

CS251 Homework 2: Sorting and Hashing

Out: Feb 2, 2018 @ 9:00 PM

Due: Feb 9, 2018 @ 11:59 PM

Submission Instructions: Please submit a typeset PDF on blackboard.

Short-Answer Questions:

- C 1. What are the worst-case time complexities of 'insert' and 'find' operations in a dictionary ADT, where n is the size of the dictionary?
- a) $O(1)$ & $O(1)$
 - b) $O(n)$ & $O(n)$
 - c) $O(1)$ & $O(n)$
 - d) $O(n)$ & $O(1)$
- D 2. Which of the following sorting algorithms has the lowest worst-case time complexity?
- a) Selection sort
 - b) Insertion sort
 - c) Bubble sort
 - d) Heap sort
- A 3. Which of the following sorting algorithms is not stable (i.e., the initial order of equal keys is maintained)?
- a) Selection sort
 - b) Insertion sort
 - c) Bubble sort
 - d) All sorting algorithms are stable
4. Suppose you have the following hash table, implemented using linear probing. The hash function we are using is the identity function, $h(x) = x \% 7$.

0	1	2	3	4	5	6
14	21	9	3	4	12	18

Which of the following is/are a possible order of insertion into the hash table? There may be multiple correct answers. Select all that apply.

- a) 9, 14, 4, 18, 12, 3, 21
- b) 12, 3, 14, 18, 4, 9, 21
- c) 12, 14, 3, 9, 4, 18, 21
- d) 9, 12, 14, 3, 4, 21, 18
- e) 12, 9, 18, 3, 14, 21, 4

5. What is the worst case running time of search with linear probing?
 - a) $O(1)$
 - b) $O(n)$
 - c) $O(\log n)$
 - d) $O(\sqrt{n})$

6. What are the 'expected running time of insertion' and 'worst case running time of deletion', in a hash table with quadratic probing to resolve collisions?
 - a) $O(\log n)$ and $O(\sqrt{n})$
 - b) $O(1)$ and $O(n)$
 - c) $O(\sqrt{n})$ and $O(\sqrt{n})$
 - d) $O(\sqrt{n})$ and $O(n)$

7. Using alphabet position, encode 'science' with the following hash map

$$p(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_{n-1} z^{n-1} \text{ with } z=19$$

8. Which of the following sorting algorithms takes linear time in the best case?
 - a. Selection sort
 - b. Insertion sort
 - c. Bubble sort
 - d. Heap sort

9. A heap with n keys has a height of:
 - a. $\theta(n)$
 - b. $\theta(\log n)$
 - c. $\theta(n \log n)$
 - d. $\theta(\log^2 n)$

10. Time complexity of getMin() and removeMin() operations on a min-heap are respectively
 - a. $\theta(1)$ & $\theta(\log n)$
 - b. $\theta(\log n)$ & $\theta(\log n)$
 - c. $\theta(\log n)$ & $\theta(1)$
 - d. $\theta(1)$ & $\theta(n)$

Long-Answer Questions:

11. Given two sets A and B represented as sorted sequences, describe an efficient algorithm for computing $A \oplus B$, which is the set of elements that are in A or B , but not in both.
12. Let S be a random permutation of n distinct integers. Argue that the expected running time of insertion-sort on S is $\Omega(n^2)$.
(Hint: Note that half of the elements ranked in the top half of a sorted version of S are expected to be in the first half of S .)
13. Come up a compression map (function) that results in no collisions for the following data for a 7-entry hash table: 55, 45, 12, 1, 4, 23, 7. Draw the resulting hash table.
14. Draw the 7-entry hash table that results from hash function, $h(i) = (2i+5)\%7$, to hash the keys 12, 34, 13, 78, 21, 91, 11, 39, 10, 6, and 15, assuming collisions are handled by chaining.
15. Does a preorder traversal of a heap list its keys in non-decreasing order? Argue that it is right or give a counterexample to disprove.