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In [5]: 1 from IPython.display import Latex
        2
        3 code = get_latex_string("InversFourier.txt")
        4
        5 Latex(f"{{{code}}}")
```

Out[5]: if $f_0, f_1, f_2, \dots, f_m$ are the frequencies of Sine wave which are used to reconstruct the signal

$$x(n) = \sum_{i=0}^{M-1} A_i * \sin(2 * \pi * f_i * n/N * \phi_i)$$

Using DFT we have proved that

$$X(f_i) = \frac{N A_i * e^{j\phi_i}}{2j} \quad \& \quad X(N - f_i) = \frac{N A_i * e^{-j\phi_i}}{2j}$$

We want to reconstruct $x(n)$ to do so multiply $X(f_i)$ by $e^{2j\pi * f_i * n/N}$

$$X(f_i) * e^{2j\pi * f_i * n/N} = \frac{N A_i * e^{2j\pi * f_i * n/N + j\phi_i}}{2j}$$

$$X(N - f_i) * e^{2j\pi * (N - f_i) * n/N} = \frac{N A_i * e^{2j\pi * (N - f_i) * n/N - j\phi_i}}{2j}$$

$$\Rightarrow \frac{N A_i * e^{-j\phi_i} * [e^{2j\pi * N * n/N} * e^{-2j\pi * f_i * n/N}]}{2j}$$

$$\Rightarrow \frac{N A_i * e^{-j\phi_i} * [e^{2j\pi * n} * e^{-2j\pi * f_i * n/N}]}{2j}$$

$$e^{2j\pi * n} = 1 \forall n \in \mathbb{Z}$$

$$\Rightarrow \frac{N A_i * e^{-j\phi_i} * [e^{-2j\pi * f_i * n/N}]}{2j}$$

lets Add $X(f_i) * e^{2j\pi * f_i * n/N}$ and $X(N - f_i) * e^{2j\pi * (N - f_i) * n/N}$

$$\Rightarrow X(f_i) * e^{2j\pi * f_i * n/N} + X(N - f_i) * e^{2j\pi * (N - f_i) * n/N} = N A_i * \frac{e^{2j\pi * n * f_i/N + j\phi_i} - e^{-2j\pi * n * f_i/N - j\phi_i}}{2j}$$

let $x = 2 * \pi * f_i * n/N + \phi_i$

$$\Rightarrow X(f_i) * e^{2j\pi * f_i * n/N} + X(N - f_i) * e^{2j\pi * (N - f_i) * n/N} = N A_i * \frac{e^{jx} - e^{-jx}}{2j}$$

$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin(\theta)$$

$$\Rightarrow X(f_i) * e^{2j\pi * f_i * n/N} + X(N - f_i) * e^{2j\pi * (N - f_i) * n/N} = N A_i * \sin(x)$$

$$\Rightarrow X(f_i) * e^{2j\pi * f_i * n/N} + X(N - f_i) * e^{2j\pi * (N - f_i) * n/N} = N A_i * \sin(2 * \pi * n * f_i/N + \phi_i)$$

so inverse discret fourier transform can be expressed as

$$x(n) = \frac{1}{N} * \sum_{k=0}^{N-1} X(k) * e^{2j\pi * k * n/N}$$