Introduction to IPOPT:

A tutorial for downloading, installing, and using IPOPT.

Revision number of this document: Revision: 661

March 14, 2006

Abstract

This document is a guide to using IPOPT 3.1 (the new C++ version of IPOPT). It includes instructions on how to obtain and compile IPOPT, a description of the interface, user options, etc.,, as well as a tutorial on how to solve a nonlinear optimization problem with IPOPT.

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1 Introduction

IPOPT (Interior Point Optimizer, pronounced "I-P-Opt") is an open source software package for largescale nonlinear optimization. It can be used to solve general nonlinear programming problems of the form

$$\min_{x \in \mathbb{R}^n} \qquad f(x) \tag{1}$$

s.t.
$$g^L \le g(x) \le g^U$$
 (2) $x^L \le x \le x^U$, (3)

$$x^L < x < x^U, \tag{3}$$

where $x \in \mathbb{R}^n$ are the optimization variables (possibly with lower and upper bounds, $x^L \in (\mathbb{R} \cup \{-\infty\})^n$ and $x^U \in (\mathbb{R} \cup \{+\infty\})^n$, $f: \mathbb{R}^n \longrightarrow \mathbb{R}$ is the objective function, and $g: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ are the general nonlinear constraints. The functions f(x) and g(x) can be linear or nonlinear and convex or non-convex (but should be twice continuously differentiable). The constraints, g(x), have lower and upper bounds, $g^L \in (\mathbb{R} \cup \{-\infty\})^n$ and $g^U \in (\mathbb{R} \cup \{+\infty\})^m$. Note that equality constraints of the form $g_i(x) = \bar{g}_i$ can be specified by setting $g_i^L = g_i^U = \bar{g}_i$.

Mathematical Background 1.1

IPOPT implements an interior point line search filter method that aims to find a local solution of (1)-(3). The mathematical details of the algorithm can be found in several publications [3, 4, 7, 6, 5].

1.2Availability

The IPOPT package is available from COIN-OR (www.coin-or.org) under the CPL (Common Public License) open-source license and includes the source code for IPOPT. This means, it is available free of charge, also for commercial purposes. However, if you give away software including IPOPT code (in source code or binary form) and you made changes to the IPOPT source code, you are required to make those changes public and to clearly indicate which modifications you made. After all, the goal of open source software is the continuous development and improvement of software. For details, please refer to the Common Public License.

Also, if you are using IPOPT to obtain results for a publication, we politely ask you to point out in your paper that you used IPOPT, and to cite the publication [7]. Writing high-quality numerical software takes a lot of time and effort, and does usually not translate into a large number of publications, therefore we believe this request is only fair:).

1.3 Prerequisites

In order to build IPOPT, some third party components are required:

- BLAS (Basic Linear Algebra Subroutines). Many vendors of compilers and operating systems
 provide precompiled and optimized libraries for these dense linear algebra subroutines. But you
 can also get the source code from www.netlib.org and have the IPOPT distribution compile it
 automatically.
- LAPACK (Linear Algebra PACKage). Also for LAPACK, some vendors offer precompiled and optimized libraries. But like with BLAS, you can get the source code from www.netlib.org and have the IPOPT distribution compile it automatically.

Note that currently LAPACK is only required if you intend to use the quasi-Newton options in IPOPT. You can compile the code without LAPACK, but an error message will then occur if you try to run the code with an option that requires LAPACK. Currently, the LAPACK routines that are used by IPOPT are only DPOTRF, DPOTRS, and DSYEV.

• A sparse symmetric indefinite linear solver. The IPOPT needs to obtain the solution of sparse, symmetric, indefinite linear systems, and for this it relies on third-party code.

Currently, the following linear solvers can be used:

```
- MA27 from the Harwell Subroutine Library (see http://www.cse.clrc.ac.uk/nag/hsl/).
```

- MA57 from the Harwell Subroutine Library (see http://www.cse.clrc.ac.uk/nag/hsl/).
- The Watson Sparse Matrix Package (WSMP) (see http://www-users.cs.umn.edu/~agupta/wsmp.html)
- The Parallel Sparse Direct Linear Solver (PARDISO) (see http://www.computational.unibas.ch/cs/scicomp/software/pardiso/).

You need to include at least one of the linear solvers above in order to run IPOPT. Currently, there is development on integrating also TAUCS and MUMPS by contributors, but this work has not yet been completed.

Interfaces to other linear solvers might be added in the future; if you are interested in contributing such an interface please contact us! Note that IPOPT requires that the linear solver is able to provide the inertia (number of positive and negative eigenvalues) of the symmetric matrix that is factorized.

• Furthermore, IPOPT can also use the Harwell Subroutine MC19 for scaling of the linear systems before they are passed to the linear solver. This may be particularly useful if IPOPT is used with MA27 or MA57. However, it is not required to have MC19 to compile IPOPT; if this routine is missing, the scaling is never performed⁴.

⁴There are more recent scaling routines in the HSL, but they have not (yet) been integrated. Contributions are welcome!

• ASL (AMPL Solver Library). The source code is available at www.netlib.org, and the IPOPT makefiles will automatically compile it for you if you put the source code into a designated space. NOTE: This is only required if you want to use IPOPT from AMPL and want to compile the IPOPT AMPL solver executable.

For more information on third-party components and how to obtain them, see Section 2.2.

Since the IPOPT code is written in C++, you will need a C++ compiler to build the IPOPT library. We tried very hard to write the code as platform and compiler independent as possible.

In addition, the configuration script currently also searches for a Fortran, since some of the dependencies above are written in Fortran. If all third party dependencies are available as self-contained libraries, those compilers are in principle not necessary. Also, it is possible to use the Fortran-to-C compiler f2c from www.netlib.org to convert Fortran code to C, and compile the resulting C files with a C compiler and create a library containing the required third party dependencies. But so far we have not tested this ourselves, and currently the configuration script for IPOPT looks for a Fortran compiler.

1.4 How to use IPOPT

If desired, the IPOPT distribution generates an executable for the modeling environment AMPL. As well, you can link your problem statement with IPOPT using interfaces for C++, C, or Fortran. IPOPT can be used with most Linux/Unix environments, and on Windows using Visual Studio .NET or Cygwin. Below in Section 3 this document demonstrates how to solve problems using IPOPT. This includes installation and compilation of IPOPT for use with AMPL as well as linking with your own code.

Finally, the IPOPT distribution includes an interface for CUTEr⁵, if you want to use IPOPT to solve problems modeled in SIF.

The old (Fortran 2.x) version of IPOPT has been interfaced with Matlab, the NLPAPI on COIN, and is also available on NEOS. The new version might be available through similar means in the future. Please check the IPOPT homepage for updates.

1.5 More Information and Contributions

More and up-to-date information can be found at the IPOPT homepage,

Here, you can find FAQs, some (hopefully useful) hints, a bug report system etc. The website is managed with Wiki, which means that every user can edit the webpages from the regular web browser. In particular, we encourage IPOPT users to share their experiences and usage hints on the "Success Stories" and "Hints and Tricks" pages⁶

IPOPT is an open source project, and we encourage people to contribute code (such as interfaces to appropriate linear solvers, modeling environments, or even algorithmic features). If you are interested in contributing code, please have a look at the COIN constributions webpage⁷, and contact the IPOPT project leader.

There is also a mailing list for IPOPT, available from the webpage

where you can subscribe to get notified of updates, and to ask general questions regarding installation and usage. (You might want to look at the archives before posting a question.)

We try to answer questions posted to the mailing list in a reasonable manner. Please understand that we cannot answer all questions in detail, and because of time constraints, we may not be able to help you

⁵see http://cuter.rl.ac.uk/cuter-www/

⁶Since we had some malicious hacker attacks destroying the content of the web pages in the past, you are now required to enter a user name and password; simply follow the instructions in the last paragraph of the Documentation section on the main project page.

⁷see http://www.coin-or.org/contributions.html

model and debug your particular optimization problem. However, if you have a challenging optimization problem and are interested in consulting services by IBM Research, please contact the IPOPT project leader, Andreas Wächter.

1.6 History of IPOPT

The original IPOPT (Fortran version) was a product of the dissertation research of Andreas Wächter [4], under Lorenz T. Biegler at the Chemical Engineering Department at Carnegie Mellon University. The code was made open source and distributed by the COIN-OR initiative, which is now a non-profit corporation. IPOPT has been actively developed under COIN-OR since 2002.

To continue natural extension of the code and allow easy addition of new features, IBM Research decided to invest in an open source re-write of IPOPT in C++. The new C++ version of the IPOPT optimization code (IPOPT 3.0.0 and beyond) is currently developed at IBM Research and remains part of the COIN-OR initiative. Future development on the Fortran version will cease with the exception of occasional bug fix releases.

2 Installing IPOPT

The following sections describe the installation procedures on UNIX/Linux systems. For installation instructions on Windows see Section 2.4.

2.1 Getting the IPOPT Code

IPOPT is available from the COIN-OR subversion repository. You can either download the code using svn (the *subversion*⁸ client similar to CVS) or simply retrieve a tarball (compressed archive file). While the tarball is an easy method to retrieve the code, using the *subversion* system allows users the benefits of the version control system, including easy updates and revision control.

2.1.1 Getting the IPOPT code via subversion

Of course, the *subversion* client must be installed on your system if you want to obtain the code this way (the executable is called svn); it is already installed by default for many recent Linux distributions. Information about *subversion* and how to download it can be found at http://subversion.tigris.org/.

To obtain the IPOPT source code via subversion, change into the directory in which you want to create a subdirectory Ipopt with the IPOPT source code. Then follow the steps below:

1. Download the code from the repository

```
$ svn co https://www.coin-or.org/svn/Ipopt/trunk Ipopt
Note: The $ indicates the command line prompt, do not type $, only the text following it.
```

- 2. Change into the root directory of the IPOPT distribution
 - \$ cd Ipopt

In the following, "\$IPOPTDIR" will refer to the directory in which you are right now (output of pwd).

2.1.2 Getting the IPOPT code as a tarball

To use the tarball, follow the steps below:

1. Download the latest tarball from http://www.coin-or.org/Tarballs. The file you should look for has the form ipopt-3.x.x.tar.gz (where "3.x.x." is the version number). Put this file in a directory under which you want to put the IPOPT installation.

⁸see http://subversion.tigris.org/

2. Issue the following commands to unpack the archive file:

```
$ gunzip ipopt-3.x.x.tar.gz
$ tar xvf ipopt-3.x.x.tar
```

Note: The \$ indicates the command line prompt, do not type \$, only the text following it.

3. Change into the root directory of the IPOPT distribution

```
$ cd ipopt-3.x.x
```

In the following, "\$IPOPTDIR" will refer to the directory in which you are right now (output of pwd).

2.2 Download External Code

IPOPT uses a few external packages that are not included in the IPOPT source code distribution, namely ASL (the AMPL Solver Library), BLAS, LAPACK. It also requires a sparse symmetric linear solver.

Since this third party software released under different licenses than IPOPT, we cannot distribute that code together with the IPOPT packages and have to ask you to go through the hassle of obtaining it yourself (even though we tried to make it as easy for you as we could). Keep in mind that it is still your responsibility to ensure that your downloading and usage if the third party components conforms with their licenses.

Note that you only need to obtain the ASL if you intend to use IPOPT from AMPL. It is not required if you want to specify your optimization problem in a programming language (C++, C, or Fortran). Also, currently, LAPACK is only required if you intend to use the quasi-Newton options implemented in IPOPT.

2.2.1 Download BLAS, LAPACK and ASL

If you have the download utility wget installed on your system, retrieving BLAS, LAPACK, and ASL is straightforward using scripts included with the ipopt distribution. These scripts download the required files from the Netlib Repository (www.netlib.org).

```
$ cd $IPOPTDIR/Extern/blas
$ ./get.blas
$ cd ../lapack
$ ./get.lapack
$ cd ../ASL
$ ./get.ASL
```

If you do not have wget installed on your system, please read the INSTALL.* files in the \$IPOPTDIR/Extern/blas, \$IPOPTDIR/Extern/lapack and \$IPOPTDIR/Extern/ASL directories for alternative instructions.

2.2.2 Download HSL Subroutines

IPOPT requires a sparse symmetric linear solver. There are different possibilities. In this section we describe how to obtain the source code for MA27 (and MC19) from the Harwell Subroutine Library (HSL). Those routines are freely available for non-commercial, academic use, but it is your responsibility to investigate the licensing of all third party code.

The use of alternative linear solvers is described in Appendix D. You do not necessarily have to use MA27 as described in this section, but at least one linear solver is required for IPOPT to function.

- 1. Go to http://hsl.rl.ac.uk/archive/hslarchive.html
- 2. Follow the instruction on the website, read the license, and submit the registration form.
- 3. Go to HSL Archive Programs, and find the package list.

- 4. In your browser window, click on MA27.
- 5. Make sure that Double precision: is checked. Click Download package (comments removed)
- 6. Save the file as ma27ad.f in \$IPOPTDIR/Extern/HSL/

Note: Some browsers append a file extension (.txt) when you save the file, in which case you have to rename it.

- 7. Go back to the package list using the back button of your browser.
- 8. In your browser window, click on MC19.
- 9. Make sure Double precision: is checked. Click Download package (comments removed)
- 10. Save the file as mc19ad.f in \$IPOPTDIR/Extern/HSL/

Note: Some browsers append a file extension (.txt) when you save the file, so you may have to rename it.

Note: Whereas currently obtaining MA27 is essential for using IPOPT, MC19 could be omitted (with the consequence that you cannot use this method for scaling the linear systems arising inside the IPOPT algorithm).

Note: If you have the source code for the linear solver MA57 (successor of MA27) in a file called ma57ad.f (including all dependencies), you can simply put it into the \$IPOPTDIR/Extern/HSL/ directory. The IPOPT configuration script will then find this file and compile it into the IPOPT library (just as is would compile MA27).

2.3 Compiling and Installing IPOPT

IPOPT can be easily compiled and installed with the usual configure, make, make install commands. Below are the basic steps that should work on most systems. For special compilations and for some troubleshooting see Appendix D and consult the IPOPT homepage before submitting a ticket or sending a message to the mailing list.

- 1. Go to the main directory of IPOPT:
 - \$ cd \$IPOPTDIR
- 2. Run the configure script
 - \$./configure

If the last output line of the script reads "configure: Configuration successful" then everything worked fine. Otherwise, look at the screen output, have a look at the config.log output file and/or consult Appendix D.

The default configure (without any options) is sufficient for most users. If you want to see the configure options, consult Appendix D.

- 3. Build the code
 - \$ make
- 4. Install IPOPT
 - \$ make install

This installs

- the IPOPT AMPL solver executable (if ASL source was downloaded) in \$IPOPTDIR/bin,
- the IPOPT library (libipopt.a) in \$IPOPTDIR/lib,

- text files ipopt_addlibs_cpp.txt and ipopt_addlibs_f.txt in \$IPOPTDIR/lib that contain a line each with additional linking flags that are required for linking code with the ipopt library, for C++ and Fortran main programs, respectively. (This is only for convenience if you want to find out what additional flags are required, for example, to include the Fortran runtime libraries with a C++ compiler.)
- the necessary header files in \$IPOPTDIR/include/ipopt.

