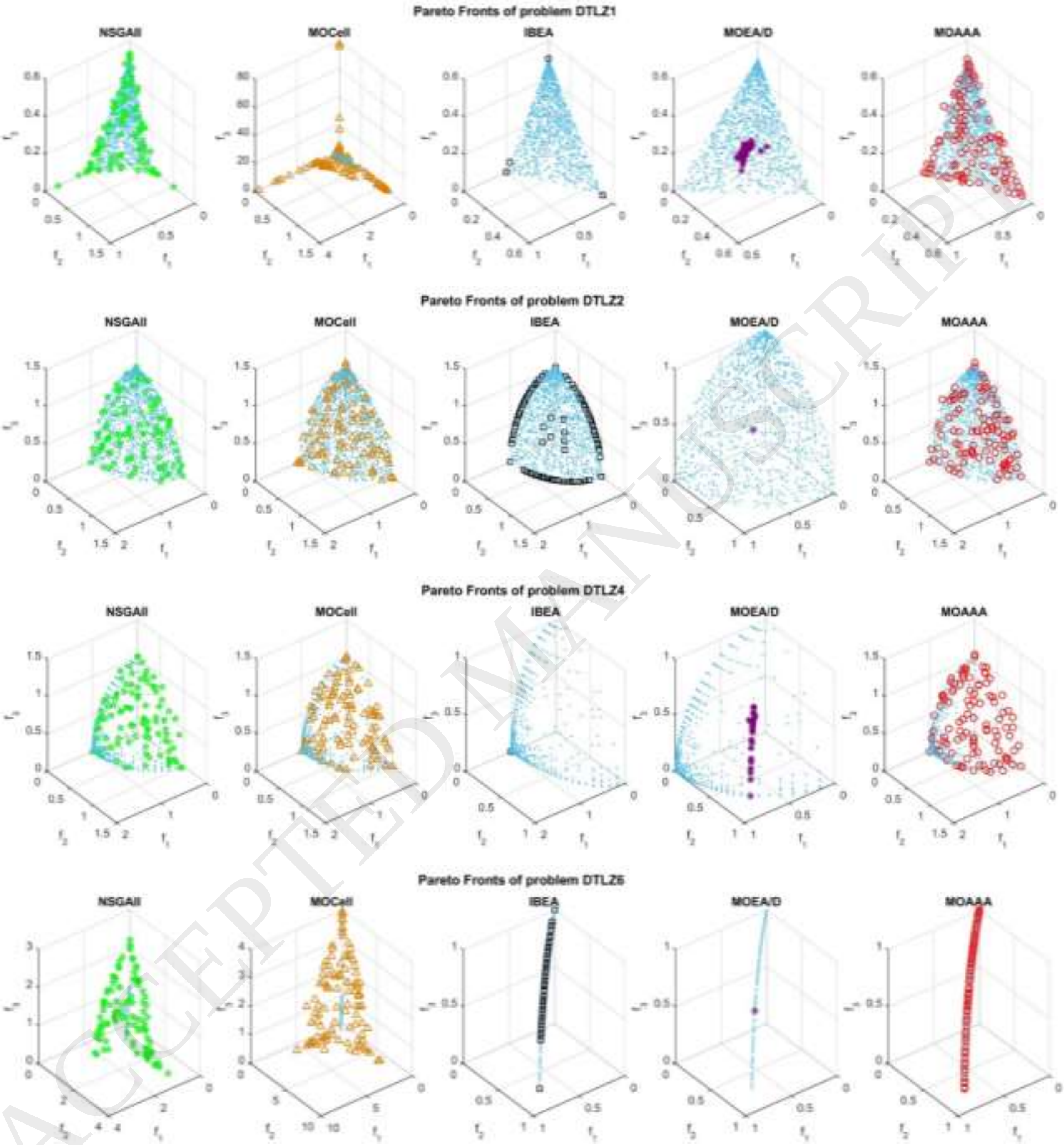
Fig. 4. The Pareto fronts of MOAAA versions for ZDT4 and DTLZ1 problems.

Aim functions $F_m(X)$ $m=1,2,\ldots,M$, $X = (x_1, x_2, \ldots, x_d)$
Define parameters (cutting force (Δ) = 2, energy loss (e) = 0.3)
Start the population consisting of *n* times algal colony with random solutions
Define the aim functions values
Set the initial bigness value of each algal colonies as 1
t = n *t: time* *n: population size*
Apply fast-nondominated-sorting and crowding-distance strategies to algae colonies population
Calculate the quality ranking (QR) value, energy (E) and friction surface (τ) of the algae colonies
While (t < $MAXFES_s$) $MAXFES_s$: *Maximum fitness evaluation number*
Create a copy of algae colonies population as TEMP_POP
For i=1: *n*
Select an algal colony (X) from TEMP_POP with the binary tournament method compared by QR
Select an other algal colony (Y) as neighbour with the same way
$OffSpring_i = X$
Select three random sizes (algae cell) (k, l, and m) for the helical motion
$$OffSpring_{im}^{t+1} = X_m^t + (Y_m^t - X_m^t)(\Delta - \tau_X)p$$
$$OffSpring_{ik}^{t+1} = X_k^t + (Y_k^t - X_k^t)(\Delta - \tau_X)\cos\alpha$$
$$OffSpring_{il}^{t+1} = X_l^t + (Y_l^t - X_l^t)(\Delta - \tau_X)\sin\beta$$
$\alpha, \beta \in [0,2\pi]$; $p \in [-1,1]$;
Apply polynominal mutation for $OffSpring_i$ to create perturbation
Check the updated sizes m, k, l and shift to boundary values if exceed
Calculate objective functions of $OffSpring_i$
$E(X) = E(X) - (\frac{e}{2})$ motion-based energy loss
if new solution ($OffSpring_i$) can not dominate old solution (X),
$E(X) = E(X) - (\frac{e}{2})$ metabolism-related energy loss
if E(X) < 0, remove X from TEMP_POP **end if**
end if
t = t+1
end for
Combine algal colonies population and OffSpring population (n + n = 2n)
Apply fast-non-dominated-sorting and crowding distance strategies to the combined population and select the best *n* solutions
Calculate the quality ranking (QR) value, energy (E) and friction surface (τ) of the algae colonies
end While
Keep the Pareto solutions

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Fig. 5. The pseudo code of MOAAA



ACCEPTED MANUSCRIPT

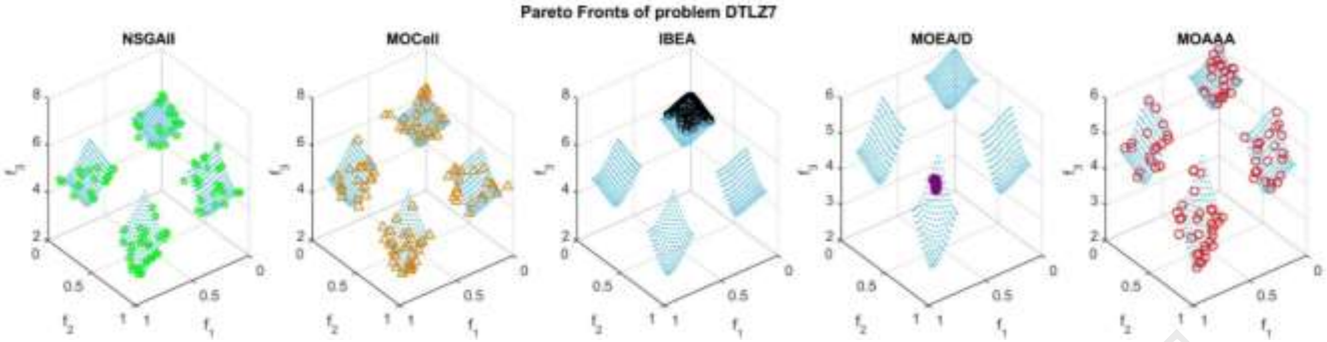
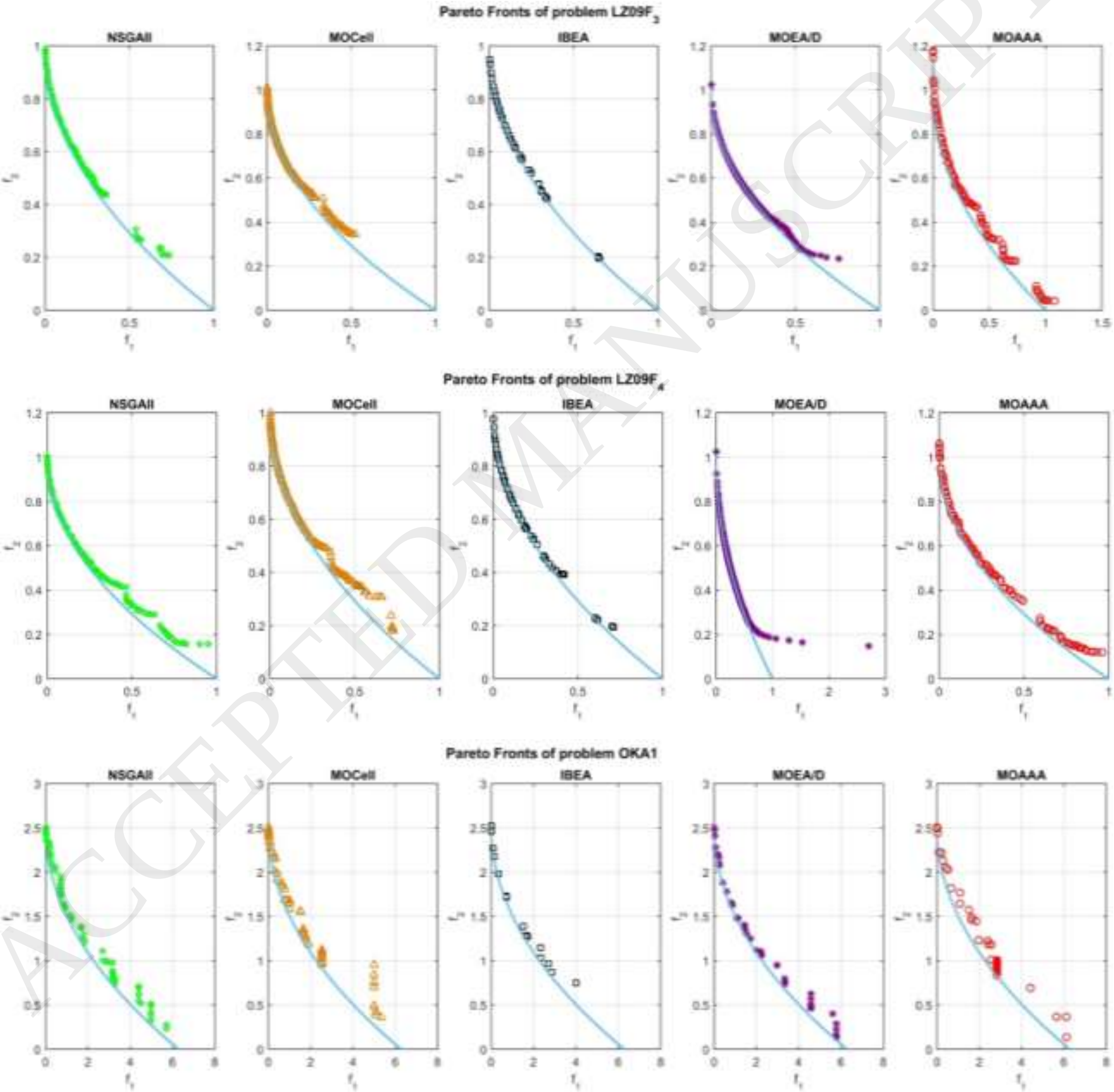


