Event-B Tutorial

赵永望
zhaoyw@buaa.edu.cn
北京航空航天大学 计算机学院

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提 纲

- 1. 形式化方法(Formal Method)
  - 形式化方法
  - B方法
  - Event-B基本思想
- 2. 基本理论
  - 集合论(Set Theory)
  - 命题逻辑(Propositional Logic)与谓词逻辑(Predicate Logic)
  - 演绎推理(Deductive Reasoning)
- 3. Event-B详解
  - 抽象机(Machine)与上下文(Context)
  - 精化(Refinement)
  - 语义(Semantics)
  - 正确性证明(Proof)
- 4. Event-B实践
  - 开发、证明

1. 形式化方法

什么叫形式化方法 Formal Method

- Formal Method vs. Informal Method
- Formal
  - 形式主义？(NO)
  - 逻辑的、数学的、正规的 (YES)

- In *computer science*, *specifically software engineering and hardware engineering*, formal methods are a particular kind of mathematically based techniques for the specification, development and verification of software and hardware systems.

  *R. W. Butler (2001-08-06), "What is Formal Methods?", http://shemesh.larc.nasa.gov/fm/fm-what.html(**NASA Langley Formal Methods**)*

- Informal method: 用word写需求，用visio画设计图，……

软件工程：问题和需要

- **最重要的问题：**
  - 软件的功能和质量的保证
  - 开发成本
  - 开发效率
- **有关计算机软件的研究都围绕着：**
  - 提高软件系统的质量和可靠性
  - 提高开发的效率
  - 降低开发的成本
  - 就是说：想让马儿快快壮，还不想让马儿多吃草

软件工程：实际情况

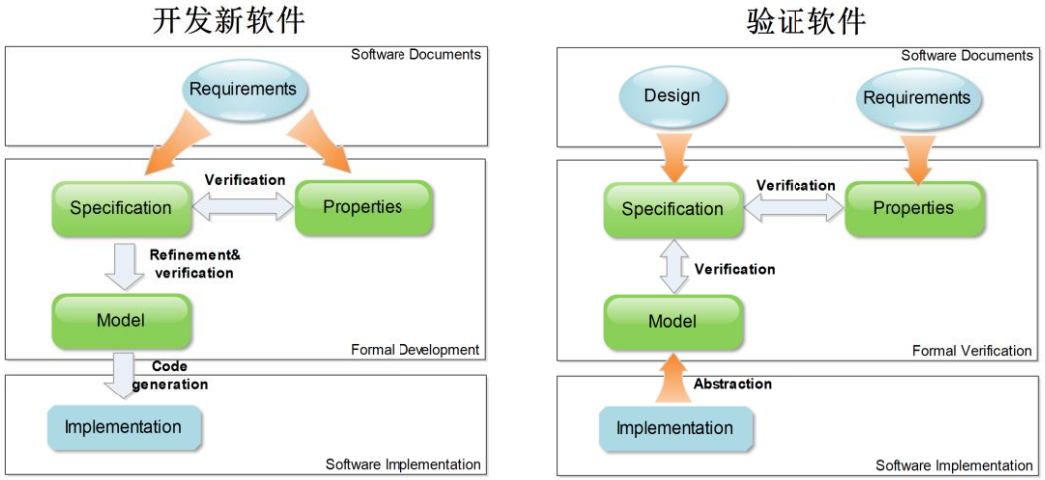
- 软件开发的实际情况并不乐观：
  - 项目经常延误，预算经常超支
  - 开发的后续阶段常发现许多前期设计错误，更正的代价高昂
  - 发布运行的软件中常常存在着许多错误，时常崩溃
  - 软件维护和更新工作的代价很高
- 一个情况：没有人愿意给软件做保险

- **软件的一个小错误，可能导致整个系统的崩溃**
- **难以估计软件出问题的可能性和造成的损失**

- 机械工程：在一间明亮的房间里面寻找一只黑猫
- 化学工程：在一间黑暗的房间里面寻找一只黑猫
- 软件工程：在一间黑暗的房间里面寻找一只黑猫，而实际上房间里并没有猫
- 系统工程：在一间黑暗的房间里面寻找一只黑猫，而实际上房间里并没有猫，但某人还大呼：我找到了
- —— 摘自形式化方法的幽默故事：http://shemesh.larc.nasa.gov/fm/fm-humor.html

形式化方法

- 形式化方法不能确保系统的可靠性，但其可以通过揭示系统的不一致性、歧义性和不完备性来增加我们对系统的理解程度，从而提高我们对系统可靠性的可信度
- 形式化方法使用的方式(针对软件)



- 形式化方法使用的3个层面
  - Level 0: formal specification
  - Level 1: formal development, formal verification: proofs of properties or refinement from the specification to a program
  - Level 2: theorem prover: fully formal machine-checked proofs
- 形式化方法包括：
  - 规约（specifying）
  - 建模（Modeling）
    - Abstraction、Refinement
  - 验证（verification）
    - 模型检测(model checking)、演绎推理/定理证明(Deductive reasoning, theorem proving)

提高软件质量的途径

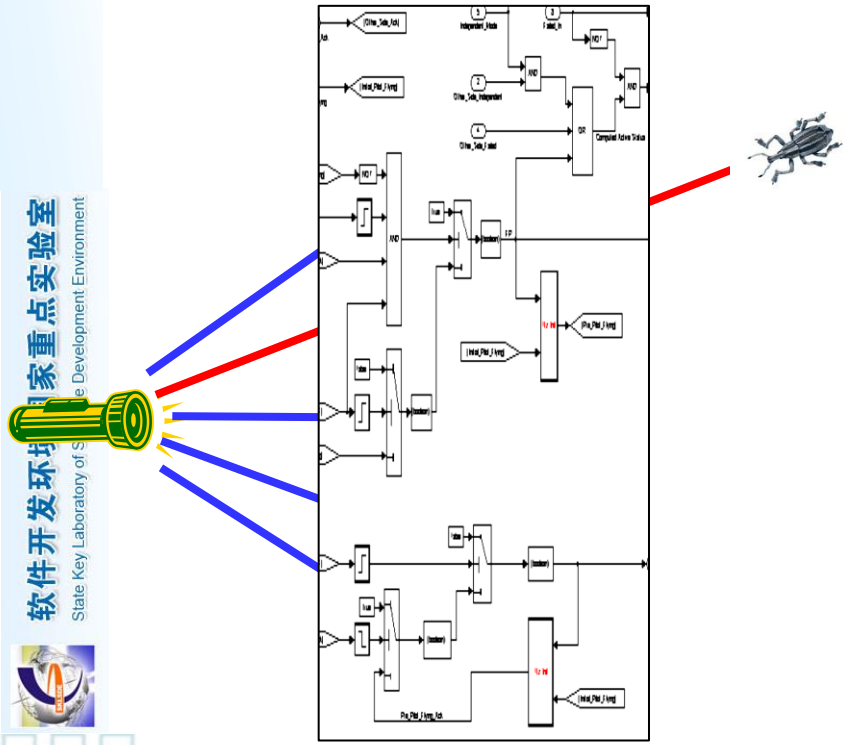
- 分析：程序和产品的分析
  - 静态分析：通过对程序代码进行扫描分析而获得程序相关性质的技术
  - 动态分析：通过对程序的执行来获得程序的相关性质，一般与程序切片、调试和测试技术相结合
- 测试
  - 以运行系统为主要手段发现系统错误
  - 只能发现系统的错误，不能证明系统没有错，无法回答系统一定没有错误这样一类问题
  - 工业界常用的途径，成本高，代价大
  - 仿真（Simulation）：以运行系统模型为主要手段发现系统错误
- 验证
  - 建立系统模型，确认系统模型是否存在错误
  - 可一以从某个角度回答系统一定没有错误这样一类问题，从而进步提高我们对系统可靠性的可信度

软件测试 vs 形式验证

测试只检测给定的情况

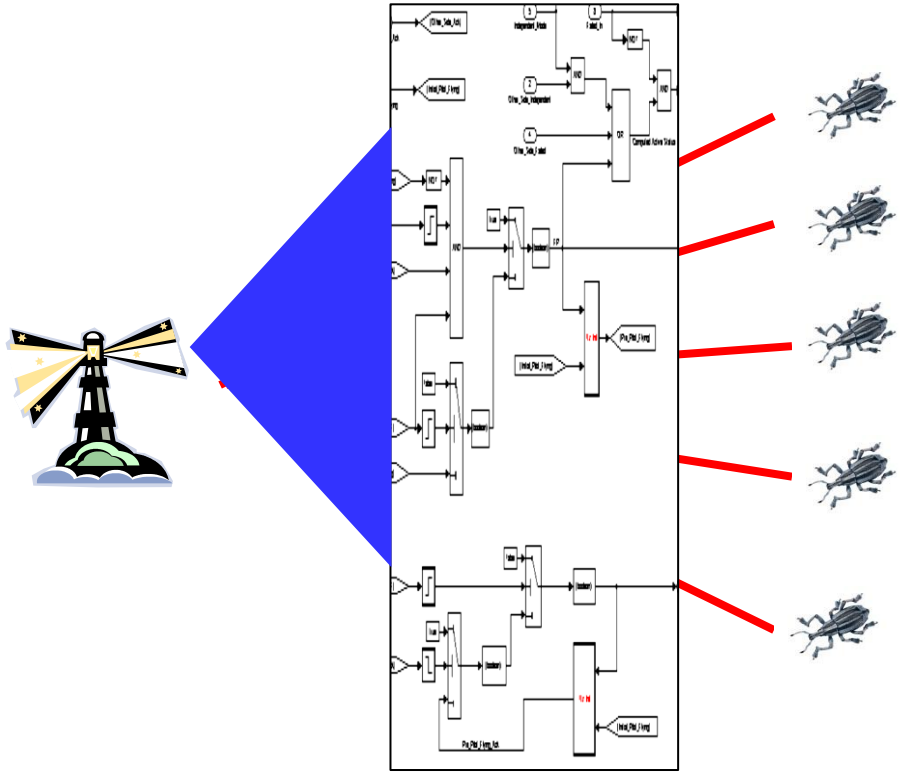
即使小规模系统也可能存在上万种待测情况

优势：简单、直观、工业级工具较多



形式验证(模型检测)自动检测所有的情况

优势：可有效发现测试难以发现的错误、并发性验证



选择何种形式化方法?

精确地
规约系统

使用精确定义的形式化语言规约系统行为

使用“一致性检测”

自动化地检测系统规约中的类型错误、空缺用例、非确定性、循环定义等错误

系统行为
模拟

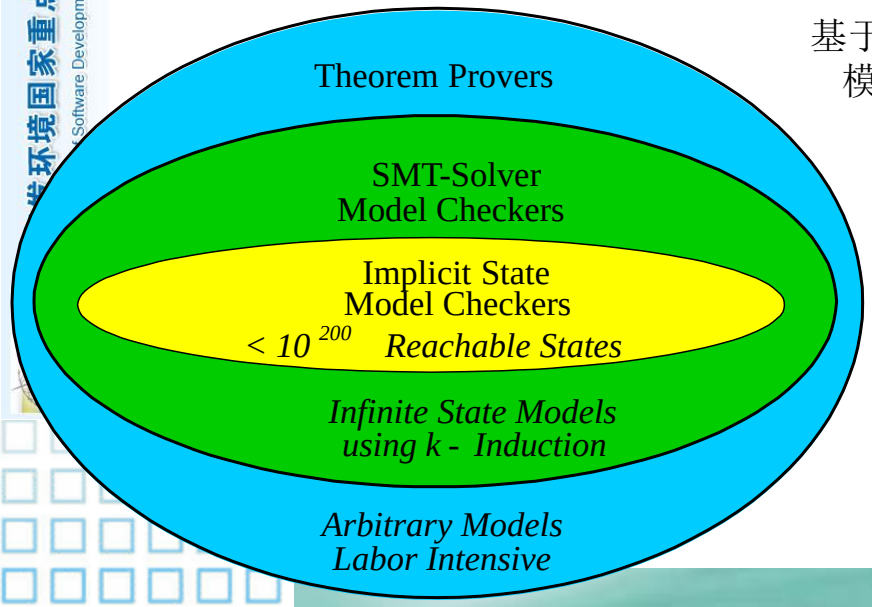
基于(可执行)规约模拟系统运行

基于模型检测的
规约验证

检测关键应用的性质

基于定理证明的
规约验证

工作量逐渐增大，
人员要求不断提高
同时，
系统规约的可信性增加



欧洲阿丽亚娜5型火箭

- On June 4, 1996, an unmanned Ariane 5 rocket launched by the European Space Agency exploded just 40 seconds after its lift-off.
  - A conversion from 64-bit floating point to 16 bit integer with a value larger than possible with Arian 4. The overflow caused a hardware trap.
- Value of rocket and cargo: $500 million

美国电力监测与控制管理系统 Blackout

- **In August, 2003, the largest blackout in our country's history occurred.**
  - **The 2003 North America blackout was triggered by a local outage that went undetetected.**
    - A *race condition* in General Electric's monitoring software prevented an alarm.
- **Estimated cost to New York City alone: $1.1 billion.**
- **多计算机系统试图同时访问同一资源引起的软件失效**

美国F-22猛禽战斗机

- 2004年12月20日，美空军第422测试评估大队的一架F-22战斗机因软件问题在起飞过程中失控坠毁
- 2007年2月9日同样因软件问题延迟在日本部署

安全关键系统必须采用形式化方法

- 航空无线电技术委员会RTCA的DO-178B《机载系统和设备认证中的软件要求》标准用于规范机载系统和设备的软件开发过程，并指导软件认证到相应的安全级别。于2012年颁布DO-178C，增加了形式化方法的附件，强调采用形式化方法开展基于模型的开发和验证

- 美国国家宇航局NASA于1995年和1997年先后发布《Formal Methods Specification and Verification Guidebook》强调采用验证和形式化方法来保证系统可靠性

- 英国国防部在90年代发行了两种与软件开发生命周期中使用形式方法有关的安全至上标准Def Stan00-55和Def Stan 00-56标准，要求在安全至上的软件开发中必须采用形式方法

机载软件的安全性要求

- DO-178B/C对机载软件的安全性要求
- DO-178C 增加了DO-333“Formal method supplement”

安全需求	严重性说明	接受频率 (发生的风险)	系统开发安全等级
高	灾难	Extremely Improbable (X <10^{-9})	Level A
	危险	Extremely remote (10^{-9} < X < 10^{-7})	Level B
	重要	remote (10^{-7} < X < 10^{-5})	Level C
	次要	Probable (X > 10^{-5})	Level D
低	无	All	Level E

（X <10^{-7} 表示100架客机按平均每天飞行8小时，33年内不发生错误）

DO-178B A级认证操作系统

- WindRiver VxWorks 653
- GreenHill INTEGRITY-178B
- LynxWorks LynxOS-178
- SYSGO PikeOS
- DDC-I Deos

安全关键系统必须采用形式化方法

- **CC对安全关键软件的要求**

国际信息技术安全评估通用标准
Common Criteria
ISO/IEC 15408

CC级别	需　求	功能规约	高层设计	低层设计	代码实现
EAL1	非形式化	非形式化	非形式化	非形式化	非形式化
EAL2	非形式化	非形式化	非形式化	非形式化	非形式化
EAL3	非形式化	非形式化	非形式化	非形式化	非形式化
EAL4	非形式化	非形式化	非形式化	非形式化	非形式化
EAL5	形式化	部分形式化	部分形式化	非形式化	非形式化
EAL6	形式化	部分形式化	部分形式化	部分形式化	非形式化
EAL7	形式化	形式化	形式化	部分形式化	非形式化
完全验证	形式化	形式化	形式化	形式化	形式化

EAL1—功能测试
EAL2—结构测试
EAL3—系统地测试和检查
EAL4—系统地设计、测试和复查
EAL5—部分形式化设计和测试
EAL6—部分形式化验证的设计和测试
EAL7—形式化验证的设计和测试

CC认证的操作系统

通过认证的操作系统	认证等级	认证时间
VMware® ESXi Server 3.5 and VirtualCenter 2.5	EAL4+	2010年2月
Microsoft Windows Mobile 6.5	EAL4+	2010年2月
Apple Mac OS X 10.6	EAL3+	2010年1月
Red Hat Enterprise Linux Ver. 5.3 on Dell 11G Family Servers	EAL4+	2009年12月
Windows Vista Enterprise; Windows Server 2008 Standard Edition; Windows Server 2008 Enterprise Edition; Windows Server 2008 Datacenter Edition	EAL4+ ALC_FLR.3	2009年8月
Oracle Enterprise Linux Version 5 Update 1	EAL4+ ALC_FLR.3	2008年10月
Green Hills Software INTEGRITY-178B Separation Kernel, comprising: INTEGRITY-178B Real Time Operating System (RTOS),	**EAL6+**	**2008年9月**

形式验证的效果

- **提高可靠性、降低安全认证成本**
  - **“Crash-Proof Code”**
    - **one of the 10 breakthrough technologies in 2011 by “MIT Technology Review”**

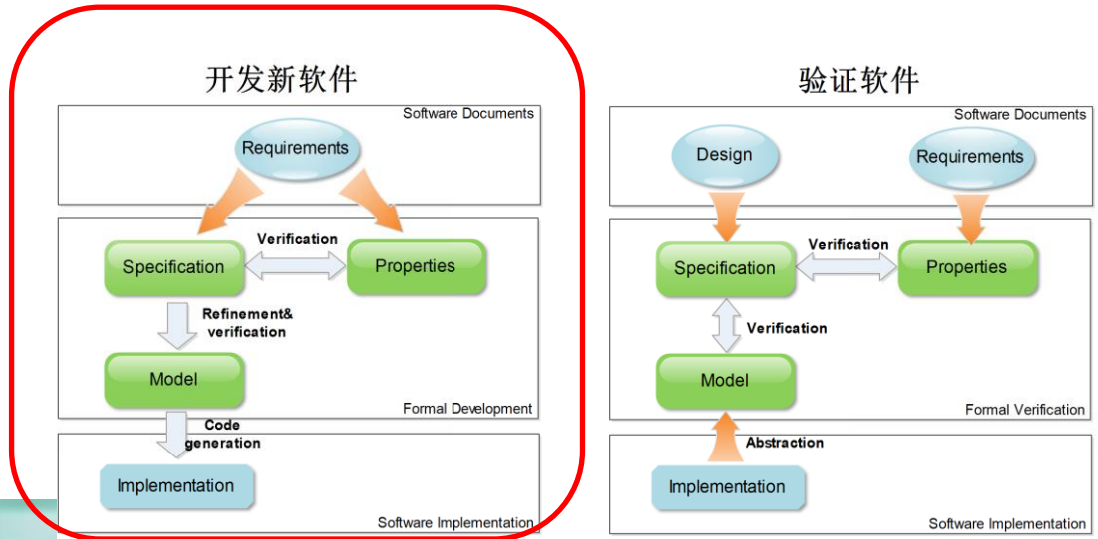
  - **澳大利亚国家信息通信技术研究中心**
  - **对seL4内核进行形式验证**
    - seL4内核：构造OS的基本机制，包括线程、消息传递、中断、虚拟内存、访问控制等
    - 内核包含8700行C代码（其中1200行为内核启动代码），600多行ARM汇编代码
  - **20万行手写Isabelle/HOL证明代码，形式验证工作量25人年**
  - **共发现460多个Bug(C代码中160多个，设计中150个，规约中150个)，而在测试中只发现16个Bug**
  - **完全形式化验证成本：600万美元**
    - 走CC EAL6级认证，10000美元/行代码成本计算，需8700万美元
    - 而且CC EAL6认证比完全形式化验证的软件可靠性确保度低

B方法

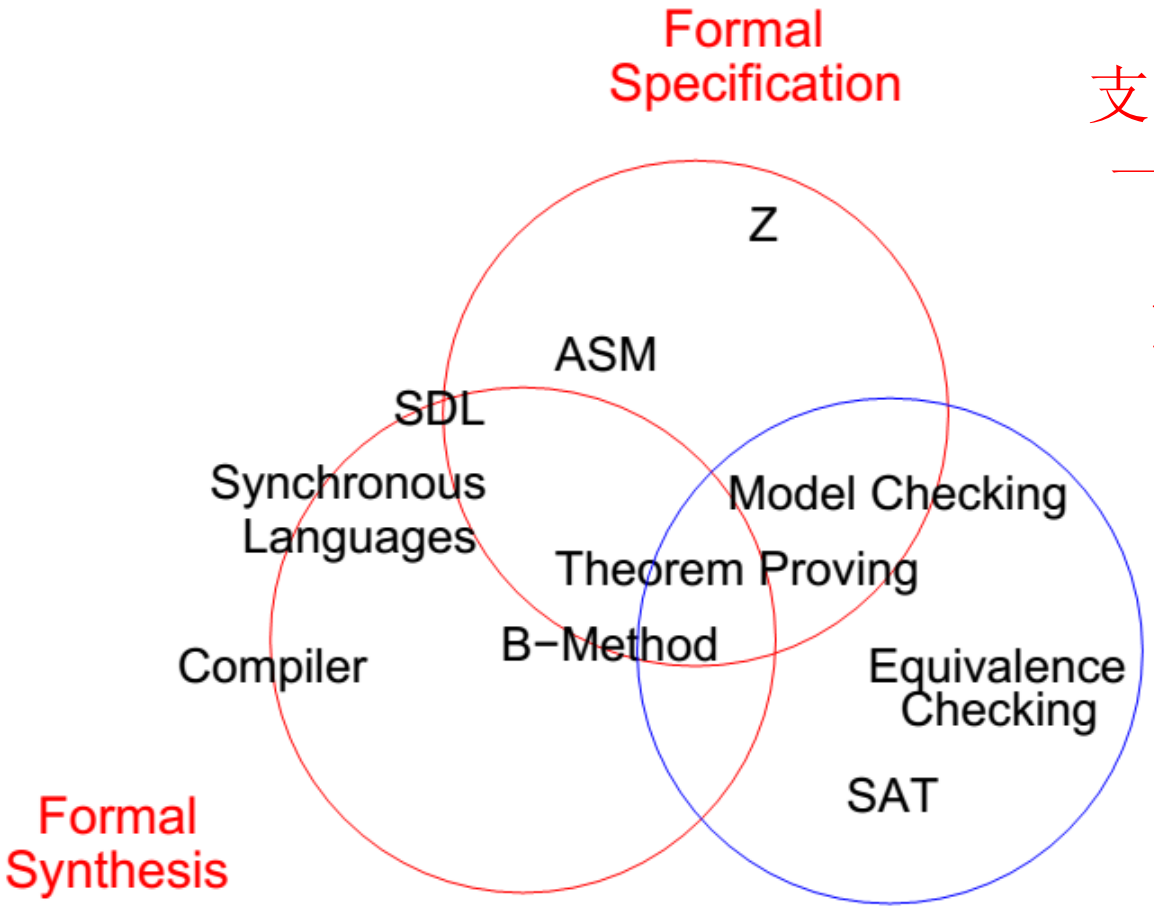
- 由Jean-Raymond Abrial提出
  - 1980s,提出Z语言
  - 1990s,提出B语言
    - 太复杂
  - 2000s, 提出Event-B
    - 替代经典B

