

- 1 **Example 1 — A Dense Example**
- 2 **Example 2 — A Dense Example**
- 3 **Example 3 — A Dense Example**
- 4 **Example 4 — A MDS Example**
- 5 **Example 5 — A MDS Example**
- 6 **Example 6 — A Convex Sparse Example**

$$\min_{x \in \mathbb{R}^n} \quad \frac{a}{4} \sum_{i=1}^n (x_i - 1)^4 \quad (1)$$

$$\text{s.t.} \quad 4ax_1 + 2ax_2 = 10a \quad (2)$$

$$5a \leq 2ax_1 + ax_3 \quad (3)$$

$$a \leq 2ax_1 + 0.5ax_i \leq 2an \quad (4)$$

$$x_1 \text{ free}, \quad (5)$$

$$0.0 \leq x_2, \quad (6)$$

$$1.5 \leq x_3 \leq 10 \quad (7)$$

$$0.5 \leq x_i, \quad \forall i = 4, \dots, n \quad (8)$$

Here  $n \geq 3$  and  $a > 0$  are parameters which can be tuned via user's inputs. If they are not specified by the user, they are set to the default values,  $n = 3$  and  $a = 1.0$ .

The analytical solution is obtained at  $x_1 = 1.75$ ,  $x_2 = x_3 = 1.5$ , and  $x_i = 1$  for  $i \geq 4$ . The objective value is 0.110352. File `nlpSparse.ex6_driver.cpp` provides more details about the usage of this example.

## 7 Example 7 — A Nonconvex Sparse Example

$$\min_{x \in \mathbb{R}^n} \quad -\frac{a}{4} \sum_{i=1}^n (x_i - 1)^4 + 0.5 \sum_{i=1}^n x_i^2 \quad (9)$$

$$\text{s.t.} \quad 4x_1 + 2x_2 = 10 \quad (10)$$

$$4x_1 + 2x_2 = 10 \quad (11)$$

$$5 \leq 2x_1 + x_3 \quad (12)$$

$$4x_1 + 2x_3 \leq 19 \quad (13)$$

$$1 \leq 2x_1 + 0.5x_i \leq 2n \quad (14)$$

$$x_1 \text{ free}, \quad (15)$$

$$0.0 \leq x_2, \quad (16)$$

$$1.5 \leq x_3 \leq 10 \quad (17)$$

$$0.5 \leq x_i, \quad \forall i = 4, \dots, n \quad (18)$$

Here  $n \geq 3$  and  $a > 0$  are parameters which can be tuned via user's inputs. If they are not specified by the user, they are set to the default values,  $n = 3$  and  $a = 0.1$ . Note that the equality constraints (10) and (11) are duplicate. As a result, the constraint Jacobian matrix is rank deficient and may introduce numerical difficulties. File `nlpSparse_ex7_driver.cpp` provides more details about the usage of this example.

## 8 Example 8 — A Primal Decomposition Example

## 9 Example 9 — A Primal Decomposition Example