Fig. 6. The Pareto fronts and the box plots produced by algorihtms for the DTLZ1, DTLZ2, DTLZ4, DTLZ6, DTLZ7 problems.



ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

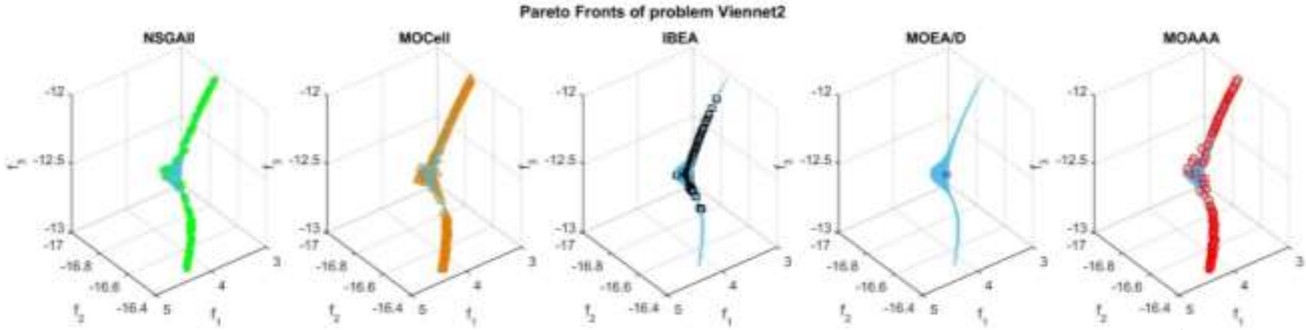


Fig. 7. The Pareto fronts produced by algorihtms for the LZ09F3, LZ09F4, OKA1, Viennet2 problems.

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

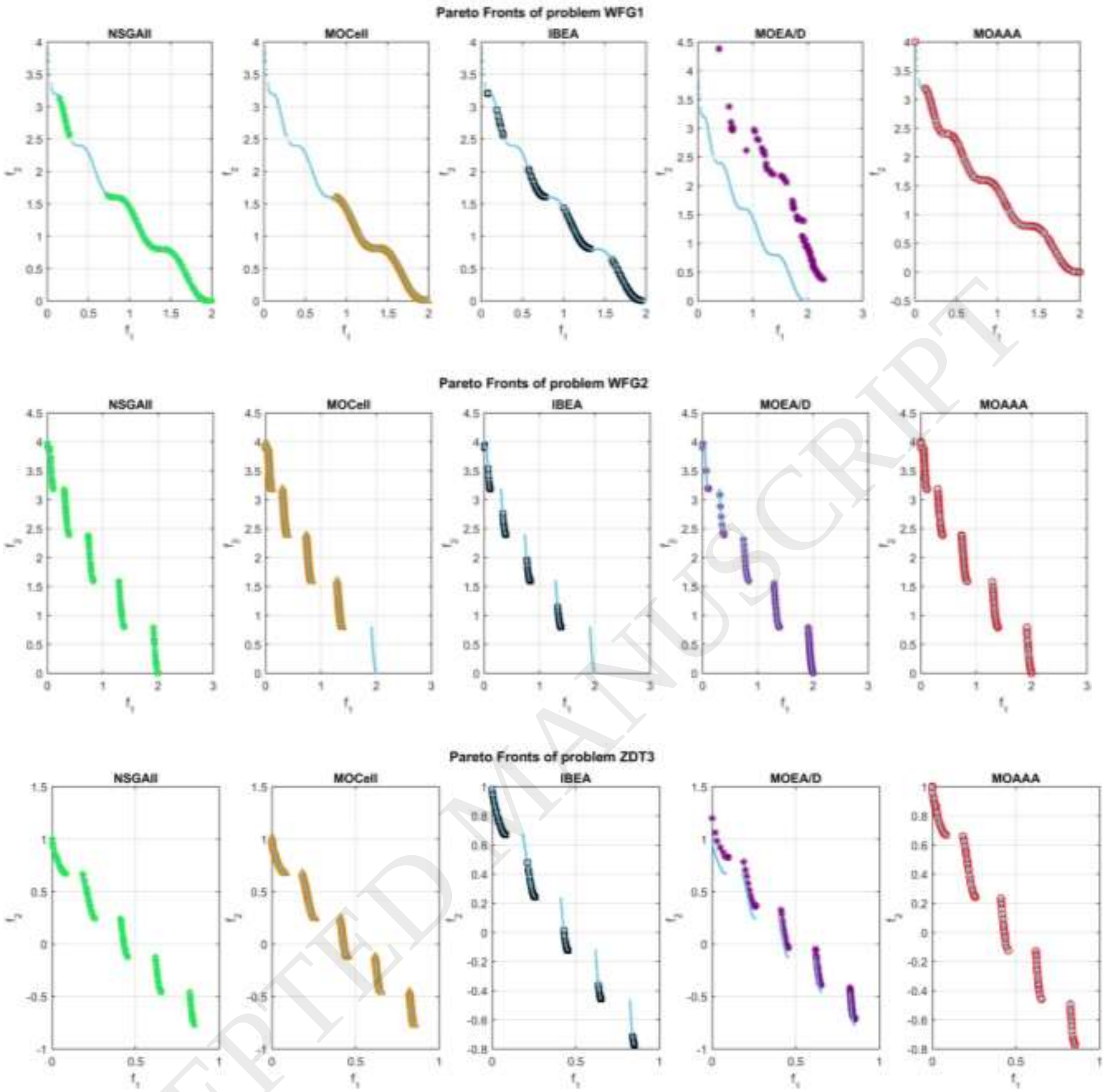
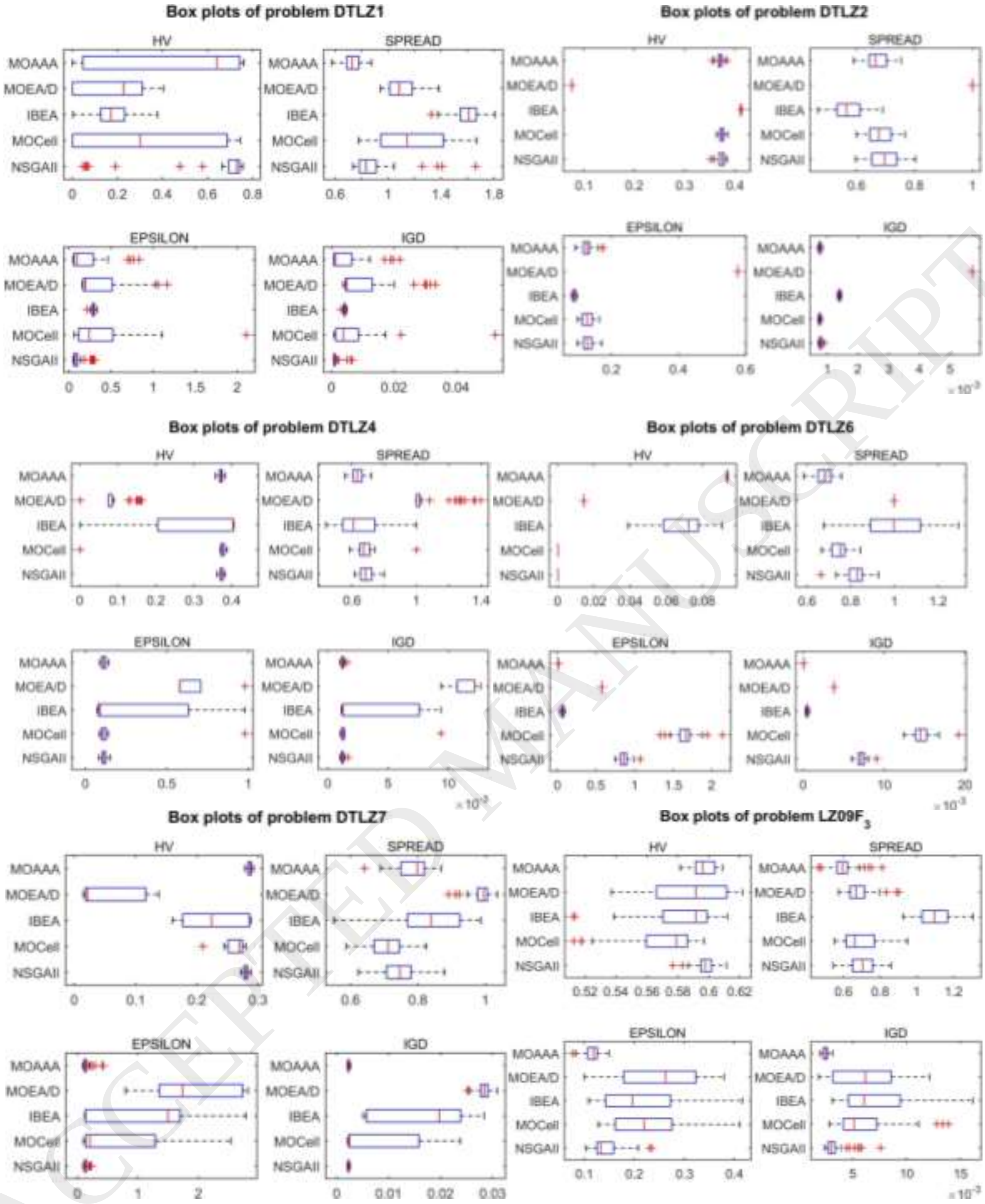