- 基于集合论、基本算术、一阶逻辑

B方法属于这一类



B方法



支持Refinement，而一般的定理证明器中没有直接的Refinement概念

B方法

- B方法是在开发巴黎地铁信号系统的过程中发展起来的，后来又用于开发巴黎的若干地铁/机场自动车的驾驶系统
- J-R Abrial 在2006 年国际软件工程大会上的特邀报告“工业开发中的形式化方法，成就、问题和未来”中介绍了B方法在工业界的广泛应用
- 特征
  - B 语言和B 方法基于一阶逻辑和集合论，有严格的数学理论基础，写出的规范简明、精确，无歧义性
  - 面向软件(classibal B)、面向系统(event-B)
    - 支持事件机制：可用于表达分区内核的中断、健康监控等
  - 支持Refinement
    - 自动证明程度高，明确的上下层可追溯性
    - 一般的定理证明器，不直接支持Refinement
  - 擅长描述规范
  - 支持代码生成

B方法

- B方法有成熟的工业应用

- 西门子开发巴黎地铁14号线自动驾驶系统
  - 107,000 lines of B; 29,000 proofs;
  - 1998投入运行
- ClearSy公司开发巴黎Roissy机场自动穿梭车
  - 28,000+155,000 lines of B; 43,000 proofs;
  - 2006年投入运行
- EADS公司开发阿丽亚娜火箭的任务调度模型

- 正在采用类似的方式进行开发的系统：纽约城市地铁，巴塞罗那地铁，布拉格地铁，巴黎地铁1号线 等

B方法开发地铁的历史

L1 Algiers

NY Flushing

SHUTTLE ROISSY AIRPORT Paris

L2 L3 Sao Paulo

New York Canarsie

L3 Paris

Lyon

Mexico

L9 Seoul

L9 Barcelona

Instanbul

Airport Express Hong Kong

L2 Budapest

Toronto

Madrid

Metro L10 Beijing

Circle Line Singapore

KVB 6000 trains France

San Juan

L1 L2 Malaga

METEOR L14 Paris

Delhi

Metro Lausanne

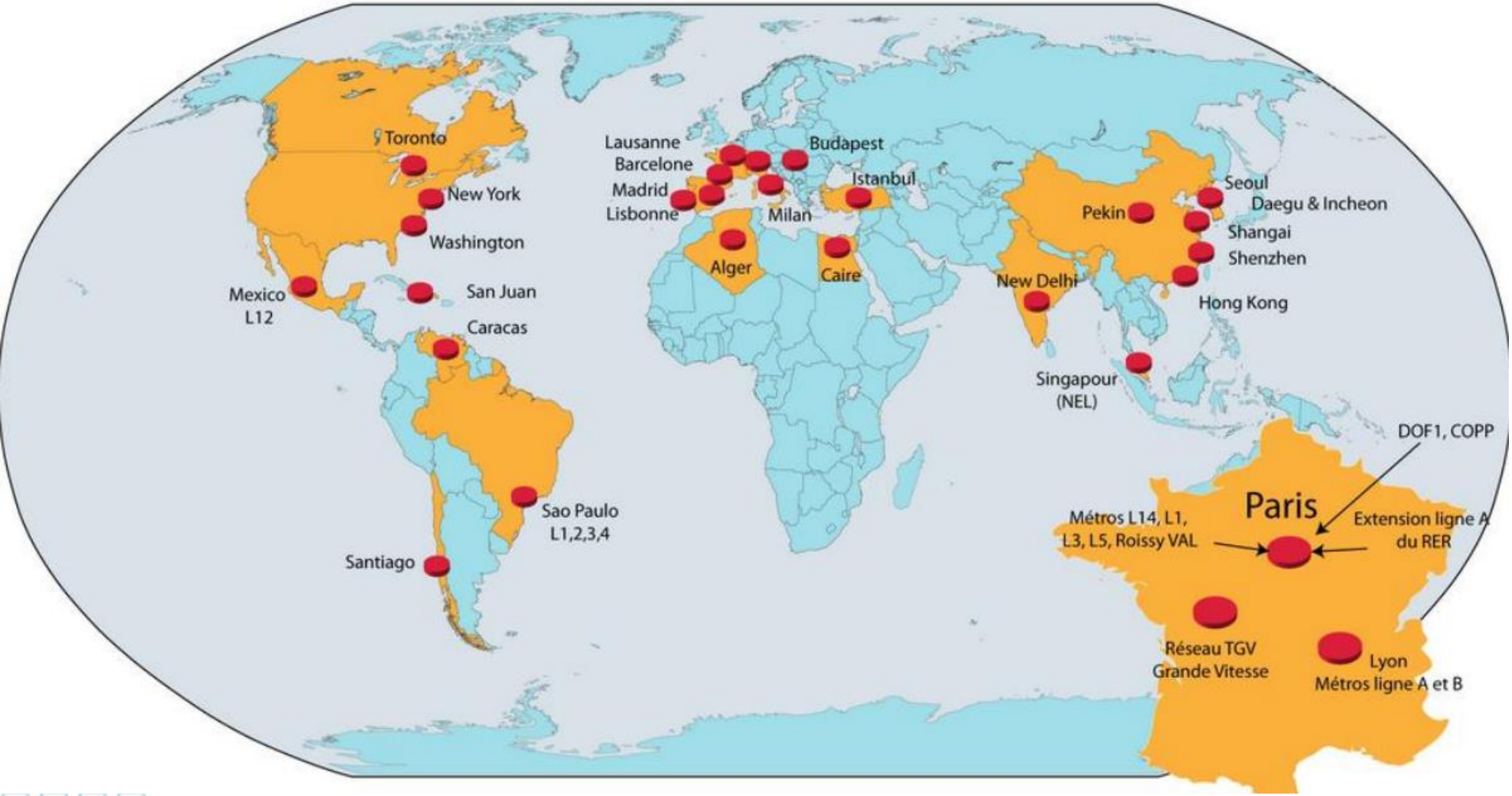
L5 Milano

1990

2000

2010

全球采用B方法开发的地铁项目



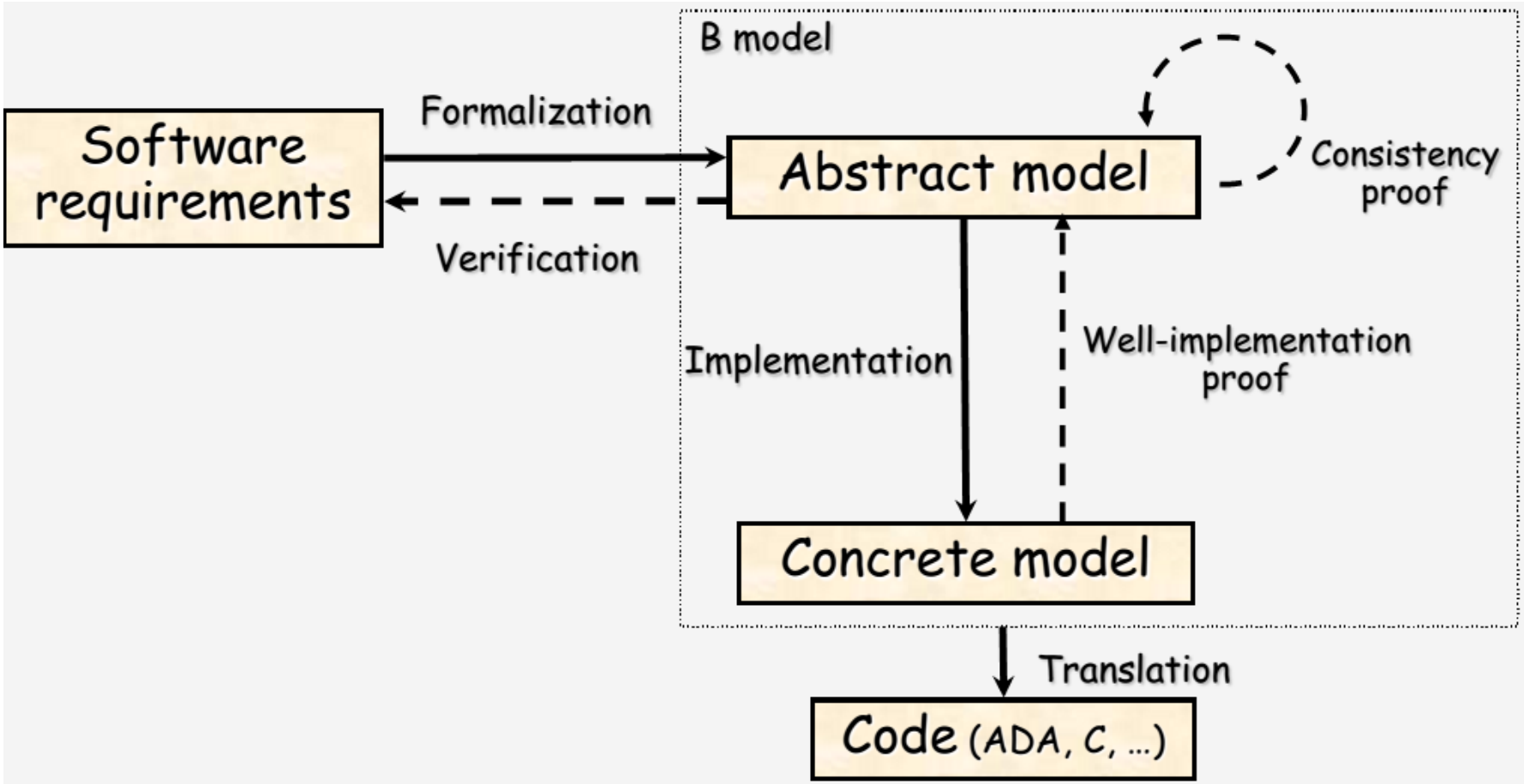
B方法应用状况

- Space
  - Space Systems Finland
    - Bepi Colombo (ESAs first mission to Mercury)
    - On-Board Satellite Software: Attitude and Orbit Control System (AOCS)
  - SSF
- Transportation
  - Siemens, Systerel: Communication-based Train Control (CBTC)
  - Battelle and ClearSy: automation of the Flushing and Culver metro lines in New York
  - Alstom: Dynamic Trusted Railway Interlocking
- Automotive
  - Bosch: Cruise-control system, Engine start-stop system
- 其他
  - SAP: Service choreography modelling
  - Critical Software Technologies: Integrated Secondary Flight Display, used on-board commercial and military aircraft; Smart Energy Grids
  - XMOS: XCore Microprocessor指令集验证
  - The CCF Display and Information System, air traffic controllers based at the London Terminal Control Centre
  - STMicroelectronics: modelling and generation of a VHDL code for a smartcard-based microcircuit
  - Dependable Systems Forum (DSF) project in Japan (NTT-Data, Fujitsu, Hitachi, NEC, Toshiba, and SCSK)

B方法应用状况

- Automotive:
  - Diagnosis (Peugeot)
  - Contactless keycard (Renault)
- Banking:
  - Reconciliation (Société Générale)
- Space:
  - Ariane 5 flight software (EADS)
- Microelectronics
  - Smartcard (STMicroelectronics)
- Nuclear
  - Control System Design (EDF)
- Industry
  - Pneumatic Press (CNAM)

用B方法开发软件系统的过程



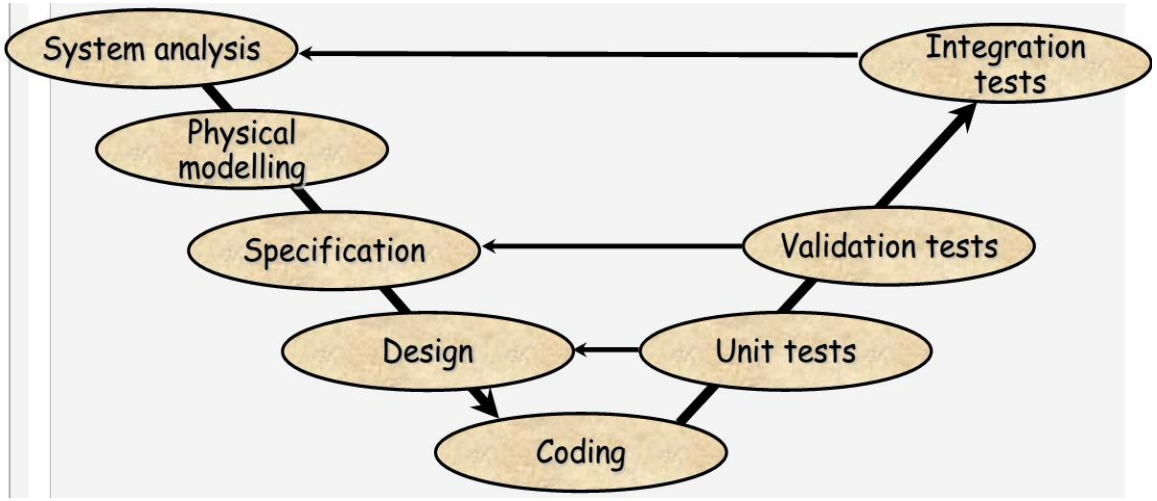
用B方法开发软件系统的过程

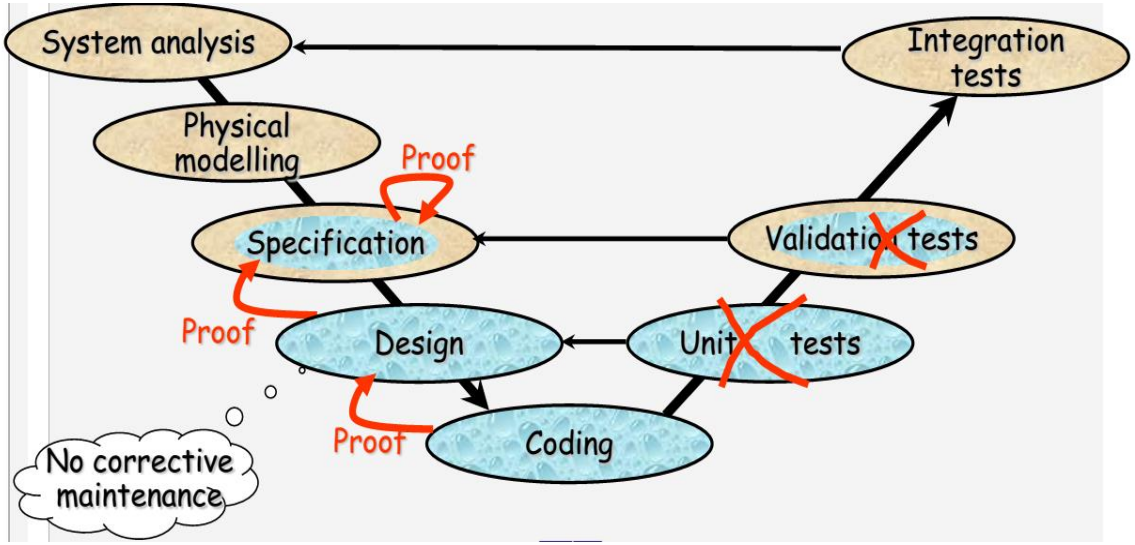
- **第一阶段：根据软件需求文档构造系统的抽象模型**
  - 基于需求文档，构造初步的抽象模型
  - 向抽象模型中加入必要的不变式，基于工具提取并证明不变式定理（证明义务）
    - 一些定理可以自动证明，有些需要人工交互证明
  - 扩充已有模型，加入更多细节，证明不变式定理
  - 不断重复上面几步，直至得到完整的抽象模型并证明所有定理可能发现需求文档中不完全/不一致，需要交流/调整文档和模型
  - 手工/自动任务
    - 建立和扩充模型，加入不变式等完全是人工的工作
    - 不变式定理（证明义务）的抽取将自动完成
    - 工具可以自动证明许多定理，会有一些定理需要交互式证明
  - 工作结果：待开发软件系统的一个完整的抽象模型

用B方法开发软件系统的过程

- **第二阶段：开发软件系统的具体模型**
  - 在已有模型中加入更多细节，使更多的操作具体化，用更接近计算机实现的数据结构代替抽象结构，得到一个更为“具体”的模式
  - 证明新的具体模型是原有模型的“精化”
    - 使用工具自动生成一组“精化关系定理”，并证明这些定理
  - 不断重复上面两步，最终做出一个可以直接对应到实现的模型
  - 精化工作主要由人做，人们也开发了做精化的辅助工具
  - 精化定理由工具自动生成，定理证明自动或交互式完成
- **第三阶段：生成系统的可执行代码**
  - 用工具检查模型是否已能翻译到编程语言代码
  - 用工具把最终模型翻译到编程语言，用编译器生成可执行程序
  - 这一阶段完全是自动的

与传统开发过程的比较



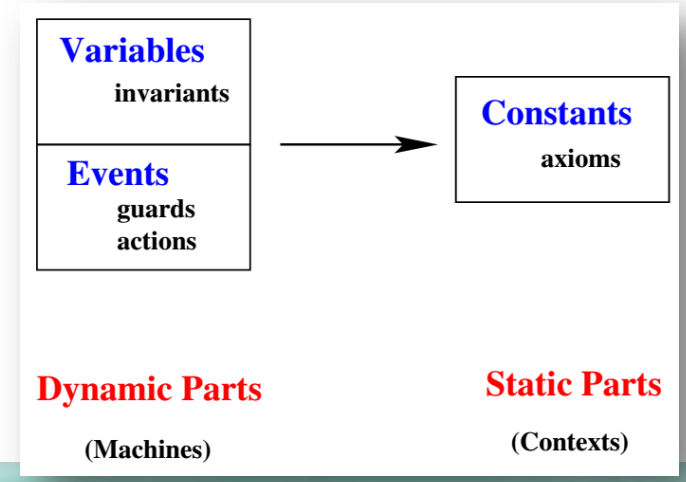


B方法支撑工具

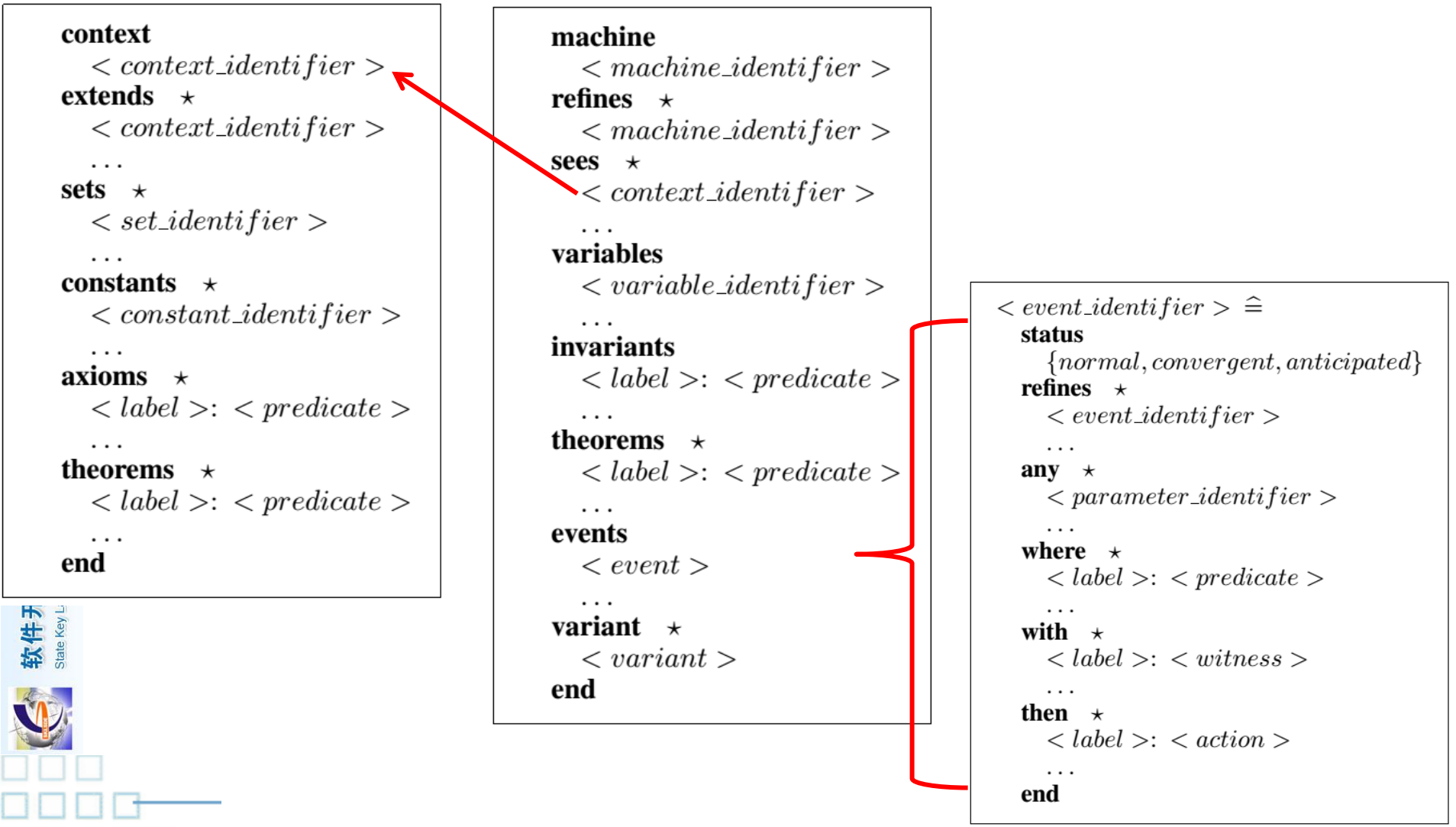
- **Atelier B (ClearSy)**
  - created to develop industrial B-Software projects
  - a set of tools integrated into a project manager tool
    - static checkers
    - automatic proof obligation generator
    - automatic provers and interactive prover
    - code translators: Ada, C, …
  - it is also used for B-System
- **Rodin platform (September 2007)**
  - a new open platform dedicated to B-System (in construction)
- **B4free (www.b4free.com)**
  - free but restricted to academic users and owners of Atelier B
  - the core tools of Atelier B + a new x-emacs interface