You can change the default installation directory (here \$IPOPTDIR) to something else (such as /usr/local) by using the --prefix switch for configure.

5. Install IPOPT for use with CUTEr

If you have CUTEr already installed on your system and you want to use IPOPT as a solver for problems modeled in SIF, type

\$ make cuter

This assumes that you have the environment variable MYCUTER defined according to the CUTEr instructions. After this, you can use the script sdipo as the CUTEr script to solve a SIF model.

Note: It is possible to compile the code in directories separate from the source files. This comes in handy when you want to compile the code with different compilers, compiler options, or different operating system that share a common file system. To use this feature, change into the directory where you want to compile the code, and then type \$IPOPTDIR/configure with all the options (replacing \$IPOPTDIR by the path to configure). For this, the directories with the IPOPT source must not have any configuration and compiled code.

2.4 Installation on Windows

There are two ways to install IPOPT on Windows systems. The first option, described in Section 2.4.1, is to use Cygwin (see www.cygwin.com), which offers a UNIX-like environment on Windows and in which the installation procedure described earlier in this section can be used. The IPOPT distribution also includes projects files for the Microsoft Visual Studio (see Section 2.4.2).

2.4.1 Installation with Cygwin

Cygwin is a Linux-like environment for Windows; if you don't know what it is you might want to have a look at the Cygwin homepage, www.cygwin.com.

It is possible to build the IPOPT AMPL solver executable in Cygwin for general use in Windows. You can also hook up IPOPT to your own program if you compile it in the Cygwin environment⁹.

If you want to compile IPOPT under Cygwin, you first have to install Cygwin on your Windows system. This is pretty straight forward; you simply download the "setup" program from www.cygwin.com and start it.

Then you do the following steps (assuming here that you don't have any complications with firewall settings etc - in that case you might have to choose some connection settings differently):

- 1. Click next
- 2. Select "install from the internet" (default) and click next
- 3. Select a directory where Cygwin is to be installed (you can leave the default) and choose all other things to your liking, then click next
- 4. Select a temp dir for Cygwin setup to store some files (if you put it on your desktop you will later remember to delete it)
- 5. Select "direct connection" (default) and click next

⁹It is also possible to build an IPOPT DLL that can be used from non-cygwin compilers, but this is not (yet?) supported.

- 6. Select some mirror site that seems close by to you and click next
- 7. OK, now comes the complicated part:

You need to select the packages that you want to have installed. By default, there are already selections, but the compilers are usually not pre-chosen. You need to make sure that you select the GNU compilers (for Fortran, C, and C++ — together with the MinGW options), the GNU Make, and Subversion. For this, click on the "Devel" branch (which opens a subtree) and select:

- gcc
- gcc-core
- gcc-g77
- gcc-g++
- gcc-mingw
- gcc-mingw-core
- gcc-mingw-g77
- gcc-mingw-g++
- make
- subversion

Then, in the "Web" branch, please select "wget" (which will make the installation of third party dependencies for IPOPT easier)

This will automatically also select some other packages.

- 8. Then you click on next, and Cygwin will be installed (follow the rest of the instructions and choose everything else to your liking). At a later point you can easily add/remove packages with the setup program.
- 9. Now that you have Cygwin, you can open a Cygwin window, which is like a UNIX shell window.
- 10. Now you just follow the instructions in the beginning of Sections 2: You download the IPOPT code into your Cygwin home directory (from the Windows explorer that is usually something like C:\Cygwin\home\your_user_name). After that you obtain the third party code (like on Linux/UNIX), type

```
./configure
```

make install

in the correct directories, and hopefully that will work. The IPOPT AMPL solver executable will be in the subdirectory bin (called "ipopt.exe").

2.4.2 Using Visual Studio

The IPOPT distribution includes project files that can be used to compile the IPOPT library and a Fortran and C++ example within the Microsoft Visual Studio. The project files have been created with Microsoft Visual C++ .NET 2003 Standard, and the Intel Visual Fortran Compiler 8.1.

In order to use those project files, download the IPOPT source code, as well as the required third party code (put it into the Extern/blas, Extern/lapack, and Extern/HSL directories—ASL is not required for the Fortran and C examples). Then open the solution file

```
$IPOPTDIR\Windows\VisualStudio_dotNET\Ipopt\Ipopt.sln
```

Note: Since the project files were created only with the Standard edition of the C++ compiler, code optimization might be disabled; for fast performance make sure you enable code optimization.

3 Interfacing your NLP to IPOPT: A tutorial example.

IPOPT has been designed to be flexible for a wide variety of applications, and there are a number of ways to interface with IPOPT that allow specific data structures and linear solver techniques. Nevertheless, the authors have included a standard representation that should meet the needs of most users.

This tutorial will discuss four interfaces to IPOPT, namely the AMPL modeling language[1] interface, and the C++, C, and Fortran code interfaces. AMPL is a 3rd party modeling language tool that allows users to write their optimization problem in a syntax that resembles the way the problem would be written mathematically. Once the problem has been formulated in AMPL, the problem can be easily solved using the (already compiled) IPOPT AMPL solver executable, ipopt. Interfacing your problem by directly linking code requires more effort to write, but can be far more efficient for large problems.

We will illustrate how to use each of the four interfaces using an example problem, number 71 from the Hock-Schittkowsky test suite [2],

$$\min_{x \in \Re^4} \qquad x_1 x_4 (x_1 + x_2 + x_3) + x_3 \tag{4}$$

s.t.
$$x_1 x_2 x_3 x_4 \ge 25$$
 (5)

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 40 (6)$$

$$1 \le x_1, x_2, x_3, x_4 \le 5,\tag{7}$$

with the starting point

$$x_0 = (1, 5, 5, 1) \tag{8}$$

and the optimal solution

 $x_* = (1.00000000, 4.74299963, 3.82114998, 1.37940829).$

You can find further, less documented examples for using IPOPT from your own source code in the Examples subdirectory.

3.1 Using IPOPT through AMPL

Using the AMPL solver executable is by far the easiest way to solve a problem with IPOPT. The user must simply formulate the problem in AMPL syntax, and solve the problem through the AMPL environment. There are drawbacks, however. AMPL is a 3rd party package and, as such, must be appropriately licensed (a free student version for limited problem size is available from the AMPL website, www.ampl.com). Furthermore, the AMPL environment may be prohibitive for very large problems. Nevertheless, formulating the problem in AMPL is straightforward and even for large problems, it is often used as a prototyping tool before using one of the code interfaces.

This tutorial is not intended as a guide to formulating models in AMPL. If you are not already familiar with AMPL, please consult [1].

The problem presented in equations (4)–(8) can be solved with IPOPT with the AMPL model file given in Figure 1.

The line, "option solver ipopt;" tells AMPL to use IPOPT as the solver. The IPOPT executable (installed in Section 2.3) must be in the PATH for AMPL to find it. The remaining lines specify the problem in AMPL format. The problem can now be solved by starting AMPL and loading the mod file:

```
$ ampl
> model hs071_ampl.mod;
.
```

The problem will be solved using IPOPT and the solution will be displayed.

At this point, AMPL users may wish to skip the sections about interfacing with code, but should read Section 5 concerning IPOPT options, and Section 6 which explains the output displayed by IPOPT.

```
\mbox{\tt\#} tell ampl to use the ipopt executable as a solver
# make sure ipopt is in the path!
option solver ipopt;
# declare the variables and their bounds,
# set notation could be used, but this is straightforward
var x1 >= 1, <= 5;</pre>
var x2 >= 1, <= 5;
var x3 >= 1, <= 5;
var x4 >= 1, <= 5;
# specify the objective function
minimize obj:
                x1 * x4 * (x1 + x2 + x3) + x3;
# specify the constraints
        inequality:
                x1 * x2 * x3 * x4 >= 25;
        equality:
                x1^2 + x2^2 + x3^2 + x4^2 = 40;
# specify the starting point
let x1 := 1;
let x2 := 5;
let x3 := 5;
let x4 := 1;
# solve the problem
solve;
# print the solution
display x1;
display x2;
display x3;
display x4;
```

Figure 1: AMPL model file hs071_ampl.mod

1. Problem dimensions

- number of variables
- number of constraints

2. Problem bounds

- variable bounds
- constraint bounds

3. Initial starting point

- Initial values for the primal x variables
- Initial values for the multipliers (only required for a warm start option)

4. Problem Structure

- number of nonzeros in the Jacobian of the constraints
- number of nonzeros in the Hessian of the Lagrangian function
- sparsity structure of the Jacobian of the constraints
- sparsity structure of the Hessian of the Lagrangian function

5. Evaluation of Problem Functions

Information evaluated using a given point (x, λ, σ_f) coming from IPOPT

- Objective function, f(x)
- Gradient of the objective $\nabla f(x)$
- Constraint function values, g(x)
- Jacobian of the constraints, $\nabla g(x)^T$
- Hessian of the Lagrangian function, $\sigma_f \nabla^2 f(x) + \sum_{i=1}^m \lambda_i \nabla^2 g_i(x)$ (this is not required if a quasi-Newton options is chosen to approximate the second derivatives)

Figure 2: Information required by IPOPT

3.2 Interfacing with IPOPT through code

In order to solve a problem, IPOPT needs more information than just the problem definition (for example, the derivative information). If you are using a modeling language like AMPL, the extra information is provided by the modeling tool and the IPOPT interface. When interfacing with IPOPT through your own code, however, you must provide this additional information.

The information required by IPOPT is shown in Figure 2. The problem dimensions and bounds are straightforward and come solely from the problem definition. The initial starting point is used by the algorithm when it begins iterating to solve the problem. If IPOPT has difficulty converging, or if it converges to a locally infeasible point, adjusting the starting point may help. Depending on the starting point, IPOPT may also converge to different local solutions.

Providing the sparsity structure of derivative matrices is a bit more involved. IPOPT is a nonlinear programming solver that is designed for solving large-scale, sparse problems. While IPOPT can be customized for a variety of matrix formats, the triplet format is used for the standard interfaces in this tutorial. For an overview of the triplet format for sparse matrices, see Appendix A. Before solving the problem, IPOPT needs to know the number of nonzero elements and the sparsity structure (row and column indices of each of the nonzero entries) of the constraint Jacobian and the Lagrangian function Hessian. Once defined, this nonzero structure MUST remain constant for the entire optimization proce-

dure. This means that the structure needs to include entries for any element that could ever be nonzero, not only those that are nonzero at the starting point.

As IPOPT iterates, it will need the values for Item 5. in Figure 2 evaluated at particular points. Before we can begin coding the interface, however, we need to work out the details of these equations symbolically for example problem (4)-(7).

The gradient of the objective f(x) is given by

$$\begin{bmatrix} x_1x_4 + x_4(x_1 + x_2 + x_3) \\ x_1x_4 \\ x_1x_4 + 1 \\ x_1(x_1 + x_2 + x_3) \end{bmatrix},$$

and the Jacobian of the constraints g(x) is

$$\left[\begin{array}{cccc} x_2x_3x_4 & x_1x_3x_4 & x_1x_2x_4 & x_1x_2x_3 \\ 2x_1 & 2x_2 & 2x_3 & 2x_4 \end{array}\right].$$

We also need to determine the Hessian of the Lagrangian¹⁰. The Lagrangian function for the NLP (4)-(7) is defined as $f(x) + g(x)^T \lambda$ and the Hessian of the Lagrangian function is, technically, $\nabla^2 f(x_k) + \sum_{i=1}^m \lambda_i \nabla^2 g_i(x_k)$. However, so that IPOPT can ask for the Hessian of the objective or the constraints independently if required, we introduce a factor (σ_f) in front of the objective term. For IPOPT then, the symbolic form of the Hessian of the Lagrangian is

$$\sigma_f \nabla^2 f(x_k) + \sum_{i=1}^m \lambda_i \nabla^2 g_i(x_k) \tag{9}$$

(with the σ_f parameter), and for the example problem this becomes

$$\sigma_{f} \begin{bmatrix} 2x_{4} & x_{4} & x_{4} & 2x_{1} + x_{2} + x_{3} \\ x_{4} & 0 & 0 & x_{1} \\ x_{4} & 0 & 0 & x_{1} \\ 2x_{1} + x_{2} + x_{3} & x_{1} & x_{1} & 0 \end{bmatrix} + \lambda_{1} \begin{bmatrix} 0 & x_{3}x_{4} & x_{2}x_{4} & x_{2}x_{3} \\ x_{3}x_{4} & 0 & x_{1}x_{4} & x_{1}x_{3} \\ x_{2}x_{4} & x_{1}x_{4} & 0 & x_{1}x_{2} \\ x_{2}x_{3} & x_{1}x_{3} & x_{1}x_{2} & 0 \end{bmatrix} + \lambda_{2} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

where the first term comes from the Hessian of the objective function, and the second and third term from the Hessian of the constraints (5) and (6), respectively. Therefore, the dual variables λ_1 and λ_2 are then the multipliers for constraints (5) and (6), respectively.

The remaining sections of the tutorial will lead you through the coding required to solve example problem (4)–(7) using, first C++, then C, and finally Fortran. Completed versions of these examples can be found in \$IPOPTDIR/Examples under hs071_cpp, hs071_c, hs071_f.

As a user, you are responsible for coding two sections of the program that solves a problem using IPOPT: the main executable (e.g., main) and the problem representation. Typically, you will write an executable that prepares the problem, and then passes control over to IPOPT through an Optimize or Solve call. In this call, you will give IPOPT everything that it requires to call back to your code whenever it needs functions evaluated (like the objective function, the Jacobian of the constraints, etc.). In each of the three sections that follow (C++, C, and Fortran), we will first discuss how to code the problem representation, and then how to code the executable.

3.3 The C++ Interface

This tutorial assumes that you are familiar with the C++ programming language, however, we will lead you through each step of the implementation. For the problem representation, we will create a class that

¹⁰If a quasi-Newton option is chosen to approximate the second derivatives, this is not required. However, if second derivatives can be computed, it is often worthwhile to let IPOPT use them, since the algorithm is then usually more robust and converges faster. More on the quasi-Newton approximation in Section 4.2.

inherits off of the pure virtual base class, TNLP (IpTNLP.hpp). For the executable (the main function) we will make the call to IPOPT through the IpoptApplication class (IpIpoptApplication.hpp). In addition, we will also be using the SmartPtr class (IpSmartPtr.hpp) which implements a reference counting pointer that takes care of memory management (object deletion) for you (for details, see Appendix B).

After "make install" (see Section 2.3), the header files are installed in \$IPOPTDIR/include/ipopt (or in \$PREFIX/include/ipopt if the switch --prefix=\$PREFIX was used for configure).

3.3.1 Coding the Problem Representation

We provide the information required in Figure 2 by coding the HSO71_NLP class, a specific implementation of the TNLP base class. In the executable, we will create an instance of the HSO71_NLP class and give this class to IPOPT so it can evaluate the problem functions through the TNLP interface. If you have any difficulty as the implementation proceeds, have a look at the completed example in the Examples/hsO71_cpp directory.