Fig. 8. The Pareto fronts produced by algorihtms for the WFG1, WFG2, ZDT3 problems.

ACCEPTED MANUSCRIPT



ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

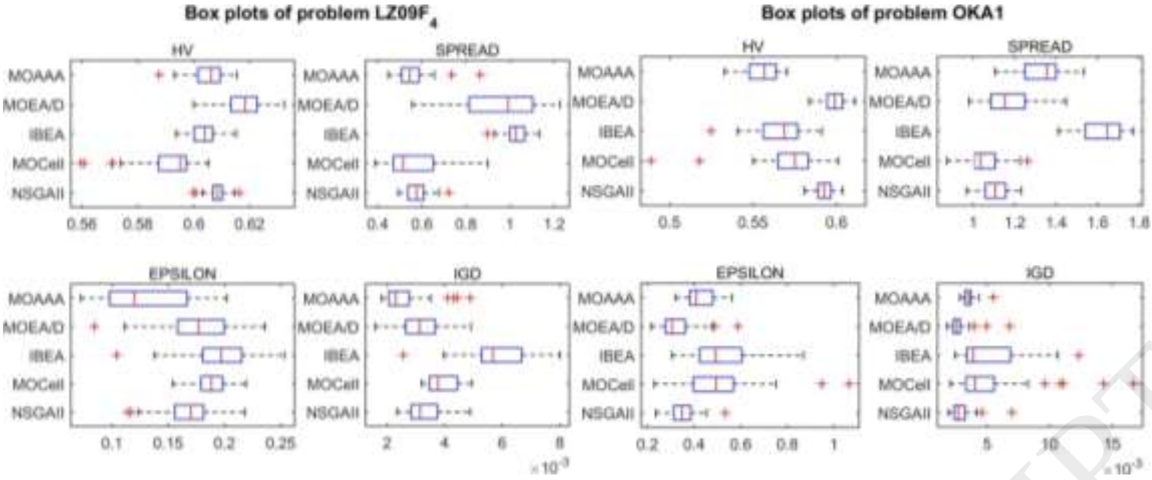


Fig. 9. The box plots produced by algorihtms for stated problems(I).

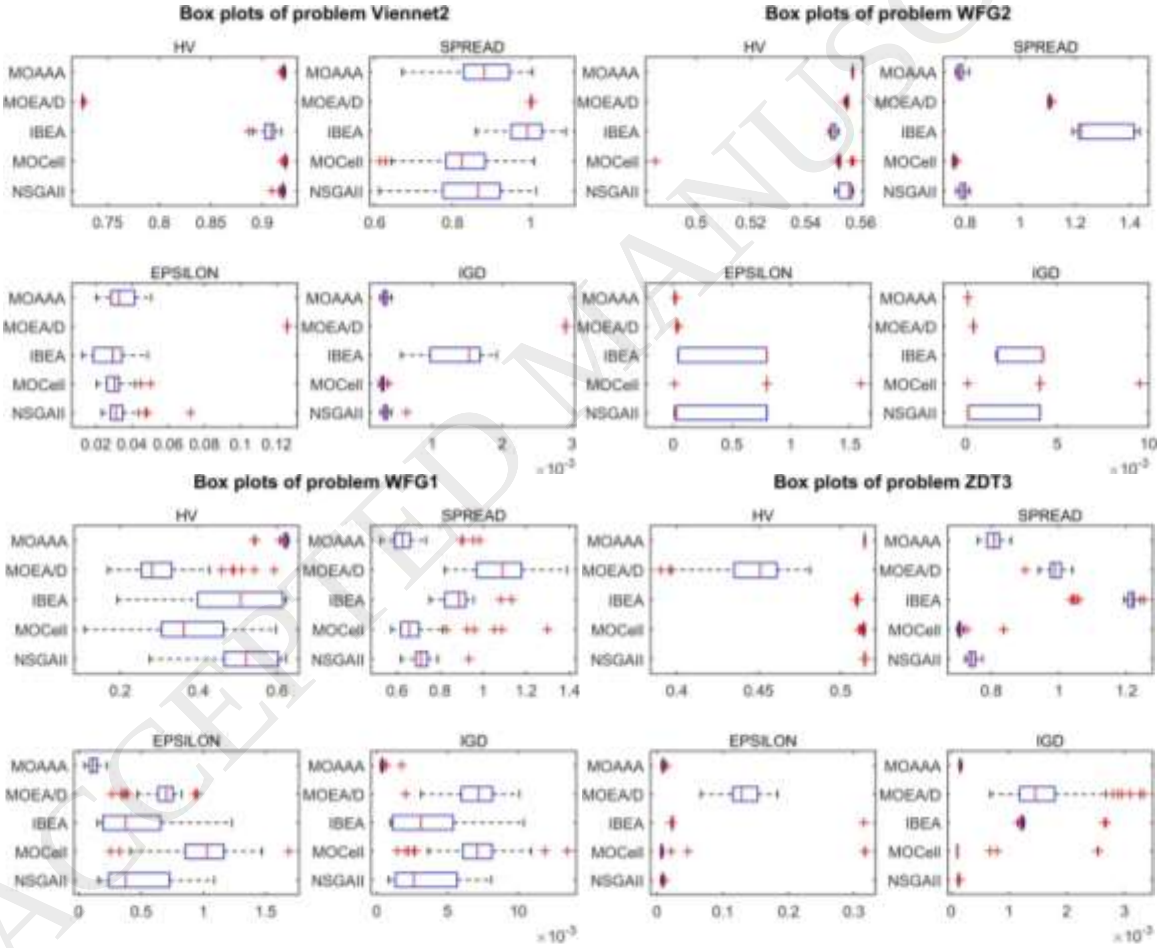


Fig. 10. The box plots produced by algorihtms for stated problems(II).