Event-B基本思想

- **Event-B模型**
  - 离散迁移系统：离散状态及迁移(事件触发)
  - 状态：带公理(axiom)的常量(constant)、带不变式(invariant)的变量(variable)
  - 事件：包括卫式(guard)和动作(action)，卫式表明事件触发条件，动作表明状态被修改的方式
  - 常量、变量、卫式和动作均采用集合论表达式来书写

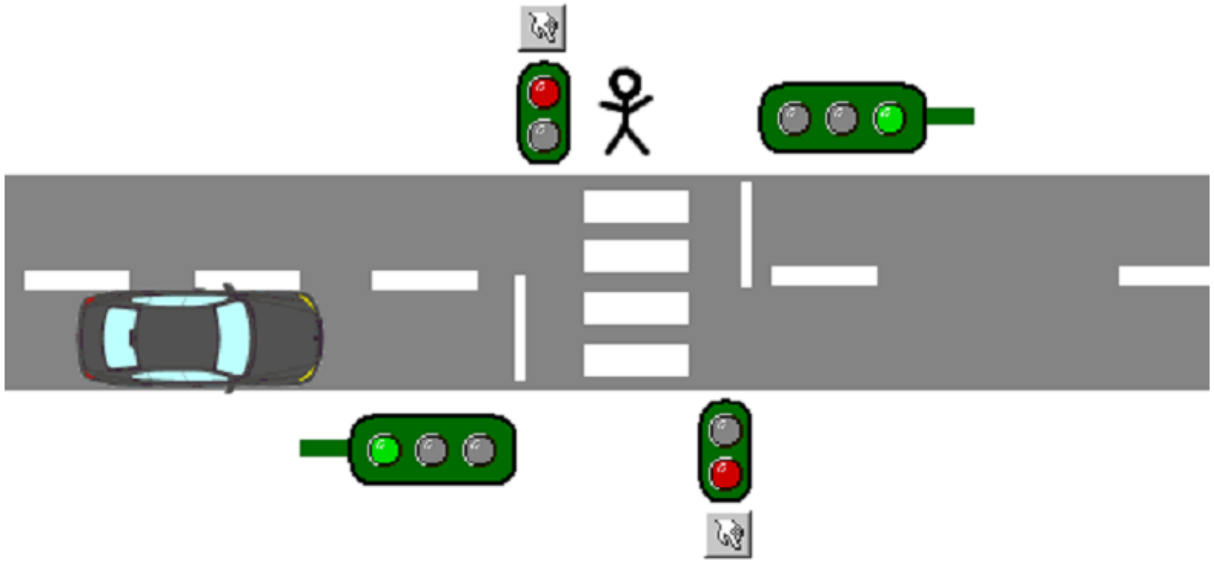


Context、Machine和Event



一个简单的例子(1)

- 交通信号灯控制器



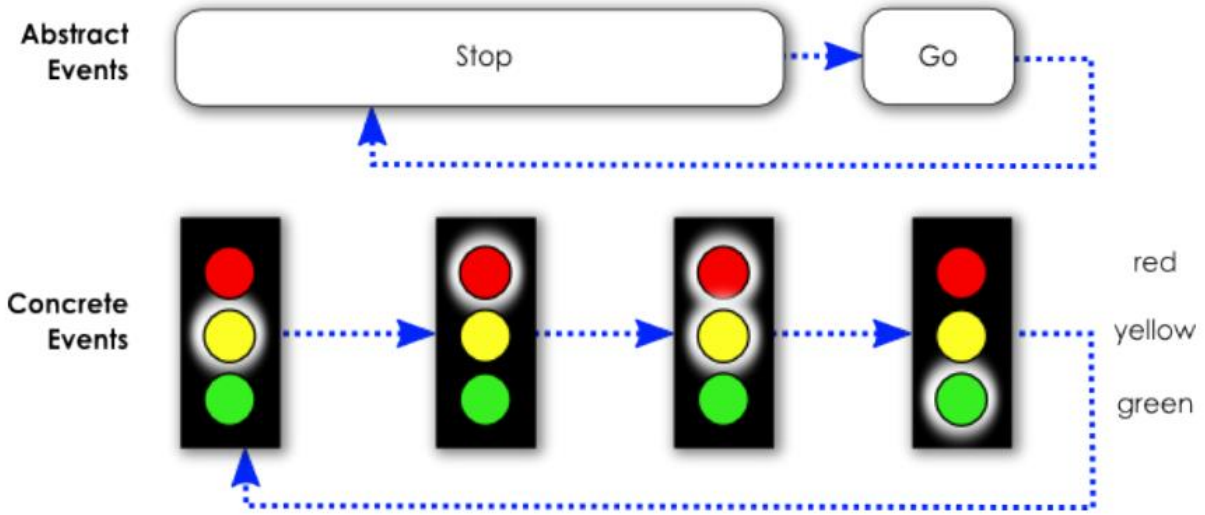
一个简单的例子(1)

MACHINE mac
VARIABLES
cars_go
peds_go
INVARIANTS
inv1 : $cars\_go \in BOOL$
inv2 : $peds\_go \in BOOL$
inv3 : $\neg(cars\_go = TRUE \wedge peds\_go = TRUE)$
EVENTS
Initialisation
begin
act1 : $cars\_go := FALSE$
act2 : $peds\_go := FALSE$
end
Event $set\_peds\_go \mathrel{\widehat{=}}$
when
grd1 : $cars\_go = FALSE$
then
act1 : $peds\_go := TRUE$
end

Event $set\_peds\_stop \mathrel{\widehat{=}}$
begin
act1 : $peds\_go := FALSE$
end
Event $set\_cars \mathrel{\widehat{=}}$
any
new_value
where
grd1 : $new\_value \in BOOL$
grd2 : $new\_value = TRUE \Rightarrow peds\_go = FALSE$
then
act1 : $cars\_go := new\_value$
end
END

一个简单的例子(2)

- 交通信号灯控制器——求精



一个简单的例子(2)

- 交通信号灯控制器——求精
  - 引入Context

CONTEXT agatha

CONSTANTS

red
yellow
green

SETS

COLOURS

AXIOMS

type : $partition(COLOURS, \{red\}, \{yellow\}, \{green\})$

一个简单的例子(2)

- 交通信号灯控制器——求精

MACHINE mac1
REFINES mac
SEES ctx1
VARIABLES
peds_colour
cars_colours
INVARIANTS
inv4 : $peds\_colour \in \{red, green\}$
inv5 : $cars\_colours \subseteq COLOURS$
gluing_peds : $peds\_go = TRUE \Leftrightarrow peds\_colour = green$
gluing_cars : $cars\_go = TRUE \Leftrightarrow green \in cars\_colours$
EVENTS
Initialisation
begin
init4 : $peds\_colour := red$
init5 : $cars\_colours := \{red\}$
end
Event $set\_peds\_green \mathrel{\widehat{=}}$
refines set_peds_go
when
grd1 : $green \notin cars\_colours$
then
act2 : $peds\_colour := green$
end

Event $set\_peds\_red \mathrel{\widehat{=}}$
refines set_peds_stop
begin
act1 : $peds\_colour := red$
end
Event $set\_cars\_colours \mathrel{\widehat{=}}$
refines set_cars
any
$new\_value\_colours$
where
grd1 : $new\_value\_colours \subseteq COLOURS$
grd2 : $green \in new\_value\_colours \Rightarrow peds\_colour = red$
grd_y_r : $cars\_colours = \{yellow\} \Rightarrow new\_value\_colours = \{red\}$
grd_r_ry : $cars\_colours = \{red\} \Rightarrow new\_value\_colours = \{red, yellow\}$
grd_ry_g : $cars\_colours = \{red, yellow\} \Rightarrow new\_value\_colours = \{green\}$
grd_g_y : $cars\_colours = \{green\} \Rightarrow new\_value\_colours = \{yellow\}$
with
new_value : $new\_value = TRUE \Leftrightarrow green \in new\_value\_colours$
then
act1 : $cars\_colours := new\_value\_colours$
end
END

最终的目的是什么

- 对上述模型进行证明，确保正确性
- 根据上述模型，生成Ada/C/Java代码，得到软件实体

- 怎么做？ 后面再讲

2. 基本理论

本部分内容

- 命题逻辑
- 集合论
- 谓词逻辑

命题逻辑

- 逻辑常量：TRUE(1), FALSE(0)
- 命题符号：P, Q, S, …
  - 命题，如：我国于2013年11月划定了东海防空识别区
- 逻辑连接词

  ∧...与

  ∨...或

  →...蕴涵

  ↔..当且仅当

  ¬...非

命题公式的解释

- 真值表

P Q	¬P	P∧Q	P∨O	P→Q	P↔Q
0 0	1	0	0	1	1
0 1	1	0	1	1	0
1 0	0	0	1	0	0
1 1	0	1	1	1	1

等价公式

- ⇔公式的等价关系，↔是逻辑连接词

双重否定律	$\neg\neg A \Leftrightarrow A$
结合律	$(A \wedge B) \wedge C \Leftrightarrow A \wedge (B \wedge C)$　　$(A \vee B) \vee C \Leftrightarrow A \vee (B \vee C)$ $(A \leftrightarrow B) \leftrightarrow C \Leftrightarrow A \leftrightarrow (B \leftrightarrow C)$
交换律	$A \vee B \Leftrightarrow B \vee A$　　$A \wedge B \Leftrightarrow B \wedge A$
分配律	$A \wedge (B \vee C) \Leftrightarrow (A \wedge B) \vee (A \wedge C)$　　$A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge (A \vee C)$
吸收律	$A \vee (A \wedge B) \Leftrightarrow A$　　$A \wedge (A \vee B) \Leftrightarrow A$
德·摩根律	$\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$　　$\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$
同一律	$A \vee F \Leftrightarrow A$　　$A \wedge T \Leftrightarrow A$
零律	$A \vee T \Leftrightarrow T$　　$A \wedge F \Leftrightarrow F$
否定律	$P \vee \neg P \Leftrightarrow T$　　$P \wedge \neg P \Leftrightarrow F$
条件转化律	$A \rightarrow B \Leftrightarrow \neg A \vee B$　　$A \rightarrow B \Leftrightarrow \neg B \rightarrow \neg A$

命题逻辑的推理

- 设A和B是两个命题公式，当且仅当$A \rightarrow B$为一永真式 (或重言式)，即$A \Rightarrow B$，称B是A的有效结论，或B可由A逻辑推出

- Logical inference is used to create new sentences that logically follow from a given set of predicate calculus sentences
- An inference rule is sound
- An inference rule is complete

推理规则

- 一些推理的正确性规则

RULE	PREMISE	CONCLUSION
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$\mathbf{A \vee B, \neg B \vee C}$	$\mathbf{A \vee C}$

证明过程

- A proof is a sequence of sentences, where each sentence is either a premise or a sentence derived from earlier sentences in the proof by one of the rules of inference.
- The last sentence is the theorem (also called goal or query) that we want to prove.
- Example for the "weather problem" given above.

1 Humid	Premise	"It is humid"
2 Humid→Hot	Premise	"If it is humid, it is hot"
3 Hot	Modus Ponens(1,2)	"It is hot"
4 (Hot∧Humid)→Rain	Premise	"If it's hot & humid, it's raining"
5 Hot∧Humid	And Introduction(1,2)	"It is hot and humid"
6 Rain	Modus Ponens(4,5)	"It is raining"

集合论

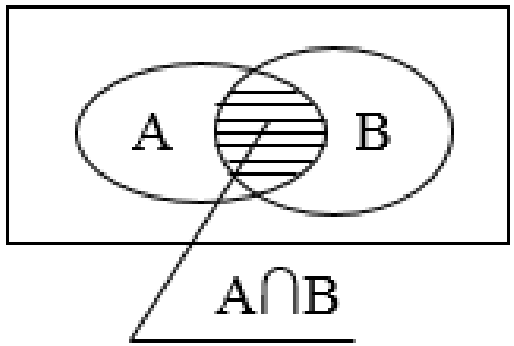
- **集合**：是由确定的对象(客体)构成的集体。
  - “确定”是指：论域内任何客体，要么属于这个集合，要么不属于这个集合，是唯一确定的。
- **元素**：集合中的对象，称之为元素
  - ∈：表示元素与集合的属于关系。
- 有限集合与无限集合
- 集合的表示
  - 列元素法　A={1, 2, 3, 4, 5}
  - 谓词表示法　B={x ｜ x∈N ∧1≤x≤5 }
- 集合的特征
  - 互异性　{1,2,3,2,4}= {1,2,3,4}
  - 无序性　{4,2,1,3 }= {1,2,3,4}
- 空集
  - 不含任何元素的集合叫做空集
  - 空集是唯一的

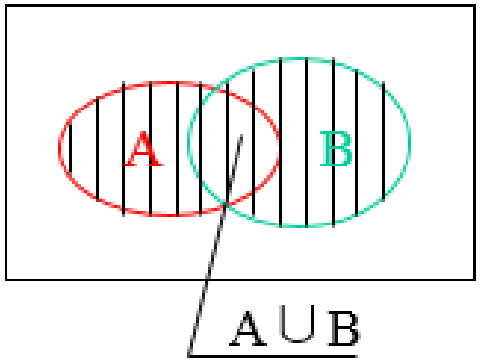
集合之间的关系

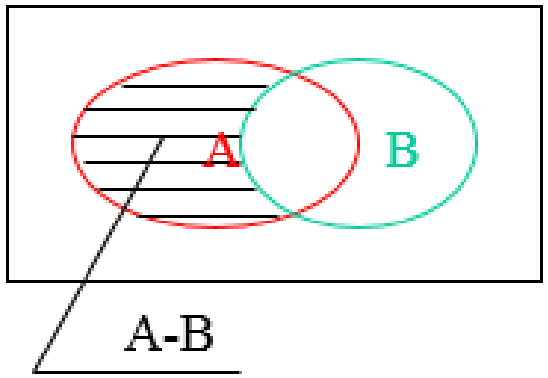
- 子集
  - 如果B中的每个元素都是A中的元素，则称B是A的子集合，简称子集
  - $B \subseteq A \Leftrightarrow \forall x(x \in B \rightarrow x \in A)$
- 相等
  - 设A，B为集合，如果$A \subseteq B$且$B \subseteq A$，则称A与B相等，记作A=B
- 真子集
  - $B \subseteq A$且$B \neq A$，则称B是A的真子集

集合的基本运算

- 交、并、差







- 各种运算的性质

集合的基本运算

- 笛卡尔积
  - $S \times T = \{x \mapsto y | x \in S \wedge y \in T\}$
  - $x \mapsto y$是偶对
- 幂集Powerset
  - $\mathbb{P}(S) = \{s | s \subseteq S\}$
- 集合的势
  - $card(S)$

集合上的谓词

- $s \in S$ (是$\in (s, S)$的习惯写法， $\in$是谓词)
- $s \notin S$
- $S \subset T$
- $S \not\subset T$
- $S \subseteq T$
- $S \not\subseteq T$
- 是否有限集$finite(S)$
- 是否集合的分割$partition(S, x, y)$
  - $S = x \cup y \wedge x \cap y = \Phi$

集合上的关系

- 关系是偶对的集合，是多对多的映射
- 集合S和T之间的关系集合：$S \leftrightarrow T = \mathbb{P}(S \times T)$
- S和T之间的某个关系：$r \in S \leftrightarrow T$
- Domain: $\mathbf{dom}(r)$
  - $\forall r.r \in S \leftrightarrow T \Longrightarrow dom(r) = \{x | (\exists y.x \mapsto y \in r)\}$
- Range: $\mathbf{ran}(r)$
  - $\forall r.r \in S \leftrightarrow T \Longrightarrow ran(r) = \{y | (\exists x.x \mapsto y \in r)\}$
- Total relation： $S \twoheadleftarrow T$
  - $r \in S \twoheadleftarrow T \Longrightarrow dom(r) = S$
- Subjective relation： $S \twoheadrightarrow T$
  - $r \in S \twoheadrightarrow T \Longrightarrow ran(r) = T$
- Total subjective relation： $S \twoheadleftarrow\!\!\!\!\twoheadrightarrow T$
  - $r \in S \twoheadleftarrow\!\!\!\!\twoheadrightarrow T \Longrightarrow dom(r) = S \wedge ran(r) = T$

关系运算

- Forward composition: $p; q$
  - $\forall p, q. p \in S \times T \wedge q \in T \times U \Longrightarrow p; q = \{x \mapsto y | (\exists z. x \mapsto z \in p \wedge z \mapsto y \in q)\}$
- Backward composition:
  - $p \circ q = q; p$
- Identity
  - $S \lhd id = \{x \mapsto x | x \in S\}$
- Domain restriction
  - $S \lhd r = \{x \mapsto y | x \mapsto y \in r \wedge x \in S\}$
- Domain subtraction
  - $S \lhd r = \{x \mapsto y | x \mapsto y \in r \wedge x \notin S\}$
- Range restriction
  - $r \rhd T = \{x \mapsto y | x \mapsto y \in r \wedge y \in T\}$
- Range subtraction
  - $r \rhd T = \{x \mapsto y | x \mapsto y \in r \wedge y \notin T\}$
- Inverse
  - $r^{-1} = \{y \mapsto x | x \mapsto y \in r\}$

关系运算

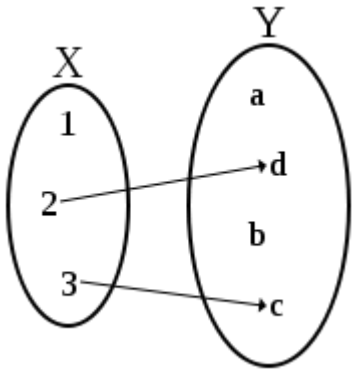
- **Relational image**
  - $r[S] = \{y | \exists x. x \in S \wedge x \mapsto y \in r\}$
- **Overriding**
  - $r_1 \trianglelefteq r_2 = r_2 \cup (dom(r_2) \lhd r_1)$
- **Direct product**
  - $p \otimes q = \{x \mapsto (y \mapsto z) | x \mapsto y \in p \wedge x \mapsto z \in q\}$
- **Parallel product**
  - $p \parallel q = \{(x \mapsto y) \mapsto (m \mapsto n) | x \mapsto m \in p \wedge y \mapsto n \in q\}$
- **Projection: prj1**
  - $(S \times T) \lhd prj1 = \{(x \mapsto y) \mapsto x | x \mapsto y \in S \times T\}$
- **Projection: prj2**
  - $(S \times T) \lhd prj2 = \{(x \mapsto y) \mapsto y | x \mapsto y \in S \times T\}$

函数

- 函数：一种特殊的关系，多对一的关系
- Partial functions
  - $S \nrightarrow T = \{r . r \in S \leftrightarrow T \wedge r^{-1}; r \subseteq T \lhd id\}$
- Total functions：定义域是S的partial function
  - $S \rightarrow T = \{f | f \in S \nrightarrow T \wedge dom(f) = S\}$
- Partial injections: *One-to-one* relations
  - $S \rightarrowtail T = \{f . f \in S \nrightarrow T \wedge f^{-1} \in T \nrightarrow S\}$
- Total injections：定义域是S的partial injection
  - $S \rightarrowtail T = S \rightarrowtail T \cap S \rightarrow T$
- Partial surjections： *Onto* relations，值域是T的partial function
  - $S \twoheadrightarrow T = \{f . f \in S \nrightarrow T \wedge ran(f) = T\}$
- Total surjections：定义域是S的partial surjection
  - $S \twoheadrightarrow T = S \twoheadrightarrow T \cap S \rightarrow T$
- Bijections：定义域是S，值域是T的partial injection
  - $S \rightarrowtail\!\!\!\!\twoheadrightarrow T = S \rightarrowtail T \cap S \twoheadrightarrow T$

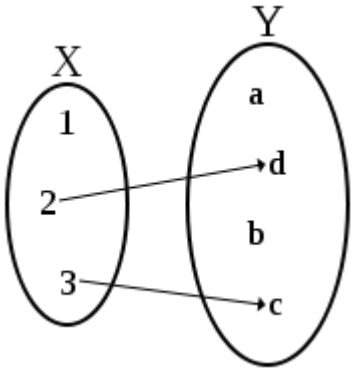
概念澄清

- 以Partial functions为例
  - 定义：$S \nrightarrow T = \{r.r \in S \leftrightarrow T \wedge r^{-1}; r \subseteq T \lhd id\}$
  - $S \nrightarrow T$是S到T的所有partial function的集合， $S \nrightarrow T$不是一个function
  - 若$r \in S \nrightarrow T$，则r是一个partial function，函数r的定义域不一定是S、值域不一定是T
  - 如：