Start by creating a new directory under Examples, called MyExample and create the files hs071_nlp.hpp and hs071_nlp.cpp. In hs071_nlp.hpp, include IpTNLP.hpp (the base class), tell the compiler that we are using the IPOPT namespace, and create the declaration of the HS071_NLP class, inheriting off of TNLP. Have a look at the TNLP class in IpTNLP.hpp; you will see eight pure virtual methods that we must implement. Declare these methods in the header file. Implement each of the methods in HS071_NLP.cpp using the descriptions given below. In hs071_nlp.cpp, first include the header file for your class and tell the compiler that you are using the IPOPT namespace. A full version of these files can be found in the Examples/hs071_cpp directory.

It is very easy to make mistakes in the implementation of the function evaluation methods, in particular regarding the derivatives. IPOPT has a feature that can help you to debug the derivative code, using finite differences, see Section 4.1.

Note that the return value of any bool-valued function should be true, unless an error occured, for example, because the value of a problem function could not be evaluated at the required point.

Method get_nlp_info with prototype

Give IPOPT the information about the size of the problem (and hence, the size of the arrays that it needs to allocate).

- n: (out), the number of variables in the problem (dimension of x).
- m: (out), the number of constraints in the problem (dimension of g(x)).
- nnz_jac_g: (out), the number of nonzero entries in the Jacobian.
- nnz_h_lag: (out), the number of nonzero entries in the Hessian.
- index_style: (out), the numbering style used for row/col entries in the sparse matrix format (C_STYLE: 0-based, FORTRAN_STYLE: 1-based; see also Appendix A).

IPOPT uses this information when allocating the arrays that it will later ask you to fill with values. Be careful in this method since incorrect values will cause memory bugs which may be very difficult to find.

Our example problem has 4 variables (n), and 2 constraints (m). The constraint Jacobian for this small problem is actually dense and has 8 nonzeros (we still need to represent this Jacobian using the sparse matrix triplet format). The Hessian of the Lagrangian has 10 "symmetric" nonzeros (i.e., nonzeros in the lower left triangular part.). Keep in mind that the number of nonzeros is the total number of elements that may *ever* be nonzero, not just those that are nonzero at the starting point. This information is set once for the entire problem.

```
bool HS071_NLP::get_nlp_info(Index& n, Index& m, Index& nnz_jac_g,
                           Index& nnz_h_lag, IndexStyleEnum& index_style)
  // The problem described in HS071_NLP.hpp has 4 variables, x[0] through x[3]
 n = 4:
 // one equality constraint and one inequality constraint
  // in this example the Jacobian is dense and contains 8 nonzeros
  nnz_jac_g = 8;
  // the Hessian is also dense and has 16 total nonzeros, but we
  // only need the lower left corner (since it is symmetric)
  nnz_h_lag = 10;
  // use the C style indexing (0-based)
  index_style = TNLP::C_STYLE;
 return true;
Method get_bounds_info with prototype
virtual bool get_bounds_info(Index n, Number* x_1, Number* x_u,
                                  Index m, Number* g_l, Number* g_u)
```

Give IPOPT the value of the bounds on the variables and constraints.

- n: (in), the number of variables in the problem (dimension of x).
- x_1: (out) the lower bounds x^L for x.
- x_u: (out) the upper bounds x^U for x.
- m: (in), the number of constraints in the problem (dimension of g(x)).
- g_1: (out) the lower bounds g^L for g(x).
- g_u: (out) the upper bounds g^U for g(x).

The values of n and m that you specified in get_nlp_info are passed to you for debug checking. Setting a lower bound to a value less than or equal to the value of the option nlp_lower_bound_inf will cause IPOPT to assume no lower bound. Likewise, specifying the upper bound above or equal to the value of the option nlp_upper_bound_inf will cause IPOPT to assume no upper bound. These options, nlp_lower_bound_inf and nlp_upper_bound_inf, are set to -10^{19} and 10^{19} , respectively, by default, but may be modified by changing the options (see Section 5).

In our example, the first constraint has a lower bound of 25 and no upper bound, so we set the lower bound of constraint [0] to 25 and the upper bound to some number greater than 10^{19} . The second constraint is an equality constraint and we set both bounds to 40. IPOPT recognizes this as an equality constraint and does not treat it as two inequalities.

```
x_1[i] = 1.0;
 }
 // the variables have upper bounds of 5
 for (Index i=0; i<4; i++) {
   x_u[i] = 5.0;
 // the first constraint g1 has a lower bound of 25
 // the first constraint g1 has NO upper bound, here we set it to 2e19.
 // Ipopt interprets any number greater than nlp_upper_bound_inf as
 // infinity. The default value of nlp_upper_bound_inf and nlp_lower_bound_inf
 // is 1e19 and can be changed through ipopt options.
 g_u[0] = 2e19;
 // the second constraint g2 is an equality constraint, so we set the
 // upper and lower bound to the same value
 g_1[1] = g_u[1] = 40.0;
 return true;
}
Method get_starting_point with prototype
virtual bool get_starting_point(Index n, bool init_x, Number* x,
                                     bool init_z, Number* z_L, Number* z_U,
                                     Index m, bool init_lambda, Number* lambda)
```

Give IPOPT the starting point before it begins iterating.

- n: (in), the number of variables in the problem (dimension of x).
- init_x: (in), if true, this method must provide an initial value for x.
- x: (out), the initial values for the primal variables, x.
- init_z: (in), if true, this method must provide an initial value for the bound multipliers z^L and z^U .
- z_L: (out), the initial values for the bound multipliers, z^L .
- z_U : (out), the initial values for the bound multipliers, z^U .
- m: (in), the number of constraints in the problem (dimension of g(x)).
- init_lambda: (in), if true, this method must provide an initial value for the constraint multipliers, λ .
- lambda: (out), the initial values for the constraint multipliers, λ .

The variables n and m are passed in for your convenience. These variables will have the same values you specified in get_nlp_info.

Depending on the options that have been set, IPOPT may or may not require bounds for the primal variables x, the bound multipliers z^L and z^U , and the constraint multipliers λ . The boolean flags <code>init_x</code>, <code>init_z</code>, and <code>init_lambda</code> tell you whether or not you should provide initial values for x, z^L , z^U , or λ respectively. The default options only require an initial value for the primal variables x. Note, the initial values for bound multiplier components for "infinity" bounds ($x_U^{(i)} = -\infty$ or $x_U^{(i)} = \infty$) are ignored.

In our example, we provide initial values for x as specified in the example problem. We do not provide any initial values for the dual variables, but use an assert to immediately let us know if we are ever asked for them.

```
bool HSO71_NLP::get_starting_point(Index n, bool init_x, Number* x,
                                  bool init_z, Number* z_L, Number* z_U,
                                  Index m, bool init_lambda,
                                  Number* lambda)
{
  // Here, we assume we only have starting values for x, if you code
  // your own NLP, you can provide starting values for the dual variables
  // if you wish to use a warmstart option
  assert(init_x == true);
  assert(init_z == false);
  assert(init_lambda == false);
  // initialize to the given starting point
  x[0] = 1.0;
  x[1] = 5.0;
  x[2] = 5.0;
  x[3] = 1.0;
  return true;
Method eval_f with prototype
virtual bool eval_f(Index n, const Number* x,
                        bool new_x, Number& obj_value)
```

Return the value of the objective function at the point x.

- n: (in), the number of variables in the problem (dimension of x).
- x: (in), the values for the primal variables, x, at which f(x) is to be evaluated.
- new_x: (in), false if any evaluation method was previously called with the same values in x, true otherwise.
- obj_value: (out) the value of the objective function (f(x)).

The boolean variable new_x will be false if the last call to any of the evaluation methods ($eval_*$) used the same x values. This can be helpful when users have efficient implementations that calculate multiple outputs at once. IPOPT internally caches results from the TNLP and generally, this flag can be ignored.

The variable n is passed in for your convenience. This variable will have the same value you specified in get_nlp_info.

For our example, we ignore the new_x flag and calculate the objective.

Return the gradient of the objective function at the point x.

• n: (in), the number of variables in the problem (dimension of x).

- x: (in), the values for the primal variables, x, at which $\nabla f(x)$ is to be evaluated.
- new_x: (in), false if any evaluation method was previously called with the same values in x, true otherwise.
- grad_f: (out) the array of values for the gradient of the objective function $(\nabla f(x))$.

The gradient array is in the same order as the x variables (i.e., the gradient of the objective with respect to x[2] should be put in $grad_f[2]$).

The boolean variable new_x will be false if the last call to any of the evaluation methods ($eval_*$) used the same x values. This can be helpful when users have efficient implementations that calculate multiple outputs at once. IPOPT internally caches results from the TNLP and generally, this flag can be ignored.

The variable n is passed in for your convenience. This variable will have the same value you specified in get_nlp_info.

In our example, we ignore the new_x flag and calculate the values for the gradient of the objective.

Return the value of the constraint function at the point x.

- n: (in), the number of variables in the problem (dimension of x).
- x: (in), the values for the primal variables, x, at which the constraint functions, g(x), are to be evaluated.
- new_x: (in), false if any evaluation method was previously called with the same values in x, true otherwise.
- m: (in), the number of constraints in the problem (dimension of g(x)).
- g: (out) the array of constraint function values, g(x).

The values returned in g should be only the g(x) values, do not add or subtract the bound values g^L or g^U .

The boolean variable new_x will be false if the last call to any of the evaluation methods ($eval_*$) used the same x values. This can be helpful when users have efficient implementations that calculate multiple outputs at once. IPOPT internally caches results from the TNLP and generally, this flag can be ignored.

The variables n and m are passed in for your convenience. These variables will have the same values you specified in get_nlp_info.

In our example, we ignore the new_x flag and calculate the values of constraint functions.

```
bool HS071_NLP::eval_g(Index n, const Number* x, bool new_x, Index m, Number* g)
{
   assert(n == 4);
   assert(m == 2);
```

Return either the sparsity structure of the Jacobian of the constraints, or the values for the Jacobian of the constraints at the point x.

- n: (in), the number of variables in the problem (dimension of x).
- x: (in), the values for the primal variables, x, at which the constraint Jacobian, $\nabla g(x)^T$, is to be evaluated.
- new_x: (in), false if any evaluation method was previously called with the same values in x, true otherwise.
- m: (in), the number of constraints in the problem (dimension of g(x)).
- n_ele_jac: (in), the number of nonzero elements in the Jacobian (dimension of iRow, jCol, and values).
- iRow: (out), the row indices of entries in the Jacobian of the constraints.
- jCol: (out), the column indices of entries in the Jacobian of the constraints.
- values: (out), the values of the entries in the Jacobian of the constraints.

The Jacobian is the matrix of derivatives where the derivative of constraint $g^{(i)}$ with respect to variable $x^{(j)}$ is placed in row i and column j. See Appendix A for a discussion of the sparse matrix format used in this method.

If the iRow and jCol arguments are not NULL, then IPOPT wants you to fill in the sparsity structure of the Jacobian (the row and column indices only). At this time, the x argument and the values argument will be NULL.

If the x argument and the values argument are not NULL, then IPOPT wants you to fill in the values of the Jacobian as calculated from the array x (using the same order as you used when specifying the sparsity structure). At this time, the iRow and jCol arguments will be NULL;

The boolean variable new_x will be false if the last call to any of the evaluation methods ($eval_*$) used the same x values. This can be helpful when users have efficient implementations that calculate multiple outputs at once. IPOPT internally caches results from the TNLP and generally, this flag can be ignored.

The variables n, m, and nele_jac are passed in for your convenience. These arguments will have the same values you specified in get_nlp_info.

In our example, the Jacobian is actually dense, but we still specify it using the sparse format.

```
iRow[0] = 0; jCol[0] = 0;
   iRow[1] = 0; jCol[1] = 1;
   iRow[2] = 0; jCol[2] = 2;
    iRow[3] = 0; jCol[3] = 3;
   iRow[4] = 1; jCol[4] = 0;
   iRow[5] = 1; jCol[5] = 1;
   iRow[6] = 1; jCol[6] = 2;
   iRow[7] = 1; jCol[7] = 3;
  }
  else {
   // return the values of the Jacobian of the constraints
   values[0] = x[1]*x[2]*x[3]; // 0,0
   values[1] = x[0]*x[2]*x[3]; // 0,1
   values[2] = x[0]*x[1]*x[3]; // 0,2
   values[3] = x[0]*x[1]*x[2]; // 0,3
   values[4] = 2*x[0]; // 1,0
   values[5] = 2*x[1]; // 1,1
   values[6] = 2*x[2]; // 1,2
   values[7] = 2*x[3]; // 1,3
 return true;
Method eval h with prototype
virtual bool eval_h(Index n, const Number* x, bool new_x,
                       Number obj_factor, Index m, const Number* lambda,
                       bool new_lambda, Index nele_hess, Index* iRow,
```

Return either the sparsity structure of the Hessian of the Lagrangian, or the values of the Hessian of the Lagrangian (9) for the given values for x, σ_f , and λ .

• n: (in), the number of variables in the problem (dimension of x).

Index* jCol, Number* values)

- x: (in), the values for the primal variables, x, at which the Hessian is to be evaluated.
- new_x: (in), false if any evaluation method was previously called with the same values in x, true otherwise.
- obj_factor: (in), factor in front of the objective term in the Hessian, sigma_f.
- m: (in), the number of constraints in the problem (dimension of g(x)).
- lambda: (in), the values for the constraint multipliers, λ, at which the Hessian is to be evaluated.
- new_lambda: (in), false if any evaluation method was previously called with the same values in lambda, true otherwise.
- nele_hess: (in), the number of nonzero elements in the Hessian (dimension of iRow, jCol, and values).
- iRow: (out), the row indices of entries in the Hessian.
- jCol: (out), the column indices of entries in the Hessian.
- values: (out), the values of the entries in the Hessian.

The Hessian matrix that IPOPT uses is defined in Eq. 9. See Appendix A for a discussion of the sparse symmetric matrix format used in this method.

If the iRow and jCol arguments are not NULL, then IPOPT wants you to fill in the sparsity structure of the Hessian (the row and column indices for the lower or upper triangular part only). In this case, the x, lambda, and values arrays will be NULL.

If the x, lambda, and values arrays are not NULL, then IPOPT wants you to fill in the values of the Hessian as calculated using x and lambda (using the same order as you used when specifying the sparsity structure). In this case, the iRow and jCol arguments will be NULL.

The boolean variables new_x and new_lambda will both be false if the last call to any of the evaluation methods (eval_*) used the same values. This can be helpful when users have efficient implementations that calculate multiple outputs at once. IPOPT internally caches results from the TNLP and generally, this flag can be ignored.

The variables n, m, and $nele_hess$ are passed in for your convenience. These arguments will have the same values you specified in get_nlp_info .

