In [2]:

```
from IPython.display import Latex

code = get_latext_string("Fourier.txt")

Latex(f"""{code}""")
```

Out[2]:

$x(n) = [x_0, x_1, \ldots, x_{N-1}]$ *which means x(n) is sampled using N points*

ith sample corresponds to (i/N)th second in time domine

Any periodic wave can be expressed as sum of various sine waves

$$x(n) = a_0 + \sum_{i=0}^{M-1} A_i * sin(2 * \pi * f_i * n/N + \phi_i)$$

Discriet Fourier Transform is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n) * exp(-2j * \pi * k * n/N)$$

Lets Try to simplify it

$$X(k) = \sum_{n=0}^{N-1} a_0 * e^{-2j*\pi*k*n/N} + \sum_{n=0}^{N-1}\sum_{i=0}^{M-1} A_i * sin(2 * \pi * f_i * n/N + \phi_i) * e^{-2j*\pi*k*n/N}$$

$$\sum_{n=0}^{N-1} a_0 * e^{-2j*\pi*k*n/N} = 0$$

$$X(k) = \sum_{n=0}^{N-1}\sum_{i=0}^{M-1} A_i * \frac{[e^{2j*\pi*f_i*n/N+\phi_i} - e^{-2j*\pi*f_i*n/N-\phi_i}]}{2j} * e^{-2j*\pi*k*n/N}$$

$$X(k) = \sum_{n=0}^{N-1}\sum_{i=0}^{M-1} A_i * \frac{[e^{2j*\pi*(f_i-k)*n/N+\phi_i} - e^{-2j*\pi*(f_i+k)*n/N-\phi_i}]}{2j}$$

$$\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f(i,j) = \sum_{j=0}^{N-1}\sum_{i=0}^{M-1} f(i,j) \textit{ Well Proven statement}$$

$$X(k) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1} A_i * \frac{[e^{2j*\pi*(f_i-k)*n/N+\phi_i} - e^{-2j*\pi*(f_i+k)*n/N-\phi_i}]}{2j}$$

$$X(k) = \sum_{i=0}^{M-1}\sum_{n=0}^{N-1} A_i * \frac{[e^{j\phi_i} * e^{2j*\pi*(f_i-k)*n/N} - e^{-j\phi_i} * e^{-2j*\pi*(f_i+k)*n/N}]}{2j}$$

$$X(k) = \sum_{i=0}^{M-1} \frac{A_i * e^{j\phi_i}}{2j} \sum_{n=0}^{N-1} e^{2j*\pi*(f_i-k)*n/N} - \sum_{i=0}^{M-1} \frac{A_i * e^{-j\phi_i}}{2j} \sum_{n=0}^{N-1} e^{-2j*\pi*(f_i+k)*n/N}$$

$$\sum_{n=0}^{N-1} e^{2j*\pi*(f_i-k)*n/N} [\textit{ This is in Geometric Progression}]$$

$$X(k) = \sum_{i=0}^{M-1} \frac{A_i * e^{j\phi_i}}{2j} * \frac{e^{2j*\pi*(f_i-k)} - 1}{e^{2j\pi*(f_i-k)/N} - 1} - \sum_{i=0}^{M-1} \frac{A_i * e^{-j\phi_i}}{2j} * \frac{e^{-2j*\pi*(f_i+k)} - 1}{e^{-2j*\pi*(f_i+k)/N} - 1}$$

$e^{2j*pi*z} = 1 \; \forall z \epsilon \mathbb{Z}$

$$\frac{e^{2j*\pi*(f_i-k)} - 1}{e^{2j\pi*(f_i-k)/N} - 1} == 0 \qquad \forall \; k! = f_i \qquad \frac{e^{-2j*\pi*(f_i+k)} - 1}{e^{-2j*\pi*(f_i+k)/N} - 1} == 0 \qquad \forall \; k + f_i! = N$$

$$\frac{e^{2j*\pi*(f_i-k)} - 1}{e^{2j\pi*(f_i-k)/N} - 1} == N \qquad \forall \; k == f_i \qquad \frac{e^{-2j*\pi*(f_i+k)} - 1}{e^{-2j*\pi*(f_i+k)/N} - 1} == N \qquad \forall \; k + f_i == N$$

$$X(f_i) = \frac{NA_i * e^{j\phi_i}}{2j}$$

$$X(N - f_i) = \frac{-NA_i * e^{-j\phi_i}}{2j}$$

so this concludes that the spikes in DFT corresponds to frequecy and length of the spike corresponds to am