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 1. Characteristics of the Test Problems

Problem	Separability	Modality	Scalability	Geometry
ZDT1	f_1 separable, f_2 non-separable	uni	DV	convex
ZDT2	f_1 separable, f_2 non-separable	uni	DV	concave
ZDT3	f_1 separable, f_2 non-separable	x	DV	Concave, disconnected
ZDT4	f_1 separable, f_2 non-separable	f_1 uni, f_2 multi	DV	convex
ZDT6	f_1 separable, f_2 non-separable	multi	DV	concave
WFG1	separable	uni	DV&OBJ	convex, mixed
WFG2	non-separable	f_1 uni, f_2 multi	DV&OBJ	convex, disconnected
WFG3	non-separable	uni	DV&OBJ	linear, degenerate
WFG4	non-separable	multi	DV&OBJ	concave
WFG5	separable	deceptive	DV&OBJ	concave
WFG6	non-separable	uni	DV&OBJ	concave
WFG7	separable	uni	DV&OBJ	concave
WFG8	non-separable	uni	DV&OBJ	concave
WFG9	non-separable	multi, deceptive	DV&OBJ	concave
DTLZ1	separable	multi	DV&OBJ	linear
DTLZ2	separable	x	DV&OBJ	concave
DTLZ4	separable	unimodal	DV&OBJ	concave
DTLZ5	f_1 to f_{m-1} non-separable	unimodal	DV&OBJ	degenerate
DTLZ6	f_1 to f_{m-1} non-separable	unimodal	DV&OBJ	degenerate
DTLZ7	separable	x	DV&OBJ	disconnected
OKA1	x	x	No	convex
OKA2	x	x	No	concave
Schaffer	x	uni	No	convex
Fonseca	x	uni	No	convex

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 2. Friedman test results of the MOAAA for tuning of *cutting force* (Δ) parameter

Experiments	Exp1	Exp2	Exp3	Exp4	Exp5
energy loss (e)	0.3	0.3	0.3	0.3	0.3
cutting force (Δ)	1	2	3	4	5
HV	3.666	4.222	3.166	2.444	1.499
SPREAD	2.944	2.333	3.027	2.944	3.749
EPSILON	3.277	1.888	2.611	3.194	4.027
IGD	2.749	1.555	2.583	3.638	4.472

Table 3. Friedman test results of the MOAAA for tuning of *energy loss* (e) parameter

Experiments	Exp6	Exp7	Exp8	Exp9	Exp10	Exp11	Exp12	Exp13	Exp14	Exp15
energy loss (e)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
cutting force (Δ)	2	2	2	2	2	2	2	2	2	2
HV	6.388	6.749	7.444	6.138	5.416	5.638	4.527	4.638	4.222	3.833
SPREAD	3.805	3.861	3.999	4.861	6.083	5.277	6.277	6.805	6.666	7.361
EPSILON	4.611	5.222	4.500	4.972	5.888	5.833	5.583	5.638	6.166	6.583
IGD	4.361	4.583	3.416	5.250	6.249	5.333	5.694	6.305	6.805	7.00

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 4. The mean and the standard deviation values of the HV metric for all the algorithms.

	NSGAII		MOCell		IBEA		MOEA/D		MOAAA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
ZDT1	6.59e-01	3.0e-04	6.61e-01	2.7e-04	6.62e-01	8.4e-05	6.41e-01	8.7e-03	6.58e-01	3.4e-04
ZDT2	3.26e-01	3.0e-04	3.28e-01	4.2e-04	3.27e-01	1.6e-04	3.10e-01	6.2e-03	3.25e-01	4.6e-04
ZDT3	5.15e-01	1.7e-04	5.14e-01	1.2e-03	5.10e-01	3.2e-04	4.46e-01	2.3e-02	5.15e-01	1.5e-04
ZDT4	6.54e-01	3.4e-03	6.57e-01	2.1e-03	2.54e-01	9.2e-02	3.00e-01	2.0e-01	5.57e-01	1.2e-01
ZDT6	3.88e-01	1.6e-03	3.94e-01	7.5e-04	3.96e-01	3.9e-04	4.01e-01	2.0e-03	3.98e-01	3.1e-04
WFG1	5.18e-01	9.1e-02	3.69e-01	1.1e-01	4.85e-01	1.2e-01	3.17e-01	9.0e-02	6.20e-01	1.6e-02
WFG2	5.54e-01	2.5e-03	5.51e-01	9.6e-03	5.50e-01	1.0e-03	5.55e-01	3.3e-04	5.57e-01	1.1e-04
WFG3	4.92e-01	5.9e-04	4.94e-01	5.9e-04	4.94e-01	3.2e-04	4.93e-01	1.3e-04	4.92e-01	3.5e-04
WFG4	2.09e-01	4.1e-04	2.10e-01	1.8e-04	2.09e-01	3.1e-04	2.03e-01	1.7e-03	2.09e-01	2.5e-04
WFG5	2.18e-01	2.6e-04	2.19e-01	3.5e-05	2.17e-01	1.9e-04	2.18e-01	1.2e-04	2.18e-01	2.5e-03
WFG6	1.98e-01	1.3e-02	1.83e-01	2.4e-02	1.98e-01	1.1e-02	2.09e-01	1.7e-04	2.08e-01	1.2e-03
WFG7	2.09e-01	2.7e-04	2.10e-01	1.2e-04	2.08e-01	1.4e-04	2.09e-01	9.4e-05	2.09e-01	3.4e-04
WFG9	2.24e-01	1.6e-03	2.25e-01	2.3e-03	2.24e-01	1.3e-03	2.22e-01	5.5e-04	2.24e-01	1.0e-03
DTLZ1	6.42e-01	2.1e-01	3.29e-01	3.1e-01	1.75e-01	7.7e-02	1.71e-01	1.5e-01	4.22e-01	3.4e-01
DTLZ2	3.73e-01	6.9e-03	3.74e-01	5.2e-03	4.12e-01	6.3e-04	7.55e-02	2.7e-06	3.71e-01	5.4e-03
DTLZ4	3.74e-01	5.1e-03	3.03e-01	1.5e-01	2.88e-01	1.4e-01	9.06e-02	3.4e-02	3.71e-01	5.3e-03
DTLZ5	9.28e-02	2.2e-04	9.40e-02	3.5e-05	9.18e-02	1.9e-04	1.43e-02	1.5e-07	9.28e-02	2.1e-04
DTLZ6	0.0e+00	0.0e+00	0.00e+00	0.0e+00	6.89e-02	1.5e-02	1.45e-02	3.3e-10	9.36e-02	1.8e-04
DTLZ7	2.80e-01	3.7e-03	2.62e-01	1.7e-02	2.29e-01	4.8e-02	5.64e-02	4.7e-02	2.86e-01	3.5e-03
LZ09F1	6.52e-01	6.7e-04	6.51e-01	2.5e-03	6.54e-01	3.2e-03	6.61e-01	1.3e-04	6.51e-01	7.1e-04
LZ09F2	5.14e-01	3.9e-02	4.47e-01	7.9e-02	4.97e-01	3.7e-02	5.33e-01	3.3e-02	5.59e-01	2.3e-02
LZ09F3	5.97e-01	6.5e-03	5.69e-01	2.2e-02	5.83e-01	2.2e-02	5.89e-01	2.4e-02	5.97e-01	7.6e-03
LZ09F4	6.09e-01	3.1e-03	5.91e-01	1.0e-02	6.04e-01	4.9e-03	6.17e-01	7.5e-03	6.05e-01	6.1e-03
LZ09F5	6.08e-01	6.3e-03	5.95e-01	1.2e-02	6.03e-01	9.9e-03	6.13e-01	1.4e-02	6.12e-01	5.2e-03
LZ09F6	1.53e-01	4.3e-02	1.48e-01	4.0e-02	8.20e-02	8.4e-02	7.39e-02	5.2e-03	1.90e-01	3.4e-02
LZ09F7	4.43e-01	4.7e-02	3.46e-01	7.4e-02	4.00e-01	6.0e-02	4.33e-01	1.2e-01	3.19e-01	1.4e-01
LZ09F8	4.19e-01	3.7e-02	3.36e-01	7.0e-02	3.80e-01	5.1e-02	3.45e-01	1.0e-01	2.72e-01	1.1e-01
LZ09F9	1.61e-01	5.8e-02	1.43e-01	5.3e-02	1.56e-01	4.0e-02	1.86e-01	4.9e-02	2.26e-01	2.4e-02
OKA1	5.93e-01	5.2e-03	5.72e-01	1.8e-02	5.66e-01	1.4e-02	5.99e-01	6.5e-03	5.56e-01	9.5e-03
OKA2	1.51e-01	2.3e-02	3.96e-02	5.9e-02	6.67e-02	4.9e-02	5.23e-02	3.5e-02	5.77e-02	1.2e-02
Viennet2	9.20e-01	2.0e-03	9.22e-01	9.6e-04	9.07e-01	6.7e-03	7.25e-01	4.2e-04	9.21e-01	1.1e-03
Viennet3	8.33e-01	5.7e-04	8.35e-01	5.2e-04	8.29e-01	3.8e-03	5.66e-01	4.7e-05	8.33e-01	5.9e-04
Poloni	9.13e-01	8.6e-05	9.13e-01	2.2e-05	9.11e-01	1.1e-03	9.11e-01	7.7e-05	9.13e-01	6.9e-05
Schaffer	5.47e-01	1.7e-01	5.48e-01	1.9e-01	5.23e-01	1.8e-01	5.03e-01	2.3e-01	4.70e-01	2.2e-01
Fonseca	3.08e-01	4.3e-04	3.12e-01	8.8e-05	3.11e-01	1.0e-04	3.12e-01	1.1e-05	3.09e-01	3.6e-04
Kursawe	4.00e-01	2.5e-04	4.01e-01	1.0e-04	3.94e-01	8.8e-04	4.00e-01	1.2e-04	4.00e-01	2.0e-04

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 5. The mean and the standard deviation values of the SPREAD metric for all the algorithms.