In our example, the Hessian is dense, but we still specify it using the sparse matrix format. Because the Hessian is symmetric, we only need to specify the lower left corner.

```
bool HS071_NLP::eval_h(Index n, const Number* x, bool new_x,
                       Number obj_factor, Index m, const Number* lambda,
                       bool new_lambda, Index nele_hess, Index* iRow,
                       Index* jCol, Number* values)
  if (values == NULL) {
    // return the structure. This is a symmetric matrix, fill the lower left
    // triangle only.
    // the Hessian for this problem is actually dense
    Index idx=0:
    for (Index row = 0; row < 4; row++) {</pre>
      for (Index col = 0; col <= row; col++) {</pre>
        iRow[idx] = row;
        jCol[idx] = col;
        idx++;
    }
    assert(idx == nele_hess);
  }
  else {
    // return the values. This is a symmetric matrix, fill the lower left
    // triangle only
    // fill the objective portion
    values[0] = obj_factor * (2*x[3]); // 0,0
    values[1] = obj_factor * (x[3]); // 1,0
    values[2] = 0;
    values[3] = obj_factor * (x[3]);
                                     // 2,0
    values[4] = 0;
                                        // 2,1
    values[5] = 0;
                                       // 2,2
    values[6] = obj_factor * (2*x[0] + x[1] + x[2]); // 3,0
    values[7] = obj_factor * (x[0]);
                                                      // 3,1
                                                      // 3,2
    values[8] = obj_factor * (x[0]);
    values[9] = 0;
                                                      // 3,3
    // add the portion for the first constraint
    values[1] += lambda[0] * (x[2] * x[3]); // 1,0
```

```
values[3] += lambda[0] * (x[1] * x[3]); // 2,0
values[4] += lambda[0] * (x[0] * x[3]); // 2,1

values[6] += lambda[0] * (x[1] * x[2]); // 3,0
values[7] += lambda[0] * (x[0] * x[2]); // 3,1
values[8] += lambda[0] * (x[0] * x[1]); // 3,2

// add the portion for the second constraint
values[0] += lambda[1] * 2; // 0,0

values[2] += lambda[1] * 2; // 1,1

values[5] += lambda[1] * 2; // 2,2

values[9] += lambda[1] * 2; // 3,3
}

return true;
```

Method finalize_solution with prototype

This is the only method that is not mentioned in Figure 2. This method is called by IPOPT after the algorithm has finished (successfully or even with most errors).

- status: (in), gives the status of the algorithm as specified in IpAlgTypes.hpp,
 - SUCCESS: Algorithm terminated successfully at a locally optimal point, satisfying the convergence tolerances (can be specified by options).
 - MAXITER_EXCEEDED: Maximum number of iterations exceeded (can be specified by an option).
 - STOP_AT_TINY_STEP: Algorithm proceeds with very little progress.
 - STOP_AT_ACCEPTABLE_POINT: Algorithm stopped at a point that was converged, not to "desired" tolerances, but to "acceptable" tolerances (see the acceptable-... options).
 - LOCAL_INFEASIBILITY: Algorithm converged to a point of local infeasibility. Problem may be infeasible.
 - USER_REQUESTED_STOP: The user call-back function intermediate_callback (see Section 3.3.4) returned flase, i.e., the user code requested a premature termination of the optimization.
 - DIVERGING_ITERATES: It seems that the iterates diverge.
 - RESTORATION_FAILURE: Restoration phase failed, algorithm doesn't know how to proceed.
 - ERROR_IN_STEP_COMPUTATION: An unrecoverable error occurred while IPOPT tried to compute
 the search direction.
 - INVALID_NUMBER_DETECTED: Algorithm received an invalid number (such as NaN or Inf) from the NLP; see also option check_derivatives_for_naninf.
 - INTERNAL_ERROR: An unknown internal error occurred. Please contact the IPOPT authors through the mailing list.
- n: (in), the number of variables in the problem (dimension of x).
- x: (in), the final values for the primal variables, x_* .

- z.L.: (in), the final values for the lower bound multipliers, z_*^L .
- z_U: (in), the final values for the upper bound multipliers, z_*^U .
- m: (in), the number of constraints in the problem (dimension of g(x)).
- g: (in), the final value of the constraint function values, $g(x_*)$.
- lambda: (in), the final values of the constraint multipliers, λ_* .
- obj_value: (in), the final value of the objective, $f(x_*)$.

This method gives you the return status of the algorithm (SolverReturn), and the values of the variables, the objective and constraint function values when the algorithm exited.

In our example, we will print the values of some of the variables to the screen.

```
void HS071_NLP::finalize_solution(SolverReturn status,
                                  Index n, const Number* x, const Number* z_L,
                                  const Number* z_U, Index m, const Number* g,
                                  const Number* lambda, Number obj_value)
  // here is where we would store the solution to variables, or write to a file, etc
  // so we could use the solution.
  // For this example, we write the solution to the console
  printf("\n\nSolution of the primal variables, x\n");
  for (Index i=0; i<n; i++) {
   printf("x[%d] = %e\n", i, x[i]);
 printf("\n\nSolution of the bound multipliers, z_L and z_U\n");
  for (Index i=0; i<n; i++) {
   printf("z_L[\%d] = \%e\n", i, z_L[i]);
  for (Index i=0; i<n; i++) {
   printf("z_U[%d] = %e\n", i, z_U[i]);
  printf("\n\nObjective value\n");
  printf("f(x*) = %e\n", obj_value);
```

This is all that is required for our HSO71_NLP class and the coding of the problem representation.

3.3.2 Coding the Executable (main)

Now that we have a problem representation, the HSO71_NLP class, we need to code the main function that will call IPOPT and ask IPOPT to find a solution.

Here, we must create an instance of our problem (HSO71_NLP), create an instance of the IPOPT solver (IpoptApplication), initialize it, and ask the solver to find a solution. We always use the SmartPtr template class instead of raw C++ pointers when creating and passing IPOPT objects. To find out more information about smart pointers and the SmartPtr implementation used in IPOPT, see Appendix B.

Create the file MyExample.cpp in the MyExample directory. Include HS071_NLP.hpp and IpIpoptApplication.hpp, tell the compiler to use the Ipopt namespace, and implement the main function.

```
#include "IpIpoptApplication.hpp"
#include "hs071_nlp.hpp"
using namespace Ipopt;
int main(int argv, char* argc[])
{
```

```
// Create a new instance of your nlp
// (use a SmartPtr, not raw)
SmartPtr<TNLP> mynlp = new HS071_NLP();
// Create a new instance of IpoptApplication
// (use a SmartPtr, not raw)
SmartPtr<IpoptApplication> app = new IpoptApplication();
// Change some options
// Note: The following choices are only examples, they might not be
         suitable for your optimization problem.
app->Options()->SetNumericValue("tol", 1e-9);
app->Options()->SetStringValue("mu_strategy", "adaptive");
app->Options()->SetStringValue("output_file", "ipopt.out");
// Intialize the IpoptApplication and process the options
app->Initialize();
// Ask Ipopt to solve the problem
ApplicationReturnStatus status = app->OptimizeTNLP(mynlp);
if (status == Solve_Succeeded) {
  printf("\n\*** The problem solved!\n");
else {
  printf("\n*** The problem FAILED!\n");
// As the SmartPtrs go out of scope, the reference count
// will be decremented and the objects will automatically
// be deleted.
return (int) status;
```

The first line of code in main creates an instance of HSO71_NLP. We then create an instance of the IPOPT solver, IpoptApplication. The call to app->Initialize(...) will inialize that object, process this options (particularly the output relatex options), and the call to app->OptimizeTNLP(...) will run IPOPT and try to solve the problem. By default, IPOPT will write to its progress to the console, and return the SolverReturn status.

3.3.3 Compiling and Testing the Example

Our next task is to compile and test the code. If you are familiar with the compiler and linker used on your system, you can build the code, including the IPOPT library libipopt.a (and other necessary libraries, as listed in the ipopt_addlibs_cpp.txt and ipopt_addlibs_f.txt files). If you are using Linux/UNIX, then a sample makefile exists already that was created by configure. Copy Examples/hs071_cpp/Makefile into your MyExample directory. This makefile was created for the hs071_cpp code, but it can be easily modified for your example problem. Edit the file, making the following changes,

```
    change the EXE variable
        EXE = my_example
    change the OBJS variable
        OBJS = HSO71_NLP.o MyExample.o
    and the problem should compile easily with,
    make
    Now run the executable,
    ./my_example
    and you should see output resembling the following,
```

```
**************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
Ipopt is released as open source code under the Common Public License (CPL).
       For more information visit http://projects.coin-or.org/Ipopt
***********************************
Number of nonzeros in equality constraint Jacobian...:
Number of nonzeros in inequality constraint Jacobian.:
                                                        4
Number of nonzeros in Lagrangian Hessian....:
                                                        10
Total number of variables....:
                   variables with only lower bounds:
                                                        0
              variables with lower and upper bounds:
                   variables with only upper bounds:
Total number of equality constraints....:
                                                        1
Total number of inequality constraints....:
                                                        1
       inequality constraints with only lower bounds:
  inequality constraints with lower and upper bounds:
                                                         0
       inequality constraints with only upper bounds:
iter
       objective
                 inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
  0 1.6109693e+01 1.12e+01 5.28e-01 0.0 0.00e+00 - 0.00e+00 0.00e+00 0
  1 1.7410406e+01 8.38e-01 2.25e+01 -0.3 7.97e-01
                                                   - 3.19e-01 1.00e+00f 1
  2 1.8001613e+01 1.06e-02 4.96e+00 -0.3 5.60e-02 2.0 9.97e-01 1.00e+00h 1
  3 1.7199482e+01 9.04e-02 4.24e-01 -1.0 9.91e-01
                                                  - 9.98e-01 1.00e+00f 1
  4 1.6940955e+01 2.09e-01 4.58e-02 -1.4 2.88e-01 - 9.66e-01 1.00e+00h 1
  5 1.7003411e+01 2.29e-02 8.42e-03 -2.9 7.03e-02 - 9.68e-01 1.00e+00h 1
  6 1.7013974e+01 2.59e-04 8.65e-05 -4.5 6.22e-03 - 1.00e+00 1.00e+00h 1
                                                  - 1.00e-00 1.00e+00h 1
  7 1.7014017e+01 2.26e-07 5.71e-08 -8.0 1.43e-04
  8 1.7014017e+01 4.62e-14 9.09e-14 -8.0 6.95e-08
                                                  - 1.00e+00 1.00e+00h 1
Number of Iterations....: 8
Number of objective function evaluations
Number of objective gradient evaluations
                                                = 9
                                                = 9
Number of equality constraint evaluations
Number of inequality constraint evaluations
Number of equality constraint Jacobian evaluations = 9
Number of inequality constraint Jacobian evaluations = 9
Number of Lagrangian Hessian evaluations = 8
Total CPU secs in IPOPT (w/o function evaluations) =
                                                       0.220
Total CPU secs in NLP function evaluations
                                                       0.000
EXIT: Optimal Solution Found.
Solution of the primal variables, \boldsymbol{\boldsymbol{x}}
x[0] = 1.000000e+00
x[1] = 4.743000e+00
x[2] = 3.821150e+00
x[3] = 1.379408e+00
Solution of the bound multipliers, z_L and z_U
z_L[0] = 1.087871e+00
z_L[1] = 2.428776e-09
z_L[2] = 3.222413e-09
z_L[3] = 2.396076e-08
z_U[0] = 2.272727e-09
z_U[1] = 3.537314e-08
z_U[2] = 7.711676e-09
z_U[3] = 2.510890e-09
```

Objective value

```
f(x*) = 1.701402e+01
```

```
*** The problem solved!
```

This completes the basic C++ tutorial, but see Section 6 which explains the standard console output of IPOPT and Section 5 for information about the use of options to customize the behavior of IPOPT.

The Examples/ScalableProblems directory contains other NLP problems coded in C++.

3.3.4 Additional methods in TNLP

The following methods are available to additional features that are not explained in the example. Default implementations for those methods are provided, so that a user can safely ignore them, unless she wants to make use of those features. These features is not yet(?) available from C or Fortran.

Method intermediate_callback with prototype

It is not required to implement (overload) this method. This method is called once per iteration (during the convergence check), and can be used to obtain information about the optimization status while IPOPT solves the problem, and also to requires a premature termination.

The information provided by the entities in the argument list corresponds to what IPOPT prints in the iteration summary (see also Section 6). Further information can be obtained from the ip_data and ip_cq objects (for experts only :).

You you let this method return false, IPOPT will terminate with the User_Requested_Stop status. If you do not implement this method (as we do in this example), the default implementation always returns true.

```
Method get_scaling_parameters with prototype
```

This method is called if the nlp_scaling_method is chosen as user-scaling. Then the user is to provide scaling factors for the objective function, as well as for the optimization variables and/or constraints. The return value should be true, unless an error occurred, and the program is to be aborted.

The value returned in obj_scaling determines, how IPOPT should interally scale the objective function. For example, if this number is chosen to be 10, then IPOPT solves internally an optimization problem that has 10 times the value of the original objective function provided by the TNLP. In particular, if this value is negative, then IPOPT will maximize the objective function instead of minimizing it.

The scaling factors for the variables can be returned in x_scaling, which has the same length as x in the other TNLP methods, and the factors are ordered like x. You need to set use_x_scaling to true, if you

want IPOPT so scale the variables. If it is false, no internal scaling of the variables is done. Similarly, the scaling factors for the constraints can be returned in g_scaling, and this scaling is activated by setting use_g_scaling to true.

As a guideline, we suggest to scale the optimization problem (either directly in the original formulation, or after using scaling factors) so that all sensitivities, i.e., all non-zero first partial derivatives, are typically of the order 0.1 - 10.

Method get_number_of_nonlinear_variables with prototype

virtual Index get_number_of_nonlinear_variables()

This method is only important if the limited-memory quasi-Newton options is used, see Section 4.2. It is to be used to return the number of variables that appear nonlinearly in the objective function or in at least one constraint function. If a negative number is returned, IPOPT assumes that all variables are nonlinear.

If the user doesn't overload this method in her implementation of the class derived from TNLP, the default implementation returns -1, i.e., then all variables are assumed to be nonlinear.

Method get_list_of_nonlinear_variables with prototype

This method is called by IPOPT only if the limited-memory quasi-Newton options is used, and if the get_number_of_nonlinear_variables method returns a positive number; this number is then identical with num_nonlin_vars and the length of the array pos_nonlin_vars. In this call, you need to list the indices of all nonlinear variables in pos_nonlin_vars, where the numbering starts with 0 order 1, depending on the numbering style determined in get_nlp_info.

3.4 The C Interface

The C interface for IPOPT is declared in the header file IpStdCInterface.h, which is found in \$IPOPTDIR/include/ipopt (or in \$PREFIX/include/ipopt if the switch --prefix=\$PREFIX was used for configure); while reading this section, it will be helpful to have a look at this file.

In order to solve an optimization problem with the C interface, one has to create an IpoptProblem¹¹ with the function CreateIpoptProblem, which later has to be passed to the IpoptSolve function.

The IpoptProblem created by CreateIpoptProblem contains the problem dimensions, the variable and constraint bounds, and the function pointers for callbacks that will be used to evaluate the NLP problem functions and their derivatives (see also the discussion of the C++ methods get_nlp_info and get_bounds_info in Section 3.3.1 for information about the arguments of CreateIpoptProblem).

The prototypes for the callback functions, Eval_F_CB, Eval_Grad_F_CB, etc., are defined in the header file IpStdCInterface.h. Their arguments correspond one-to-one to the arguments for the C++ methods discussed in Section 3.3.1; for example, for the meaning of n, x, new_x, obj_value in the declaration of Eval_F_CB see the discussion of "eval_f". The callback functions should return TRUE, unless there was a problem doing the requested function/derivative evaluation at the given point x (then it should return FALSE).