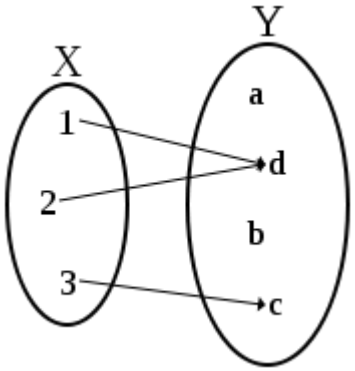


  - $r \in X \nrightarrow Y, r = \{2 \mapsto d, 3 \mapsto c\}, dom(r) = \{2,3\} \neq X, ran(r) = \{c, d\} \neq Y$

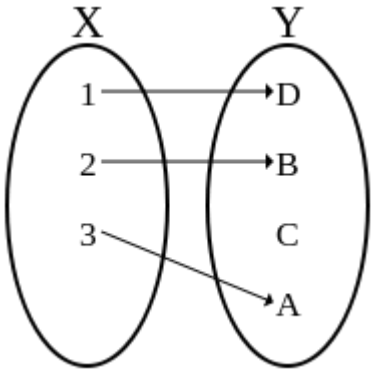
函数图示



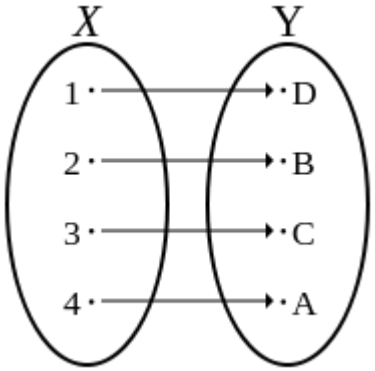
partial function, partial injection



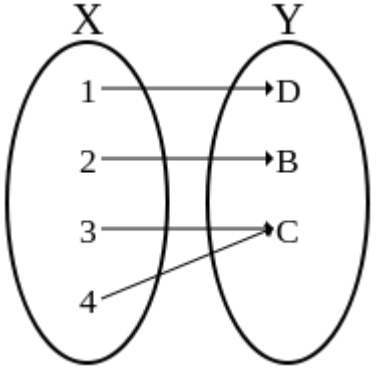
total function, but not injection)



Total injection, but not surjection



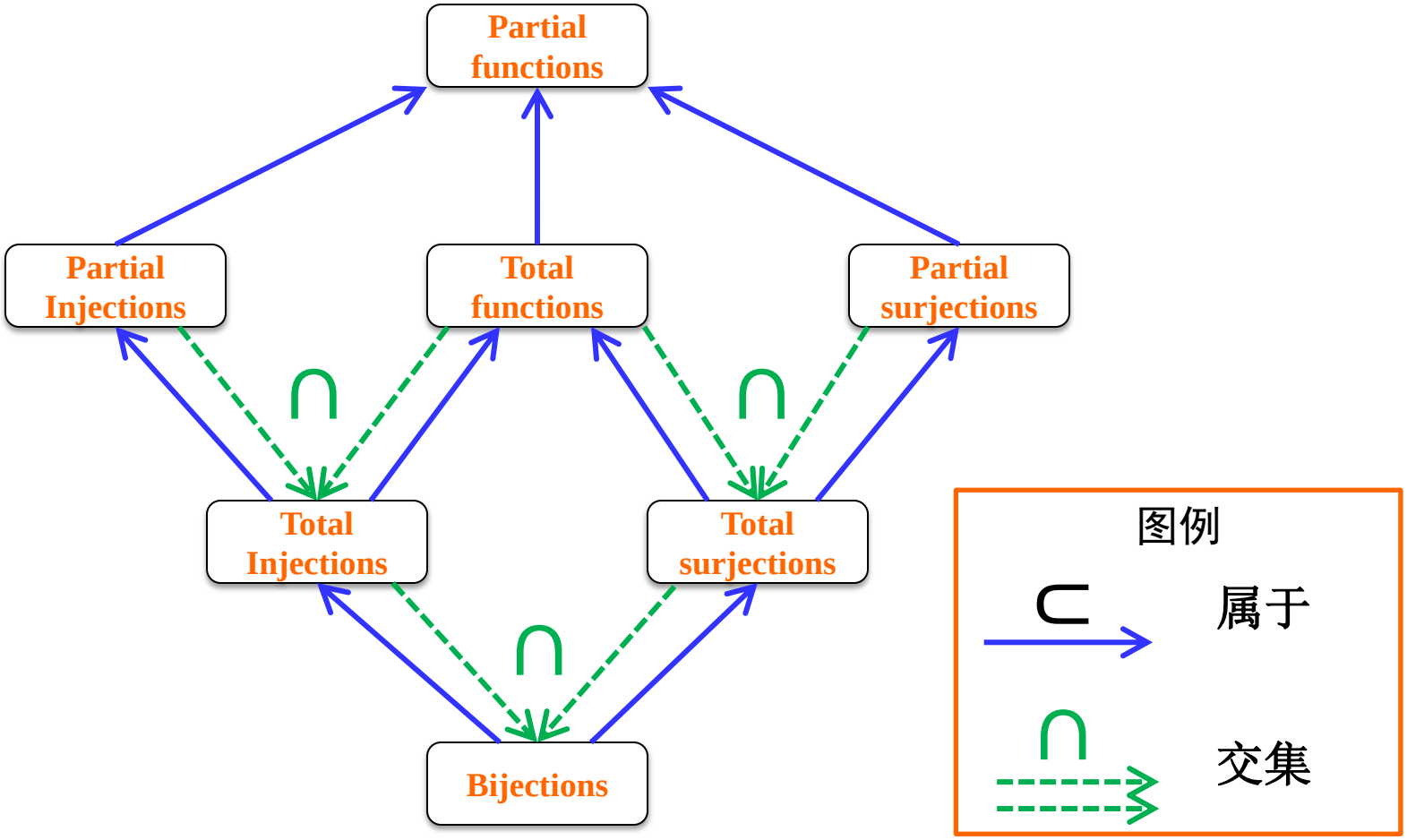
bijection



Total surjections

不同函数间的关系

- 集合S到T的不同函数



谓词逻辑

- **命题逻辑**
  - 与、或、非、命题符合
  - 没有量词
    - “All poodles are dogs.”
    - “There is at least one black swan.”
    - “Only supervisors are allowed to fill in the form.”
    - “No person can solve this problem.”
  - 著名的苏格拉底三段论：
    - “人都是要死的，苏格拉底是人，所以苏格拉底是要死的。”
    - $\forall x(\text{person}(x) \longrightarrow \text{dead}(x)), \text{person}(s)$
    - $\text{person}(s) \longrightarrow \text{dead}(s), \text{person}(s)$
    - $\text{dead}(s)$
- **谓词**
  - 谓词是一个*bool*值的函数： $f \in S \to \{true, false\}$或写成 $f: S \to \{true, false\}$

谓词逻辑

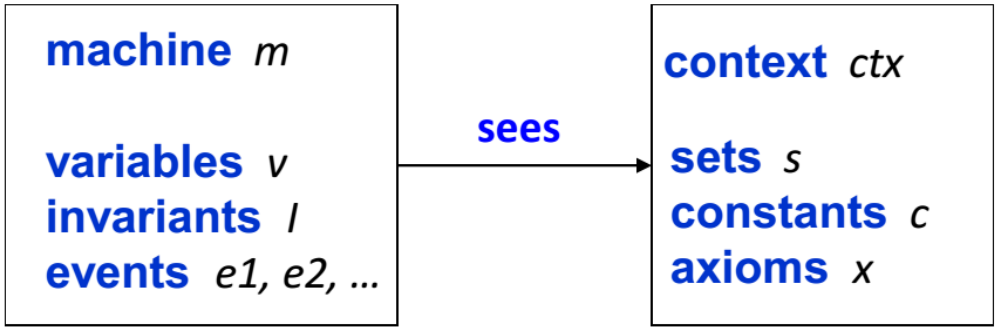
- 变量：x,y,…
- 谓词：female(x), sister(x,y),…
- 量词：$\forall x, \exists x$
- 逻辑连接词：与命题逻辑一样

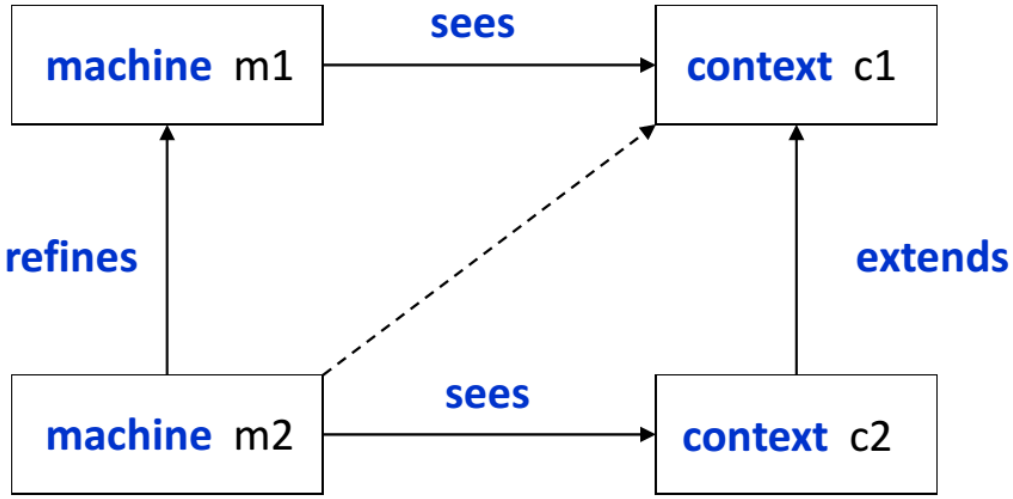
3. Event-B详解

本部分内容

- 抽象机与上下文
- 求精
- 正确性证明

Event-B的基本Component





Context

- SETS：声明用户自定义的、规约中使用的抽象类型
- CONSTANTS：不可修改的逻辑变量，即常量，常量的类型须在AXIOMS中声明
- AXIOMS：逻辑谓词，context中元素的规则。在开发模型时这些谓词是真的，并作为证明的前提
  - 某些axiom可以标记为theorem，表明这些theorem可以通过之前的axiom来证明，一旦被证明，这些theorem在后续的证明中就跟axiom一样

CONTEXT agatha

CONSTANTS

red
yellow
green

SETS

COLOURS

AXIOMS

type : $partition(COLOURS, \{red\}, \{yellow\}, \{green\})$

Machine

MACHINE mac
VARIABLES
cars_go
peds_go
INVARIANTS
inv1 : $cars\_go \in BOOL$
inv2 : $peds\_go \in BOOL$
inv3 : $\neg(cars\_go = TRUE \land peds\_go = TRUE)$
EVENTS
Initialisation
begin
act1 : $cars\_go := FALSE$
act2 : $peds\_go := FALSE$
end
Event $set\_peds\_go \mathrel{\widehat{=}}$
when
grd1 : $cars\_go = FALSE$
then
act1 : $peds\_go := TRUE$
end

Event $set\_peds\_stop \mathrel{\widehat{=}}$
begin
act1 : $peds\_go := FALSE$
end
Event $set\_cars \mathrel{\widehat{=}}$
any
new_value
where
grd1 : $new\_value \in BOOL$
grd2 : $new\_value = TRUE \Rightarrow peds\_go = FALSE$
then
act1 : $cars\_go := new\_value$
end
END

Machine

- 一个machine通过变量以及修改变量值的事件，描述一个模型的动态行为
- Refines: 后续再讨论
- Sees
  - 引用context，可以表明该machine中可以使用context的sets和constants
  - 并且在machine的每个证明中，axioms都是前提假设(hypotheses)
- Variables
  - 状态变量
  - 变量的值由initialisation事件确定，并可被其他event修改
  - 变量表明了machine的状态(state)
  - 每个变量的类型，必须在INRATIANTS中声明
- Invariants
  - 声明machine的每个可达状态下必须为真的谓词
- Events
  - 可为变量赋新值
  - 事件的*guards*子句表明了该事件可执行的条件
  - 每个machine唯一的initialisation是一个特殊的事件

Event

- Any
  - 事件的参数列表
- Where(when)
  - 该事件可执行的条件
- With
  - Witness：在refinement中介绍
- Then
  - 事件执行的动作

事件内的动作

- skip空动作
- 赋值动作
  - $z := E$
- 通过集合赋值
  - $x :\in S$
  - “becomes in” : 从集合S内任取一个元素赋给x
- 通过谓词赋值
  - $z :| P$
  - “becomes such that”: 为z内的变量赋任意、满足谓词P的值
- 函数赋值
  - $f(x) := E$
  - 将函数f中x的对应值赋为E
  - $f(x) := E = f := f \trianglelefteq \{x \mapsto E\}$

Event-B模型的语义

- 事件的语义
  - 系统动态行为由卫式命令（事件）表示

WHEN *guard* **THEN** *action* **END**

- Machine的语义
  - 系统总体行为是系统事件的无限循环

forever do

Event1 or

Event2 or

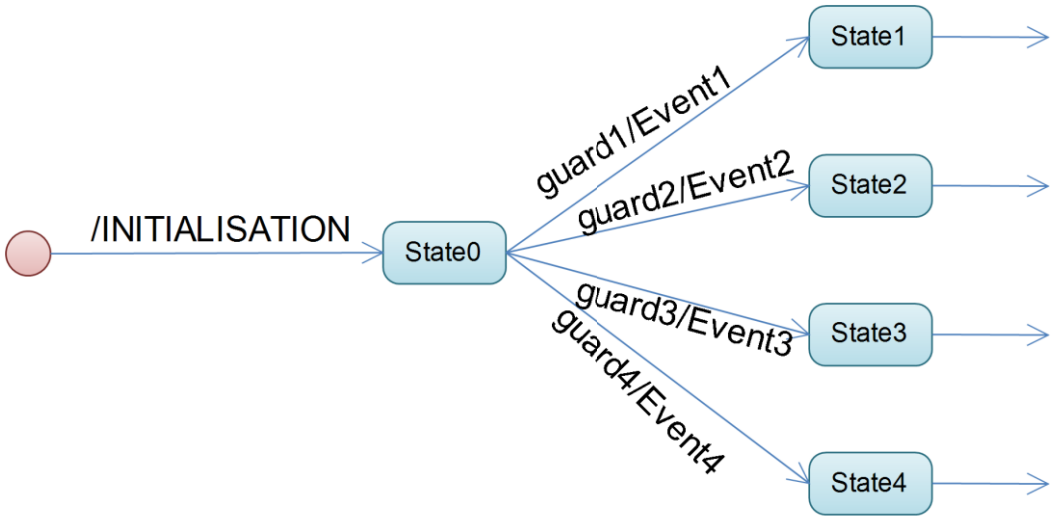
Event3 or ...

end

  - 模型不变式定义了一个允许状态的集合，每个事件都必须保持不变式

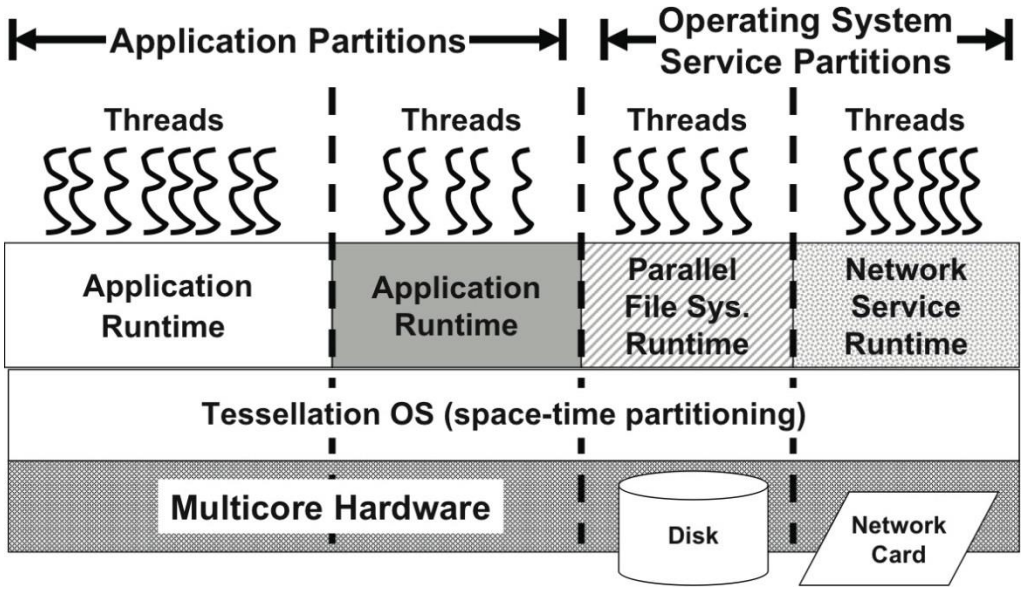
Event-B模型的语义

- 状态机语义



基于集合的建模

- Event-B建模方式：基于集合
- 分区操作系统内核（Partitioning Kernel）



分区内核的集合

```
context Kernel_ctx

sets Partitions
    AllThreads // the thread including used and unused threads
    PartitionModes

    ThreadStates
    CriticalityLevels
    SystemState // system state of HM
    DeadlineType
    ……
```

分区的集合

所有线程的集合

分区运行模式类型的集合

线程状态类型的集合

分区关键等级类型的集合

线程deadline类型的集合

分区内核的集合

-

constants

PM_IDLE PM_NORMAL PM_WARM_START PM_COLD_START *// partition operating mode, according to ARINC653*

TS_Dormant TS_Ready TS_Waiting TS_Suspend TS_WaitandSuspend TS_Running *//thread state*

CRITILITY_LEVEL_A CRITILITY_LEVEL_B CRITILITY_LEVEL_C CRITILITY_LEVEL_D CRITILITY_LEVEL_E *// denotes criticality level of partition, Each partition is assigned a DO-178B criticality level from level A to E*

Critical_Level */* denotes criticality level of partition, Each partition is assigned a DO-178B criticality level from level A to E*/*

Period_of_Partition

/ defines the activation period of the partition, and is used to determine the partition's runtime placement within the core module's overall time frame. */*

Duration_of_Partition *// the amount of processor time given to the partition every period of the partition.*

System_Partition *// denotes the system partition running the kernel*

System_HM_Table Module_HM_Table Partition_HM_Table

DEADLINE_HARD DEADLINE_SOFT

分区状态的类型常量

线程状态的类型常量

关键等级的类型常量

每个分区的关键等级(函数)

分区内核的集合

axioms

@axm_finite_partitions finite(**Partitions**)

分区集合是有限集

@axm_partition_partitionmodes partition(**PartitionModes**,{**PM_IDLE**}, {**PM_NORMAL**}, {**PM_WARM_START**},{**PM_COLD_START**})

分区模式类型集合的分割

@axm_partition_threadstates partition(**ThreadStates**,{**TS_Dormant**}, {**TS_Ready**}, {**TS_Waiting**}, {**TS_Suspend**}, {**TS_WaitandSuspend**}, {**TS_Running**})

@axm_partition_nums card(**Partitions**) ≥ **2** *// at least two partition, the kernal partition and a application partition*

分区集合的势

@axm_periodofpart **Period_of_Partition** ∈ **Partitions** → ℕ
@axm_durationofpart **Duration_of_Partition** ∈ **Partitions** → ℕ

都是Total function

@axm_criticallevel **Critical_Level** ∈ **Partitions** → **CriticalityLevels**
@axm_syspart **System_Partition** ∈ **Partitions**

是分区集合的一个元素

@axm_partition_criticallevel partition(**CriticalityLevels**,{**CRITILITY_LEVEL_A**}, {**CRITILITY_LEVEL_B**}, {**CRITILITY_LEVEL_C**}, {**CRITILITY_LEVEL_D**},{**CRITILITY_LEVEL_E**})

@axm_syshmtable **System_HM_Table**∈**ErrorCode**×**SystemState** ⇸ **ErrorLevel**

@axm_modulehmtable **Module_HM_Table** ∈ **ErrorCode**×**SystemState** ⇸ **ModuleActionType**

@axm_parhmtable **Partition_HM_Table** ∈ **Partitions** → (**ErrorCode**×**SystemState** ⇸ **PartitionActionType**)

@axm_partition_deadlinetype partition(**DeadlineType**,{**DEADLINE_HARD**}, {**DEADLINE_SOFT**})

分区内核的健康监控

- **System HM table**
  - 根据检测到的错误及系统状态，定义一个错误的等级

System HM Table Mappings

ML = Module Level Error
PL = Partition Level Error

System States: State Symbolic										
System States: State Codes										
Detected Error		module init	Module Function	partition switching	partition init	partition process management	partition error handler	Process Execution	OS Execution	...
Symbolic Name	Error ID	state 1	state 2	state 3	state 4	state 5	state 6	state 7	state 8	
configuration error	1	ML								
module config error	2	ML								
partition config error	3				PL					
partition init error	4				PL					
segmentation error	5	ML	ML	ML	PL	PL	PL	MEMORY_VIOLATION	PL	
time duration exceeded	6	ML	ML	ML	PL	PL	PL	DEADLINE_MISSED	PL	
invalid OS call	7	ML	ML	ML	PL	PL	PL	ILLEGAL_REQUEST		
divide by 0	8	ML	ML	ML	PL	PL	PL	NUMERIC_ERROR	PL	
overflow	9	ML	ML	ML	PL	PL	PL	NUMERIC_ERROR	PL	
floating point error	10	ML	ML	ML	PL	PL	PL	NUMERIC_ERROR	PL	
application reports error	11							APPLICATION_ERROR		
stack overflow	12	ML	ML	ML	PL	PL	PL	STACK_OVERFLOW	PL	
parity error	13	ML	ML	ML	PL	PL	PL	HARDWARE_FAULT	PL	
io access error	14	ML	ML	ML	PL	PL	PL	HARDWARE_FAULT	PL	
supervisor priv. violation	15				ML	PL	PL	MEMORY_VIOLATION	PL	
power interrupt	16	ML	ML	ML	ML	PL	PL	POWER_FAILURE	PL	
power failure	17	ML	ML	ML	ML	ML	ML	ML	ML	
...	...									