	NSGAII		MOCell		IBEA		MOEA/D		MOAAA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
ZDT1	3.69e-01	3.2e-02	8.26e-02	1.1e-02	3.03e-01	1.8e-02	3.68e-01	5.6e-02	6.40e-01	5.1e-02
ZDT2	3.71e-01	3.0e-02	8.71e-02	1.4e-02	3.39e-01	2.2e-02	3.20e-01	8.8e-02	6.79e-01	6.5e-02
ZDT3	7.45e-01	1.4e-02	7.12e-01	2.6e-02	1.20e+00	5.6e-02	9.90e-01	2.6e-02	8.08e-01	2.4e-02
ZDT4	3.88e-01	3.8e-02	1.35e-01	5.1e-02	1.12e+00	5.9e-02	9.75e-01	1.3e-01	5.85e-01	1.7e-01
ZDT6	3.58e-01	2.8e-02	1.22e-01	1.2e-01	4.05e-01	4.7e-02	1.53e-01	9.8e-03	7.86e-01	3.5e-02
WFG1	7.17e-01	4.7e-02	6.99e-01	1.4e-01	8.78e-01	7.4e-02	1.09e+00	1.5e-01	6.46e-01	9.6e-02
WFG2	7.93e-01	1.3e-02	7.58e-01	3.7e-03	1.28e+00	9.7e-02	1.11e+00	3.6e-03	7.82e-01	1.0e-02
WFG3	3.72e-01	3.0e-02	5.62e-02	1.0e-02	2.55e-01	3.1e-02	3.45e-01	7.5e-04	3.16e-01	2.4e-02
WFG4	3.70e-01	3.1e-02	1.22e-01	1.3e-02	5.14e-01	2.6e-02	5.13e-01	4.9e-02	3.27e-01	2.5e-02
WFG5	3.94e-01	2.8e-02	1.23e-01	1.6e-02	5.92e-01	6.2e-02	4.54e-01	1.0e-02	4.29e-01	3.3e-02
WFG6	3.87e-01	2.9e-02	1.48e-01	3.0e-02	5.26e-01	3.1e-02	4.14e-01	5.7e-03	3.30e-01	2.9e-02
WFG7	3.84e-01	3.1e-02	1.19e-01	1.7e-02	5.19e-01	4.4e-02	4.15e-01	7.4e-03	3.44e-01	2.9e-02
WFG9	3.63e-01	2.6e-02	1.13e-01	1.7e-02	5.03e-01	3.2e-02	4.48e-01	1.4e-02	3.29e-01	2.6e-02
DTLZ1	8.91e-01	1.8e-01	1.17e+00	2.7e-01	1.61e+00	1.1e-01	1.10e+00	1.0e-01	7.38e-01	6.7e-02
DTLZ2	6.94e-01	4.7e-02	6.84e-01	4.3e-02	5.75e-01	4.9e-02	1.00e+00	1.7e-05	6.70e-01	3.9e-02
DTLZ4	6.90e-01	4.4e-02	7.30e-01	1.4e-01	6.59e-01	1.5e-01	1.06e+00	1.2e-01	6.40e-01	3.6e-02
DTLZ5	4.57e-01	5.6e-02	1.45e-01	4.6e-02	6.61e-01	4.4e-02	1.00e+00	3.0e-06	4.45e-01	4.7e-02
DTLZ6	8.21e-01	4.7e-02	7.48e-01	3.8e-02	9.94e-01	1.6e-01	1.00e+00	0.0e+00	6.77e-01	4.4e-02
DTLZ7	7.41e-01	5.3e-02	7.08e-01	5.4e-02	8.24e-01	1.2e-01	9.89e-01	2.9e-02	7.86e-01	4.7e-02
LZ09F1	5.22e-01	1.2e-01	5.05e-01	1.8e-01	7.59e-01	5.0e-02	3.18e-01	4.4e-02	3.90e-01	1.2e-01
LZ09F2	1.46e+00	1.4e-01	1.38e+00	1.9e-01	1.47e+00	1.4e-01	9.45e-01	1.4e-01	1.15e+00	1.2e-01
LZ09F3	7.03e-01	7.0e-02	6.97e-01	1.1e-01	1.10e+00	9.5e-02	6.87e-01	6.8e-02	5.95e-01	7.0e-02
LZ09F4	5.76e-01	5.0e-02	5.55e-01	1.2e-01	1.03e+00	4.9e-02	9.54e-01	1.8e-01	5.57e-01	7.1e-02
LZ09F5	6.45e-01	6.9e-02	5.60e-01	8.4e-02	1.07e+00	7.8e-02	6.54e-01	7.3e-02	5.53e-01	5.5e-02
LZ09F6	9.26e-01	8.3e-02	8.57e-01	8.4e-02	1.59e+00	4.7e-01	9.93e-01	2.4e-02	9.26e-01	1.0e-01
LZ09F7	1.35e+00	1.3e-01	1.33e+00	1.9e-01	1.13e+00	1.5e-01	1.26e+00	1.0e-01	1.29e+00	2.2e-01
LZ09F8	1.27e+00	8.5e-02	1.31e+00	1.9e-01	1.18e+00	1.8e-01	1.25e+00	8.1e-02	1.25e+00	2.9e-01
LZ09F9	1.61e+00	1.9e-01	1.59e+00	1.5e-01	1.68e+00	1.2e-01	9.71e-01	1.3e-01	1.31e+00	2.0e-01
OKA1	1.11e+00	6.2e-02	1.05e+00	8.2e-02	1.63e+00	1.0e-01	1.16e+00	1.0e-01	1.33e+00	9.8e-02
OKA2	1.52e+00	8.8e-02	1.09e+00	1.7e-01	1.47e+00	3.1e-01	1.54e+00	3.2e-01	1.55e+00	1.2e-01
Viennet2	8.48e-01	9.5e-02	8.29e-01	9.1e-02	9.86e-01	5.5e-02	1.00e+00	6.5e-04	8.71e-01	8.2e-02
Viennet3	7.18e-01	5.2e-02	6.85e-01	5.4e-02	8.24e-01	8.5e-02	1.00e+00	2.8e-05	7.00e-01	6.0e-02
Poloni	8.07e-01	2.3e-02	7.53e-01	2.0e-02	1.16e+00	3.7e-02	1.27e+00	3.7e-03	7.93e-01	1.8e-02
Schaffer	6.18e-01	2.5e-01	5.70e-01	1.9e-01	6.37e-01	2.6e-01	1.16e+00	3.4e-02	6.43e-01	2.8e-01
Fonseca	4.01e-01	3.5e-02	7.74e-02	1.1e-02	3.99e-01	2.2e-02	1.46e-01	9.7e-04	2.89e-01	2.2e-02
Kursawe	5.72e-01	2.5e-02	4.17e-01	5.9e-03	8.65e-01	2.6e-02	7.32e-01	4.2e-03	4.98e-01	1.9e-02

ACCEPTED MANUSCRIPT

Table 6. The mean and the standard deviation values of the EPSILON metric for all the algorithms.