Note the additional argument of type UserDataPtr in the callback functions. This pointer argument is available for you to communicate information between the main program that calls IpoptSolve and any of the callback functions. This pointer is simply passed unmodified by IPOPT among those functions. For example, you can use this to pass constants that define the optimization problem and are computed before the optimization in the main C program to the callback functions.

¹¹ IpoptProblem is a pointer to a C structure; you should not access this structure directly, only through the functions provided in the C interface.

After an IpoptProblem has been created, you can set algorithmic options for IPOPT (see Section 5) using the AddIpopt...Option functions. Finally, the IPOPT algorithm is called with IpoptSolve, giving IPOPT the IpoptProblem, the starting point, and arrays to store the solution values (primal and dual variables), if desired. Finally, after everything is done, you should call FreeIpoptProblem to release internal memory that is still allocated inside IPOPT.

In the remainder of this section we discuss how the example problem (4)–(7) can be solved using the C interface. A completed version of this example can be found in Examples/hs071_c.

In order to implement the example problem on your own, create a new directory MyCExample and create a new file, hs071_c.c. Here, include the interface header file IpStdCInterface.h, along with other necessary header files, such as stdlib.h and assert.h. Add the prototypes and implementations for the five callback functions. Have a look at the C++ implementation for eval_f, eval_g, eval_grad_f, eval_jac_g, and eval_h in Section 3.3.1. The C implementations have somewhat different prototypes, but are implemented almost identically to the C++ code. See the completed example in Examples/hs071_c/hs071_c.c if you are not sure how to do this.

We now need to implement the main function, create the IpoptProblem, set options, and call IpoptSolve. The CreateIpoptProblem function requires the problem dimensions, the variable and constraint bounds, and the function pointers to the callback routines. The IpoptSolve function requires the IpoptProblem, the starting point, and allocated arrays for the solution. The main function from the example is shown next, and discussed below.

```
int main()
{
 Index n=-1:
                                       /* number of variables */
 Index m=-1;
                                       /* number of constraints */
 Number* x_L = NULL;
                                       /* lower bounds on x */
 Number* x_U = NULL;
                                      /* upper bounds on x */
 Number* g_L = NULL;
                                      /* lower bounds on g */
 Number* g_U = NULL;
                                      /* upper bounds on g */
 IpoptProblem nlp = NULL;
                                      /* IpoptProblem */
 enum ApplicationReturnStatus status; /* Solve return code */
 Number* x = NULL;
                                      /* starting point and solution vector */
 Number* mult_x_L = NULL;
                                       /* lower bound multipliers
 at the solution */
 Number* mult_x_U = NULL;
                                       /* upper bound multipliers
 at the solution */
 Number obj;
                                       /* objective value */
 Index i:
                                       /* generic counter */
 /* set the number of variables and allocate space for the bounds */
 x_L = (Number*)malloc(sizeof(Number)*n);
 x_U = (Number*)malloc(sizeof(Number)*n);
 /* set the values for the variable bounds */
 for (i=0; i<n; i++) {
   x_L[i] = 1.0;
   x_U[i] = 5.0;
 /* set the number of constraints and allocate space for the bounds */
 m=2;
 g_L = (Number*)malloc(sizeof(Number)*m);
 g_U = (Number*)malloc(sizeof(Number)*m);
  /* set the values of the constraint bounds */
 g_L[0] = 25; g_U[0] = 2e19;
 g_L[1] = 40; g_U[1] = 40;
 /* create the IpoptProblem */
 nlp = CreateIpoptProblem(n, x_L, x_U, m, g_L, g_U, 8, 10, 0,
   &eval_f, &eval_g, &eval_grad_f,
```

```
&eval_jac_g, &eval_h);
/* We can free the memory now - the values for the bounds have been
   copied internally in CreateIpoptProblem */
free(x L):
free(x_U);
free(g_L);
free(g_U);
/* set some options */
AddIpoptNumOption(nlp, "tol", 1e-9);
AddIpoptStrOption(nlp, "mu_strategy", "adaptive");
/* allocate space for the initial point and set the values */
x = (Number*)malloc(sizeof(Number)*n);
x[0] = 1.0;
x[1] = 5.0;
x[2] = 5.0;
x[3] = 1.0;
/* allocate space to store the bound multipliers at the solution */
mult_x_L = (Number*)malloc(sizeof(Number)*n);
mult_x_U = (Number*)malloc(sizeof(Number)*n);
/* solve the problem */
status = IpoptSolve(nlp, x, NULL, &obj, NULL, mult_x_L, mult_x_U, NULL);
if (status == Solve_Succeeded) {
  printf("\n\nSolution of the primal variables, x\n");
  for (i=0; i<n; i++) {
   printf("x[%d] = %e\n", i, x[i]);
 printf("\n\nSolution of the bound multipliers, z_L and z_U\n");
  for (i=0; i<n; i++) {
   printf("z_L[%d] = %e\n", i, mult_x_L[i]);
  for (i=0; i<n; i++) {
   printf("z_U[\%d] = \%e\n", i, mult_x_U[i]);
 printf("\n\nObjective value\n");
 printf("f(x*) = %e\n", obj);
/* free allocated memory */
FreeIpoptProblem(nlp);
free(x);
free(mult_x_L);
free(mult_x_U);
return 0;
```

Here, we declare all the necessary variables and set the dimensions of the problem. The problem has 4 variables, so we set n and allocate space for the variable bounds (don't forget to call free for each of your malloc calls before the end of the program). We then set the values for the variable bounds.

The problem has 2 constraints, so we set m and allocate space for the constraint bounds. The first constraint has a lower bound of 25 and no upper bound. Here we set the upper bound to 2e19. IPOPT interprets any number greater than or equal to nlp_upper_bound_inf as infinity. The default value of nlp_lower_bound_inf and nlp_upper_bound_inf is -1e19 and 1e19, respectively, and can be changed through IPOPT options. The second constraint is an equality with right hand side 40, so we set both the upper and the lower bound to 40.

We next create an instance of the IpoptProblem by calling CreateIpoptProblem, giving it the problem dimensions and the variable and constraint bounds. The arguments nele_jac and nele_hess are the number of elements in Jacobian and the Hessian, respectively. See Appendix A for a description of the sparse matrix format. The index_style argument specifies whether we want to use C style indexing for the row and column indices of the matrices or Fortran style indexing. Here, we set it to 0 to indicate C style. We also include the references to each of our callback functions. IPOPT uses these function pointers to ask for evaluation of the NLP when required.

After freeing the bound arrays that are no longer required, the next two lines illustrate how you can change the value of options through the interface. IPOPT options can also be changed by creating a ipopt.opt file (see Section 5). We next allocate space for the initial point and set the values as given in the problem definition.

The call to IpoptSolve can provide us with information about the solution, but most of this is optional. Here, we want values for the bound multipliers at the solution and we allocate space for these.

We can now make the call to IpoptSolve and find the solution of the problem. We pass in the IpoptProblem, the starting point x (IPOPT will use this array to return the solution or final point as well). The next 5 arguments are pointers so IPOPT can fill in values at the solution. If these pointers are set to NULL, IPOPT will ignore that entry. For example, here, we do not want the constraint function values at the solution or the constraint multipliers, so we set those entries to NULL. We do want the value of the objective, and the multipliers for the variable bounds. The last argument is a void* for user data. Any pointer you give here will also be passed to you in the callback functions.

The return code is an ApplicationReturnStatus enumeration, see the header file ReturnCodes_inc.h which is installed along IpStdCInterface.h in the IPOPT include directory.

After the optimizer terminates, we check the status and print the solution if successful. Finally, we free the IpoptProblem and the remaining memory, and return from main.

3.5 The Fortran Interface

The Fortran interface is essentially a wrapper of the C interface discussed in Section 3.4. The way to hook up IPOPT in a Fortran program is very similar to how it is done for the C interface, and the functions of the Fortran interface correspond one-to-one to the those of the C and C++ interface, including their arguments. You can find an implementation of the example problem (4)–(7) in \$IPOPTDIR/Examples/hs071_f.

The only special things to consider are:

- The return value of the function IPCREATE is of an INTEGER type that must be large enough to capture a pointer on the particular machine. This means, that you have to declare the "handle" for the IpoptProblem as INTEGER*8 if your program is compiled in 64-bit mode. All other INTEGER-type variables must be of the regular type.
- For the call of IPSOLVE (which is the function that is to be called to run IPOPT), all arrays, including those for the dual variables, must be given (in contrast to the C interface). The return value IERR of this function indicates the outcome of the optimization (see the include file IpReturnCodes.inc in the IPOPT include directory).
- The return IERR value of the remaining functions has to be set to zero, unless there was a problem during execution of the function call.
- The callback functions (EV_* in the example) include the arguments IDAT and DAT, which are INTEGER and DOUBLE PRECISION arrays that are passed unmodified between the main program calling IPSOLVE and the evaluation subroutines EV_* (similarly to UserDataPtr arguments in the C interface). These arrays can be used to pass "private" data between the main program and the user-provided Fortran subroutines.

The last argument of the EV-* subroutines, IERR, is to be set to 0 by the user on return, unless there was a problem during the evaluation of the optimization problem function/derivative for the given point X (then it should return a non-zero value).

4 Special Features

4.1 Derivative Checker

When writing code for the evaluation of derivatives it is very easy to make mistakes (much easier than writing it correctly the first time:). As a convenient feature, IPOPT provides the option to run a simple derivative checker, based on finite differences, before the optimization is started.

To use the derivative checker, you need to use the option derivative_test. By default, this option is set to none, i.e., no finite difference test is performed, It is set to first-order, then the first derivatives of the objective function and the constraints are verified, and for the setting second-order, the second derivatives are tested as well.

The verification is done by a simple finite differences approximation, where each component of the user-provided starting point is perturbed one of the other. The relative size of the perturbation is determined by the option derivative_test_perturbation. The default value (10⁻⁸, about the square root of the machine precision) is probably fine in most cases, but if you believe that you see wrong warnings, you might want to play with this parameter. When the test is performed, IPOPT prints out a line for every partial derivative, for which the user-provided derivative value deviates too much from the finite difference approximation. The relative tolerance for deciding when a warning should be issued, is determined by the option derivative_test_tol. If you want to see the user-provided and estimated derivative values with the relative deviation for each single partial derivative, you can switch the derivative_test_print_all option to yes.

A typical output is:

Starting derivative checker.

```
* grad_f[
                 ~ -6.5559997134793468e+02
                                                                      [ 6.101e-03]
 jac_g [
           4,
                      0.000000000000000e+00
                                                2.2160643690464592e-02
           4,
                 5] =
                      1.3798494268463347e+01 v
                                                1.3776333629422766e+01
 jac_g [
                                             ~ 1.3776333629422766e+01
           6.
                     1.4776333636790881e+01 v
* jac_g [
```

Derivative checker detected 4 error(s).

The start ("*") in the first column indicates that this line corresponds to some partial derivative for which the error tolerance was exceeded. Next, we see which partial derivative is concerned in this output line. For example, in the first line, it is the second component of the objective function gradient (or the third, if the C_STYLE numbering is used, i.e., when counting of indices starts with 0 instead of 1). The first floating point number is the value given by the user code, and the second number (after "~") is the finite differences estimation. Finally, the number in square brackets is the relative difference between those two numbers.

For constraints, the first index after <code>jac_g</code> is the number of the constraint, and the second one corresponds to the variable number (again, the choice of the numbering style matters).

Since also the sparsity structure of the constraint Jacobian has to be provided by the user, it can be faulty as well. For this, the "v" after a user-provided derivative value indicates that this component of the Jacobian is part of the user provided sparsity structure. If there is no "v", it means that the user did not include this partial derivative in the list of non-zero elements. In the above output, the partial derivative "jac_g[4,4]" is non-zero (based on the finite difference approximation), but it is not included in the list of non-zero elements (missing "v"), so that user probably made a mistake in the sparsity structure. The other two Jacobian entries are provided in the non-zero structure but their values seem to be off.

For second derivatives, the output lines look like:

There, the first line shows the deviation of the user-provided partial second derivative in the Hessian for the objective function, and the second line show an error in a partial derivative for the Hessian of the third constraint (again, the numbering style matters).

Since the second derivatives are approximates by finite differences of the first derivatives, you should first correct errors for the first derivatives. Also, since the finite difference approximations are quite expensive, you should try to debug a small instance of your problem if you can. Finally, it is of course also a good idea to run your code through some memory checker, such as valgrind on Linux.

4.2 Quasi-Newton Approximation of Second Derivatives

IPOPT has an option to approximate the Hessian of the Lagrangian by a limited-memory quasi-Newton method (L-BFGS). You can use this feature using the hessian_information and limited_memory... options. In this case, it is not necessary to implement the Hessian computation method eval_h in TNLP. If you are using the C or Fortran interface, you still need to implement these functions, but they should return false or IERR=1, respectively, and don't need to do anything else.

In general, when second derivatives can be computed with reasonable computational effort, it is usually a good idea to use them, since then IPOPT normally converges in fewer iterations and is more robust. An exception here might be the case, where your optimization problem has a dense Hessian, and then using the quasi-Newton approximation might be better, even if it the number of iterations increases, since the computation time per iteration might be significantly higher due to the very large number of non-zero elements in the linear systems that IPOPT solves in order to compute the search direction.

Since the Hessian of the Lagrangian is zero for all variables that appear only linearly in the objective and constraint functions, the Hessian approximation should only take place in the space of all nonlinear variables. By default, it is assumed that all variables are nonlinear, but you can tell IPOPT explicitly which variables are nonlinear, using the get_number_of_nonlinear_variables and get_list_of_nonlinear_variables method of the TNLP class, see Section 3.3.4. (Those methods have been implemented for the AMPL interface, so you would automatically only approximate the Hessian in the space of the nonlinear variables, if you are using the quasi-Newton option for AMPL models.)

5 IPOPT Options

Ipopt has many (maybe too many) options that can be adjusted for the algorithm. Options are all identified by a string name, and their values can be of one of three types: Number (real), Integer, or String. Number options are used for things like tolerances, integer options are used for things like maximum number of iterations, and string options are used for setting algorithm details, like the NLP scaling method. Options can be set through code, through the AMPL interface if you are using AMPL, or by creating a ipopt.opt file in the directory you are executing IPOPT.

The ipopt.opt file is read line by line and each line should contain the option name, followed by whitespace, and then the value. Comments can be included with the # symbol. Don't forget to ensure you have a newline at the end of the file. For example,

```
# This is a comment
```

```
# Turn off the NLP scaling
nlp_scaling_method none
```

```
# Change the initial barrier parameter
mu_init 1e-2
```

```
# Set the max number of iterations
max_iter 500
```

is a valid ipopt.opt file.

Options can also be set in code. Have a look at the examples to see how this is done.