Partition HM Tables define the actions

Module HM Table defines the actions

分区内核的健康监控

- **Module HM Table**
  - 定义模块级错误发生时的动作

System States		Module HM Table Mappings								
➡	State Symbolic									
➡	State Codes									
⬇ Detected Error ⬇		module init	Module Function	partition switching	partition init	partition process management	partition error handler	Process Execution	OS Execution	
Symbolic Name	Error ID	state 1	state 2	state 3	state 4	state 5	state 6	state 7	state 8	...
configuration error	1	SH								
module config error	2	SH								
partition config error	3									
partition init error	4									
segmentation error	5	SH	SH	SH						
time duration exceeded	6	IG	IG	IG						
invalid OS call	7	IG	IG	IG						
divide by 0	8	IG	IG	IG						
overflow	9	IG	IG	IG						
floating point error	10	IG	IG	IG						
application reports error	11									
stack overflow	12	RS	RS	RS						
parity error	13	RS	RS	RS						
io access error	14	IG	IG	IG						
supervisor priv. violation	15				RS					
power interrupt	16	IG	IG	IG	IG					
power failure	17	RS	RS	RS	RS	RS	RS	RS	RS	
...	...									
Module HM Callback	callback name / addr									

Module Actions
RS = RESET
SH = SHUTDOWN
IG = IGNORE

分区内核的健康监控

- **Partition HM Table**
  - 定义分区级错误发生时的动作

System States		Partition 1 HM Table Mappings								
➡	State Symbolic									
➡	State Codes									
⬇ Detected Error ⬇		module init	system function execution	partition switching	partition init	partition process management	partition error handler	Process Execution	OS Execution	
Symbolic Name	Error ID	state 1	state 2	state 3	state 4	state 5	state 6	state 7	state 8	...
configuration error	1									
module config error	2									
partition config error	3				ID					
partition init error	4				CS					
segmentation error	5				CS	ID	ID	ID	ID	
time duration exceeded	6				IG	IG	IG	WS	IG	
invalid OS call	7				IG	WS	ID	IG		
divide by 0	8				IG	IG	WS	ID	ID	
overflow	9				IG	IG	WS	ID	ID	
floating point error	10				IG	IG	WS	ID	ID	
application reports error	11							IG		
stack overflow	12				CS	ID	ID	ID	ID	
parity error	13				CS	IG	IG	ID	ID	
io access error	14				CS	IG	IG	ID	ID	
supervisor priv. violation	15					ID	ID	ID	ID	
power interrupt	16					WS	WS	ID	ID	
power failure	17									
...	...									

Partition Actions
ID = IDLE
WS = WARM START
CS = COLD START
IG = IGNORE

Partition 1 HM Callback	callback name

分区内核的健康监控定义

两个集合笛卡尔积到ErrorLeve集合的Partial function

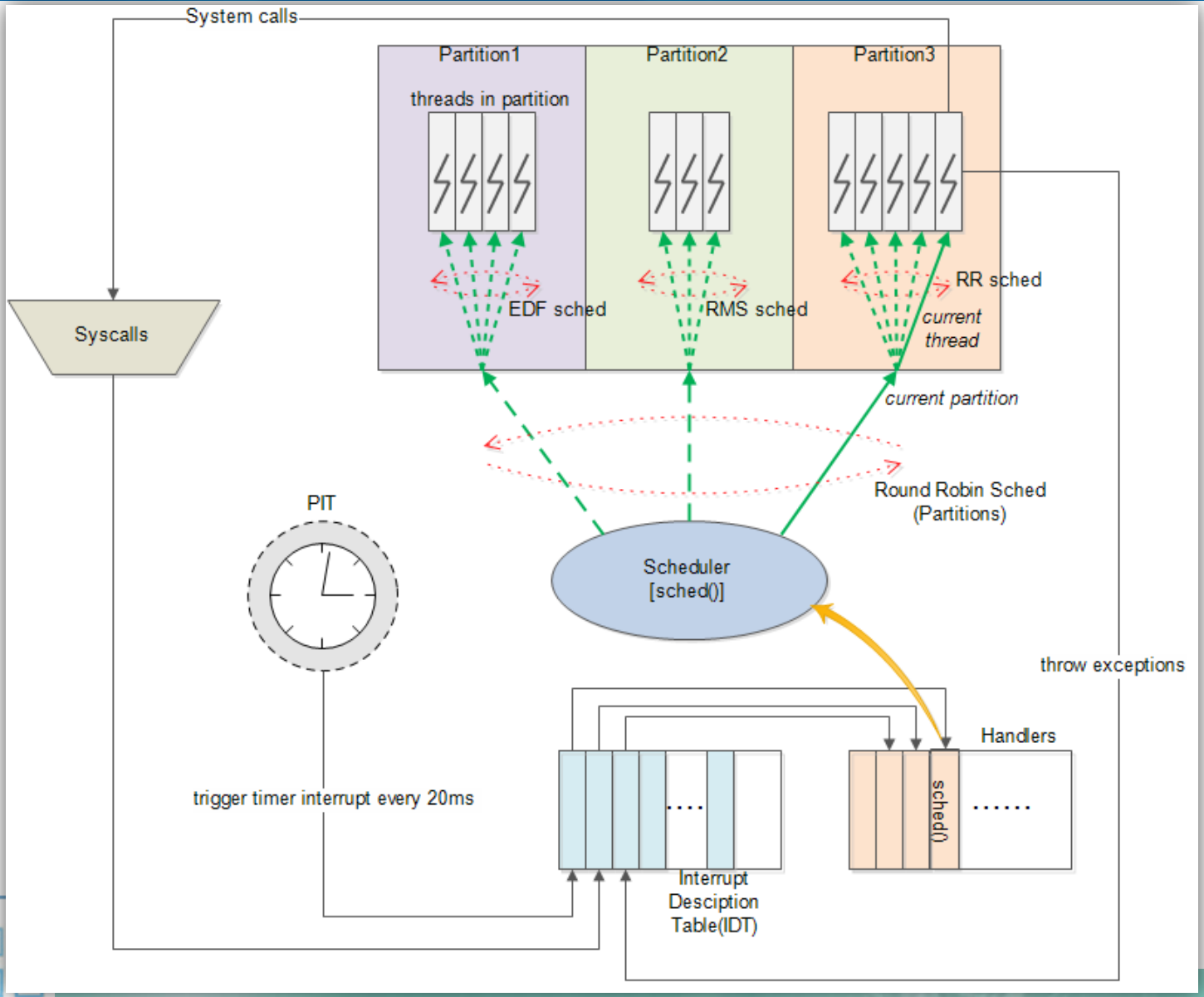
@axm_syshmtable **System_HM_Table**∈**ErrorCode**×**SystemState** ⇸ **ErrorLevel**

@axm_modulehmtable **Module_HM_Table** ∈ **ErrorCode**×**SystemState** ⇸ **ModuleActionType**

@axm_parhmtable **Partition_HM_Table** ∈ **Partitions** → (**ErrorCode**×**SystemState** ⇸ **PartitionActionType**)

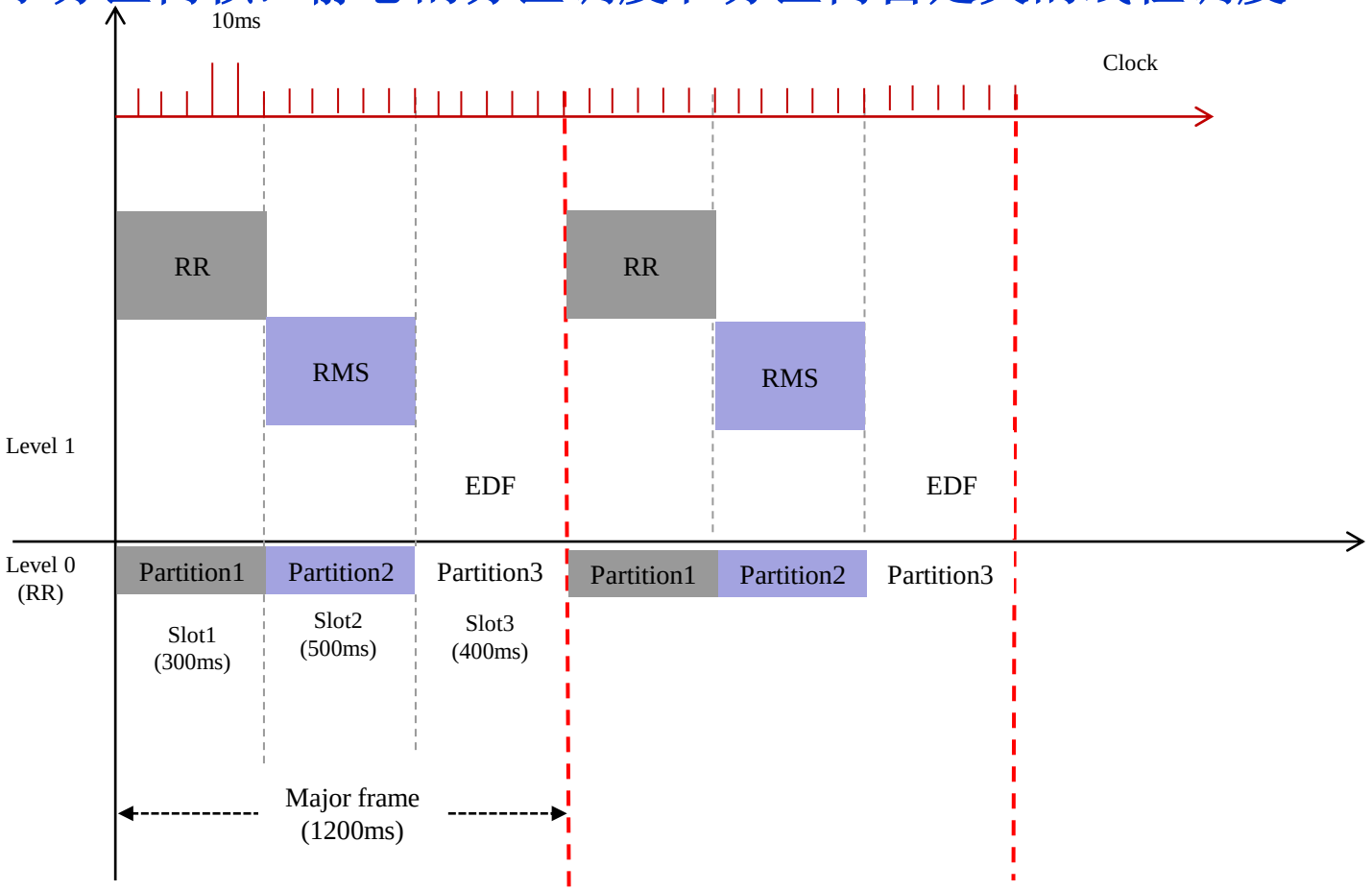
每个分区都有一个健康监控表，是Total function

分区内核的动态行为



分区内核的动态行为

- **分区管理和分区调度**
  - 分区管理包括分区内线程，调度是两级调度
  - 对于分区内核，静态的分区调度和分区内自定义的线程调度



分区内核的动态行为

- 分区调度

context Kernel_ctx

Constants

schedulingFrameofPartition *// the scheduling time frame of each partition*
schedulingFrames *// the scheduling time frames*
majorFrame *// the total time of all partitions*

axioms

@axm_scheduleframes **schedulingFrames** ∈ ℙ1(ℕ×ℕ) *// each <x |->y> : schedulingFrames, x is the start time of the frame and y is the end time*

@axm_scheduleframes2 ∀x·(x ∈ dom(**schedulingFrames**) ⇒ **x ≤ schedulingFrames(x)**) *// the end time should be larger than the start time of each frame*

@axm_scheduleframes3 ∀x,y·(x ∈ dom(**schedulingFrames**) ∧ **y ∈ dom(schedulingFrames)** ⇒ **(x ≥schedulingFrames(y)** ∨ **y ≥ schedulingFrames(x))**) *// scheduling frames are disjoint*

@axm_scheduleframeofpar **schedulingFrameofPartition** ∈ **schedulingFrames** ⇻ **Partitions** *// in a major frame, each partition should have one or more than one scheduling frame*

@axm_majorframe **majorFrame** ∈ ℕ1 *// majorFrame should be larger than sum of all schedulingframe. there may be spare time space between scheduling frames*

分区内核的动态行为

machine Kernel_Machine **sees** Kernel_ctx

系统运行态的线程集合

分区内的线程集合(函数)

variables

Threads
threadsOfPartition *// threads of each partition*
partitionstate
threadstate
current_partition
current_thread

lock_level *// denotes the current lock level of the partition*
start_condition *// denotes the reason the partition is started*

error_thread *// for health monitoring, a error handler process could be created in a partition*
basepriority_threads *// Denotes the capability of the process to manipulate other processes.*
period_threads *// Identifies the period of activation for a periodic process. A distinct and unique value should be specified to designate the process as aperiodic*
timecapacity_threads *// Defines the elapsed time within which the process should complete its execution.*
deadline_threads *// Specifies the type of deadline relating to the process, and may be "hard" or "soft".*
currentpriority_threads *// Defines the priority with which the process may access and receive resources. It is set to base priority at initialization time and is dynamic at runtime.*
deadlinetime_threads *// The deadline time is periodically evaluated by the operating system to determine whether the process is satisfactorily completing its processing within the allotted time.*

分区内核的动态行为

invariants

@inv_threadstype **Threads** ∈ **AllThreads** → **BOOL** *// true: the thread has been created, false: the thread has not been used*

@inv_partstate **partitionstate** ∈ **Partitions** → **PartitionModes**
@inv_threadofpart **threadsOfPartition** ∈ **AllThreads** ⇻ **Partitions** *// partial surjection, each used thread is belonging to a partition, each partition has one thread at least*

分区内的线程集合，partial surjection，已创建的线程肯定属于某个分区，每个分区肯定有线程

@inv_threadstate **threadstate** ∈ **AllThreads** → **ThreadStates**
@inv_currpart **current_partition** ∈ **Partitions**
@inv_currthread **current_thread** ∈ **AllThreads** ∧ **Threads**(**current_thread**) = **TRUE** ∧ **threadsOfPartition**(**current_thread**) = **current_partition**

@inv_atleastonerunnningthread **card**(**threadstate** ▷ {**TS_Running**}) ≤ **1**

对于单处理器系统，只有一个线程处于执行态，Range restriction

@inv_locklevel **lock_level** ∈ **Partitions** → ℕ
@inv_start_condition **start_condition** ∈ **Partitions** → **PartitionStartCondition**
@inv_error_thread **error_thread** ∈ **Partitions** ⇻ **AllThreads** *// partial injection. at most one error thread for each partition*

每个分区的错误处理线程，partial injection，每个分区最多一个处理线程，而且不同分区不能是同一个线程

@inv_basepriority_threads **basepriority_threads** ∈ **AllThreads** → ℕ
@inv_period_threads **period_threads** ∈ **AllThreads** → ℕ

分区内核的动态行为

- **集合的操作**

$\forall t \cdot t \in AllThreads \Rightarrow card(Threads'[\{t\}]) = 1$

$threadsOfPartition' \subseteq AllThreads \times Partitions$

$dom(threadsOfPartition') = Threads'^{\sim}[\{TRUE\}]$

$ran(threadsOfPartition') = Partitions$

$\forall t \cdot t \in Threads'^{\sim}[\{TRUE\}] \Rightarrow card(\{t\} \lhd threadsOfPartition') = 1$

$threadstate = ((AllThreads \setminus \{current\_thread\}) \times \{TS\_Dormant\}) \cup \{current\_thread \mapsto TS\_Running\}$

$deadline\_threads \coloneqq AllThreads \times \{DEADLINE\_HARD\}$

分区内核的动态行为

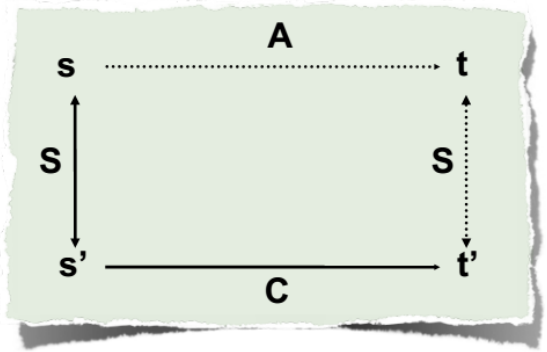
- 事件

```
event schedule
   any p t
   where
     @grd1 p ∈ Partitions
     @grd11 t ∈ AllThreads ∧ Threads(t) = TRUE
     @grd2 threadsOfPartition(t) = p
     @grd4 threadstate(t) = TS_Ready
     @grd3 partitionstate(p) = PM_NORMAL // @grd5 t ≠ current_thread
     @grd6 ∃x,y,n·(x∈ℕ1 ∧ y ∈ ℕ1 ∧ n ∈ ℕ1 ∧ x↦y ∈ schedulingFrames
          ∧ schedulingFrameofPartition(x↦y) = p
          ∧ (x + n*majorFrame) < clock_tick ∧ clock_tick < (y + n*majorFrame)) // current
tick is in the scheduling frame of the partition
     @grd0 need_reschedule = TRUE
   then
     @act1 threadstate ≔ (threadstate ⩤ (threadstate~[{TS_Running}] × {TS_Ready})) ⩤ {t ↦
TS_Running}
     @act5 current_partition ≔ p
     @act6 current_thread ≔ t
     @act7 need_reschedule ≔ FALSE
end
```

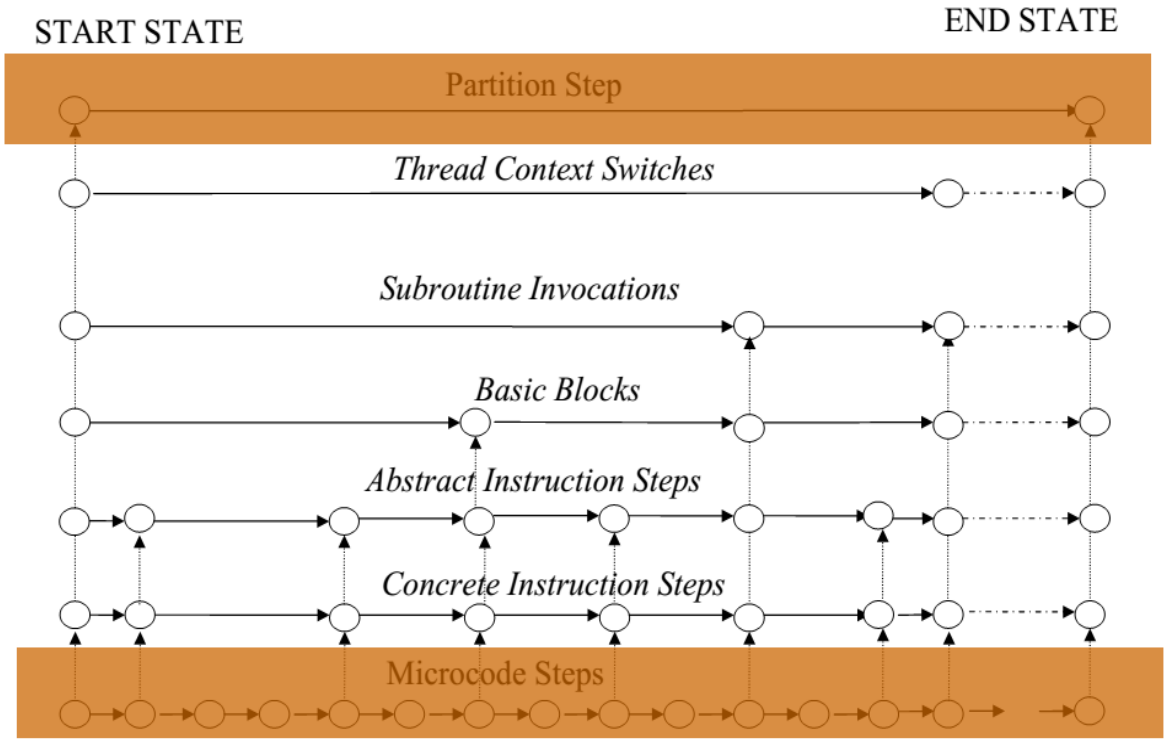
关系的override
将当前执行线程的状态置为ready
将被调度线程t的状态置为running

求精Refinement

- C是对A的求精，若C的所有行为均包含在A中，或被A所允许



- 举例：

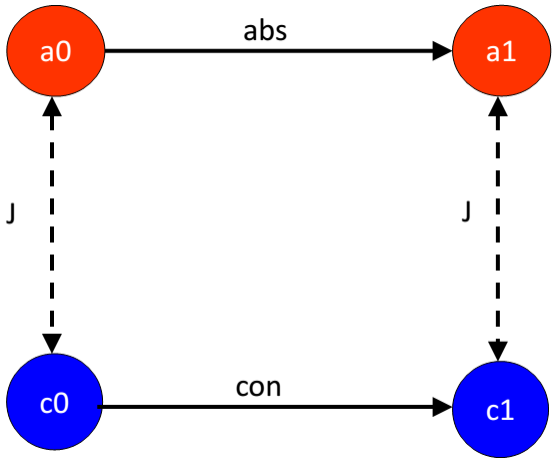


Event-B中模型求精的类型

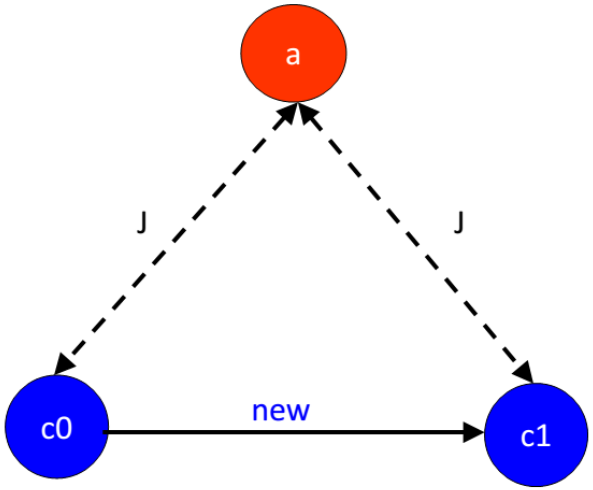
- 1. 扩展:
  - 增加新变量，及其不变式
  - 扩展已有事件，使其作用于新增的变量
  - 增加新事件，使其作用于新增的变量
- 2. 带卫式条件修改的扩展:
  - 与基本扩展一样，不同在于修改了已有事件的卫式条件
- 3. 变量替换/数据具体化:
  - 用新的变量替换某些变量，比如用具体变量替换抽象变量
  - 修改已有事件，增加新事件
- 4. 变量删除:
  - 先前引入新变量后，某些已有变量变得冗余了，删除这些冗余变量