	NSGAII		MOCell		IBEA		MOEA/D		MOAAA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
ZDT1	1.31e-02	2.2e-03	6.74e-03	4.4e-04	8.72e-03	8.5e-04	2.30e-02	7.1e-03	1.42e-02	2.2e-03
ZDT2	1.32e-02	2.1e-03	5.93e-03	3.4e-04	1.59e-02	1.4e-03	4.43e-02	1.7e-02	1.44e-02	2.4e-03
ZDT3	8.61e-03	1.2e-03	5.73e-02	1.1e-01	5.83e-02	9.5e-02	1.33e-01	2.9e-02	9.04e-03	1.6e-03
ZDT4	1.53e-02	2.9e-03	1.06e-02	1.0e-02	7.30e-01	1.1e-01	4.56e-01	2.8e-01	9.23e-02	9.4e-02
ZDT6	1.57e-02	2.1e-03	8.12e-03	5.5e-04	1.26e-02	1.1e-03	5.25e-03	1.3e-03	1.32e-02	2.3e-03
WFG1	4.75e-01	2.7e-01	9.81e-01	2.9e-01	4.51e-01	2.8e-01	6.77e-01	1.4e-01	1.18e-01	4.2e-02
WFG2	3.91e-01	3.9e-01	7.02e-01	3.0e-01	5.55e-01	3.5e-01	3.05e-02	4.0e-03	1.50e-02	2.6e-03
WFG3	3.88e-02	5.7e-03	1.68e-02	1.4e-03	2.33e-02	1.6e-03	2.77e-02	1.3e-03	3.67e-02	7.3e-03
WFG4	3.47e-02	7.4e-03	1.50e-02	8.8e-04	4.29e-02	4.9e-03	6.73e-02	1.7e-02	3.12e-02	4.1e-03
WFG5	5.68e-02	2.0e-02	5.24e-02	2.9e-02	4.85e-02	2.0e-02	7.63e-02	7.6e-03	5.92e-02	1.8e-02
WFG6	4.91e-02	2.2e-02	6.00e-02	4.4e-02	6.08e-02	2.0e-02	2.41e-02	7.4e-04	3.27e-02	5.6e-03
WFG7	3.38e-02	4.2e-03	1.46e-02	5.4e-04	4.27e-02	5.3e-03	2.50e-02	5.1e-04	3.49e-02	8.2e-03
WFG9	3.59e-02	8.6e-03	1.78e-02	3.3e-03	3.38e-02	5.1e-03	3.17e-02	1.2e-03	3.50e-02	5.7e-03
DTLZ1	1.03e-01	7.1e-02	3.39e-01	3.4e-01	2.92e-01	2.0e-02	3.78e-01	2.9e-01	2.15e-01	2.0e-01
DTLZ2	1.30e-01	1.9e-02	1.30e-01	1.7e-02	8.88e-02	4.6e-03	5.77e-01	1.2e-05	1.27e-01	1.7e-02
DTLZ4	1.14e-01	1.8e-02	2.80e-01	3.5e-01	3.75e-01	3.4e-01	6.41e-01	9.2e-02	1.12e-01	1.5e-02
DTLZ5	1.10e-02	1.9e-03	5.26e-03	1.3e-03	3.18e-02	1.6e-03	5.77e-01	1.7e-06	1.07e-02	1.6e-03
DTLZ6	8.62e-01	6.4e-02	1.66e+00	1.4e-01	6.36e-02	2.0e-02	5.77e-01	0.0e+00	1.01e-02	1.7e-03
DTLZ7	1.31e-01	2.9e-02	6.65e-01	7.0e-01	1.28e+00	9.7e-01	1.91e+00	6.6e-01	1.43e-01	6.6e-02
LZ09F1	1.87e-02	1.6e-03	4.55e-02	2.4e-02	4.26e-02	2.9e-02	8.82e-03	9.3e-04	1.83e-02	1.8e-03
LZ09F2	2.11e-01	5.7e-02	3.20e-01	1.3e-01	2.40e-01	6.5e-02	1.88e-01	6.4e-02	1.52e-01	3.9e-02
LZ09F3	1.44e-01	2.9e-02	2.25e-01	6.9e-02	2.12e-01	7.9e-02	2.45e-01	8.7e-02	1.15e-01	1.8e-02
LZ09F4	1.67e-01	2.1e-02	1.88e-01	1.5e-02	1.96e-01	2.7e-02	1.77e-01	3.4e-02	1.28e-01	3.8e-02
LZ09F5	1.24e-01	2.6e-02	1.80e-01	6.3e-02	1.67e-01	5.5e-02	1.65e-01	6.4e-02	9.04e-02	1.9e-02
LZ09F6	3.44e-01	9.4e-02	4.52e-01	1.9e-01	4.78e-01	1.4e-01	6.89e-01	8.3e-02	2.95e-01	6.0e-02
LZ09F7	3.65e-01	1.0e-01	5.44e-01	1.4e-01	4.49e-01	1.2e-01	3.85e-01	1.9e-01	4.09e-01	1.1e-01
LZ09F8	3.37e-01	9.3e-02	5.40e-01	1.4e-01	4.52e-01	1.1e-01	3.97e-01	1.5e-01	4.17e-01	8.4e-02
LZ09F9	2.48e-01	6.1e-02	3.74e-01	1.2e-01	2.82e-01	7.9e-02	2.12e-01	6.7e-02	1.63e-01	3.7e-02
OKA1	3.47e-01	5.7e-02	5.08e-01	1.5e-01	5.18e-01	1.3e-01	3.26e-01	6.8e-02	4.25e-01	5.9e-02
OKA2	4.63e-01	9.4e-02	8.72e-01	2.2e-01	7.09e-01	1.9e-01	6.97e-01	1.8e-01	6.29e-01	4.0e-02
Viennet2	3.33e-02	8.2e-03	3.05e-02	6.1e-03	2.79e-02	9.7e-03	1.26e-01	6.8e-05	3.41e-02	7.6e-03
Viennet3	4.88e-02	1.1e-02	5.24e-02	1.0e-02	6.69e-02	2.2e-02	4.78e-01	1.8e-04	5.11e-02	1.1e-02
Poloni	1.49e-01	2.6e-02	7.11e-02	6.7e-03	4.16e-01	4.5e-02	2.22e-01	6.5e-03	1.29e-01	3.3e-02
Schaffer	1.47e+00	8.3e-01	1.59e+00	1.4e+00	1.73e+00	1.5e+00	1.76e+00	1.3e+00	1.95e+00	1.5e+00
Fonseca	1.27e-02	1.4e-03	6.62e-03	4.6e-04	8.26e-03	7.4e-04	6.33e-03	3.4e-05	1.14e-02	1.4e-03
Kursawe	7.71e-02	8.5e-03	4.42e-02	5.2e-03	2.72e-01	1.7e-02	7.78e-02	5.1e-03	6.93e-02	1.0e-02

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 7. The mean and the standard deviation values of the IGD metric for all the algorithms.

	NSGAII		MOCell		IBEA		MOEA/D		MOAAA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
ZDT1	1.88e-04	1.0e-05	1.41e-04	1.8e-06	1.64e-04	4.4e-06	5.08e-04	1.9e-04	2.30e-04	9.8e-06
ZDT2	1.91e-04	6.9e-06	1.41e-04	2.3e-06	5.31e-04	3.3e-05	4.51e-04	1.3e-04	2.38e-04	1.4e-05
ZDT3	1.32e-04	6.7e-06	5.19e-04	8.9e-04	1.40e-03	4.7e-04	1.62e-03	6.8e-04	1.58e-04	9.7e-06
ZDT4	2.44e-04	5.4e-05	1.99e-04	1.5e-04	2.17e-02	3.1e-03	1.08e-02	7.1e-03	2.40e-03	2.9e-03
ZDT6	3.29e-04	3.0e-05	2.07e-04	1.3e-05	2.53e-04	7.8e-06	1.44e-04	3.0e-05	2.51e-04	1.3e-05
WFG1	3.35e-03	2.1e-03	7.02e-03	2.4e-03	3.62e-03	2.5e-03	6.80e-03	1.7e-03	4.33e-04	2.9e-04
WFG2	2.02e-03	2.0e-03	3.62e-03	1.6e-03	3.42e-03	1.2e-03	4.33e-04	9.1e-06	1.25e-04	4.6e-06
WFG3	1.55e-04	6.8e-06	1.11e-04	2.4e-06	1.30e-04	2.7e-06	1.36e-04	4.1e-07	1.47e-04	7.4e-06
WFG4	1.63e-04	9.7e-06	1.21e-04	1.3e-06	5.26e-04	3.4e-05	2.34e-04	3.2e-05	1.54e-04	4.9e-06
WFG5	1.26e-04	5.7e-06	9.56e-05	5.9e-06	3.61e-04	3.5e-05	1.24e-04	2.4e-06	1.35e-04	1.3e-05
WFG6	4.51e-04	3.4e-04	8.86e-04	7.9e-04	8.36e-04	2.0e-04	2.28e-04	1.1e-06	2.30e-04	1.5e-05
WFG7	1.56e-04	7.0e-06	1.14e-04	1.6e-06	5.03e-04	2.7e-05	1.48e-04	7.3e-07	1.52e-04	9.0e-06
WFG9	2.36e-04	2.1e-05	1.82e-04	2.7e-05	4.75e-04	3.9e-05	2.42e-04	4.7e-06	2.30e-04	1.3e-05
DTLZ1	1.49e-03	1.7e-03	6.60e-03	8.4e-03	4.16e-03	3.3e-04	9.95e-03	8.5e-03	4.59e-03	5.4e-03
DTLZ2	7.75e-04	3.8e-05	7.57e-04	3.3e-05	1.40e-03	3.3e-05	5.72e-03	9.7e-08	7.61e-04	3.6e-05
DTLZ4	1.21e-03	1.2e-04	2.84e-03	3.3e-03	4.00e-03	3.2e-03	1.15e-02	9.9e-04	1.23e-03	1.1e-04
DTLZ5	2.04e-05	1.3e-06	1.42e-05	3.3e-07	1.01e-04	4.8e-06	1.48e-03	3.5e-09	1.96e-05	9.1e-07
DTLZ6	7.17e-03	5.8e-04	1.47e-02	1.2e-03	4.64e-04	1.2e-04	3.82e-03	0.0e+00	5.70e-05	3.0e-06
DTLZ7	2.27e-03	1.3e-04	7.95e-03	7.4e-03	1.68e-02	8.8e-03	2.82e-02	1.4e-03	2.27e-03	1.6e-04
LZ09F1	4.36e-04	1.9e-05	7.05e-04	2.9e-04	7.73e-04	6.2e-04	2.37e-04	1.5e-05	4.38e-04	2.3e-05
LZ09F2	6.08e-03	1.7e-03	1.09e-02	4.7e-03	8.11e-03	2.8e-03	4.70e-03	2.3e-03	3.43e-03	1.0e-03
LZ09F3	3.28e-03	1.0e-03	6.25e-03	2.7e-03	7.14e-03	3.3e-03	6.24e-03	3.1e-03	2.43e-03	2.9e-04
LZ09F4	3.27e-03	5.8e-04	3.96e-03	5.2e-04	5.84e-03	1.0e-03	3.21e-03	7.6e-04	2.53e-03	6.9e-04
LZ09F5	2.71e-03	9.2e-04	4.62e-03	2.5e-03	5.04e-03	2.3e-03	3.71e-03	1.9e-03	1.89e-03	3.0e-04
LZ09F6	7.53e-03	1.9e-03	8.72e-03	2.9e-03	1.66e-02	2.9e-03	1.92e-02	8.5e-04	6.19e-03	1.2e-03
LZ09F7	1.11e-02	3.7e-03	1.78e-02	5.8e-03	1.84e-02	5.8e-03	9.39e-03	4.7e-03	1.15e-02	3.9e-03
LZ09F8	9.49e-03	3.4e-03	1.86e-02	6.3e-03	1.73e-02	5.6e-03	1.10e-02	4.6e-03	1.23e-02	2.8e-03
LZ09F9	8.29e-03	3.3e-03	1.09e-02	3.3e-03	9.46e-03	2.1e-03	4.40e-03	1.5e-03	3.89e-03	1.2e-03
OKA1	2.91e-03	8.5e-04	5.16e-03	3.0e-03	5.19e-03	2.6e-03	2.69e-03	7.9e-04	3.54e-03	4.4e-04
OKA2	8.71e-03	2.0e-03	2.60e-02	1.0e-02	1.85e-02	8.6e-03	1.69e-02	9.4e-03	1.17e-02	5.6e-04
Viennet2	3.42e-04	5.4e-05	2.94e-04	2.9e-05	1.36e-03	4.0e-04	2.91e-03	1.4e-06	3.24e-04	3.5e-05
Viennet3	1.72e-04	1.6e-05	1.63e-04	1.4e-05	4.35e-03	2.7e-04	7.50e-03	1.7e-06	1.76e-04	2.6e-05
Poloni	1.00e-04	4.7e-06	7.08e-05	1.2e-06	1.01e-03	5.7e-04	5.59e-04	1.9e-05	9.42e-05	5.0e-06
Schaffer	2.24e-02	1.2e-02	2.53e-02	2.0e-02	2.69e-02	2.2e-02	2.60e-02	1.7e-02	3.13e-02	2.3e-02
Fonseca	3.17e-04	1.2e-05	2.16e-04	3.1e-06	2.57e-04	6.5e-06	2.04e-04	5.3e-07	2.82e-04	1.0e-05
Kursawe	1.82e-04	1.0e-05	1.25e-04	2.0e-06	1.28e-03	1.8e-04	1.74e-04	1.3e-06	1.59e-04	8.1e-06