A subset of IPOPT options are available through AMPL. To set options through AMPL, use the internal AMPL command options. For example,

```
options ipopt "nlp_scaling_method=none mu_init=1e-2 max_iter=500"
```

is a valid options command in AMPL. The most common options are referenced in Appendix C. These are also the options that are available through AMPL using the options command **TODO:** CHECK IF **THAT IS CORRECT**. To specify other options when using AMPL, you can always create ipopt.opt. Note, the ipopt.opt file is given preference when setting options. This way, you can easily override any options set in a particular executable or AMPL model by specifying new values in ipopt.opt.

For a short list of the valid options, see the Appendix C. You can print the documentation for all IPOPT options by adding the option,

```
print_options_documentation yes
```

and running IPOPT (like the AMPL solver executable, for instance). This will output all of the options documentation to the console.

6 IPOPT Output

This section describes the standard IPOPT console output with the default setting for print_level. The output is designed to provide a quick summary of each iteration as IPOPT solves the problem.

Before IPOPT starts to solve the problem, it displays the problem statistics (number of nonzeroelements in the matrices, number of variables variables, etc.). Note that if you have fixed variables (both upper and lower bounds are equal), IPOPT may remove these variables from the problem internally and not include them in the problem statistics.

Following the problem statistics, IPOPT will begin to solve the problem and you will see output resembling the following,

```
iter objective inf_pr inf_du lg(mu) ||d|| lg(rg) alpha_du alpha_pr ls
0 1.6109693e+01 1.12e+01 5.28e-01 0.0 0.00e+00 - 0.00e+00 0.00e+00 0
1 1.8029749e+01 9.90e-01 6.62e+01 0.1 2.05e+00 - 2.14e-01 1.00e+00f 1
2 1.8719906e+01 1.25e-02 9.04e+00 -2.2 5.94e-02 2.0 8.04e-01 1.00e+00h 1
```

and the columns of output are defined as,

iter: The current iteration count. This includes regular iterations and iterations while in restoration phase. If the algorithm is in the restoration phase, the letter r' will be appended to the iteration number.

objective: The unscaled objective value at the current point. During the restoration phase, this value remains the unscaled objective value for the original problem.

inf_pr: The scaled primal infeasibility at the current point. During the restoration phase, this value is the primal infeasibility of the original problem at the current point.

inf_du: The scaled dual infeasibility at the current point. During the restoration phase, this is the value of the dual infeasibility for the restoration phase problem.

 ${\tt lg(mu):}\ \log_{10}$ of the value of the barrier parameter mu.

||d||: The infinity norm (max) of the primal step (for the original variables x and the internal slack variables s). During the restoration phase, this value includes the values of additional variables, p and n (see Eq. (30) in [7]).

lg(rg): log_{10} of the value of the regularization term for the Hessian of the Lagrangian in the augmented system.

alpha_du: The stepsize for the dual variables.

alpha_pr: The stepsize for the primal variables.

1s: The number of backtracking line search steps.

When the algorithm terminates, IPOPT will output a message to the screen based on the return status of the call to Optimize. The following is a list of the possible return codes, their corresponding output message to the console, and a brief description.

Solve_Succeeded:

Console Message: EXIT: Optimal Solution Found.

This message indicates that IPOPT found a (locally) optimal point within the desired tolerances.

Solved_To_Acceptable_Level:

Console Message: EXIT: Solved To Acceptable Level.

This indicates that the algorithm did not converge to the "desired" tolerances, but that it was able to obtain a point satisfying the "acceptable" tolerance level as specified by acceptable-* options. This may happen if the desired tolerances are too small for the current problem.

Infeasible_Problem_Detected:

Console Message: EXIT: Converged to a point of local infeasibility. Problem may be infeasible.

The restoration phase converged to a point that is a minimizer for the constraint violation (in the ℓ_1 -norm), but is not feasible for the original problem. This indicates that the problem may be infeasible (or at least that the algorithm is stuck at a locally infeasible point). The returned point (the minimizer of the constraint violation) might help you to find which constraint is causing the problem. If you believe that the NLP is feasible, it might help to start the optimization from a different point.

Search_Direction_Becomes_Too_Small:

Console Message: EXIT: Search Direction is becoming Too Small.

This indicates that IPOPT is calculating very small step sizes and making very little progress. This could happen if the problem has been solved to the best numerical accuracy possible given the current scaling.

Maximum_Iterations_Exceeded:

Console Message: EXIT: Maximum Number of Iterations Exceeded.

This indicates that IPOPT has exceeded the maximum number of iterations as specified by the option max_iter.

Restoration_Failed:

Console Message: EXIT: Restoration Failed!

This indicates that the restoration phase failed to find a feasible point that was acceptable to the filter line search for the original problem. This could happen if the problem is highly degenerate, does not satisfy the constraint qualification, or if your NLP code provides incorrect derivative information.

${\tt Invalid_Option:}$

Console Message: (details about the particular error will be output to the console)

This indicates that there was some problem specifying the options. See the specific message for details.

Not_Enough_Degrees_Of_Freedom:

Console Message: EXIT: Problem has too few degrees of freedom.

This indicates that your problem, as specified, has too few degrees of freedom. This can happen

if you have too many equality constraints, or if you fix too many variables (IPOPT removes fixed variables).

Invalid_Problem_Definition:

Console Message: (no console message, this is a return code for the C and Fortran interfaces only.) This indicates that there was an exception of some sort when building the IpoptProblem structure in the C or Fortran interface. Likely there is an error in your model or the main routine.

Unrecoverable_Exception:

Console Message: (details about the particular error will be output to the console)

This indicates that IPOPT has thrown an exception that does not have an internal return code. See the specific message for details.

NonIpopt_Exception_Thrown:

Console Message: Unknown Exception caught in Ipopt

An unknown exception was caught in IPOPT. This exception could have originated from your model or any linked in third party code.

Insufficient_Memory:

Console Message: EXIT: Not enough memory.

An error occurred while trying to allocate memory. The problem may be too large for your current memory and swap configuration.

Internal Error:

Console Message: EXIT: INTERNAL ERROR: Unknown SolverReturn value - Notify IPOPT Authors. An unknown internal error has occurred. Please notify the authors of IPOPT.

A Triplet Format for Sparse Matrices

IPOPT was designed for optimizing large sparse nonlinear programs. Because of problem sparsity, the required matrices (like the constraints Jacobian or Lagrangian Hessian) are not stored as dense matrices, but rather in a sparse matrix format. For the tutorials in this document, we use the triplet format. Consider the matrix

$$\begin{bmatrix}
1.1 & 0 & 0 & 0 & 0 & 0 & 0.5 \\
0 & 1.9 & 0 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 2.6 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 7.8 & 0.6 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.5 & 2.7 & 0 & 0 \\
1.6 & 0 & 0 & 0 & 0.4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.9 & 1.7
\end{bmatrix}$$
(10)

A standard dense matrix representation would need to store $7 \cdot 7=49$ floating point numbers, where many entries would be zero. In triplet format, however, only the nonzero entries are stored. The triplet format records the row number, the column number, and the value of all nonzero entries in the matrix. For the matrix above, this means storing 14 integers for the rows, 14 integers for the columns, and 14 floating point numbers for the values. While this does not seem like a huge space savings over the 49 floating point numbers stored in the dense representation, for larger matrices, the space savings are very dramatic¹².

The option index_style in get_nlp_info tells IPOPT if you prefer to use C style indexing (0-based, i.e., starting the counting at 0) for the row and column indices or Fortran style (1-based). Tables 1 and 2 below show the triplet format for both indexing styles, using the example matrix (10).

row	col	value
iRow[0] = 1	jCol[0] = 1	values[0] = 1.1
iRow[1] = 1	jCol[1] = 7	values[1] = 0.5
iRow[2] = 2	jCol[2] = 2	values[2] = 1.9
iRow[3] = 2	jCol[3] = 7	values[3] = 0.5
iRow[4] = 3	jCol[4] = 3	values[4] = 2.6
iRow[5] = 3	jCol[5] = 7	values[5] = 0.5
iRow[6] = 4	jCol[6] = 3	values[6] = 7.8
iRow[7] = 4	jCol[7] = 4	values[7] = 0.6
iRow[8] = 5	jCol[8] = 4	values[8] = 1.5
iRow[9] = 5	jCol[9] = 5	values[9] = 2.7
iRow[10] = 6	jCol[10] = 1	values[10] = 1.6
iRow[11] = 6	jCol[11] = 5	values[11] = 0.4
iRow[12] = 7	jCol[12] = 6	values[12] = 0.9
iRow[13] = 7	jCol[13] = 7	values[13] = 1.7

Table 1: Triplet Format of Matrix (10) with index_style=FORTRAN_STYLE

The individual elements of the matrix can be listed in any order, and if there are multiple items for the same nonzero position, the values provided for those positions are added.

The Hessian of the Lagrangian is a symmetric matrix. In the case of a symmetric matrix, you only need to specify the lower left triangual part (or, alternatively, the upper right triangular part). For

 $^{^{12}}$ For an $n \times n$ matrix, the dense representation grows with the square of n, while the sparse representation grows linearly in the number of nonzeros.

row	col	value
iRow[0] = 0	jCol[0] = 0	values[0] = 1.1
iRow[1] = 0	jCol[1] = 6	values[1] = 0.5
iRow[2] = 1	jCol[2] = 1	values[2] = 1.9
iRow[3] = 1	jCol[3] = 6	values[3] = 0.5
iRow[4] = 2	jCol[4] = 2	values[4] = 2.6
iRow[5] = 2	jCol[5] = 6	values[5] = 0.5
iRow[6] = 3	jCol[6] = 2	values[6] = 7.8
iRow[7] = 3	jCol[7] = 3	values[7] = 0.6
iRow[8] = 4	jCol[8] = 3	values[8] = 1.5
iRow[9] = 4	jCol[9] = 4	values[9] = 2.7
iRow[10] = 5	jCol[10] = 0	values[10] = 1.6
iRow[11] = 5	jCol[11] = 4	values[11] = 0.4
iRow[12] = 6	jCol[12] = 5	values[12] = 0.9
iRow[13] = 6	jCol[13] = 6	values[13] = 1.7

Table 2: Triplet Format of Matrix (10) with index_style=C_STYLE

example, given the matrix,

$$\begin{bmatrix} 1.0 & 0 & 3.0 & 0 & 2.0 \\ 0 & 1.1 & 0 & 0 & 5.0 \\ 3.0 & 0 & 1.2 & 6.0 & 0 \\ 0 & 0 & 6.0 & 1.3 & 9.0 \\ 2.0 & 5.0 & 0 & 9.0 & 1.4 \end{bmatrix}$$

$$(11)$$

the triplet format is shown in Tables 3 and 4.

Table 3: Triplet Format of Matrix (10) with $index_style=FORTRAN_STYLE$ row col value

row	col	value
iRow[0] = 1	jCol[0] = 1	values[0] = 1.0
iRow[1] = 2	jCol[1] = 1	values[1] = 1.1
iRow[2] = 3	jCol[2] = 1	values[2] = 3.0
iRow[3] = 3	jCol[3] = 3	values[3] = 1.2
iRow[4] = 4	jCol[4] = 3	values[4] = 6.0
iRow[5] = 4	jCol[5] = 4	values[5] = 1.3
iRow[6] = 5	jCol[6] = 1	values[6] = 2.0
iRow[7] = 5	jCol[7] = 2	values[7] = 5.0
iRow[8] = 5	jCol[8] = 4	values[8] = 9.0
iRow[9] = 5	jCol[9] = 5	values[9] = 1.4

Table 4: Triplet Format of Matrix (10) with index_style=C_STYLE

row	col	value
iRow[0] = 0	jCol[0] = 0	values[0] = 1.0
iRow[1] = 1	jCol[1] = 0	values[1] = 1.1
iRow[2] = 2	jCol[2] = 0	values[2] = 3.0
iRow[3] = 2	jCol[3] = 2	values[3] = 1.2
iRow[4] = 3	jCol[4] = 2	values[4] = 6.0
iRow[5] = 3	jCol[5] = 3	values[5] = 1.3
iRow[6] = 4	jCol[6] = 0	values[6] = 2.0
iRow[7] = 4	jCol[7] = 1	values[7] = 5.0
iRow[8] = 4	jCol[8] = 3	values[8] = 9.0
iRow[9] = 4	jCol[9] = 4	values[9] = 1.4

B The Smart Pointer Implementation: SmartPtr<T>

The SmartPtr class is described in IpSmartPtr.hpp. It is a template class that takes care of deleting objects for us so we need not be concerned about memory leaks. Instead of pointing to an object with a raw C++ pointer (e.g. HS071_NLP*), we use a SmartPtr. Every time a SmartPtr is set to reference an object, it increments a counter in that object (see the ReferencedObject base class if you are interested). If a SmartPtr is done with the object, either by leaving scope or being set to point to another object, the counter is decremented. When the count of the object goes to zero, the object is automatically deleted. SmartPtr's are very simple, just use them as you would a standard pointer.

It is very important to use SmartPtr's instead of raw pointers when passing objects to IPOPT. Internally, IPOPT uses smart pointers for referencing objects. If you use a raw pointer in your executable, the object's counter will NOT get incremented. Then, when IPOPT uses smart pointers inside its own code, the counter will get incremented. However, before IPOPT returns control to your code, it will decrement as many times as it incremented earlier, and the counter will return to zero. Therefore, IPOPT will delete the object. When control returns to you, you now have a raw pointer that points to a deleted object.

This might sound difficult to anyone not familiar with the use of smart pointers, but just follow one simple rule; always use a SmartPtr when creating or passing an IPOPT object.

C Options Reference

Options can be set using ipopt.opt, through your own code, or through the AMPL ipopt_options command. See Section 5 for an explanation of how to use these commands. Shown here is a short list of the most common options for Ipopt. To view the full list of options, run the ipopt executable with the option,

print_options_documentation yes

The most common options are:

print_level: Output verbosity level.

Sets the default verbosity level for console output. The larger this value the more detailed is the output. The valid range for this integer option is $0 \le print_level \le 11$ and its default value is 4.

print_options_documentation: Switch to print all algorithmic options.

If selected, the algorithm will print the list of all available algorithmic options with some documentation before solving the optimization problem. The default value for this string option is "no". Possible values:

• no: don't print list

• yes: print list

output_file: File name of desired output file (leave unset for no file output).

NOTE: This option only works when read from the ipopt.opt options file! An output file with this name will be written (leave unset for no file output). The verbosity level is by default set to "print_level", but can be overridden with "file_print_level". The file name is changed to use only small letters. The default value for this string option is "".

Possible values:

• *: Any acceptable standard file name

file_print_level: Verbosity level for output file.

NOTE: This option only works when read from the ipopt.opt options file! Determines the verbosity level for the file specified by "output_file". By default it is the same as "print_level". The valid range for this integer option is $0 \le \texttt{file_print_level} \le 11$ and its default value is 4.

tol: Desired convergence tolerance (relative).

Determines the convergence tolerance for the algorithm. The algorithm terminates successfully, if the (scaled) NLP error becomes smaller than this value, and if the (absolute) criteria according to "dual_inf_tol", "primal_inf_tol", and "cmpl_inf_tol" are met. (This is epsilon_tol in Eqn. (6) in implementation paper). See also "acceptable_tol" as a second termination criterion. Note, some other algorithmic features also use this quantity to determine thresholds etc. The valid range for this real option is 0 < tol < +inf and its default value is $1 \cdot 10^{-08}$.

max_iter: Maximum number of iterations.