事件求精的类型

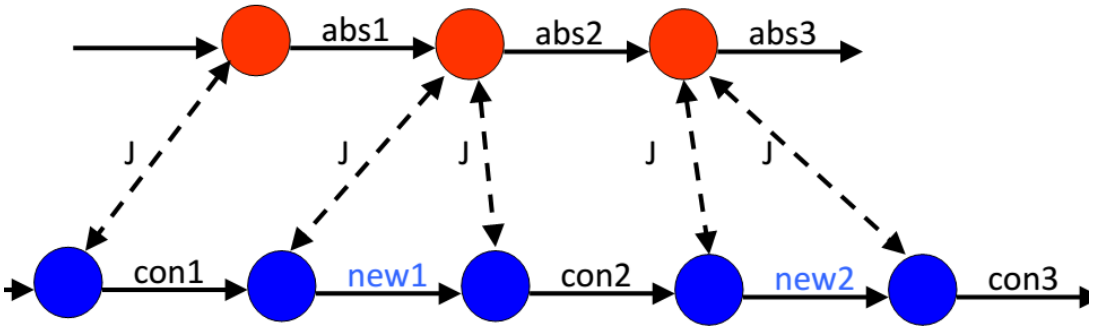
Simulation: maintaining a gluing relation



New concrete events refine skip (stuttering step)

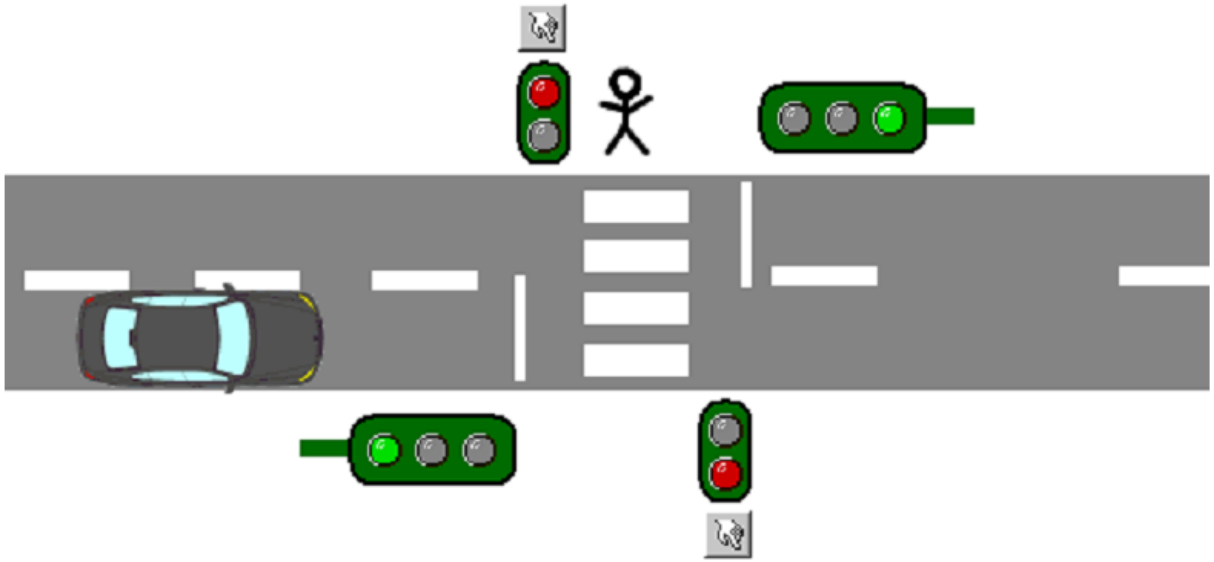


Refining traces



一个简单的例子(1)

- 交通信号灯控制器



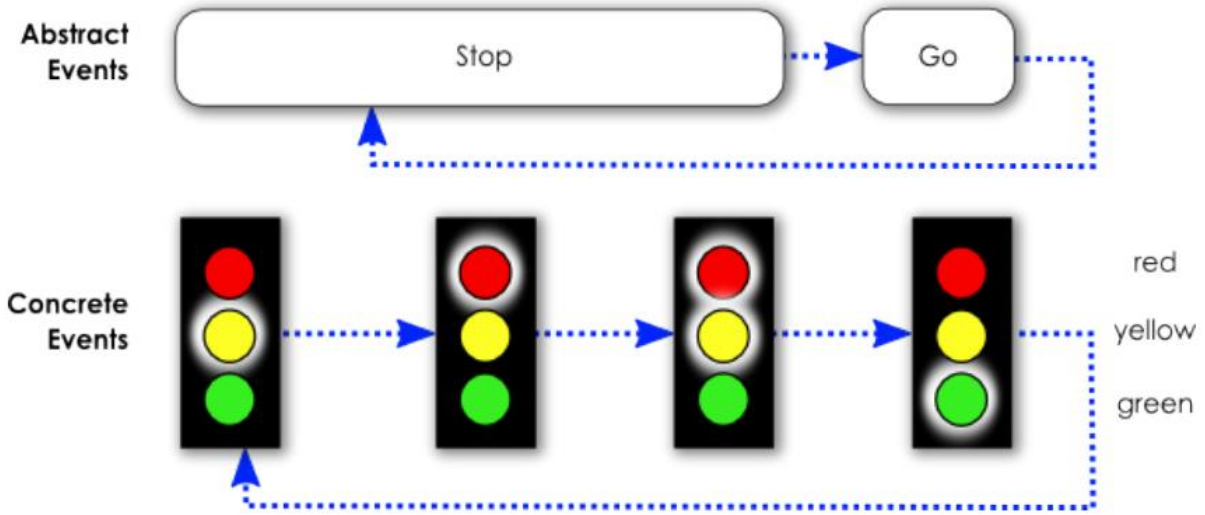
一个简单的例子(1)

MACHINE mac
VARIABLES
cars_go
peds_go
INVARIANTS
inv1 : $cars\_go \in BOOL$
inv2 : $peds\_go \in BOOL$
inv3 : $\neg(cars\_go = TRUE \wedge peds\_go = TRUE)$
EVENTS
Initialisation
begin
act1 : $cars\_go := FALSE$
act2 : $peds\_go := FALSE$
end
Event $set\_peds\_go \mathrel{\widehat{=}}$
when
grd1 : $cars\_go = FALSE$
then
act1 : $peds\_go := TRUE$
end

Event $set\_peds\_stop \mathrel{\widehat{=}}$
begin
act1 : $peds\_go := FALSE$
end
Event $set\_cars \mathrel{\widehat{=}}$
any
new_value
where
grd1 : $new\_value \in BOOL$
grd2 : $new\_value = TRUE \Rightarrow peds\_go = FALSE$
then
act1 : $cars\_go := new\_value$
end
END

一个简单的例子(2)

- 交通信号灯控制器——求精



一个简单的例子(2)

- 交通信号灯控制器——求精
  - 引入Context

CONTEXT agatha

CONSTANTS

red
yellow
green

SETS

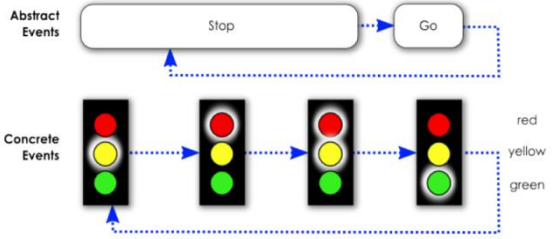
COLOURS

AXIOMS

type : $partition(COLOURS, \{red\}, \{yellow\}, \{green\})$

增加了常量和集合

一个简单的例子(2)



- 交通信号灯控制器——求精

MACHINE mac1
REFINES mac
SEES ctx1
VARIABLES
peds_colour
cars_colours

替换原有的peds_go和cars_go变量

为新变量增加新的不变式

INVARIANTS
inv4 : $peds\_colour \in \{red, green\}$
inv5 : $cars\_colours \subseteq COLOURS$
gluing_peds : $peds\_go = TRUE \Leftrightarrow peds\_colour = green$
gluing_cars : $cars\_go = TRUE \Leftrightarrow green \in cars\_colours$

为新旧变量之间的关联不变式

EVENTS
Initialisation
begin
init4 : $peds\_colour := red$
init5 : $cars\_colours := \{red\}$
end

Event $set\_peds\_green \widehat{=}$
refines set_peds_go
when
grd1 : $green \notin cars\_colours$
then
act2 : $peds\_colour := green$
end

引入新变量后，修改已有事件

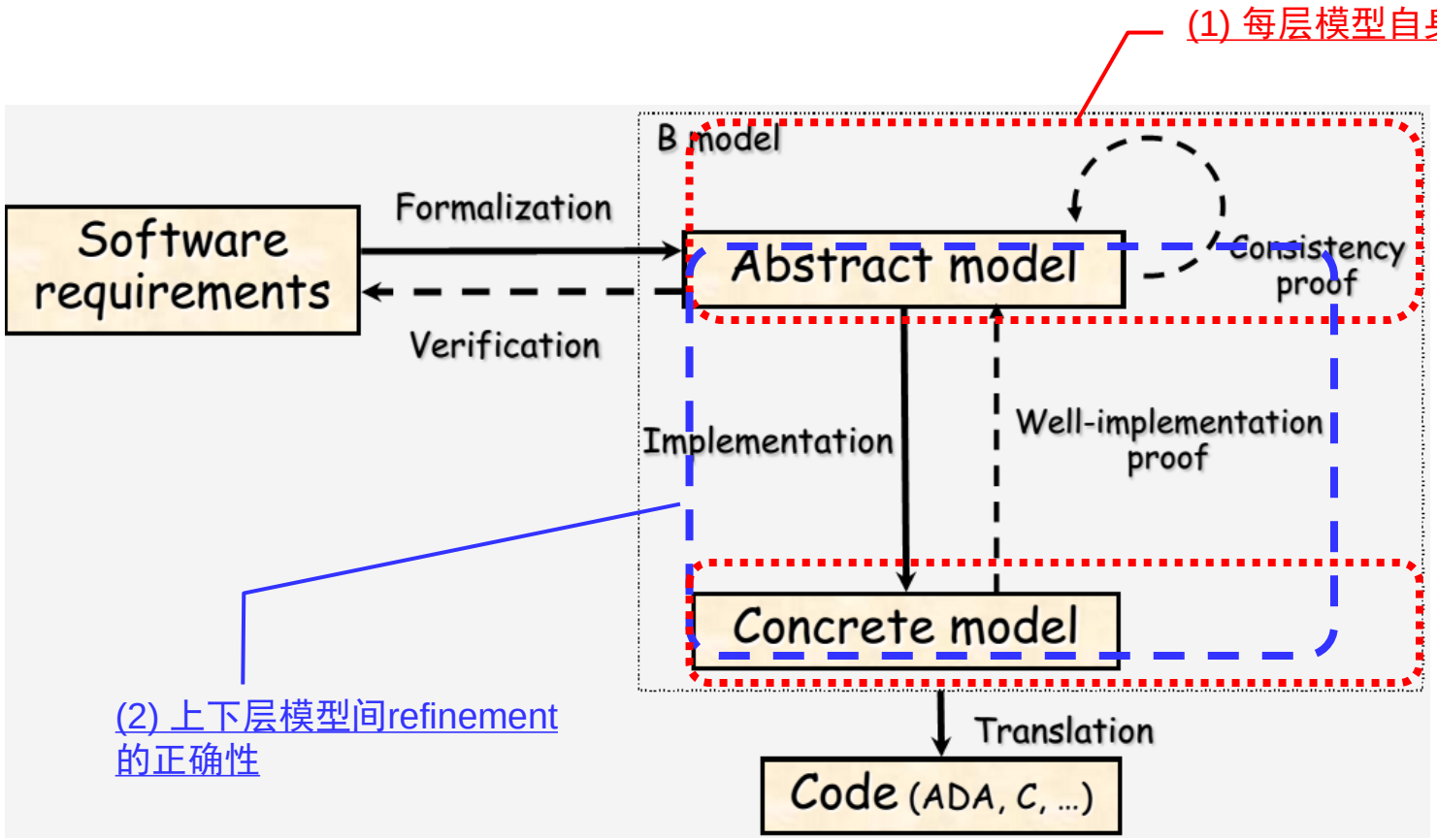
Event $set\_peds\_red \widehat{=}$
refines set_peds_stop
begin
act1 : $peds\_colour := red$
end

引入新变量后，修改已有事件

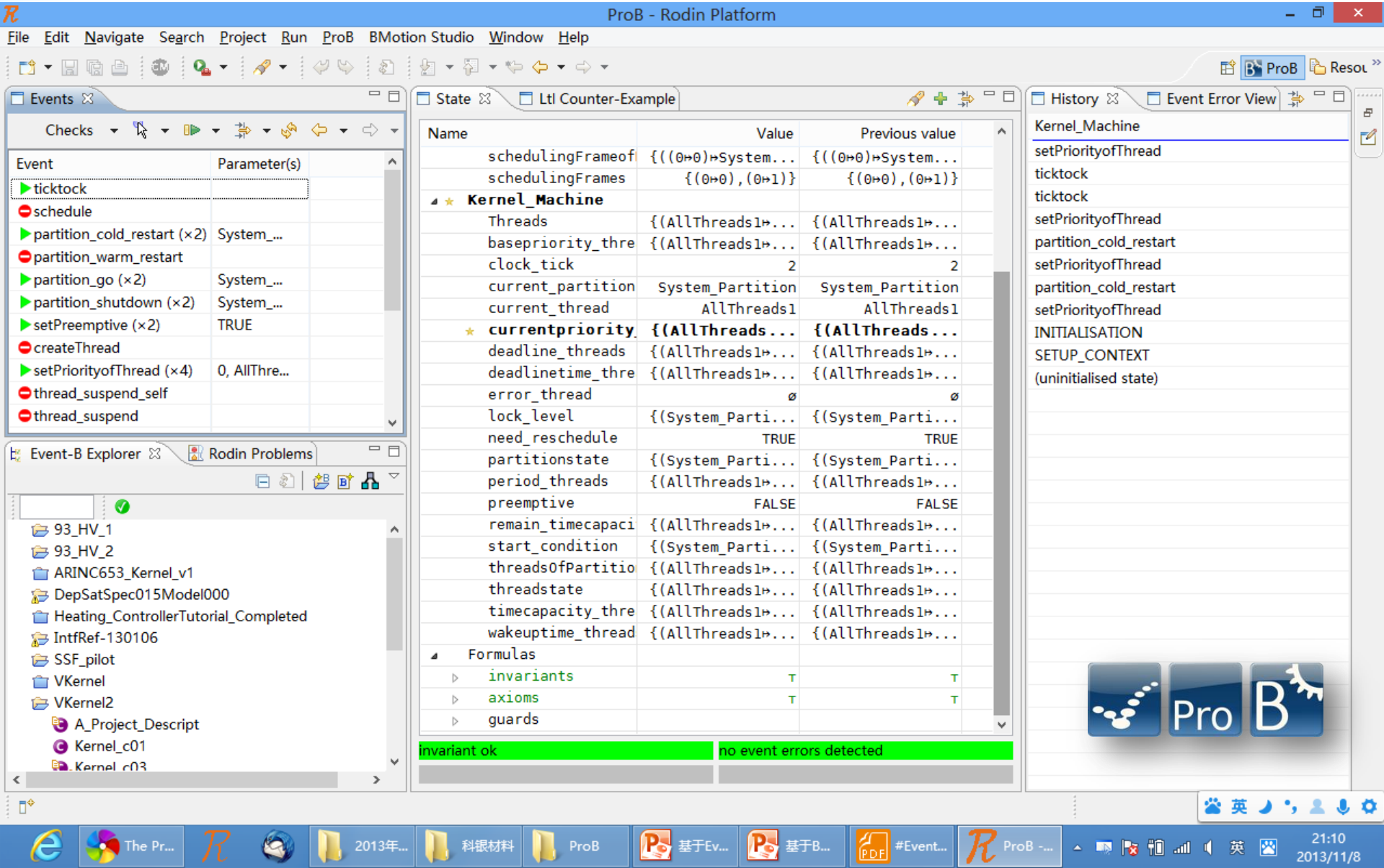
Event $set\_cars\_colours \widehat{=}$
refines set_cars
any
$new\_value\_colours$
where
grd1 : $new\_value\_colours \subseteq COLOURS$
grd2 : $green \in new\_value\_colours \Rightarrow peds\_colour = red$
grd_y_r : $cars\_colours = \{yellow\} \Rightarrow new\_value\_colours = \{red\}$
grd_r_ry : $cars\_colours = \{red\} \Rightarrow new\_value\_colours = \{red, yellow\}$
grd_ry_g : $cars\_colours = \{red, yellow\} \Rightarrow new\_value\_colours = \{green\}$
grd_g_y : $cars\_colours = \{green\} \Rightarrow new\_value\_colours = \{yellow\}$
with
new_value : new_value = TRUE ⇔ green ∈ new_value_colours
then
act1 : $cars\_colours := new\_value\_colours$
end
END

Witness是新旧事件参数间的关联不变式

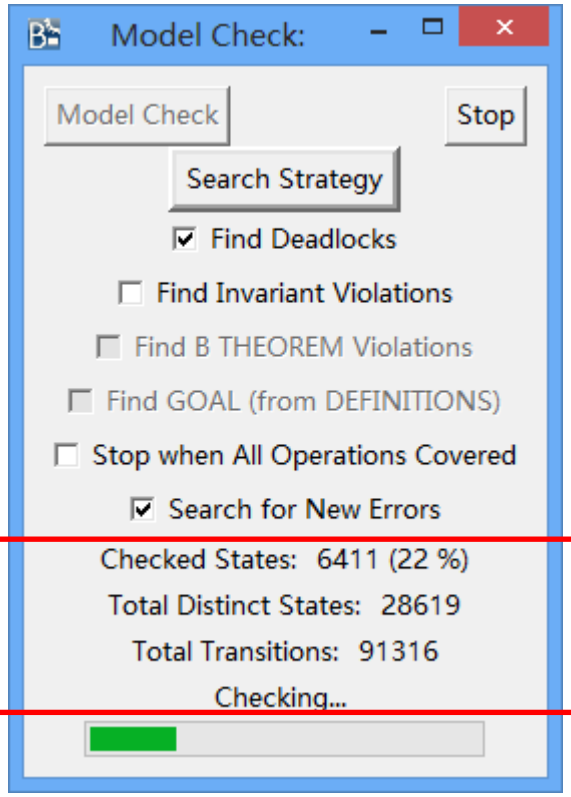
Event-B模型的正确性

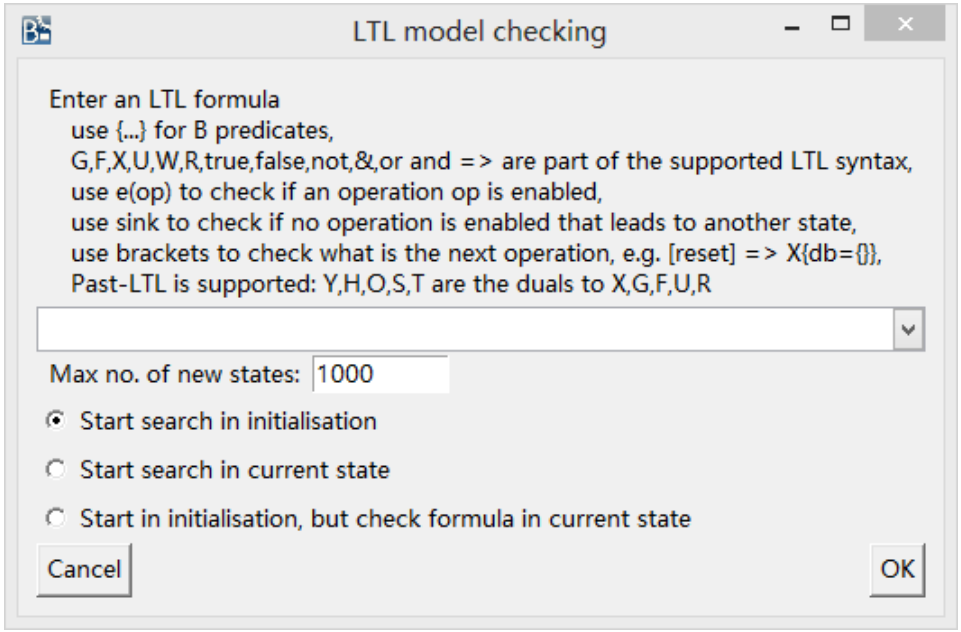


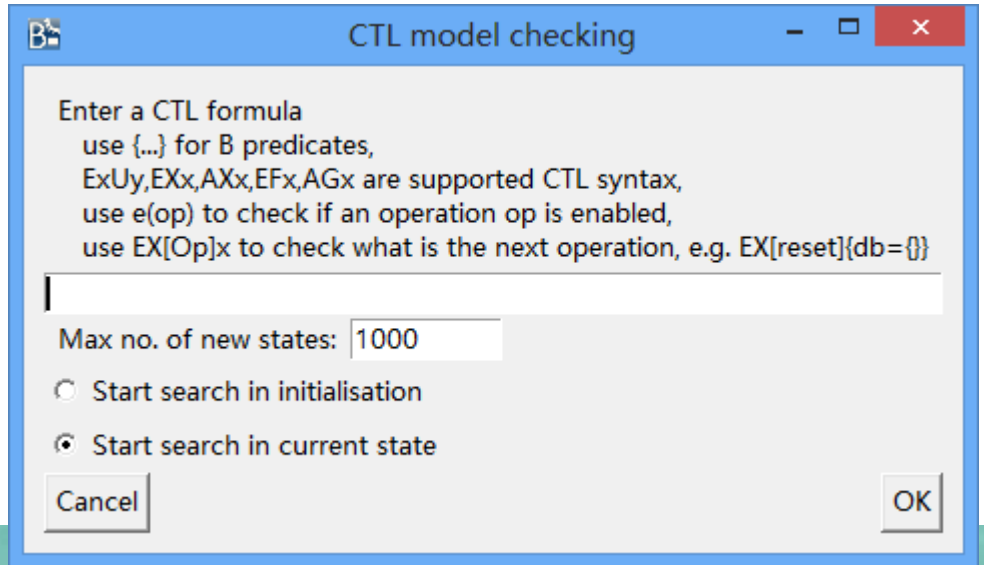
正确性验证(1)—模拟simulation



正确性验证(2)—模型检测







正确性验证(3)—演绎推理(Deductive reasoning)