Table 8. The average rankings of the algorithms for all the metrics

Algorithm	Ranking (HV)	Ranking (SPREAD)	Ranking (EPSILON)	Ranking (IGD)
NSGAII	3.319	3.138	2.583	2.611
MOCell	3.180	1.722	3.027	2.777
IBEA	2.527	3.972	3.555	4.111
MOEA/D	2.611	3.638	3.444	3.194
MOAAA	3.361	2.527	2.388	2.305

ACCEPTED MANUSCRIPT

Table 9. Wilcoxon's Rank Sum Test Results of MOAAA vs other algorithms for HV and SPREAD indicators

MOAAA vs	Indicator HV								Indicator SPREAD							
	NSGAII		MOCell		IBEA		MOEA/D		NSGAII		MOCell		IBEA		MOEA/D	
	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.
ZDT1	1.20E-16	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	9.54E-18	+
ZDT2	3.98E-15	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	2.07E-17	+
ZDT3	0.88761	-	7.71E-08	+	7.07E-18	+	7.07E-18	+	2.62E-17	+	1.05E-15	+	7.07E-18	+	7.07E-18	+
ZDT4	5.56E-12	+	1.25E-14	+	4.96E-15	+	1.29E-10	+	3.12E-09	+	1.37E-17	+	7.07E-18	+	2.05E-15	+
ZDT6	7.07E-18	+	7.07E-18	+	7.07E-18	+	1.35E-16	+	7.07E-18	+	1.35E-16	+	7.07E-18	+	7.07E-18	+
WFG1	7.08E-16	+	1.01E-17	+	1.15E-12	+	8.46E-18	+	1.62E-10	+	6.14E-03	+	6.88E-14	+	5.98E-17	+
WFG2	2.86E-15	+	2.94E-13	+	7.07E-18	+	7.07E-18	+	2.49E-05	+	1.01E-17	+	7.07E-18	+	7.07E-18	+
WFG3	3.79E-06	+	1.94E-15	+	1.01E-17	+	1.45E-17	+	5.15E-13	+	7.07E-18	+	9.42E-14	+	8.68E-10	+
WFG4	0.40225	-	7.07E-18	+	6.32E-09	+	7.07E-18	+	9.46E-10	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
WFG5	0.09872	-	4.73E-09	+	2.54E-16	+	7.75E-01	-	4.76E-07	+	7.07E-18	+	1.63E-17	+	3.32E-06	+
WFG6	2.44E-11	+	1.39E-11	+	4.21E-15	+	2.29E-04	+	8.53E-13	+	7.07E-18	+	7.07E-18	+	9.53E-17	+
WFG7	0.14106	-	7.07E-18	+	4.12E-07	+	5.31E-02	-	6.07E-09	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
WFG9	0.75901	-	0.000703	+	0.008713	+	3.39E-11	+	1.03E-08	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
DTLZ1	0.01318	+	0.009162	+	0.049807	+	0.000358	+	1.28E-09	+	1.20E-16	+	7.07E-18	+	7.07E-18	+
DTLZ2	0.01890	+	0.00427	+	7.07E-18	+	7.07E-18	+	0.00983	+	0.103019	-	9.42E-14	+	7.07E-18	+
DTLZ4	0.01722	+	0.001761	+	0.492709	-	7.06E-18	+	1.36E-07	+	2.97E-05	+	3.72E-01	-	7.07E-18	+
DTLZ5	0.41397	-	7.07E-18	+	7.07E-18	+	7.07E-18	+	0.47553	-	7.07E-18	+	7.50E-18	+	7.07E-18	+
DTLZ6	3.31E-20	+	3.31E-20	+	7.07E-18	+	5.45E-18	+	1.35E-16	+	2.81E-11	+	4.76E-16	+	1.12E-18	+
DTLZ7	3.90E-11	+	9.54E-18	+	1.74E-08	+	7.07E-18	+	3.58E-05	+	4.12E-10	+	4.66E-03	+	7.07E-18	+
LZ09F1	0.00011	+	0.759015	-	5.42E-13	+	7.07E-18	+	3.24E-11	+	3.07E-04	+	7.08E-16	+	9.52E-09	+
LZ09F2	3.95E-10	+	3.38E-16	+	2.14E-14	+	8.64E-05	+	1.06E-14	+	1.52E-09	+	3.57E-15	+	4.71E-10	+
LZ09F3	0.56021	-	1.76E-13	+	1.02E-03	+	3.43E-01	-	2.02E-10	+	9.02E-07	+	7.07E-18	+	5.14E-10	+
LZ09F4	0.00208	+	1.22E-13	+	7.82E-02	-	2.81E-11	+	0.018908	+	0.218492	-	7.07E-18	+	2.27E-16	+
LZ09F5	0.00130	+	2.51E-14	+	1.70E-07	+	1.35E-02	+	8.31E-10	+	8.99E-01	-	7.07E-18	+	7.12E-11	+
LZ09F6	4.16E-05	+	1.42E-06	+	4.86E-08	+	7.07E-18	+	0.964259	-	0.001098	+	4.27E-07	+	1.08E-04	+
LZ09F7	7.05E-07	+	9.81E-01	-	8.89E-03	+	2.57E-05	+	0.053148	-	0.36831	-	9.14E-05	+	6.17E-01	-
LZ09F8	4.20E-13	+	6.81E-03	+	1.26E-07	+	1.40E-04	+	0.033441	+	0.336206	-	0.541794	-	0.040276	+
LZ09F9	6.40E-10	+	3.26E-13	+	1.58E-13	+	2.72E-06	+	1.58E-09	+	9.88E-10	+	5.02E-14	+	1.43E-13	+
OKA1	7.07E-18	+	2.12E-10	+	8.64E-05	+	7.07E-18	+	2.54E-16	+	1.07E-16	+	7.55E-17	+	6.49E-11	+
OKA2	8.46E-18	+	1.42E-03	+	2.13E-01	-	2.90E-01	-	0.029117	+	1.09E-15	+	9.31E-01	-	1.27E-03	+
Viennet2	0.00256	+	6.49E-11	+	8.46E-18	+	7.07E-18	+	0.239836	-	0.012947	+	3.09E-11	+	1.35E-16	+
Viennet3	9.34E-07	+	1.73E-17	+	1.12E-10	+	7.07E-18	+	0.268528	-	0.10014	-	1.07E-10	+	7.07E-18	+
Poloni	5.70E-13	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	0.000546	+	2.65E-13	+	7.07E-18	+	7.07E-18	+
Schaffer	0.06410	-	0.049763	+	0.299372	-	0.247895	-	0.681654	-	0.22369	-	0.991749	-	1.29E-17	+
Fonseca	2.27E-16	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	8.99E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
Kursawe	2.51E-14	+	7.07E-18	+	7.07E-18	+	1.18E-04	+	1.45E-17	+	7.07E-18	+	7.07E-18	+	7.07E-18	+

ACCEPTED MANUSCRIPT

Table 10. Wilcoxon's Rank Sum Test Results of MOAAA vs other algorithms for EPSILON and IGD indicators