The algorithm terminates with an error message if the number of iterations exceeded this number. The valid range for this integer option is $0 \le \text{max_iter} < +\text{inf}$ and its default value is 3000.

compl_inf_tol: Desired threshold for the complementarity conditions.

Absolute tolerance on the complementarity. Successful termination requires that the max-norm of the (unscaled) complementarity is less than this threshold. The valid range for this real option is $0 < compl_inf_tol < +inf$ and its default value is 0.0001.

dual_inf_tol: Desired threshold for the dual infeasibility.

Absolute tolerance on the dual infeasibility. Successful termination requires that the max-norm of the (unscaled) dual infeasibility is less than this threshold. The valid range for this real option is $0 < dual_inf_tol < +inf$ and its default value is 0.0001.

constr_viol_tol: Desired threshold for the constraint violation.

Absolute tolerance on the constraint violation. Successful termination requires that the max-norm of the (unscaled) constraint violation is less than this threshold. The valid range for this real option is $0 < \text{constr_viol_tol} < +\text{inf}$ and its default value is 0.0001.

acceptable_tol: "Acceptable" convergence tolerance (relative).

Determines which (scaled) overall optimality error is considered to be "acceptable." There are two levels of termination criteria. If the usual "desired" tolerances (see tol, dual_inf_tol etc) are satisfied at an iteration, the algorithm immediately terminates with a success message. On the other hand, if the algorithm encounters "acceptable_iter" many iterations in a row that are considered "acceptable", it will terminate before the desired convergence tolerance is met. This is useful in cases where the algorithm might not be able to achieve the "desired" level of accuracy. The valid range for this real option is $0 < \text{acceptable_tol} < + \text{inf}$ and its default value is $1 \cdot 10^{-06}$.

acceptable_compl_inf_tol: "Acceptance" threshold for the complementarity conditions.

Absolute tolerance on the complementarity. "Acceptable" termination requires that the max-norm of the (unscaled) complementarity is less than this threshold; see also acceptable_tol. The valid range for this real option is $0 < acceptable_compl_inf_tol < +inf$ and its default value is 0.01.

acceptable_constr_viol_tol: "Acceptance" threshold for the constraint violation.

Absolute tolerance on the constraint violation. "Acceptable" termination requires that the max-norm of the (unscaled) constraint violation is less than this threshold; see also acceptable_tol. The valid range for this real option is $0 < acceptable_constr_viol_tol < +inf$ and its default value is 0.01.

acceptable_dual_inf_tol: "Acceptance" threshold for the dual infeasibility.

Absolute tolerance on the dual infeasibility. "Acceptable" termination requires that the (max-norm of the unscaled) dual infeasibility is less than this threshold; see also acceptable_tol. The valid range for this real option is $0 < acceptable_dual_inf_tol < +inf$ and its default value is 0.01.

diverging_iterates_tol: Threshold for maximal value of primal iterate.

If any component of the primal iterates exceeded this value (in absolute terms), the optimization is aborted with the exit message that the iterates seem to be diverging. The valid range for this real option is $0 < \text{diverging_iterates_tol} < + \text{inf}$ and its default value is $1 \cdot 10^{+20}$.

barrier_tol_factor: Factor for mu in barrier stop test.

The convergence tolerance for each barrier problem in the monotone mode is the value of the barrier parameter times "barrier_tol_factor". This option is also used in the adaptive mu strategy during the monotone mode. (This is kappa_epsilon in implementation paper). The valid range for this real option is $0 < \text{barrier_tol_factor} < + \text{inf}$ and its default value is 10.

obj_scaling_factor: Scaling factor for the objective function.

This option sets a scaling factor for the objective function. The scaling is seen internally by Ipopt but the unscaled objective is reported in the console output. If additional scaling parameters are computed (e.g. user-scaling or gradient-based), both factors are multiplied. If this value is chosen to be negative, Ipopt will maximize the objective function instead of minimizing it. The valid range for this real option is $-\inf < obj_scaling_factor < +\inf$ and its default value is 1.

nlp_scaling_method: Select the technique used for scaling the NLP.

Selects the technique used for scaling the problem internally before it is solved. For user-scaling, the parameters come from the NLP. If you are using AMPL, they can be specified through suffixes ("scaling_factor") The default value for this string option is "gradient-based". Possible values:

- none: no problem scaling will be performed
- user-scaling: scaling parameters will come from the user
- gradient-based: scale the problem so the maximum gradient at the starting point is scaling_max_gradient

nlp_scaling_max_gradient: Maximum gradient after NLP scaling.

This is the gradient scaling cut-off. If the maximum gradient is above this value, then gradient based scaling will be performed. Scaling parameters are calculated to scale the maximum gradient back to this value. (This is g_max in Section 3.8 of the implementation paper.) Note: This option is only used if "nlp_scaling_method" is chosen as "gradient-based". The valid range for this real option is $0 < \text{nlp_scaling_max_gradient} < + \text{inf}$ and its default value is 100.

bound_relax_factor: Factor for initial relaxation of the bounds.

Before start of the optimization, the bounds given by the user are relaxed. This option sets the factor for this relaxation. If it is set to zero, then then bounds relaxation is disabled. (See Eqn.(35) in implementation paper.) The valid range for this real option is $0 \le bound_relax_factor < +inf$ and its default value is $1 \cdot 10^{-08}$.

honor_original_bounds: Indicates whether final points should be projected into original bounds. Ipopt might relax the bounds during the optimization (see, e.g., option "bound_relax_factor"). This option determines whether the final point should be projected back into the user-provide original bounds after the optimization. The default value for this string option is "yes". Possible values:

- no: Leave final point unchanged
- yes: Project final point back into original bounds

check_derivatives_for_naninf: Indicates whether it is desired to check for Nan/Inf in derivative matrices

Activating this option will cause an error if an invalid number is detected in the constraint Jacobians or the Lagrangian Hessian. If this is not activated, the test is skipped, and the algorithm might proceed with invalid numbers and fail. The default value for this string option is "no".

- Possible values:
 - no: Don't check (faster).
 - yes: Check Jacobians and Hessian for Nan and Inf.

mu_strategy: Update strategy for barrier parameter.

Determines which barrier parameter update strategy is to be used. The default value for this string option is "monotone".

Possible values:

- monotone: use the monotone (Fiacco-McCormick) strategy
- adaptive: use the adaptive update strategy

mu_oracle: Oracle for a new barrier parameter in the adaptive strategy.

Determines how a new barrier parameter is computed in each "free-mode" iteration of the adaptive barrier parameter strategy. (Only considered if "adaptive" is selected for option "mu_strategy"). The default value for this string option is "quality-function".

Possible values:

- probing: Mehrotra's probing heuristic
- loqo: LOQO's centrality rule
- quality-function: minimize a quality function

quality_function_max_section_steps: Maximum number of search steps during direct search procedure determining the optimal centering parameter.

The golden section search is performed for the quality function based mu oracle. (Only used if option "mu_oracle" is set to "quality-function".) The valid range for this integer option is $0 \le \text{quality_function_max_section_s} + \text{inf}$ and its default value is 8.

fixed_mu_oracle: Oracle for the barrier parameter when switching to fixed mode.

Determines how the first value of the barrier parameter should be computed when switching to the "monotone mode" in the adaptive strategy. (Only considered if "adaptive" is selected for option "mu_strategy".) The default value for this string option is "average_compl".

Possible values:

- probing: Mehrotra's probing heuristic
- logo: LOQO's centrality rule
- quality-function: minimize a quality function
- average_compl: base on current average complementarity

mu_init: Initial value for the barrier parameter.

This option determines the initial value for the barrier parameter (mu). It is only relevant in the monotone, Fiacco-McCormick version of the algorithm. (i.e., if "mu_strategy" is chosen as "monotone") The valid range for this real option is $0 < mu_init < +inf$ and its default value is 0.1.

mu_max_fact: Factor for initialization of maximum value for barrier parameter.

This option determines the upper bound on the barrier parameter. This upper bound is computed as the average complementarity at the initial point times the value of this option. (Only used if option "mu_strategy" is chosen as "adaptive".) The valid range for this real option is $0 < mu_max_fact < +inf$ and its default value is 1000.

mu_max: Maximum value for barrier parameter.

This option specifies an upper bound on the barrier parameter in the adaptive mu selection mode. If this option is set, it overwrites the effect of mu_max_fact. (Only used if option "mu_strategy" is chosen as "adaptive".) The valid range for this real option is $0 < \text{mu_max} < +\text{inf}$ and its default value is 100000.

mu_min: Minimum value for barrier parameter.

This option specifies the lower bound on the barrier parameter in the adaptive mu selection mode. By default, it is set to min("tol","compl_inf_tol")/("barrier_tol_fact- or"+1), which should be a reasonable value. (Only used if option "mu_strategy" is chosen as "adaptive".) The valid range for this real option is $0 < mu_min < +inf$ and its default value is $1 \cdot 10^{-09}$.

mu_linear_decrease_factor: Determines linear decrease rate of barrier parameter.

For the Fiacco-McCormick update procedure the new barrier parameter mu is obtained by taking the minimum of mu*"mu_linear_decrease_factor" and mu*superlinear_decrease_power". (This is kappa_mu in implementation paper.) This option is also used in the adaptive mu strategy during the monotone mode. The valid range for this real option is $0 < mu_linear_decrease_factor < 1$ and its default value is 0.2.

mu_superlinear_decrease_power: Determines superlinear decrease rate of barrier parameter.

For the Fiacco-McCormick update procedure the new barrier parameter mu is obtained by taking the minimum of mu*"mu_linear_decrease_factor" and mu $^{\circ}$ superlinear_decrease_power". (This is theta_mu in implementation paper.) This option is also used in the adaptive mu strategy during the monotone mode. The valid range for this real option is $1 < mu_superlinear_decrease_power < 2$ and its default value is 1.5.

bound_frac: Desired minimum relative distance from the initial point to bound.

Determines how much the initial point might have to be modified in order to be sufficiently inside the bounds (together with "bound_push"). (This is kappa_2 in Section 3.6 of implementation paper.) The valid range for this real option is $0 < bound_frac \le 0.5$ and its default value is 0.01.

bound_mult_init_val: Initial value for the bound multipliers.

All dual variables corresponding to bound constraints are initialized to this value. The valid range for this real option is $0 < bound_mult_init_val < +inf$ and its default value is 1.

bound_push: Desired minimum absolute distance from the initial point to bound.

Determines how much the initial point might have to be modified in order to be sufficiently inside the bounds (together with "bound_frac"). (This is kappa_1 in Section 3.6 of implementation paper.) The valid range for this real option is $0 < bound_push < +inf$ and its default value is 0.01.

constr_mult_init_max: Maximum allowed least-square guess of constraint multipliers.

Determines how large the initial least-square guesses of the constraint multipliers are allowed to be (in max-norm). If the guess is larger than this value, it is discarded and all constraint multipliers are set to zero. This options is also used when initializing the restoration phase. By default, "resto.constr_mult_init_max" (the one used in RestoIterateInitializer) is set to zero. The valid range for this real option is $0 \le \text{constr_mult_init_max} < +\text{inf}$ and its default value is 1000.

bound_mult_init_val: Initial value for the bound multipliers.

All dual variables corresponding to bound constraints are initialized to this value. The valid range for this real option is $0 < bound_mult_init_val < +inf$ and its default value is 1.

warm_start_init_point: Warm-start for initial point

Indicates whether this optimization should use a warm start initialization, where values of primal and dual variables are given (e.g., from a previous optimization of a related problem.) The default value for this string option is "no".

Possible values:

- no: do not use the warm start initialization
- yes: use the warm start initialization

warm_start_bound_push: same as bound_push for the regular initializer.

The valid range for this real option is $0 < warm_start_bound_push < +inf$ and its default value is 0.001.

warm_start_bound_frac: same as bound_frac for the regular initializer.

The valid range for this real option is $0 < warm_start_bound_frac \le 0.5$ and its default value is 0.001.

warm_start_mult_bound_push: same as mult_bound_push for the regular initializer.

The valid range for this real option is $0 < warm_start_mult_bound_push < +inf$ and its default value is 0.001.

warm_start_mult_init_max: Maximum initial value for the equality multipliers.

The valid range for this real option is $-\inf < \text{warm_start_mult_init_max} < +\inf$ and its default value is $1 \cdot 10^{+06}$.

alpha_for_y: Method to determine the step size for constraint multipliers.

This option determines how the step size (alpha_y) will be calculated when updating the constraint multipliers. The default value for this string option is "primal".

Possible values:

- primal: use primal step size
- bound_mult: use step size for the bound multipliers (good for LPs)
- min: use the min of primal and bound multipliers
- max: use the max of primal and bound multipliers
- full: take a full step of size one
- min_dual_infeas: choose step size minimizing new dual infeasibility
- safe_min_dual_infeas: like "min_dual_infeas", but safeguarded by "min" and "max"

recalc_y: Tells the algorithm to recalculate the equality and inequality multipliers as least square estimates.

This asks the algorithm to recompute the multipliers, whenever the current infeasibility is less than recalc_y_feas_tol. Choosing yes might be helpful in the quasi-Newton option. However, each recalculation requires an extra factorization of the linear system. If a limited memory quasi-Newton option is chosen, this is used by default. The default value for this string option is "no".

Possible values:

- no: use the Newton step to update the multipliers
- yes: use least-square multiplier estimates

recalc_y_feas_tol: Feasibility threshold for recomputation of multipliers.

If recalc_y is chosen and the current infeasibility is less than this value, then the multipliers are recomputed. The valid range for this real option is $0 < \text{recalc_y_feas_tol} < + \text{inf}$ and its default value is $1 \cdot 10^{-06}$.

max_soc: Maximum number of second order correction trial steps at each iteration.

Choosing 0 disables the second order corrections. (This is pmax of Step A-5.9 of Algorithm A in implementation paper.) The valid range for this integer option is $0 \le \max_{soc} < +\inf$ and its default value is 4.

watchdog_shortened_iter_trigger: Number of shortened iterations that trigger the watchdog. If the number of successive iterations in which the backtracking line search did not accept the first trial point exceeds this number, the watchdog procedure is activated. Choosing "0" here disables the watchdog procedure. The valid range for this integer option is $0 \le \text{watchdog_shortened_iter_trigger} < +\text{inf}$ and its default value is 10.

watchdog_trial_iter_max: Maximum number of watchdog iterations.

This option determines the number of trial iterations allowed before the watchdog procedure is aborted and the algorithm returns to the stored point. The valid range for this integer option is $1 \le \texttt{watchdog_trial_iter_max} < + \texttt{inf}$ and its default value is 3.

expect_infeasible_problem: Enable heuristics to quickly detect an infeasible problem.

This options is meant to activate heuristics that may speed up the infeasibility determination if you expect that there is a good chance for the problem to be infeasible. In the filter line search procedure, the restoration phase is called more quickly than usually, and more reduction in the constraint violation is enforced before the restoration phase is left. If the problem is square, this option is enabled automatically. The default value for this string option is "no".