- 正确性的必要条件
  - 一个模型是否正确，其必要条件由证据义务(proof obligation) 给出
  - 证据义务：就是一系列需证明的命题
  - 对于一个模型，证据义务可由Event-B工具自动生成

- 对于一个Machine
  - 事件的可行性Feasibility
  - 事件的不变式保持Invariant preservation

- 对于一个Refined Machine
  - 事件的可行性Feasibility
  - 事件的不变式保持Invariant preservation
  - Machine的无死锁deadlockfree
  - 事件的求精正确性Correct refinement
    - 事件的卫式条件被加强或保持不变
    - 事件的动作模拟被求精事件的动作（求精后每个迁移都被抽象模型所允许）
    - 使用关联不变式(gluing invariant)来连接新旧模型的状态

Event-B模型的Proof Obligation类型

generated in contexts		
well-definedness of an axiom	label/WD	3.2.5
axiom as theorem	label/THM	3.2.6
generated for machine consistency		
well-definedness of an invariant	label/WD	3.2.5
invariant as theorem	label/THM	3.2.6
well-definedness of a guard	event/guardlabel/WD	3.2.5
guard as theorem	event/guardlabel/THM	3.2.6
well-definedness of an action	event/actionlabel/WD	3.2.5
feasibility of a non-det. action	event/actionlabel/FIS	3.2.4
invariant preservation	event/invariantlabel/INV	3.2.4
generated for refinements		
guard strengthening	event/abstract_grd_label/GRD	3.2.4
action simulation	event/abstract_act_label/SIM	3.2.4
equality of a preserved variable	event/variable/EQL	3.2.4
guard strengthening (merge)	event/MRG	3.2.4
well definedness of a witness	event/identifier/WWD	3.2.5
feasibility of a witness	event/identifier/WFIS	3.2.4
generated for termination proofs		
well definedness of a variant	VWD	3.2.5
finiteness for a set variant	FIN	3.2.4
natural number for a numeric variant	event/NAT	3.2.4
decreasing of variant	event/VAR	3.2.4

Well-definedness PO (WD)

$\mathbf{inter}\,(S)$	$S \neq \varnothing$
$\bigcap\, x \cdot x \in S \;\wedge\; P(x) \mid T(x)$	$\exists\, x \cdot x \in S \;\wedge P(x)$
$f(E)$	f is a partial function $E \in \mathrm{dom}(f)$
E/F	$F \neq 0$
$E \bmod F$	$F \neq 0$
$\mathbf{card}(S)$	$\mathbf{finite}(S)$
$\mathbf{min}(S)$	$S \subseteq \mathbb{Z}$ $\exists x \cdot x \in \mathbb{Z} \;\wedge\; (\forall n \cdot n \in S \;\Rightarrow\; x \leq n)$
$\mathbf{max}(S)$	$S \subseteq \mathbb{Z}$ $\exists x \cdot x \in \mathbb{Z} \;\wedge\; (\forall n \cdot n \in S \;\Rightarrow\; x \geq n)$

Context Theorem PO (THM)

context
ctx
extends
$\ldots$
sets
s
constants
c
axioms
$A(s, c)$
theorems
$\ldots$
$thm: \quad P(s, c)$
$\ldots$
end

s: sets
c: constants
$A(s, c)$: axioms和被证明的theorems
$P(s, c)$: 特殊的theorem

Axioms ⊢ Theorem	thm/THM

$A(s, c)$
⊢
$P(s, c)$

Machine Theorem PO (THM)

machine
$m0$
refines
...
sees
...
variables
v
invariants and thms.
$I(s, c, v)$
theorems
...
$thm: \quad P(s, c, v)$
...
events
...
end

s: sets
c: constants
v: variables
$A(s, c)$: axioms和theorems
$I(s, c, v)$: invariants和被证明的theorems
$P(s, c, v)$: 特殊的theorem

Axioms Invariants ⊢ Theorem	thm/THM

$A(s, c)$
$I(s, c, v)$
⊢
$P(s, c, v)$

Invariant Preservation (INV)

evt
any x where
$G(x, s, c, v)$
then
$v :| BAP(x, s, c, v, v')$
end

s: sets
c: constants
v: variables
$A(s, c)$: axioms和theorems
$I(s, c, v)$: invariants和theorems
x: 事件的参数
$G(x, s, c, v)$: 事件的卫式条件
$BAP(x, s, c, v, v')$: 事件的前后谓词
$inv(s, c, v')$: 该事件内修改的特殊不变式

Axioms Invariants Guards of the event Before-after predicate of the event ⊢ Modified Specific Invariant	$evt/inv/$INV

$A(s, c)$
$I(s, c, v)$
$G(x, s, c, v)$
$BAP(x, s, c, v, v')$
⊢
$inv(s, c, v')$

Feasibility PO (FIS)

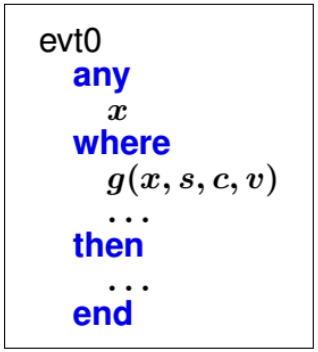
evt
any x where
$G(x, s, c, v)$
then
$v :| BAP(x, s, c, v, v')$
end

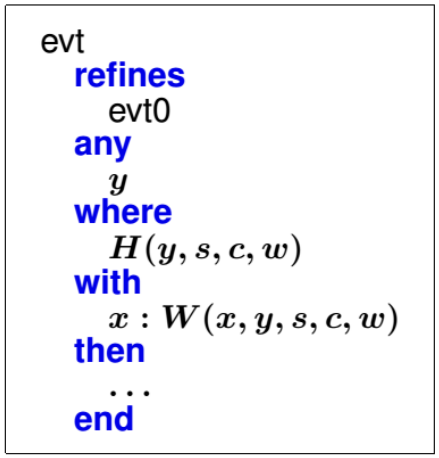
s: sets
c: constants
v: variables
$A(s, c)$: axioms和theorems
$I(s, c, v)$: invariants和theorems
x: 事件的参数
$G(x, s, c, v)$: 事件的卫式条件
$BAP(x, s, c, v, v')$: 事件的前后谓词

Axioms Invariants Guards of the event ⊢ $\exists v' \cdot$ Before-after predicate	$evt/act/$FIS

$A(s, c)$
$I(s, c, v)$
$G(x, s, c, v)$
⊢
$\exists v' . BAP(x, s, c, v, v')$

Guard Strengthening PO (GRD)





s:	sets
c:	constants
v:	抽象variables
w:	具体variables
$A(s, c)$:	axioms和theorems
$I(s, c, v)$:	抽象invariants和theorems
$J(s, c, v, w)$:	具体invariants和theorems
x:	抽象事件的参数
y:	具体事件的参数
$g(x, s, c, v)$:	抽象事件的卫式条件
$H(y, s, c, w)$:	具体事件的卫式条件
$W(x, y, s, c, w)$:	具体事件的witness谓词

Axioms Abstract invariants and thms. Concrete invariants and thms. Concrete event guards witness predicate ⊢ Abstract event specific guard	$evt/grd/$GRD

$A(s, c)$
$I(s, c, v)$
$J(s, c, v, w)$
$H(y, s, c, w)$
⊢
$g(x, s, c, v)$

Simulation PO (SIM)

evt0
any
x
where
$\ldots$
then
$v :| BA1(v, v', \ldots)$
end

evt
refines
evt0
any
y
where
$H(y, s, c, w)$
with
$x : W1(x, y, s, c, w)$
$v' : W2(y, v', s, c, w)$
then
$w :| BA2(w, w', \ldots)$
end

s:	sets
c:	constants
v:	抽象variables
w:	具体variables
$A(s, c)$:	axioms和theorems
$I(s, c, v)$:	抽象invariants和theorems
$J(s, c, v, w)$:	具体invariants和theorems
x:	抽象事件的参数
y:	具体事件的参数
$BA1(v, v')$:	抽象事件的动作
$BA2(w, w')$:	具体事件的动作

Axioms Abstract invariants and thms. Concrete invariants and thms. Concrete event guards witness predicate witness predicate Concrete before-after predicate ⊢ Abstract before-after predicate	$evt/act/$SIM

$A(s, c)$
$I(s, c, v)$
$J(s, c, v, w)$
$W1(x, y, s, c, w)$
$W2(y, v', s, c, w)$
$BA2(w, w')$
⊢
$BA1(v, v')$

Grd. Strengthening PO (MRG)

when Merging Abs. Events

evt01
any
x
where
$G1(x, s, c, v)$
then
A
end

evt02
any
x
where
$G2(x, s, c, v)$
then
A
end

evt
refines
evt01
evt02
any
x
where
$H(x, s, c, v)$
then
A
end

s: sets
c: constants
v: variables
$A(s, c)$: axioms和theorems
$I(s, c, v)$: invariants和theorems
x: 事件的参数
$H(x, s, c, v)$: 具体事件的卫式条件
$G1(x, s, c, v)$: 抽象事件1的卫式条件
$G2(x, s, c, v)$: 抽象事件2的卫式条件
A: 事件的动作

Axioms
Abstract invariants and thms.
Concrete event guards
⊢
Disjunction of abstract guards

evt/MRG

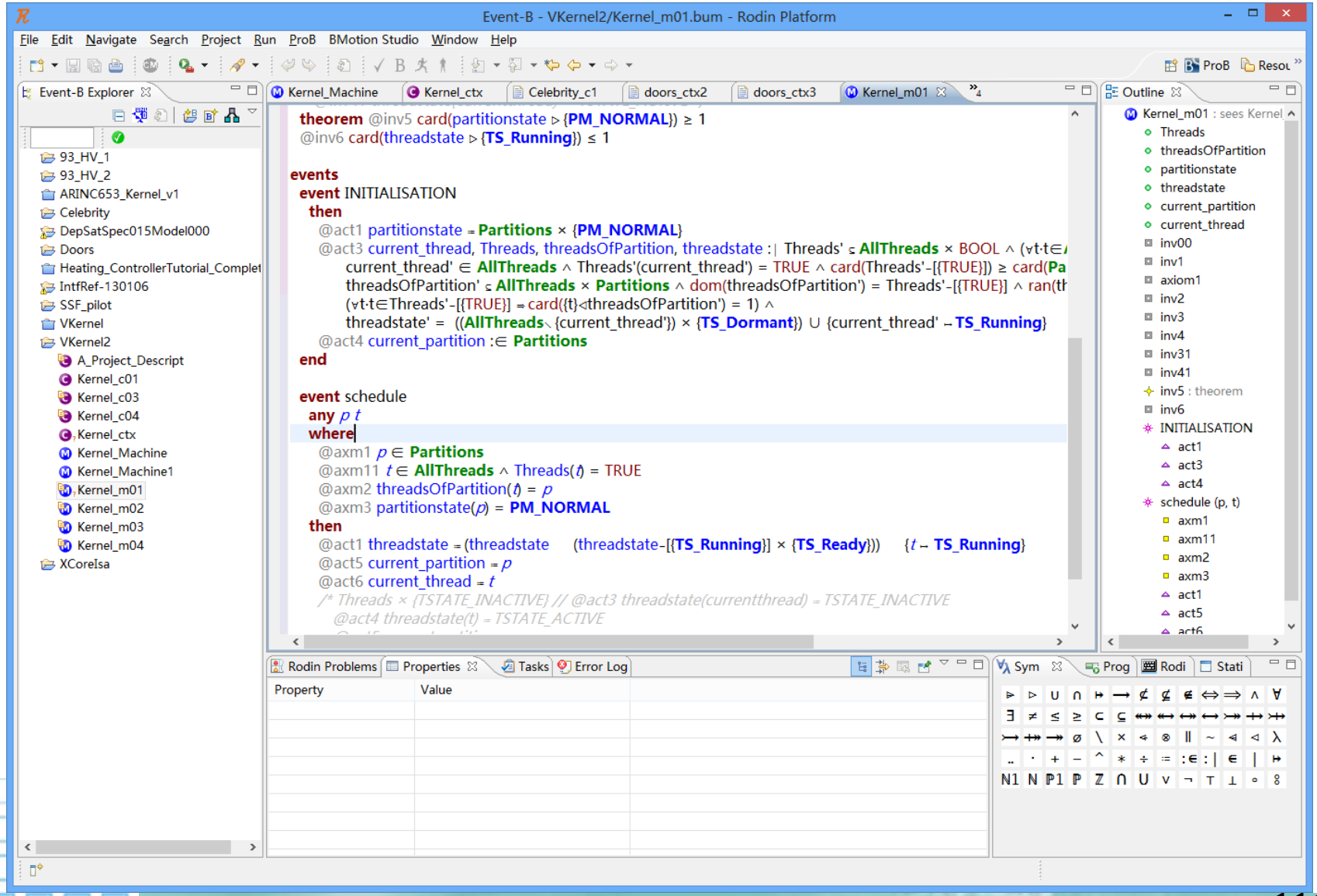
$A(s, c)$
$I(s, c, v)$
$H(x, s, c, v)$
⊢
$G1(x, s, c, v) \vee G2(x, s, c, v)$

4. Event-B实践

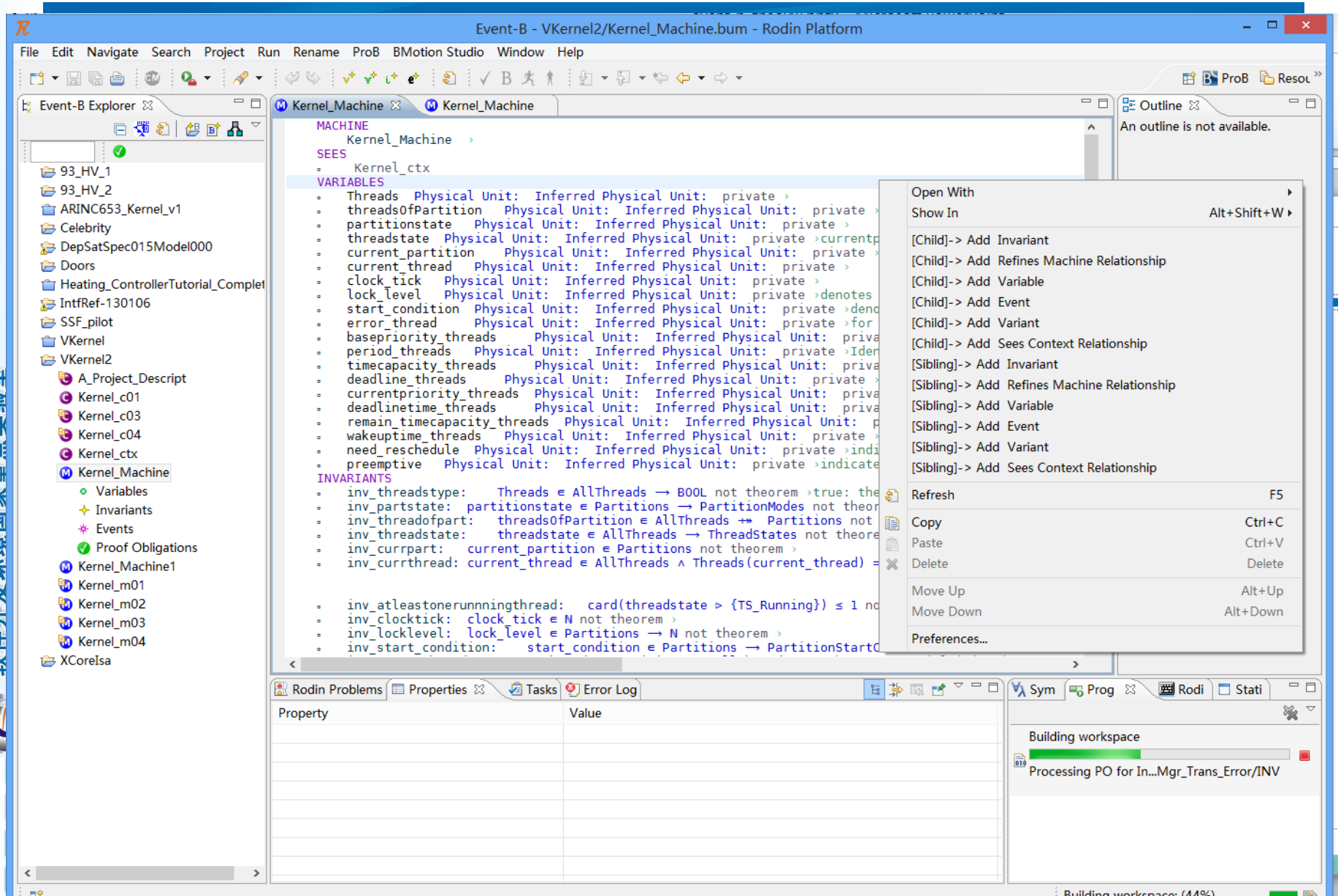
RODIN

- RODIN
  - Rigorous Open Development Environment for Complex Systems
  - RODIN是一个建模开发环境
    - 基于Eclipse平台
    - 采用定理证明作为主要的验证技术
    - 以Event-B作为主要形式化语言
- 受欧盟项目的支持
  - RODIN project (2004-2007)
  - DEPLOY project (2008-2012)
  - ADVANCE project (2011-2014)