MOAAA vs	Indicator EPSILON								Indicator IGD							
	NSGAII		MOCell		IBEA		MOEA/D		NSGAII		MOCell		IBEA		MOEA/D	
	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.	p-value	Sig.
ZDT1	0.00306	+	7.07E-18	+	7.07E-18	+	8.94E-14	+	3.13E-17	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
ZDT2	0.00889	+	7.07E-18	+	2.11E-04	+	1.51E-16	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	2.71E-15	+
ZDT3	0.31252	-	8.59E-06	+	7.07E-18	+	7.07E-18	+	5.64E-17	+	2.38E-07	+	7.07E-18	+	7.07E-18	+
ZDT4	9.01E-12	+	6.34E-17	+	7.07E-18	+	2.16E-13	+	4.16E-12	+	3.38E-15	+	7.07E-18	+	1.52E-11	+
ZDT6	3.42E-08	+	7.50E-18	+	1.61E-01	-	3.97E-17	+	1.37E-17	+	2.20E-17	+	3.98E-01	-	1.35E-16	+
WFG1	3.32E-17	+	7.07E-18	+	1.64E-15	+	7.07E-18	+	8.48E-17	+	7.97E-18	+	5.32E-17	+	7.07E-18	+
WFG2	1.89E-08	+	3.78E-10	+	7.07E-18	+	7.07E-18	+	2.29E-15	+	5.61E-10	+	7.07E-18	+	7.07E-18	+
WFG3	0.01998	+	7.07E-18	+	8.46E-18	+	1.17E-15	+	3.83E-07	+	7.07E-18	+	1.01E-17	+	2.29E-15	+
WFG4	0.01597	+	7.07E-18	+	1.47E-15	+	7.07E-18	+	8.59E-06	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
WFG5	0.30823	-	0.233406	-	0.105196	-	1.46E-16	+	0.000148	+	7.07E-18	+	7.07E-18	+	8.55E-11	+
WFG6	2.29E-06	+	3.23E-02	+	3.02E-16	+	7.49E-16	+	5.16E-11	+	2.07E-06	+	7.07E-18	+	5.98E-01	-
WFG7	0.63185	-	7.07E-18	+	2.75E-09	+	7.07E-18	+	0.00102	+	7.07E-18	+	7.07E-18	+	3.83E-02	+
WFG9	0.89851	-	9.54E-18	+	3.47E-01	-	2.45E-03	+	0.166907	-	1.36E-13	+	7.07E-18	+	1.03E-09	+
DTLZ1	0.07587	-	0.002083	+	4.55E-05	+	2.69E-04	+	0.030679	+	0.010644	+	0.231666	-	5.12E-05	+
DTLZ2	0.50588	-	0.361016	-	1.63E-17	+	7.07E-18	+	0.074741	-	0.641693	-	7.07E-18	+	7.07E-18	+
DTLZ4	0.58365	-	0.833464	-	0.532699	-	7.07E-18	+	0.319174	-	0.497112	-	0.006673	+	7.07E-18	+
DTLZ5	0.50588	-	2.20E-17	+	7.07E-18	+	7.07E-18	+	0.002083	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
DTLZ6	7.07E-18	+	7.07E-18	+	7.06E-18	+	4.37E-18	+	7.07E-18	+	7.07E-18	+	7.07E-18	+	7.99E-19	+
DTLZ7	0.99725	-	5.69E-07	+	2.05E-09	+	7.07E-18	+	0.681672	-	1.08E-06	+	7.07E-18	+	7.07E-18	+
LZ09F1	0.13738	-	5.15E-13	+	7.92E-16	+	7.07E-18	+	0.602727	-	2.38E-07	+	8.49E-14	+	7.07E-18	+
LZ09F2	8.98E-08	+	1.55E-15	+	2.31E-12	+	8.03E-03	+	6.88E-14	+	8.99E-18	+	2.20E-17	+	4.56E-04	+
LZ09F3	7.71E-08	+	5.32E-17	+	1.85E-13	+	7.62E-10	+	2.12E-10	+	8.46E-18	+	8.46E-18	+	3.69E-09	+
LZ09F4	1.11E-06	+	6.43E-12	+	1.04E-12	+	3.29E-08	+	6.32E-09	+	8.49E-14	+	3.32E-17	+	1.37E-06	+
LZ09F5	3.90E-11	+	1.20E-16	+	4.76E-16	+	7.34E-13	+	1.72E-12	+	3.53E-17	+	8.99E-18	+	1.04E-11	+
LZ09F6	1.57E-05	+	6.62E-08	+	6.89E-15	+	8.99E-18	+	3.21E-06	+	7.15E-09	+	7.07E-18	+	7.07E-18	+
LZ09F7	0.03703	+	2.22E-06	+	1.14E-01	-	9.83E-03	+	0.555578	-	9.91E-09	+	4.18E-09	+	1.30E-03	+
LZ09F8	9.16E-06	+	6.22E-06	+	1.49E-01	-	5.31E-02	-	1.00E-06	+	1.17E-07	+	5.46E-06	+	3.35E-03	+
LZ09F9	2.56E-11	+	3.74E-17	+	3.02E-15	+	5.18E-04	+	3.11E-14	+	7.50E-18	+	7.50E-18	+	1.14E-01	-
OKA1	1.17E-08	+	1.50E-03	+	3.42E-04	+	3.56E-11	+	1.17E-07	+	3.80E-04	+	3.07E-03	+	6.31E-13	+
OKA2	5.88E-14	+	2.42E-05	+	2.24E-01	-	7.34E-02	-	4.76E-14	+	4.16E-05	+	4.09E-11	+	5.79E-01	-
Viennet2	0.45864	-	0.018219	+	0.003584	+	7.07E-18	+	0.075872	-	3.37E-05	+	7.07E-18	+	7.07E-18	+
Viennet3	0.25968	-	0.607539	-	0.000262	+	7.07E-18	+	0.969755	-	0.01568	+	7.07E-18	+	7.07E-18	+
Poloni	3.27E-05	+	7.07E-18	+	7.50E-18	+	1.35E-16	+	8.98E-08	+	7.07E-18	+	7.07E-18	+	7.07E-18	+
Schaffer	0.13556	-	0.067204	-	0.346702	-	0.357403	-	0.01926	+	0.036414	+	0.144829	-	0.107462	-
Fonseca	1.18E-05	+	7.07E-18	+	2.33E-17	+	7.07E-18	+	5.98E-17	+	7.07E-18	+	7.12E-17	+	7.07E-18	+
Kursawe	1.01E-05	+	1.14E-17	+	7.07E-18	+	1.52E-07	+	4.01E-16	+	7.07E-18	+	7.07E-18	+	3.38E-16	+

ACCEPTED MANUSCRIPT

ACCEPTED MANUSCRIPT

Table 11. Computational time of all the algorithms in second (mean of 50 runs)

	NSGAII	MOCell	IBEA	MOEA/D	MOAAA
ZDT1	0.187	0.308	1.765	0.107	0.168
ZDT2	0.186	0.300	1.791	0.099	0.164
ZDT3	0.185	0.295	1.783	0.101	0.158
ZDT4	0.156	0.163	1.819	0.074	0.167
ZDT6	0.152	0.177	1.963	0.076	0.160
WFG1	0.167	0.215	2.043	0.104	0.192
WFG2	0.136	0.184	2.063	0.077	0.157
WFG3	0.129	0.227	2.097	0.069	0.144
WFG4	0.146	0.243	2.099	0.080	0.153
WFG5	0.137	0.284	2.072	0.072	0.151
WFG6	0.133	0.219	2.105	0.069	0.145
WFG7	0.147	0.276	2.105	0.082	0.159
WFG9	0.178	0.281	2.140	0.106	0.181
DTLZ1	0.178	0.190	2.578	0.074	0.204
DTLZ2	0.187	0.584	2.572	0.079	0.193
DTLZ4	0.202	0.422	2.611	0.087	0.207
DTLZ5	0.166	0.343	2.540	0.076	0.165
DTLZ6	0.206	0.393	2.574	0.090	0.188
DTLZ7	0.228	0.556	2.639	0.097	0.208
LZ09F1	0.193	0.220	2.183	0.096	0.177
LZ09F2	0.322	0.397	2.436	0.181	0.245
LZ09F3	0.323	0.428	2.403	0.182	0.242
LZ09F4	0.320	0.441	2.394	0.187	0.241
LZ09F5	0.359	0.493	2.431	0.217	0.274
LZ09F6	0.235	0.384	2.665	0.112	0.219
LZ09F7	0.223	0.196	2.186	0.108	0.202
LZ09F8	0.232	0.197	2.208	0.110	0.203
LZ09F9	0.333	0.413	2.382	0.190	0.250
OKA1	0.148	0.089	2.085	0.062	0.158
OKA2	0.152	0.084	2.101	0.065	0.162
Viennet2	0.151	0.303	2.511	0.065	0.179
Viennet3	0.152	0.262	2.497	0.072	0.180
Poloni	0.151	0.258	2.051	0.077	0.169
Schaffer	0.181	0.025	2.106	0.054	0.198
Fonseca	0.139	0.178	2.084	0.067	0.162
Kursawe	0.156	0.184	2.097	0.076	0.180