Possible values:

- no: the problem probably be feasible
- yes: the problem has a good chance to be infeasible

expect_infeasible_problem_ctol: Threshold for disabling "expect_infeasible_problem" option. If the constraint violation becomes smaller than this threshold, the "expect_infeasible_problem" heuristics in the filter line search are disabled. If the problem is square, this options is set to 0. The valid range for this real option is $0 \le \text{expect_infeasible_problem_ctol} < +\text{inf}$ and its default value is 0.001.

start_with_resto: Tells algorithm to switch to restoration phase in first iteration.

Setting this option to "yes" forces the algorithm to switch to the feasibility restoration phase in the first iteration. If the initial point is feasible, the algorithm will abort with a failure. The default value for this string option is "no".

Possible values:

- no: don't force start in restoration phase
- yes: force start in restoration phase

soft_resto_pderror_reduction_factor: Required reduction in primal-dual error in the soft restoration phase.

The soft restoration phase attempts to reduce the primal-dual error with regular steps. If the damped primal-dual step (damped only to satisfy the fraction-to-the-boundary rule) is not decreasing the primal-dual error by at least this factor, then the regular restoration phase is called. Choosing "0" here disables the soft restoration phase. The valid range for this real option is $0 \le soft_resto_pderror_reduction_factor < +inf$ and its default value is 0.9999.

required_infeasibility_reduction: Required reduction of infeasibility before leaving restoration phase.

The restoration phase algorithm is performed, until a point is found that is acceptable to the filter and the infeasibility has been reduced by at least the fraction given by this option. The valid range for this real option is $0 \le \text{required_infeasibility_reduction} < 1$ and its default value is 0.9.

bound_mult_reset_threshold: Threshold for resetting bound multipliers after the restoration phase. After returning from the restoration phase, the bound multipliers are updated with a Newton step for complementarity. Here, the change in the primal variables during the entire restoration phase is taken to be the corresponding primal Newton step. However, if after the update the largest bound multiplier exceeds the threshold specified by this option, the multipliers are all reset to 1. The valid range for this real option is $0 \le bound_mult_reset_threshold < +inf and its default value is 1000.$

constr_mult_reset_threshold: Threshold for resetting equality and inequality multipliers after restoration phase.

After returning from the restoration phase, the constraint multipliers are recomputed by a least square estimate. This option triggers when those least-square estimates should be ignored. The valid range for this real option is $0 \le \text{constr_mult_reset_threshold} < + \text{inf}$ and its default value is 0.

evaluate_orig_obj_at_resto_trial: Determines if the original objective function should be evaluated at restoration phase trial points.

Setting this option to "yes" makes the restoration phase algorithm evaluate the objective function of the original problem at every trial point encountered during the restoration phase, even if this value is not required. In this way, it is guaranteed that the original objective function can be evaluated without error at all accepted iterates; otherwise the algorithm might fail at a point where the restoration phase accepts an iterate that is good for the restoration phase problem, but not the original problem. On the other hand, if the evaluation of the original objective is expensive, this might be costly. The default value for this string option is "yes".

Possible values:

• no: skip evaluation

• yes: evaluate at every trial point

linear_solver: Linear solver used for step computations.

Determines which linear algebra package is to be used for the solution of the augmented linear system (for obtaining the search directions). Note, the code must have been compiled with the linear solver you want to choose. Depending on your Ipopt installation, not all options are available. The default value for this string option is "ma27".

Possible values:

• ma27: use the Harwell routine MA27

• ma57: use the Harwell routine MA57

• pardiso: use the Pardiso package

• wsmp: use WSMP package (not yet working)

• taucs: use TAUCS package (not yet working)

linear_system_scaling: Method for scaling the linear system.

Determines the method used to compute symmetric scaling factors for the augmented system. This scaling is independent of the NLP problem scaling. The default value for this string option is "mc19". Possible values:

• none: no scaling will be performed

• mc19: use the Harwell routine mc19

linear_scaling_on_demand: Flag indicating that linear scaling is only done if it seems required.

This option is only important if a linear scaling method (e.g., mc19) is used. If you choose "no", then the scaling factors are computed for every linear system from the start. This can be quite expensive. Choosing "yes" means that the algorithm will starting the scaling method only when the solutions to the linear system seem not good, and then use it until the end. The default value for this string option is "yes".

Possible values:

- no: Always scaling the linear system.
- yes: Start using linear system if solutions seem not good.

max_refinement_steps: Maximum number of iterative refinement steps per linear system solve. Iterative refinement (on the full unsymmetric system) is performed for each right hand side. This option determines the maximum number of iterative refinement steps. The valid range for this integer option is $0 \le max_refinement_steps < +inf$ and its default value is 10.

min_refinement_steps: Minimum number of iterative refinement steps per linear system solve. Iterative refinement (on the full unsymmetric system) is performed for each right hand side. This option determines the minimum number of iterative refinements (i.e. at least "min_refinement_steps" iterative refinement steps are enforced per right hand side.) The valid range for this integer option is $0 \le \min_{refinement_steps} < +\inf_{refinement_steps} < +\int_{refinement_steps} < +\int_{refinement$

max_hessian_perturbation: Maximum value of regularization parameter for handling negative curvature.

In order to guarantee that the search directions are indeed proper descent directions, Ipopt requires that the inertia of the (augmented) linear system for the step computation has the correct number of negative and positive eigenvalues. The idea is that this guides the algorithm away from maximizers and makes Ipopt more likely converge to first order optimal points that are minimizers. If the inertia is not correct, a multiple of the identity matrix is added to the Hessian of the Lagrangian in the augmented system. This parameter gives the maximum value of the regularization parameter. If a regularization of that size is not enough, the algorithm skips this iteration and goes to the restoration phase. (This is delta_wmax in the implementation paper.) The valid range for this real option is $0 < \max_{n} \text{hessian_perturbation} < + \inf_{n} \text{ and its default value is } 1 \cdot 10^{+20}$.

min_hessian_perturbation: Smallest perturbation of the Hessian block.

The size of the perturbation of the Hessian block is never selected smaller than this value, unless no perturbation is necessary. (This is delta_wm̂in in implementation paper.) The valid range for this real option is $0 \le \min_{n} \text{hessian_perturbation} < +\inf_{n} \text{ and its default value is } 1 \cdot 10^{-20}$.

first_hessian_perturbation: Size of first x-s perturbation tried.

The first value tried for the x-s perturbation in the inertia correction scheme. (This is delta_0 in the implementation paper.) The valid range for this real option is $0 < first_hessian_perturbation < +inf$ and its default value is 0.0001.

perturb_inc_fact_first: Increase factor for x-s perturbation for very first perturbation.

The factor by which the perturbation is increased when a trial value was not sufficient - this value is used for the computation of the very first perturbation and allows a different value for the first perturbation than that used for the remaining perturbations. (This is $bar_kappa_w^+$ in the implementation paper.) The valid range for this real option is $1 < perturb_inc_fact_first < +inf$ and its default value is 100.

perturb_inc_fact: Increase factor for x-s perturbation.

The factor by which the perturbation is increased when a trial value was not sufficient - this value is used for the computation of all perturbations except for the first. (This is kappa_w $\hat{+}$ in the implementation paper.) The valid range for this real option is $1 < \text{perturb_inc_fact} < + \text{inf}$ and its default value is 8.

perturb_dec_fact: Decrease factor for x-s perturbation.

The factor by which the perturbation is decreased when a trial value is deduced from the size of the most recent successful perturbation. (This is kappa_w- in the implementation paper.) The valid range for this real option is $0 < \text{perturb_dec_fact} < 1$ and its default value is 0.3333333.

jacobian_regularization_value: Size of the regularization for rank-deficient constraint Jacobians. (This is bar delta_c in the implementation paper.) The valid range for this real option is $0 \le jacobian_regularization_value$ +inf and its default value is $1 \cdot 10^{-08}$.

hessian_approximation: Indicates what Hessian information is to be used.

This determines which kind of information for the Hessian of the Lagrangian function is used by the algorithm. The default value for this string option is "exact".

Possible values:

- exact: Use second derivatives provided by the NLP.
- limited-memory: Perform a limited-memory quasi-Newton approximation

limited_memory_max_history: Maximum size of the history for the limited quasi-Newton Hessian approximation.

This option determines the number of most recent iterations that are taken into account for the limited-memory quasi-Newton approximation. The valid range for this integer option is $0 \le limited_memory_max_history < +inf$ and its default value is 6.

limited_memory_max_skipping: Threshold for successive iterations where update is skipped. If the update is skipped more than this number of successive iterations, we quasi-Newton approximation is reset. The valid range for this integer option is $1 \leq limited_memory_max_skipping < +inf$ and its default value is 2.

derivative_test: Enable derivative checker

If this option is enabled, a (slow) derivative test will be performed before the optimization. The test is performed at the user provided starting point and marks derivative values that seem suspicious The default value for this string option is "none".

Possible values:

- $\bullet\,$ none: do not perform derivative test
- first-order: perform test of first derivatives at starting point
- second-order: perform test of first and second derivatives at starting point

derivative_test_perturbation: Size of the finite difference perturbation in derivative test. This determines the relative perturbation of the variable entries. The valid range for this real option is $0 < \text{derivative_test_perturbation} < + \text{inf}$ and its default value is $1 \cdot 10^{-08}$.

derivative_test_tol: Threshold for indicating wrong derivative.

If the relative deviation of the estimated derivative from the given one is larger than this value, the corresponding derivative is marked as wrong. The valid range for this real option is $0 < \texttt{derivative_test_tol} < + \texttt{inf}$ and its default value is 0.0001.

derivative_test_print_all: Indicates whether information for all estimated derivatives should be printed.

Determines verbosity of derivative checker. The default value for this string option is "no". Possible values:

- no: Print only suspect derivatives
- yes: Print all derivatives

ma27_pivtol: Pivot tolerance for the linear solver MA27.

A smaller number pivots for sparsity, a larger number pivots for stability. The valid range for this real option is 0 < ma27-pivtol < 1 and its default value is $1 \cdot 10^{-08}$.

ma27_pivtolmax: Maximum pivot tolerance for the linear solver MA27.

Ipopt may increase pivtol as high as pivtolmax to get a more accurate solution to the linear system. The valid range for this real option is $0 < ma27_pivtolmax < 1$ and its default value is 0.0001.

ma27_liw_init_factor: Integer workspace memory for MA27.

The initial integer workspace memory = $liw_init_factor * memory required by unfactored system. Ipopt will increase the workspace size by meminc_factor if required. The valid range for this real option is <math>1 \le ma27_liw_init_factor < +inf$ and its default value is 5.

ma27_la_init_factor: Real workspace memory for MA27.

The initial real workspace memory = la_init_factor * memory required by unfactored system. Ipopt will increase the workspace size by meminc_factor if required. The valid range for this real option is $1 \le ma27_la_init_factor < +inf$ and its default value is 5.

ma27_meminc_factor: Increment factor for workspace size for MA27.

If the integer or real workspace is not large enough, Ipopt will increase its size by this factor. The valid range for this real option is $1 \le \text{ma27_meminc_factor} < +\text{inf}$ and its default value is 10.

D Detailed Installation Information

The configuration script and Makefiles in the IPOPT distribution have been created using GNU's autoconf and automake. They attempt to automatically adapt the compiler settings etc. to the system they are running on. We tested the provided scripts for a number of different machines, operating systems and compilers, but you might run into a situation where the default setting does not work, or where you need to change the settings to fit your particular environment.

In general, you can see the list of options and variables that can be set for the **configure** script by typing **configure** --help. Below a few particular options are discussed:

• The configure script tries to determine automatically, if you have BLAS and/or LAPACK already installed on your system (trying a few default libraries), and if it does not find them, it makes sure that you put the source code in the required place.

However, you can specify a BLAS library (such as your local ATLAS library¹³) explicitly, using the --with-blas flag for configure. For example,

```
./configure --with=blas="-L$HOME/lib -latlas"
```

To tell the configure script to compile and use the downloaded BLAS source files even if a BLAS library is found on your system, specify --with-blas=BUILD.

Similarly, you can use the --with-lapack switch to specify the location of your LAPACK library, or use the keyword BUILD to force the IPOPT makefiles to compile LAPACK together with IPOPT.

- Similarly, if you have a precompiled library containing the Harwell Subroutines, you can specify its location with the --with-hsl flag. And the location of the AMPL solver library (with the ASL header files) can be specified with --with-asldir. TODO Other linear solvers
- If you want to specify that you want to use particular compilers, you can do so by adding the variables definitions for CXX, CC, and F77 to the ./configure command line, to specify the C++, C, and Fortran compiler, respectively. For example,

```
./configure CXX=g++ CC=gcc F77=g77
```

In order to set the compiler flags, you should use the variables CXXFLAGS, CFLAGS, FFLAGS. Note, that the IPOPT code uses "dynamic_cast". Therefore it is necessary that the C++ code is compiled including RTTI (Run-Time Type Information). Some compilers need to be given special flags to do that (e.g., "-qrtti=dyna" for the AIX xlC compiler).

• If you want to link the IPOPT library with a main program written in C or Fortran, the C and Fortran compiler doing the linking of the executable needs to be told about the C++ runtime libraries. Unfortunately, the current version of autoconf does not provide the automatic detection of those libraries. We have hard-coded some default values for some systems and compilers, but this might not work all the time.

If you have problems linking your Fortran or C code with the IPOPT library libipopt.a and the linker complains about missing symbols from C++ (e.g., the standard template library), you should specify the C++ libraries with the CXXLIBS variable. To find out what those libraries are, it is probably helpful to link a simple C++ program with verbose compiler output.

For example, for the Intel compilers on a Linux system, you might need to specify something like

• Compilation in 64bit mode sometimes requires some special consideration. For example, for compilation of 64bit code on AIX, we recommend the following configuration

¹³see http://math-atlas.sourceforge.net/

• To build library/archive files (with the ending .a) including C++ code in some environments, it is necessary to use the C++ compiler instead of ar to build the archive. This is for example the case for some older compilers on SGI and SUN. For this, the configure variables AR, ARFLAGS, and AR_X are provided. Here, AR specifies the command for the archiver for creating an archive, and ARFLAGS specifies additional flags. AR_X contains the command for extracting all files from an archive. For example, the default setting for SUN compilers for our configure script is

```
AR='CC -xar' ARFLAGS='-o' AR_X='ar x'
```

- It is possible to compile the IPOPT library in a debug configuration, by specifying --enable-debug. Then the compilers will use the debug flags (unless the compilation flag variables are overwritten in the configure command line), and additional debug checks are compiled into the code (see IpDebug.hpp). This usually leads to a significant slowdown of the code, but might be helpful when debugging something.
- It is not necessary to produce the binary files in the directories where the source files are. If you want to compile the code on different systems or with different compilers/options on a shared file system, you can keep one single copy of the source files in one directory, and the binary files for each configuration in separate directories. For this, simply run the configure script in the directory where you want the base directory for the IPOPT binary files. For example:

```
$ mkdir $HOME/Ipopt-objects
$ cd $HOME/Ipopt-objects
$ $HOME/Ipopt/configure (or $HOME/ipopt-3.1.0/configure)
```

References

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