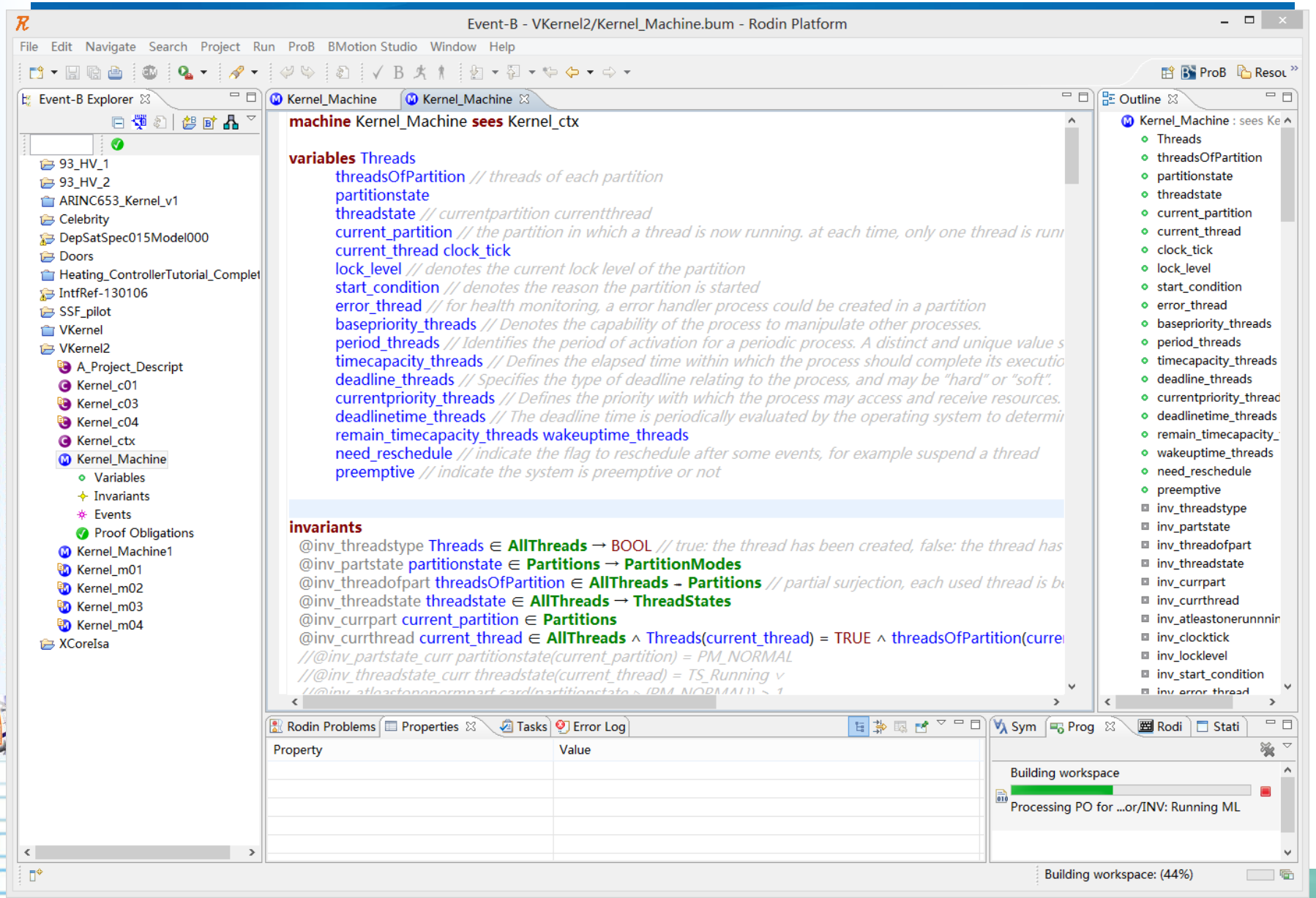
RODIN平台界面



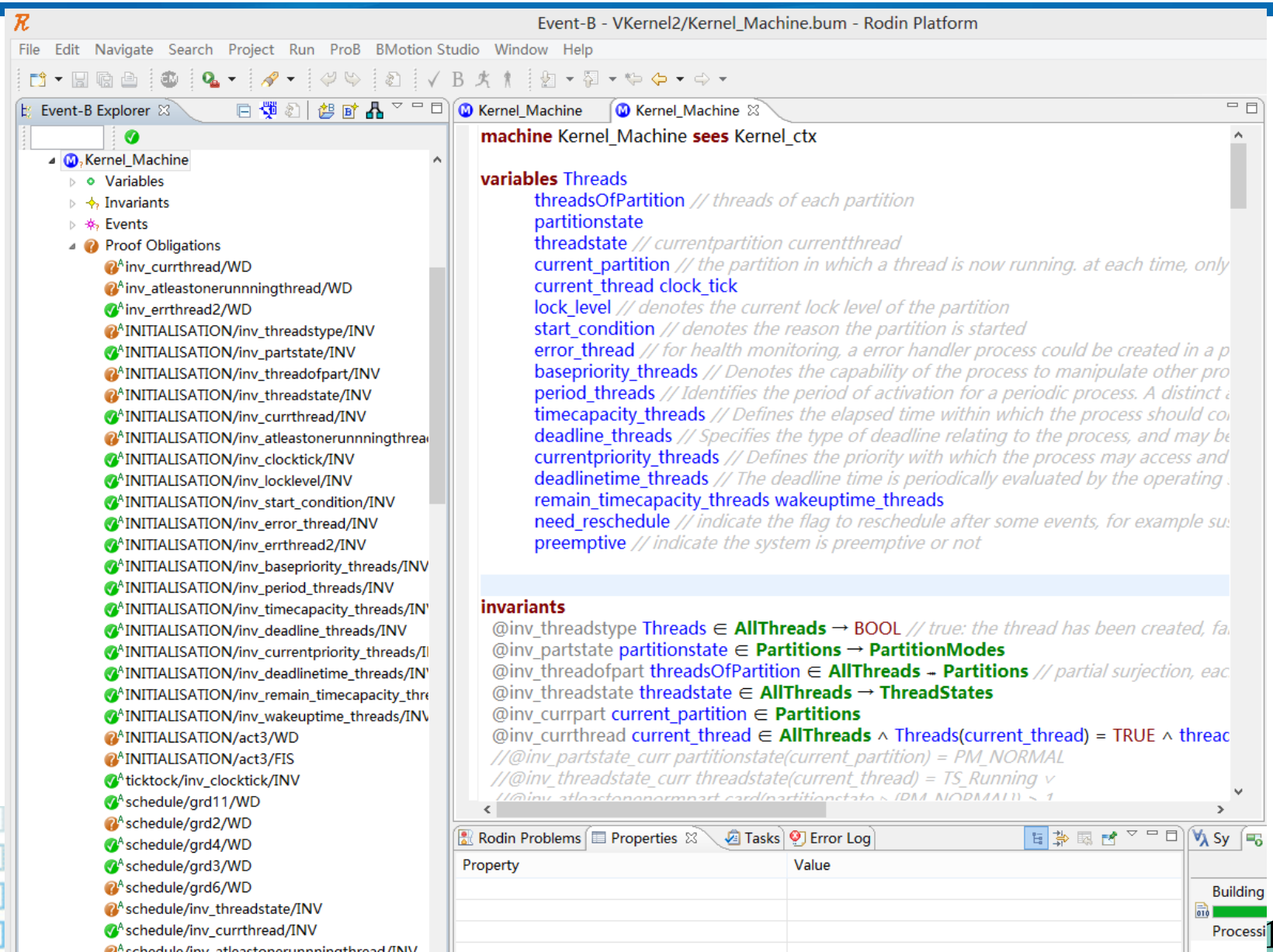
RODIN编辑器



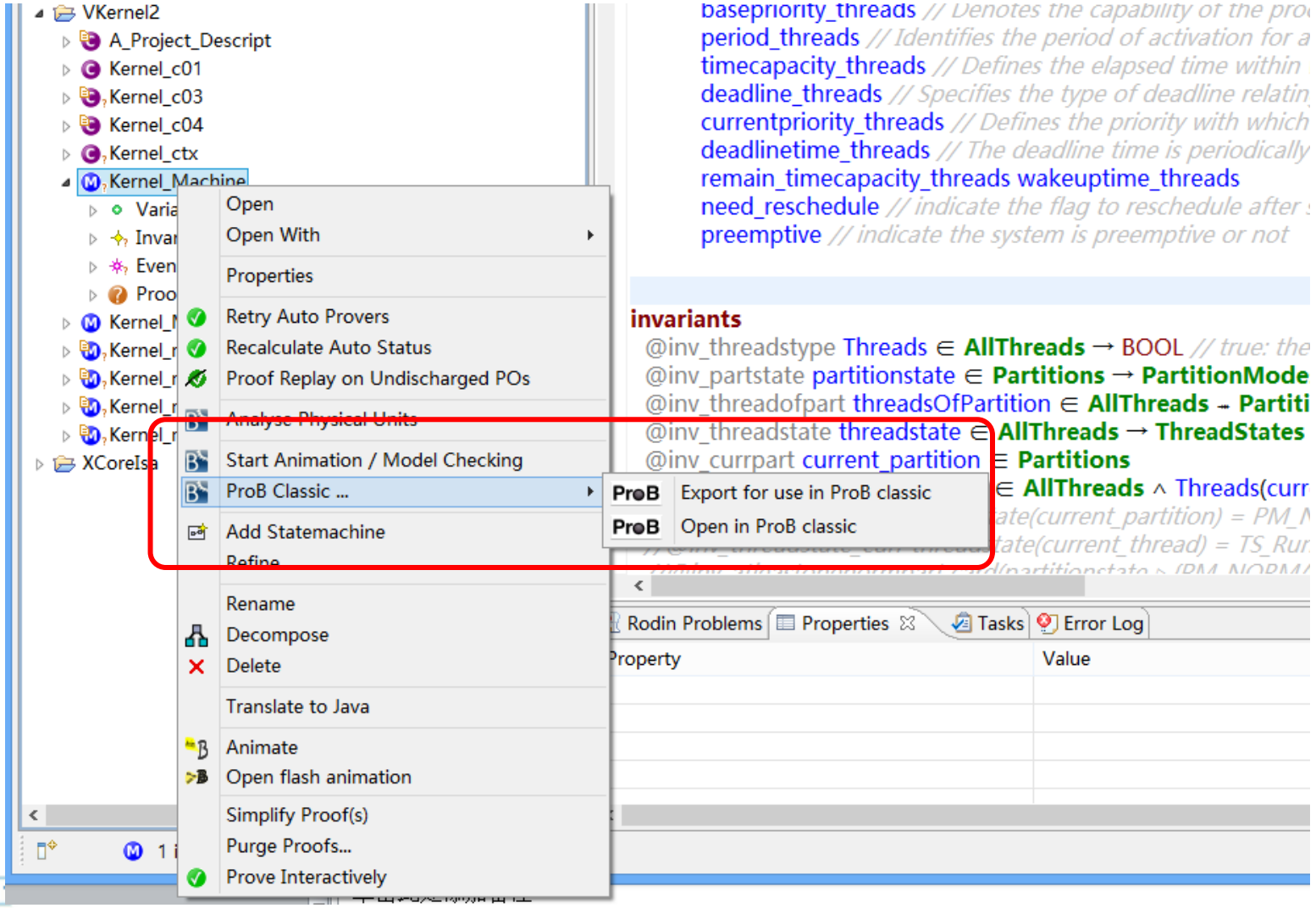
Camille编辑器(插件)



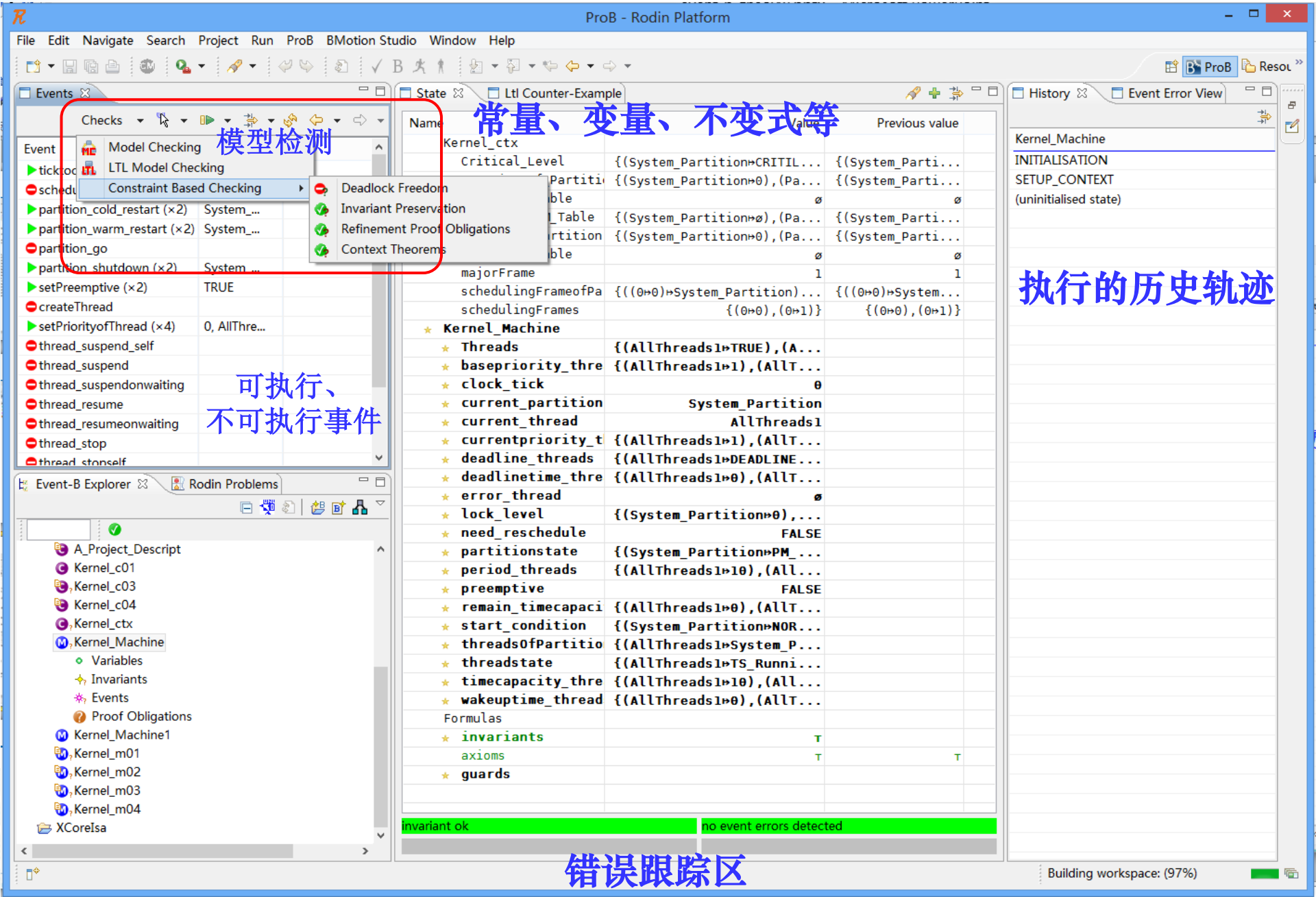
Proof Obligation自动生成



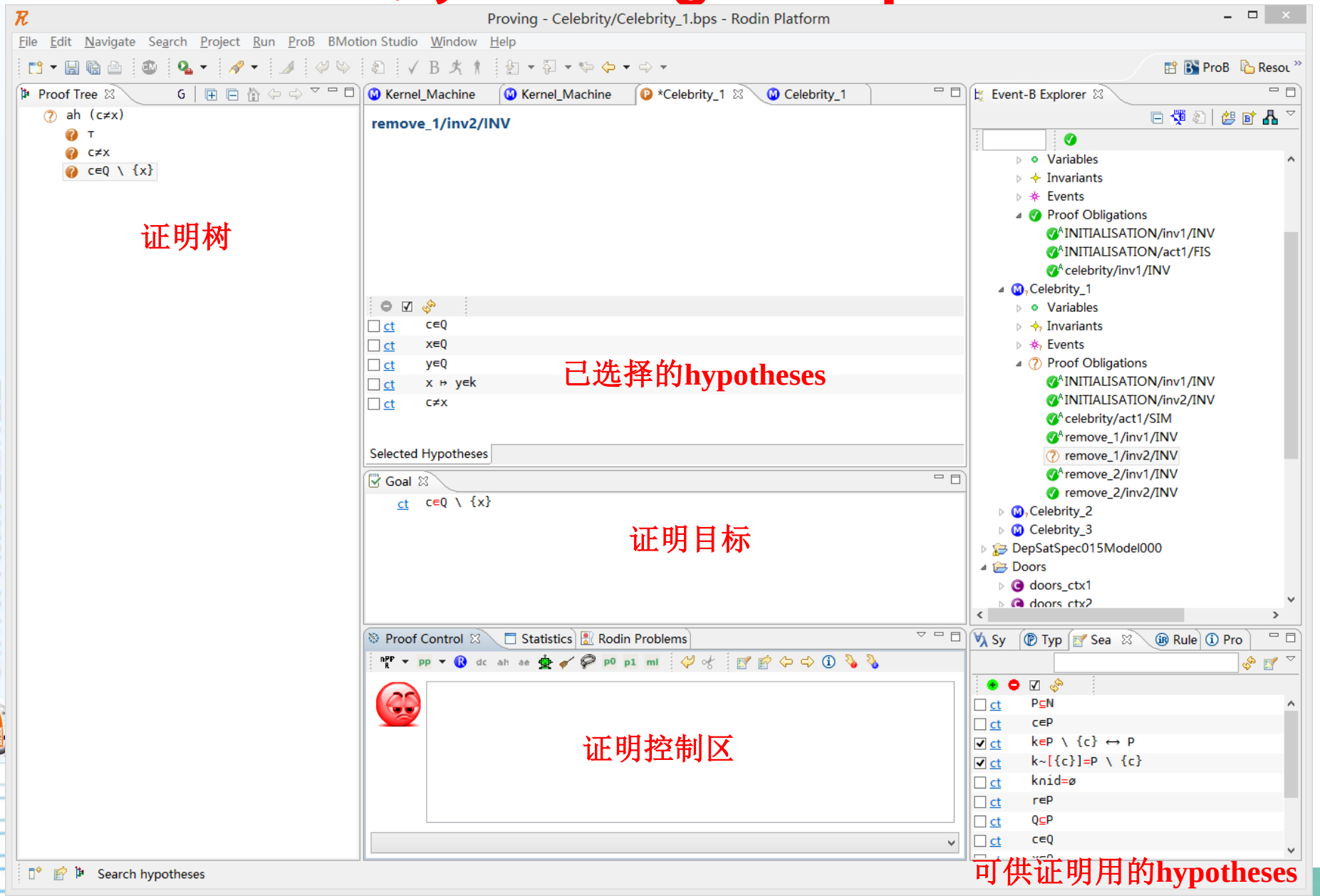
使用ProB工具


VKernel2
A_Project_Descript
Kernel_c01
Kernel_c03
Kernel_c04
Kernel_ctx
Kernel_Machine
XCoreIsa
Open
Open With
Properties
Retry Auto Provers
Recalculate Auto Status
Proof Replay on Undischarged POs
Analyse Physical Units
Start Animation / Model Checking
ProB Classic ...
Add Statemachine
Refine
Rename
Decompose
Delete
Translate to Java
Animate
Open flash animation
Simplify Proof(s)
Purge Proofs...
Prove Interactively
ProB Export for use in ProB classic
ProB Open in ProB classic
basepriority_threads // Denotes the capability of the pro
period_threads // Identifies the period of activation for a
timecapacity_threads // Defines the elapsed time within
deadline_threads // Specifies the type of deadline relatin
currentpriority_threads // Defines the priority with which
deadlinetime_threads // The deadline time is periodically
remain_timecapacity_threads wakeuptime_threads
need_reschedule // indicate the flag to reschedule after
preemptive // indicate the system is preemptive or not
invariants
@inv_threadstype Threads ∈ AllThreads → BOOL // true: the
@inv_partstate partitionstate ∈ Partitions → PartitionMode
@inv_threadofpart threadsOfPartition ∈ AllThreads ⇸ Partiti
@inv_threadstate threadstate ∈ AllThreads → ThreadStates
@inv_currpart current_partition ∈ Partitions
∈ AllThreads ∧ Threads(curr
Rodin Problems
Properties
Tasks
Error Log
Property
Value

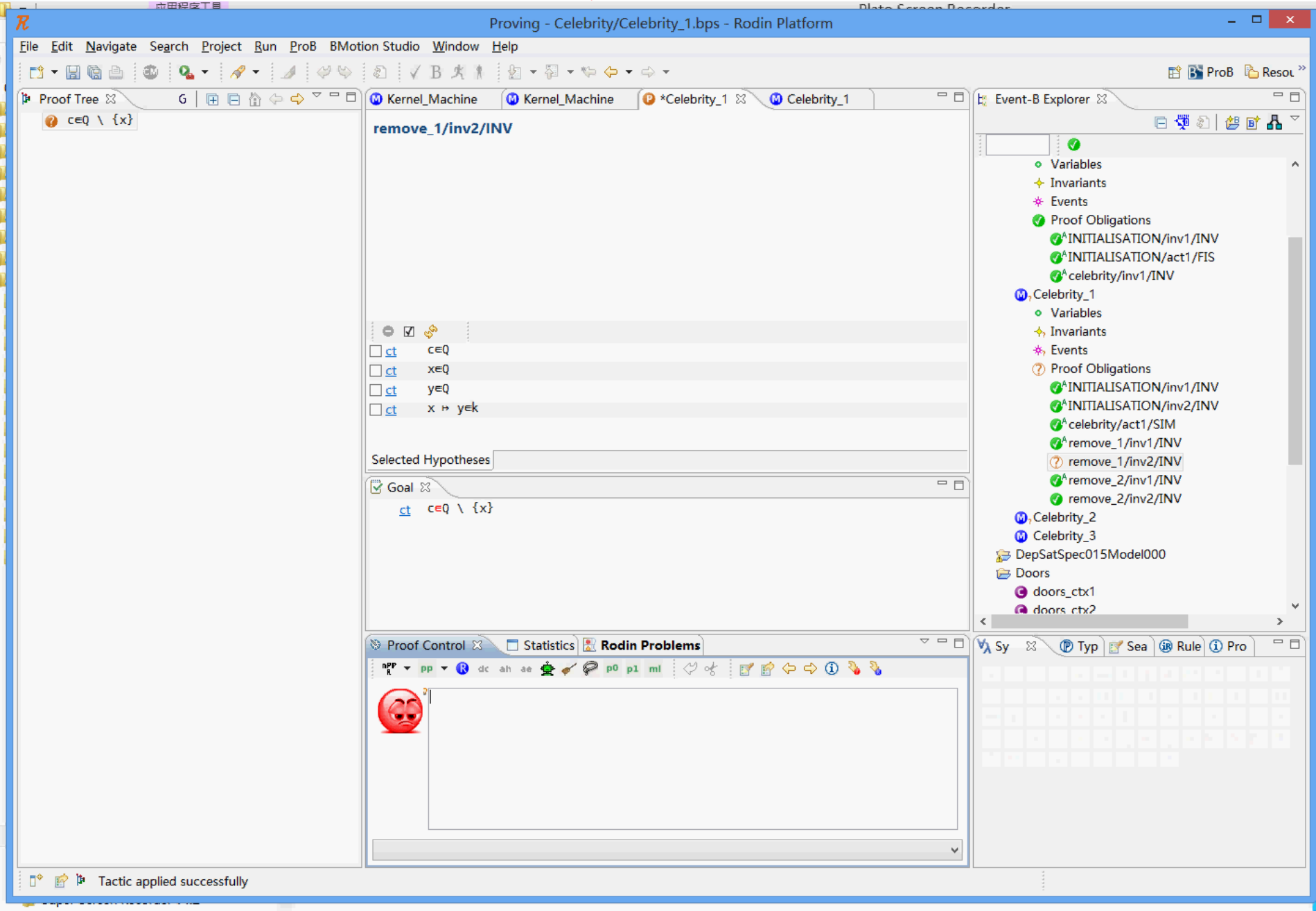
ProB:模拟和模型检测



RODIN的Proving Perspective



一个简单的证明过程


Proving - Celebrity/Celebrity_1.bps - Rodin Platform
File Edit Navigate Search Project Run ProB BMotion Studio Window Help
Proof Tree
c∈Q \ {x}
Kernel_Machine
Kernel_Machine
*Celebrity_1
Celebrity_1
remove_1/inv2/INV
ct c∈Q
ct x∈Q
ct y∈Q
ct x ↦ y∈k
Selected Hypotheses
Goal
ct c∈Q \ {x}
Event-B Explorer
Variables
Invariants
Events
Proof Obligations
INITIALISATION/inv1/INV
INITIALISATION/act1/FIS
celebrity/inv1/INV
Celebrity_1
Variables
Invariants
Events
Proof Obligations
INITIALISATION/inv1/INV
INITIALISATION/inv2/INV
celebrity/act1/SIM
remove_1/inv1/INV
remove_1/inv2/INV
remove_2/inv1/INV
remove_2/inv2/INV
Celebrity_2
Celebrity_3
DepSatSpec015Model000
Doors
doors_ctx1
Proof Control
Statistics
Rodin Problems
Tactic applied successfully

小 结

- 本报告内容
  - 形式化方法概述
  - 基本理论：命题逻辑、集合论、谓词逻辑等
  - Event-B详解
  - 实践

谢谢！

请批评指正！